

CUBE & CUBOID

Corner cubes(3 surface painted) = 8

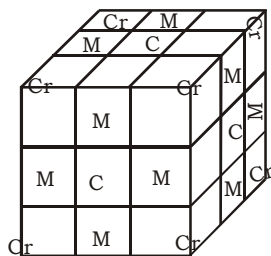
Middle cubes(2 surface painted) = 12
(n - 2)

Central cubes(One surface painted) = 6 (n - 2)²

Inner central cubes (colourless) = (n - 2)³

Total no of smaller cubes = n³

$$n = \text{Ratio} = \frac{l_1}{l_2} \rightarrow \begin{matrix} \text{Bigger cube length} \\ \text{smaller cube length} \end{matrix}$$



cr - corner cube
m - middle cube
c - centre cube

1. A 15 cm coloured cube is cut into 3 cm smaller cubes, then how many cubes are formed which have only one face painted.

Sol. $n = \frac{15}{3}$

One surface painted

▮ $6(n - 2)^2$

▮ $6(5 - 2)^2$

▮ $6 \times 9 = 54$ Ans.

2. A coloured cube is cut into 64 smaller cubes then how many smaller cubes are formed which have minimum two face painted.

Sol. $n^3 = 64$

$n = 4$

Two surface painted

▮ $12(n - 2)$

▮ $12(4 - 2)$

▮ $12 \times 2 = 24$

- ▮ At least two

$$\begin{array}{|c|c|} \hline 2 & 3 \\ \hline \end{array}$$

$24 + 8 = 32$ Ans.

- ▮ at least two

$$\begin{array}{|c|c|} \hline 2 & 3 \\ \hline \end{array}$$

▮ $\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array}$

- ▮ Not more than 2

$$\begin{array}{|c|c|c|} \hline 0 & +1 & +2 \\ \hline \end{array}$$

3. Coloured cube is cut into 125 smaller cubes of length 2 cm

- ▮ 1 What is the length as bigger cubes
2 How many smaller cubes are there which have one surface painted.

Sol. $n^3 = 125$ [Total = n^3]
 $n = 5$

$n = \frac{l_1}{l_2} \rightarrow \begin{matrix} \text{Bigger cube length} \\ \text{smaller cube length} \end{matrix}$

$5 = \frac{l_1}{l_2}$

$l_1 = 10$ cm

2nd :

at most one $n = 5$

$$\begin{array}{|c|c|c|} \hline 0 & + & 1 \\ \hline \end{array}$$

$(n - 2)^3 + 6(n - 2)^2$

$(5 - 2)^3 + 6(5 - 2)^2$

$27 + 6 \times 9$

$27 + 54 = 81$ Ans.

4. A coloured cube is cut into smaller cubes if there are 8 colourless cubes, then how many smaller cubes are there?

Sol. Colourless cube - $(n - 2)^3$

$(n - 2)^3 = 8$

$n - 2 = 2$

$n = 4$

Total = $n^3 = (4)^3 = 64$ Ans.

5. A 8 cm³ cube is cut into 125 smaller cubes then find the side of big cube?

Sol. Total = n^3

$125 = n^3$

▮ $n = 5$

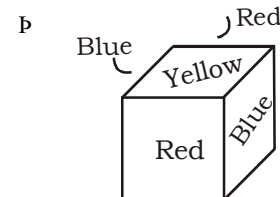
volume ▮ $8 \text{ cm}^3 = 2$

$\frac{8}{125} = \frac{l_1^3}{l_2^3}$

$5 = \frac{l_1}{l_2}$

$l_1 = 10$ cm

Ans.



- ▮ A cube is shown by figure.

6. A coloured cube is cut into 216 smaller cubes then how many cubes are formed which have only 4 faces painted.

Sol. 0 Zero

7. Then how many small cubes are formed which have only 3 faces painted from different colour.

Sol. 3

8. when two faces are painted.

$2 = 12(n - 2)$ $n^3 = 216$

$= 12(6 - 2)$ $n = 6$

$12 \times 4 = 48$

Sol. 48

9. when three face are painted

$3 = 6(n - 2)^2$

$= 6(6 - 2)^2$

$6 \times 16 = 96$ Ans.

