Dual Nature of Radiation and Matter

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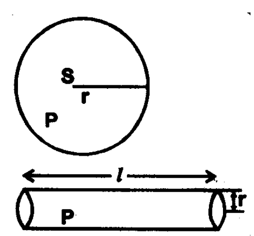
PROPERTIES OF PHOTONS

* Photon is a packet of energy emitted from a source of radiation. Photons are carrier particle of electromagnetic interaction.
* Photons travel in straight lines with speed of light $c = 3 \times 10^8$ m/s.
* The energy of photons is given as $E = h\nu = \frac{hc}{\lambda} = mc^2$
  where ν is frequency, λ is wavelength, h is Planck's constant.
* The effective or motional mass of photon is given as $m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$
* The momentum of a photon is given as $p = mc = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$
* The photon is a charge less particle of zero rest mass from relativity $E^2 = p^2c^2 + m_o{}^2c^4$
  so $m_0 = \frac{1}{c}\left[\frac{E^2}{c^2} - p^2\right]^{1/2} = 0$ (as $p = \frac{E}{c}$)
* Photons are electrically neutral. They are not deflected by electric and magnetic fields.
* If E is the energy of source in joule then number of photons emitted is
  $n = \frac{\text{total energy radiated}}{\text{energy of each photon}} = \frac{E}{h\nu} = \frac{E\lambda}{hc}$
* Intensity of photons is defined as amount of energy carried per unit area per unit time. or power carried per unit area
  Intesity $(I_p) = \frac{\text{Energy}}{\text{area} \times \text{time}} = \frac{\text{Power}}{\text{area}}$,
  $I_p = nh\nu = \frac{N}{4\pi r^2}P$
  where n = number of photons per unit area per unit time
  N = number of photons, P = power of source



e.g.

(a) For a point source $I_p = nh\nu = \frac{N}{4\pi r^2}P$

(b) For a line source $I_p = nh\nu = \frac{N}{2\pi r\ell}P$

Solved Examples

Ex.1 Find the number of photons in 6.62 joule of radiation energy of frequency 10^{12} Hz?

Sol. No. of photons $n = \frac{E}{h\nu} = \frac{6.62}{6.62\times10^{-34}\times10^{12}} = 10^{22}$

Ex.2 Calculate the energy and momentum of a photon of wavelength 6600Å.

Sol. energy of photon E

$$= \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{6600\times10^{-10}} = 3\times10^{-19}\,J$$

momentum of photon p

$$= \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{6600\times10^{-10}} = 10^{-27}\,kg\ m/sec$$

HISTORICAL FACTS ABOUT PHOTO ELECTRIC EFFECT

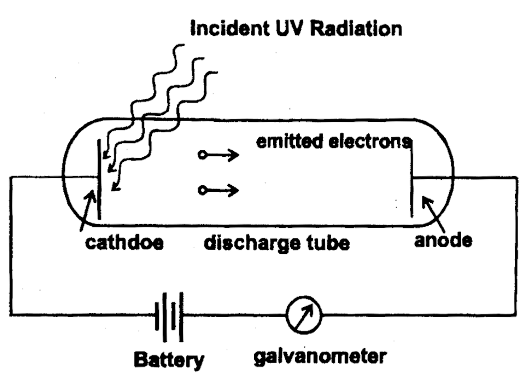
* In **1887 Hertz** observed that when UV radiation falls on cathode of an electric discharge tube then conduction takes place easily. This shows that electrons are ejected from a metal surface when irradiated by radiation of suitable wavelength?

* In **1888 Hallwach** confirmed these observations. He found that

(a) When UV radiation fall on a negatively charged cathode there was loss of negative charge and a current was recorded in galvanometer.

(b) When UV radiations fall on an uncharged plate; it acquired a positive charge and current was recorded in galvanometer.

(c) When UV radiations fall on a positively charged anode no current was recorded in galvanometer.



* **Lenard explained these observations as**

(a) When UV radiations fall on negatively charged cathode electrons are ejected causing a loss in negative charge. Similarly when they fall on uncharged plate electrons are ejected making it positively charged. In both cases electrons when reach the anode cause current to flow.

(b) When UV radiations fall on positively charged anode electrons are emitted but they are unable to reach cathode due to negative charge so no current flows.

* **Photo electric effect**

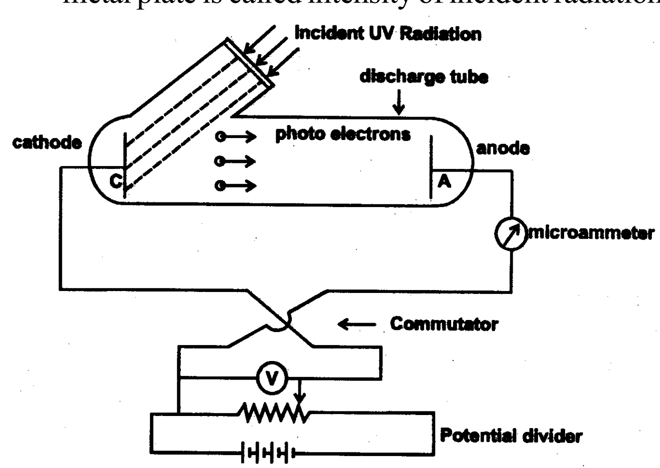
The phenomenon of emission of electrons from the surface of metal when light of suitable frequency falls on it is called photo electric effect.

1. The ejected electrons are called photo electrons
2. The current produced due to emitted electrons is called photo current.
3. Photo electric effect proves quantum nature of radiation.
4. The classical electromagnetic theory fails to explain photo electric effect.
5. Einstein explained photo electric effect using quantum nature of radiation.
6. Hallwach is credited with discovery of photo electric effect.