10. when he one face is painted

$0 = (n - 2)^3$

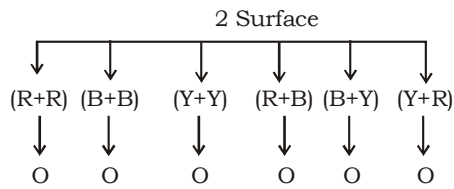
$= (6 - 2)^3$ ▮ $(4)^3 = 64$

Sol. 64

11. Then find how many faces are formed which have only two faces

painted

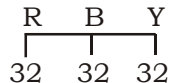
Sol. on the basis of 2 surface



$$2 \text{ Surface} = \frac{48}{3}$$

on the basis of 1 surface

- How many cubes are formed which have only one face painted.



Cuboid

- **Corner cubes(3 surface painted)= 8**
- **Middle cubes(2 surface painted) = $4(n_1 - 2) + 4(n_2 - 2) + 4(n_3 - 2)$**
- **Central cubes(One surface painted)**
 $= 2(n_1 - 2)(n_2 - 2) + 2(n_2 - 2)(n_3 - 2) + 2(n_3 - 2)(n_1 - 2)$
• Inner central cubes (colourless)
 $= (n_1 - 2)(n_2 - 2)(n_3 - 2)$

• Total no of smaller cubes = $n_1 \times n_2 \times n_3$

- Q. A cuboid of size $4 \times 8 \times 10 \text{ cm}^3$ is cut into smaller cubes of 2 cm length.

Sol. $\frac{4}{2} \frac{8}{2} \frac{10}{2}$

$$n_1 = 2, n_2 = 4, n_3 = 5$$

$$2 \times 4 \times 5 = 40$$

$$\text{Cuboid} = [T = n_1 \times n_2 \times n_3]$$

1. 3 surface = 8 Ans.
 2. 4 surface = 0 Ans.
 3. 1 Surface = 12 Ans
- Colourless : 0 Ans.

- Q. A cuboid of $9 \times 15 \times 18 \text{ cm}^3$ is cut into 3 cm length of smaller cubes, how many smaller cubes?

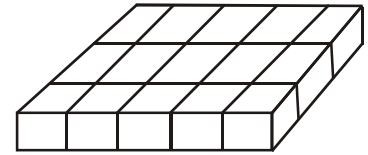
Sol. $\frac{9}{3}, \frac{15}{3}, \frac{18}{3}$

$$n_1 = 3, n_2 = 5, n_3 = 6$$

$$3 \times 5 \times 6 = 90$$

Ans. 90

- Q. A cuboid of $2 \times 6 \times 10 \text{ cm}^3$ is cut into 2 cm length of smaller cubes, then many smaller cube are there which have only 4 surface painted?



Sol. Total : $\frac{2}{2}, \frac{6}{2}, \frac{10}{2}$

$$n_1 = 1, n_2 = 3, n_3 = 5$$

$$\text{Total : } n_1 \times n_2 \times n_3 : 1 \times 3 \times 5 = 15 \text{ Ans.}$$

if $n = 1$

4 surface are a

Corner : (4)

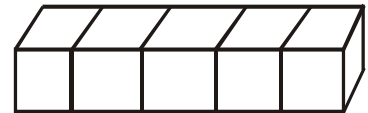
Middle: (3)

Central (2)

$$1 = 0$$

$$0 = 0$$

- Q A cuboid $3 \times 3 \times 15 \text{ cm}^3$ is cut into 3 cm of smaller cubes



p $\frac{3}{3}, \frac{3}{3}, \frac{15}{3}$

$$\text{Total : } n_1 \times n_2 \times n_3 = 5$$



EXERCISE

Directions (Q. 1-4): Read the following informations and answer the questions based on it.

- I. The length, breadth and height of a rectangular piece of wood are 4 cm, 3 cm and 5 cm respectively.
- II. Opposite sides of $5\text{ cm} \times 4\text{ cm}$ piece are coloured in red.
- III. Opposite sides of $4\text{ cm} \times 3\text{ cm}$ are coloured in blue.
- IV. Rest sides of $5\text{ cm} \times 3\text{ cm}$ are coloured on green in both sides.
- V. Now the piece is cut in such a way that cubes of $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$ will be made.
1. How many cubes shall have all the three colours?
a) 8 b) 10
c) 12 d) 14
e) None of these
 2. How many cubes shall not have any colour?
a) No any b) 2
c) 4 d) 6
e) None of these
 3. How many cubes shall have only two colours red and green on their two sides?
a) 8 b) 12
c) 16 d) 20
e) None of these
 4. How many cubes shall have only one colour?
a) 12 b) 16
c) 22 d) 28
e) None of these

Directions (Q. 5-7): The six faces of a cube are coloured, each with a different colour.