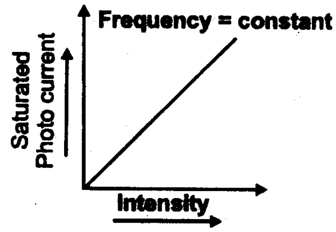
EXPERIMENTAL STUDY OF PHOTO ELECTRIC EFFECT

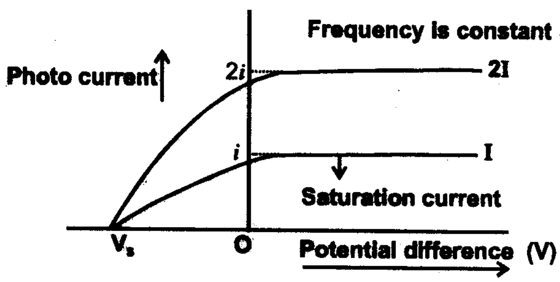
* **EFFECT OF INTENSITY OF INCIDENT RADIATION**

(a) The number of incident photons per second on a metal plate is called intensity of incident radiation.



(b) For a fixed incident frequency the saturation photo current is directly proportional to intensity of incident radiation.





e.g. when intensity of radiation is doubled at constant frequency saturation photo current is also doubled.

(c) Saturation current When all photo electrons produced reach anode the photo current becomes maximum and is independent of applied potential difference. This current is called saturation or maximum current.

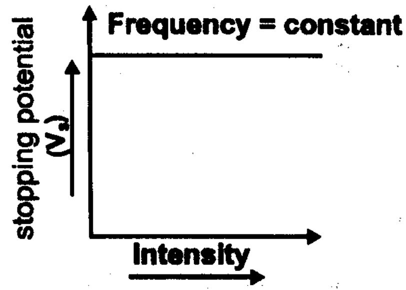
* **EFFECT OF POTENTIAL**

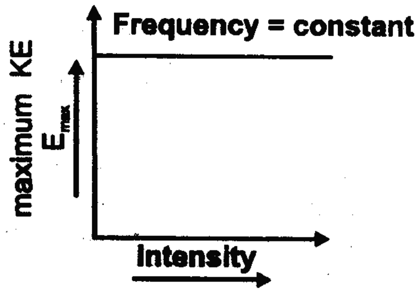
(a) When polarity of electrodes is reversed with commutator the current is reduced but does not become zero. This shows that emitted photo electrons have kinetic energy.

(b) The negative potential of anode at which the photo current becomes zero is called stopping potential (V_s). At this potential the electrons with maximum kinetic energy are stopped from reaching the anode.

(c) No photo current is produced even on increasing the intensity of incident radiation when anode is at stopping potential. Thus stopping potential is independent of intensity of incident radiation.

(d) The stopping potential is a measure of maximum kinetic energy of photo electrons. $E_{max} = eV_s$



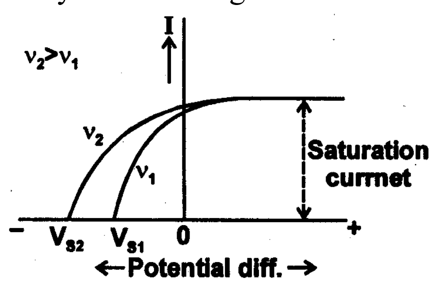


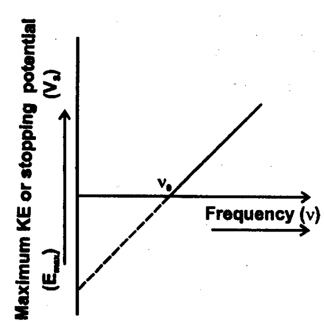
* **EFFECT OF FREQUENCY OF INCIDENT RADIATION**

a. Threshold frequency The minimum frequency of incident radiation which can eject photo electrons from a metal is known as threshold frequency (ν_o)

b. At stopping potential if frequency of incident radiation is increased then current starts flowing again. This can be made zero by increasing stopping potential.

c. Thus maximum kinetic energy of photo electron or stopping potential increases with increase in frequency of incident radiation. The maximum kinetic energy of photo electron increases linearly with increase in frequency of incident light.



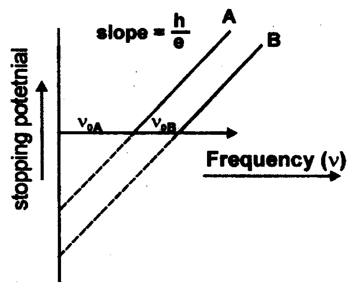


* **EFFECT OF MATERIAL OF CATHODE**

The stopping potential, work function and threshold frequency depend on nature of material of cathode.

Work function: The minimum energy required for emission of electrons from a metal is called work function

work function $\phi = h\nu_0$ where ν_0 is threshold frequency.



LAWS OF PHOTOELECTRIC EMISSION

* For a given metal and frequency of incident radiation the number of photo electrons emitted per second is directly proportional to intensity of incident radiation.
* For a given a metal there is a minimum frequency of incident radiation below which no photon emission is possible. This is called threshold frequency (ν_0)
* Above threshold frequency the maximum kinetic energy of emitted photo electron is independent of intensity of incident radiation but is directly proportional to frequency of incident radiation.
* The photo electric emission is an instantaneous process. The time lag between incidence of radiation and emission of photo electrons is less than 10^{-9} second.

EINSTEIN'S PHOTO ELECTRIC EQUATION

* Einsten explained photo electric effect using quantum nature of radiation.
* Emission of a photo electron is a result of interaction of one photon with a loosely bound electron in which photon is completely absorbed by electron.
* Some part of incident energy equal to work function is used to remove an electron from metal and remaining is given to electron as its kinetic energy.

$$h\nu = W + E_{max} = W + \frac{1}{2}mv^2_{max} = h\nu_0 + \frac{1}{2}mv^2_{max}$$

$$\frac{1}{2}mv^2_{max} = h(\nu - \nu_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = eV_S \quad ....(1)$$

equation 1 is called Einstein photo electric equation.

maximum velocity of emitted electron

$$v_{max} = \sqrt{\frac{2h(\nu - \nu_0)}{m}} = \sqrt{\frac{2hc(\lambda_0 - \lambda)}{m\lambda\lambda_0}} = \frac{\sqrt{2eV_S}}{m}$$

The stopping potential $V_S = \frac{h(\nu - \nu_0)}{e} = \frac{hc(\lambda_0 - \lambda)}{e\lambda\lambda_0}$

* The Einstein's photo electric equation is in accordance with conservation of energy. Here light energy is converted into electric energy.
* The equation explains the laws of photo electric emission.

(a) The increase in intensity increases the number of photons with same energy $h\nu$. So number of photo electrons will proportionally increase.

(b) If $\nu < \nu_0$ then KE will become negative which is not possible so in this condition photoemission is not possible.

(c) If $\nu > \nu_0$ Then KE $\propto (\nu - \nu_0)$ so maximum kinetic energy or stopping potential increases linearly with frequency of incident radiation.

IMPORTANT POINTS

* In photoelectric effect all photoelectrons do not have same kinetic energy. Their KE ranges from zero to E_{max} which depends on frequency of incident radiation and nature of cathode.
* The photo electric effect takes place only when photons strike bound electrons because for free electrons energy and momentum conservations do not hold together.
* Cesium is the best photo sensitive material.
* Efficiency of a photoemission η =

$$\frac{\text{Number of photoelectrons emitted per unit area per unit time}}{\text{Number of photons incident per unit area per unit time}}$$

$$= \frac{n_e}{n_p} \qquad \eta = \frac{\text{Intensity of emitted electrons}}{\text{Intensity of incident radiation}} = \frac{I_e}{I_p}$$

Therefore $\eta = \frac{n_e}{n_p} = \frac{I_e}{I_p}$

Solved Examples

Ex.3 A light of wavelength 5000Å and intensity 4.68 mW/cm^2 is incident on a light sensitive surface. If only 5% of incident photons produce photoelectron. Find the number of electrons emitted per unit area per unit time.

Sol. Energy of photon E = hν = $\frac{h\nu}{\lambda}$

$= \frac{6.6\times10^{-34}\times3\times10^{8}}{5\times10^{-7}}$ = 3.96 × 10^{-19} J

Intensity of radiation I_p = 4.68 mW/cm^2

$= \frac{4.68\times10^{-3}}{10^{-4}}\ \frac{W}{m^2} = 46.8\ \ W/m^2$

Number of incident photons per unit area per unit time $n_p = \frac{I_p}{h\nu} = \frac{46.8}{3.96\times10^{-19}} = 11.8\times10^{19}$

Intensity of emitted photoelectron $I_e = n_p \times \eta$

= 11.8 × 10^{19} × $\frac{5}{100} = 59\times10^{17}$

Ex.4 Light of wavelength 3500Å is incident on two metals A and B. Which metal will emit photoelectron if their work functions are 4.2 eV and 1.9 eV respectively?

Sol. Energy of incident light E = hν

$= \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{3500\times10^{-10}\times1.6\times10^{-19}}eV = 3.546\ eV$

Incident light will eject electrons if E > ϕ (work function)

since E > ϕ_B (1.9 eV) so metal B will emit photo electrons.

Ex.5 Find frequency of light which ejects electrons from a metal surface fully stopped by a retarding potential of 3 volt. The photoelectric effect begins at a frequency of 6 × 10^{14} Hz. Find work function of metal.

Sol. $\frac{1}{2}mv^2{}_{max} = eV_S,\quad h\nu = h\nu_0 + eV_S$

So $\nu = \nu_0 + \frac{eV_S}{h} = 6\times10^{4} + \frac{1.6\times10^{-19}\times3}{6.62\times10^{-34}}$

$= 13.25\times10^{14}Hz$ work function

$\phi = h\nu_0 = \frac{6.62\times10^{-34}\times6\times10^{14}}{1.6\times10^{-19}} = 2.48eV$

Ex.6 When light of wavelength 400 nm is incident on the cathode of photo cell the stopping potential recorded is 6V. If the wavelength is increased to 600 nm calculate new stopping voltage.

Sol. $eV_{S1} = \frac{hc}{\lambda_1} - W \qquad eV_{S2} = \frac{hc}{\lambda_2} - W$

$e[V_{S1} - V_{S2}] = hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$

or $V_{S2} = V_{S1} - \frac{hc}{e}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$

$= 6 - \frac{6.62\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}}$

$\left[\frac{1}{4000} - \frac{1}{6000}\right]eV = 4.97\,volt$

Ex.7 Find the number of photons emitted per second by a 25 watt source of monochromatic light of wavelength 6000 Å

Sol. Energy of one photon

$E = h\nu = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{6000\times10^{-10}} = 3.315\times10^{-19}J$

No of photons emitted per second

$= \frac{\text{total energy emitted per second}}{\text{energy of one photon}} = \frac{P}{E}$

$= \frac{25}{3.315\times10^{-19}} = 7.54\times10^{19}$

Ex.8 Lithium has a work function of 2.3 eV. It is exposed to light of wavelength 4.8 x 10^{-7} m. Find maximum KE of photo electron and what is the longest wavelength which can produce them.

Sol. $E_{max} = h\nu - h\nu_0 = h\nu - \phi = \frac{hc}{\lambda} - \phi$

$= \frac{6.62\times10^{-34}\times3\times10^{8}}{4.8\times10^{-7}\times1.6\times10^{-19}} - 2.3 = 0.28\ \ eV$

For longest wavelength $\frac{hc}{\lambda} = \phi$ or $\lambda = \frac{hc}{\phi}$

$= \frac{6.62\times10^{-34}\times3\times10^{8}}{2.3\times1.6\times10^{-19}} = 5.38\times10^{7}m$

PHOTO ELECTRIC CELL

* A photoelectric cell is a device which converts light energy into electric energy.
* This is a device which works on principle of photo electric effect.
* It consists of a semicylindrical plate coated with a photo sensitive material of low work function which works as cathode. The anode is in the form of a wire. The area of anode is very small as compared to cathode.
* We have vacuum inside the photo cell or an inert gas is filled in it.
* Electrons are emitted from photo sensitive surface when light falls on it. The photo current depends on

a. Potential difference between electrodes. On increasing the voltage the photocurrent first increases and then becomes constant.

b. Intensity of incident light.

c. Nature of surface producing photoelectron.

d. distance of photo cell from the source. If the source is at a distance r from the photo cell then the saturated photo current (I) is inversely proportional to square of distance

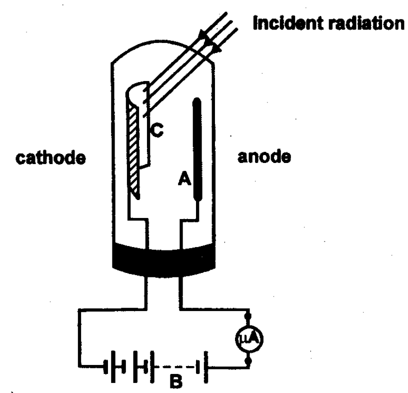
from the source $I \propto \frac{1}{r^2}$

The different type of photo cells are

a. **Photo emissive cell:** These work on photo electric effect.

b. **Photo conducting cell:** This is based on increase of electrical conductivity of material like selenium when they are exposed to light e.g. LDR (light dependent resistance)

c. **Photo voltaic cell:** When the region of contact between two specially prepared conducting surfaces is illuminated a current begins to flow. e.g. photo diode



APPLICATIONS OF PHOTO ELECTRIC CELLS

* Photocells are used in television cameras for telecasting scenes and photo telegraphy.
* Photocells are used in reproduction of sound in motion pictures.
* Photocells are used to switch on and off the street lights automatically.
* These are used to obtain electric energy from sun light during space travel.
* These are used to control temperature in furnaces and chemical reactions.
* These are used in fire and burglar's alarm, to open and close the doors automatically and in counting devices
* These are used to compare illuminating power of two sources.
* Photocells are used to detect opacity of solids, defects in materials etc.

COMPTON EFFECT

The scattering of a photon by an electron in which the wavelength of scattered photon is greater than wavelength of incident photon is called Compton effect.

Conservation of energy gives

$h\nu + m_0 c^2 = h\nu' + mc^2$

Conservation of momentum along y-axis gives

$$0 = \frac{h\nu'}{c}\sin\theta - p\sin\theta$$

Conservation of momentum along x-axis gives

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\phi$$

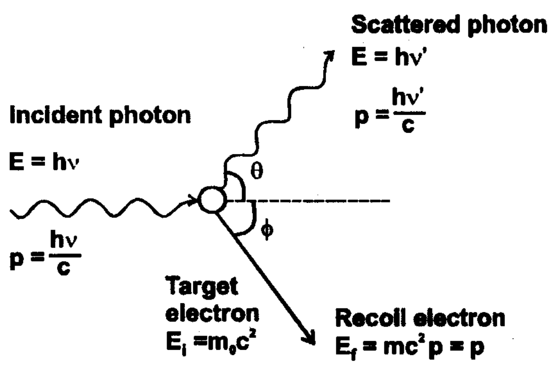
Compton shift $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$

The quantity $\lambda_c = \frac{h}{m_0 c} = 2.42 \times 10^{12}$ m is called Compton wavelength of electron.

For maximum shift

$$\theta = \pi \quad \text{so} \quad (\Delta\lambda)_{max} = \frac{2h}{m_0 c} = 4.48 \times 10^{-12} \text{m}$$

Compton effect shows the quantum concept and particle nature of photon.



DO NOT FORGET US

1. Conditions for photo electric effect can be written as -

(i) E > W (ii) $n > n_0$ (iii) $I < I_0$

2. Eienstein's equation for photo electric effect can be written as -

$$KE_{max} = E - W = h\,(n - n_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Find the - Maximum speed of photo electron V_{max}

Maximum momentum of photo electron P_{max}

Watch the dependence of KE_{max}, V_{max} and P_{max}

3. Stopping potential $V_0 = \frac{KE_{max}}{e}$

Watch the dependence of stopping potential w.r.t. energy of the incident light, intensity of incident light and the material of the metal.

4. Watch the following graphs

(i) V_0 v/s n Take the decision about n_0, I_0 and work function W from these graphs

(ii) V_0 v/s $\frac{1}{\lambda}$

5. Photo tube and its V-I characterizes

– Non ohmic nature of photo tube

– Saturation current

– Cut off current

– Stopping potential

6. Working of photo cell

DUAL NATURE OF RADIATION

* The phenomena of dispersion, interference, diffraction and polarization can only be explained on basis of wave nature of radiation because they involve with interaction of a wave with itself.

* The phenomena of Compton effect, Raman effect, photoelectric effect can only be explained on basis of particle nature of radiation because they involve with interaction of a wave with particle.

* The phenomena of rectilinear propagation, reflection, refraction etc. can be explained on basis of both wave and particle nature because there is neither interaction of wave with itself nor with the particle.

Thus radiation has dual nature i.e. wave and particle nature.

BOHR'S COMPLIMENTARY PRINCIPLE

The complete description of radiation requires both the wave and particle nature. So still both natures are not exhibited simultaneously in one experiment.

DE BROGLIE HYPOTHESIS

The hypothesis is based on following observations

* Nature loves symmetry

* The entire universe is made up of radiation and matter

* Since radiation has dual nature so matter must also have dual nature.

According to de-Broglie a wave can be associated with each moving particle whose wavelength is given by

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}} \quad ....(1)$$

This wave is called de-Broglie or matter wave. The equation one is called de-Broglie relation.

IMPORTANT POINTS

* The de-Broglie relation connects momentum, a particle characteristic with wavelength which is characteristic of wave.
* $\lambda \propto \frac{1}{v}$, so smaller is speed of the particle, larger will be the wavelength. If v = 0 then $\lambda = \infty$,so matter waves can only be associated with moving particles.
* $\lambda \propto \frac{1}{m}$, so smaller is the mass, larger will be the wavelength
* $\lambda \propto \frac{1}{p}$, so smaller is the momentum larger will be the wavelength.
* The wavelength is independent of charge and nature of particle.
* The matter waves can propagate in vacuum so they are not mechanical waves.
* These waves are not electromagnetic because they can be associated with electrically neutral moving particles.
* Matter waves are probability waves as they represent the probability of finding a particle in space.
* Matter waves propagate in the form of a wave packet moving with group velocity dω/dk.
* The phase velocity of these waves can be greater than c. $[v_p = c^2/v_g]$

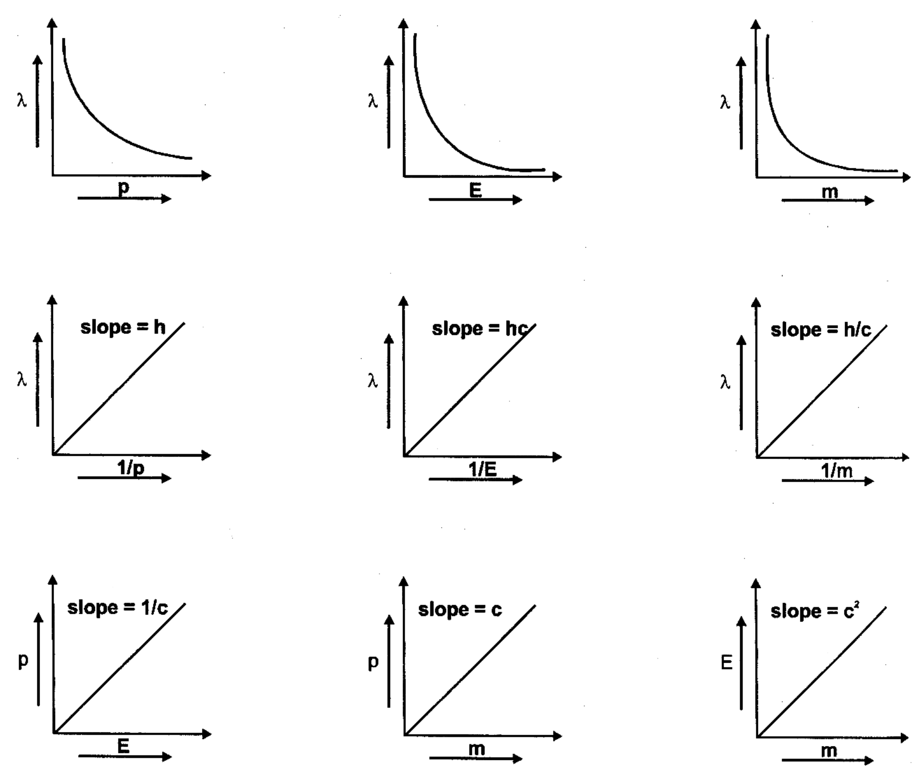
ENERGY, MOMENTUM AND WAVELENGTH OF WAVES ASSOCIATED WITH PHOTON

* Energy of photon $E = h\nu$ h = planck constant ; ν = frequency
* If m is the effective mass of photon then according to Einstein's mass energy relation.

$$E = mc^2 = h\nu = \frac{hc}{\lambda}$$

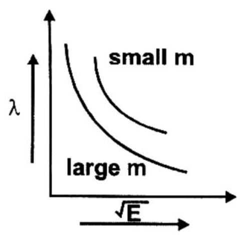
So effective mass $m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$

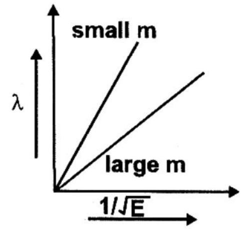
* The momentum of photon $p = mc = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$
* Photon is a chargeless, massless particle with spin $\frac{h}{2\pi}$ and travels with a velocity c.
* The wavelength of wave associated with photon is $\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{h}{mc}$
* The different graphs related to photon are

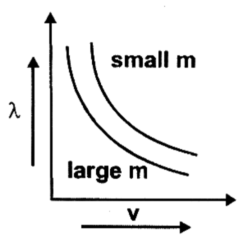


ENERGY, MOMENTUM AND WAVE-LENGTH OF WAVES ASSOCIATED WITH MOVING PARTICLES

* kinetic energy of a particle $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
* momentum of a particle $p = mv = \sqrt{2mE}$
* wavelength of associated wave $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$







The wave properties of macroscopic bodies cannot be observed because the associated wavelengths are very small ($\lambda \sim 10^{-34}$m)

The wave properties of microscopic particles can be observed because the associated wavelengths are larger ($\lambda \sim 10^{-10}$ m)

Solved Examples

Ex.9 Determine the momentum of photon of wavelength 5000Å

Sol. ccording to de-Broglie relation $p = \frac{h}{\lambda}$

$$\therefore\ p = \frac{6.6\times10^{-34}}{5000\times10^{-10}} = 1.32\times10^{-27}\,\text{kg}-\text{m/sec}$$

Ex.10 Calculate the effective mass of photon of wavelength 10Å and compare it with rest mass of electron?

Sol. effective mass of photon

$$m = \frac{h}{\lambda c} = \frac{6.6\times10^{-34}}{10\times10^{-10}\times3\times10^{8}} = 2.2\times10^{-33}\ \text{kg}$$

rest mass of electron $m_0 = 9.1 \times 10^{-31}$ kg

$$\therefore\ \frac{m}{m_0} = \frac{2.2\times10^{-33}}{9.1\times10^{-31}} = 0.024$$

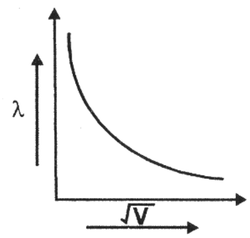
DE-BROGLIE WAVELENGTH ASSOCIATED WITH CHARGED PARTICLES

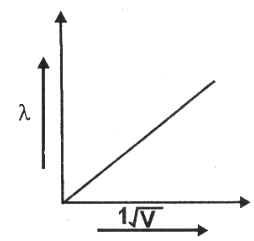
The energy acquired by charged particle when accelerated by potential difference V is

$$E = qV = \frac{1}{2}mv^2$$

so $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2qV}{m}}$

$$\therefore\ \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$





* for electrons ($m_e = 9.1 \times 10^{-31}$ kg, $q_e = 1.6 \times 10^{-19}$ C)

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times V}} = \frac{12.27}{\sqrt{V}}\text{A}$$

* for proton ($m_p = 1.67 \times 10^{-27}$ kg, $q_p = 1.6 \times 10^{-19}$ C)

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}\times V}} = \frac{0.286}{\sqrt{V}}\text{Å}$$

* for deuterons ($m_d = 2 \times 1.67 \times 10^{-27}$ kg $q_d = 1.6 \times 10^{-19}$ C)

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times2\times1.67\times10^{-27}\times1.6\times10^{-19}\times V}} = \frac{0.202}{\sqrt{V}}\text{Å}$$

* for α particle ($m_\alpha = 4 \times 1.67 \times 10^{-27}$ kg, $q_\alpha = 2 \times 1.6 \times 10^{-19}$ C)

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times4\times1.67\times10^{-27}\times2\times1.6\times10^{-19}\times V}} = \frac{0.101}{\sqrt{V}}\text{Å}$$

Ex.11 Find the ratio of de-Broglie wavelength of proton and α particle which have been accelerated through same potential difference?

Sol. de-Broglie wavelength $\lambda = \frac{h}{\sqrt{m\,q\,V}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_pq_pV}} \times \frac{\sqrt{2m_\alpha q_\alpha V}}{h} = \sqrt{\frac{m_\alpha}{m_p}\cdot\frac{q_\alpha}{q_p}} = \sqrt{\frac{4m}{m}\cdot\frac{2e}{e}} = 2\sqrt{2}$$

Ex.12 An α particle moves in circular path of radius 0.83 cm in field of 0.25 Wb/m². Find associated de-Broglie wavelength.

Sol. $\frac{mv^2}{r} = qvB$ or mv = qBr

de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{qBr}$

$\lambda = \frac{6.6\times10^{-34}}{2\times1.6\times10^{-19}\times0.25\times0.83\times10^{-2}}$

$= 9.94 \times 10^{-13} \approx 0.01$ Å

Ex.13 The de-Broglie wavelength for a neutron is 0.1Å. Calculate its momentum and energy.

Sol. momentum of neutron $P = \frac{h}{\lambda}$

$= \frac{6.6\times10^{-34}}{0.1\times10^{-10}} = 6.6\times10^{-23}\,kg\,m/s$

energy $E = \frac{p^2}{2m} = \frac{(6.6\times10^{-23})^2}{2\times1.67\times10^{-27}} = 13.12\times10^{-19}$ joule = 8.2 eV

Ex.14 What will be the de-Broglie wavelength of electron having kinetic energy of 500eV?

Sol. de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

$= \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times500\times1.6\times10^{-19}}} = 0.5467$ Å

Ex.15 An electron and proton posses same kinetic energy. Which has a greater wavelength?

Sol. $E = \frac{p^2}{2m}$, $p = \frac{h}{\lambda}$ so $E = \frac{h^2}{2m\lambda^2}$

as per question $E_e = E_p$

so $\frac{h^2}{2m_e\lambda_e^2} = \frac{h^2}{2m_p\lambda_p^2}$

$\frac{\lambda_e^2}{\lambda_p^2} = \frac{m_p}{m_e} > 1$ so $\lambda_e > \lambda_p$

hence electron has larger wavelength.

DE-BROGLIE WAVELENGTH ASSOCIATED WITH UNCHARGED PARTICLES

* for neutrons ($m_n = 1.67 \times 10^{-27}$ kg)

$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}E}} = \frac{0.286}{\sqrt{E(eV)}}$ Å

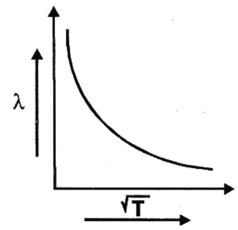
* for thermal neutron E = kT

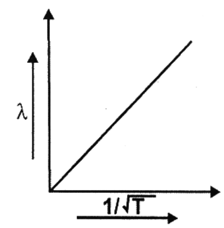
$\therefore \lambda = \frac{h}{\sqrt{2mkT}} = \frac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.38\times10^{-23}T}} = \frac{30.286}{\sqrt{T}}$ Å

* for gas molecules $\lambda = \frac{h}{mC_{rms}}$ & $E = \frac{3}{2}kT$

C_{rms} = rms velocity of gas molecules, E is energy at T K

$\therefore \lambda = \frac{h}{\sqrt{3mkT}}$





Solved Examples

Ex.16 Find the ratio of de-Broglie wavelength of molecules of hydrogen and helium which are at 27° C and 127°C respectively.

Sol. de-Broglie wavelength $\lambda = \frac{h}{\sqrt{3mkT}}$

$\frac{\lambda_H}{\lambda_{He}} = \frac{h}{\sqrt{3m_HkT_H}} \times \frac{\sqrt{3m_{He}kT_{He}}}{h} = \sqrt{\frac{m_{He}}{m_H}.\frac{T_{He}}{T_H}}$

$= \sqrt{\frac{4m}{m}\frac{(127+273)}{(27+273)}} = \sqrt{\frac{8}{3}}$

DAVISSON AND GERMER EXPERIMENT

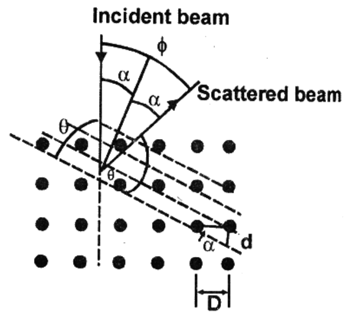
* The experiment demonstrates the diffraction of electron beam by crystal surfaces
* The experiment provides first experimental evidence for wave nature of the material particles.
* The electrons are diffracted like X-rays. The Bragg's law of diffraction are

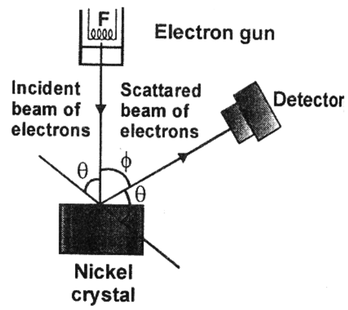
D sin ϕ = nλ and 2d sin θ = nλ

where D = interatomic distance and d = interplanar distance

θ = angle between scattering plane and incident beam

ϕ = scattering angle $2\theta + \phi = 180°$



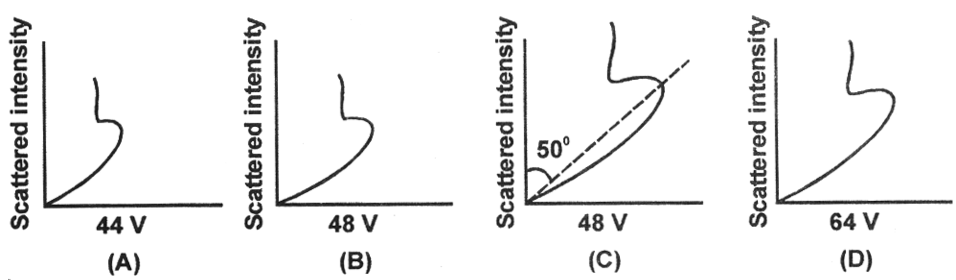


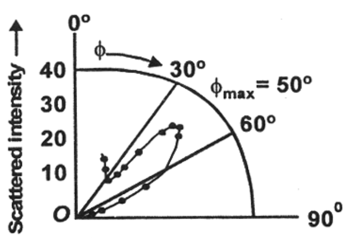
* The electrons are produced and accelerated into a beam by electron gun.

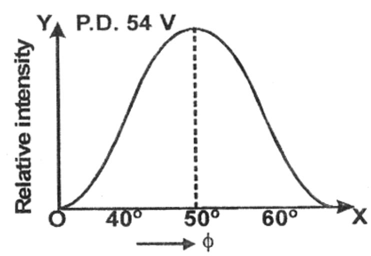
  The energy of electrons is given as $E = \frac{1}{2}mv^2 = eV$

* The accelerated electron beam is made to fall on a Ni crystal. The scattered electrons are detected by a detector

* The experimental results are shown in form of polar graphs plotted between scattering angle ϕ and intensity of scattered electron beam at different accelerating voltage. The distance of curve from point O is proportional to intensity of scattered electron beam







IMPORTANT RESULTS

* Intensity of scattered electrons depends on scattering angle ϕ
* The kink at $\phi = 50°$ is observed at all accelerating voltage
* The size of kink becomes maximum at 54 volt.
* For $\phi = 50°$ For $\theta = \frac{180-\phi}{2} = 65°$

  $D = \sin\phi = n\lambda$ $2d\sin\theta = n\lambda$

  $D = 2.15Å$ & $n = 1$ (for Ni)

  $n = 1$ and $d = 0.91Å$ (for Ni)

  $\therefore \lambda = 2.15 \sin 50 = 1.65Å$

  $\lambda = 2 \times 0.91 \sin 65 = 1.65Å$

* For V = 54 volt de-Broglie

  wavelength $\lambda = \frac{12.27}{\sqrt{54}}Å = 1.67Å$

  This value of λ is in close agreement with experimental value. Thus this experiment verifies de-Broglie's hypothesis.

Solved Examples

Ex.17 An electron beam of energy 10 KeV is incident on metallic foil. If the interatomic distance is 0.55Å. Find the angle of diffraction.

Sol. $\lambda = D\sin\phi$ and $\lambda = \frac{12.27}{\sqrt{V}}$Å so $\frac{12.27}{\sqrt{V}} = D\sin\phi$

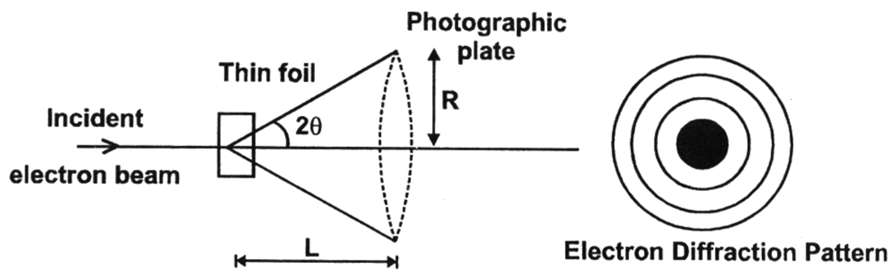
$\therefore \frac{12.27\times10^{-10}}{\sqrt{10\times10^3}} = 0.55\times10^{-10}\sin\phi$

$\sin\phi = \frac{12.27}{0.53\times100} = 0.2231$

or $\phi = \sin^{-1}(0.2231) \approx 12.89°$

G P THOMSON EXPERIMENT

When a beam of electrons is made incident on thin metallic foil then they produce a diffraction pattern identical to X-rays. The pattern consists of concentric circles around a central dark spot. The wavelength of diffracted electron waves can be calculated by measuring the radius of rings. This shows that electrons are diffracted like X-rays.



$\tan 2\theta \sim 2\theta = R/L$

$2d\sin\theta = n\lambda$ $\sin\lambda \sim \lambda$ (if λ is small)

$2\theta d = n\lambda$ n is order of diffraction

or $\frac{R}{L}d = n\lambda$ d is interplanar distance and R = radius of ring

If d and L are constant the wavelength depends on radius of ring formed in diffraction pattern.

BOHR'S QUANTIZATION CONDITION

According to de-Broglie electrons revolve around the nucleus in form of stationary waves i.e. wave packet

Electron can revolve only in those circular orbits whose circumference is an intergral multiple of de-Broglie wavelength

$2\pi r_n = n\lambda = h\frac{h}{mv}$

or $mvr_n = nh/2\pi$ which is Bohr's quantisation condition.

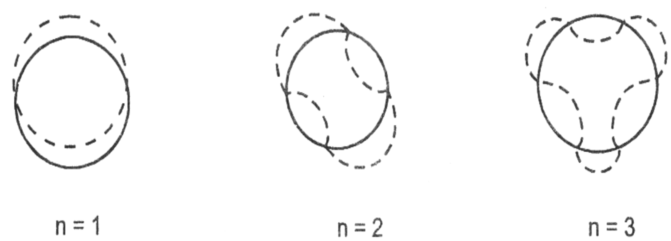
for n = 1

$\lambda_1 = 2\pi r_1$ thus only one wave is formed in K shell

for n = 2

$2\lambda_2 = 2\pi r_2$ thus two waves are formed in L shell

The stationary wave formed in various orbits are shown as.



DE-BROGLIE WAVELENGTH ASSOCIATED WITH RELATIVISTIC MOTION

If $v \approx c$ then motion is defined on basis of relativity

The mass varies as $m = m_0/\sqrt{1-v^2/c^2}$

The relation between energy and momentum is $E^2 = p^2c^2 + m_0^2c^4$

The total energy (E) = kinetic energy (K) + rest mass energy (m_0c^2)

$E = K + m_0c^2$

$\therefore pc = (E^2 - m_0^2c^4)^{1/2}$

$= \left[(K+m_0c^2) - m_0^2c^4\right]^{1/2} = [K(K+2m_0c^2)]^{1/2}$

de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{hc}{[K(K+2m_0c^2)]^{1/2}}$

SOME IMPORTANT POINTS

* If the de-Broglie wavelength of proton and α particle are equal then ratio of their velocities is

$$\lambda_p = \lambda_\alpha \quad \text{or} \quad \frac{h}{m_p v_p} = \frac{h}{m_\alpha v_\alpha}$$

$$\text{or} \quad \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m}{m} = \frac{4}{1}$$

* If de-Broglie wavelength of proton and α particle is equal then ratio of potential difference is

$$\lambda_p = \lambda_\alpha \quad \text{or} \quad \frac{h}{\sqrt{2m_p q_p V_p}} = \frac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}}$$

$$\frac{V_p}{V_\alpha} = \frac{m_\alpha}{m_p}\frac{q_\alpha}{q_p} = \frac{4m}{m}\frac{2e}{e} = \frac{8}{1}$$

* If proton and α particle are accelerated by same potential then ratio of their wavelengths is

$$\lambda_p = \frac{h}{\sqrt{2m_p q_p V_p}} \quad \lambda_\alpha = \frac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}} \quad \text{given } V_\alpha = V_p$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}\frac{q_\alpha}{q_p}} = \sqrt{\frac{4m}{m}\frac{2e}{e}} = \frac{\sqrt{8}}{1}$$

* The ratio of de-Broglie wavelength for photon and electron both of energy E is

$$\lambda_{ph} = \frac{hc}{E} \quad \lambda_e = \frac{h}{\sqrt{2mE}}$$

$$\text{so} \quad \frac{\lambda_{ph}}{\lambda_e} = \frac{hc}{E}\sqrt{\frac{2mE}{h}} = \sqrt{\frac{2mc^2}{E}}$$

* When a photon of wavelength λ is emitted from an atom then recoil energy of atom is

$$E_r = \frac{p^2}{2M} = \frac{h^2}{2M\lambda^2} \text{ joule}$$

Solved Examples

Ex.18 If the momentum of proton is changed by p_0 then de-Broglie wavelength is changed by 0.25%. The initial momentum of proton is

Sol. $p = \frac{h}{\lambda}$ so $\Delta p = \frac{-h}{\lambda^2}\Delta\lambda = \frac{-h}{\lambda}\frac{\Delta\lambda}{\lambda} = \frac{-p\Delta\lambda}{\lambda}$

In magnitude $\frac{\Delta p}{p} = \frac{\Delta\lambda}{\lambda}$ so $\frac{p_0}{p} = \frac{\Delta\lambda}{\lambda} = \frac{0.25}{100}$

or $p = \frac{100P_0}{0.25} = 400p_0$

Ex.19 If the kinetic energies of proton and electron are equal. Find the ratio of associated wavelength?

Sol. $\lambda_e = \frac{h}{\sqrt{2m_e E}} \quad \lambda_p = \frac{h}{\sqrt{2m_p E}}$

$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1836}{1}}$ or $\lambda_e : \lambda_p = \sqrt{1836} : 1$

DO NOT FORGET US

1. Matter wave are stationary waves.
2. When an electron move in an orbit, matter waves are produced due to its motion.
3. Wavelength of matter waves associated with macroscopic moving particles are very small, therefore they cannot be measured and it appears that they exhibit particle nature alone.
4. $\frac{\lambda_1}{\lambda_2} = \frac{P_2}{P_1}$ 5. $\frac{P_1}{P_2} = \sqrt{\frac{E_{k_1}}{E_{k_2}}}$ 6. $\frac{P_1}{P_2} = \sqrt{\frac{V_1}{V_2}}$
7. In davission Germer's experiment maximum peak is obtained in I-V curve at angle of diffraction of 50° and accelerating potential 54 volt.
8. Matter waves are probabilistic waves
9. The phase velocity of matter waves can be greater than that of light.
10. Matter waves are not mechanical waves
11. The wavelength of matter wave does not and on the nature and charge of the particle.
12. If the proton and α-particle have same kinetic energy then $\lambda_p/\lambda_\alpha = 2/1$
13. Electron microscope works on the principle of matter waves.
14. Resolving power of an electron microscope is inversely proportional to wavelength and therefore can be increased by increasing the accelerating potential.
15. The electron and photon have same energy E and mass of the electron is m then $\frac{\lambda_{photon}}{\lambda_e} = C\sqrt{\frac{2m}{E}}$
16. If the wavelength of de-Broglie waves associated with a proton and an α-particle are same then the potential difference are in ratio $\frac{V_p}{V_\alpha} = \frac{8}{1}$