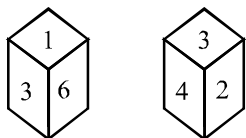
- I. The white face is between yellow and green.
- II. The red face is adjacent to brown.
- III. The green face is opposite the yellow side.

- IV. The blue face is adjacent to red.
- V. The yellow face is the top face of the cube.
5. The faces adjacent to white bear the colours
a) Yellow, green, brown and red
b) Yellow, brown, blue and green
c) Yellow, green, blue and red
d) Can't be determined
e) None of these
 6. The face opposite the red face is
a) green b) white
c) blue d) brown
e) None of these
 7. The colour of the bottom face of the cube is
a) red b) brown
c) green d) blue
e) None of these

Directions (Q. 8-13): A solid cube of each side 15 cm has been painted green, blue and yellow on pairs of opposite faces. It is then cut into cubical blocks of each side 3 cm.

8. How many cubes have no face painted?
a) 27 b) 54
c) 36 d) 50
e) 18
9. How many cubes have only one face painted?
a) 27 b) 54
c) 36 d) 50
e) 18
10. How many cubes have only two faces painted?
a) 27 b) 54
c) 36 d) 50
e) 12
11. How many cubes have two faces painted green and blue and all other faces unpainted?
a) 12 b) 16
c) 18 d) 8

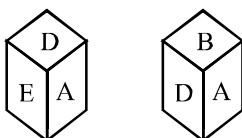
- e) 24
12. How many cubes have one face painted yellow and other faces unpainted?
a) 12 b) 16
c) 18 d) 8
e) 24
 13. How many cubes have atleast one face green?
a) 27 b) 54
c) 36 d) 60
e) 50
 14. How many cubes have at most one face painted?
a) 27 b) 45
c) 54 d) 63
e) 36
 15. How many cubes have painted both green and yellow but not blue?
a) 36 b) 16
c) 12 d) 24
e) 27
 16. How many cuboids of the dimensions $3\text{ cm} \times 4\text{ cm} \times 6\text{ cm}$ are required to form a cube of the least possible size?
a) 72 b) 36
c) 24 d) 12
e) None of these
 17. A cube has been cut into cuboids of size $2\text{ cm} \times 3\text{ cm} \times 4\text{ cm}$. What is the least possible length of the edge of the cube and how many such cuboids are obtained from this cube?
a) 12, 36 b) 24, 144
c) 24, 72 d) 12, 72
e) 12, 144
 18. The six faces of a dice are marked as faces 1, 2, 3, 4, 5 and 6. Given below are two different views of the same dice



Which face is opposite the face marked as 5?

- a) 1 b) 4
c) 6 d) 3
e) 2

19. The six faces of a dice and marked as faces A, B, C, D, E and F. Given below are two different views of the same dice.

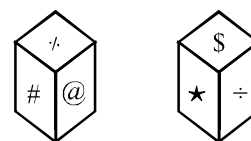


Which face is opposite the face marked as B?

- a) E b) C
c) F d) Can't
 e) None of
 these

20. The faces of a dice are marked as faces #, @, ★, & and %. Given below are two different views of the

same dice.



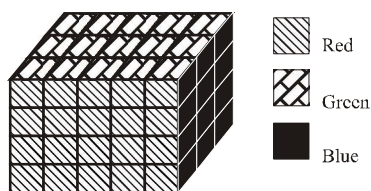
Which face is opposite the face marked as @?

- a) \$ b) &
c) ★
d) Can't determined
e) None of these

SOLUTION

Answers and explanations (Cube, Cuboid and Dice) - I

(1-4):



1. a; Three surfaces coloured is constantly 8.
 2. d; No. surface coloured = $(1 - 2)(b - 2)(h - 2) = 3 \times 1 \times 2 = 6$
 3. b; There are three cubes on each red-green interface (barring corner cubes). So, $4 \times 3 = 12$ cubes.
 4. c; One surface coloured = $2(1 - 2)(b - 2) + 2(1 - 2)(h - 2) + 2(b - 2)(h - 2) = 2\{3 \times 1 + 3 \times 2 + 1 \times 2\} = 22$
- (5-7): From the given information we deduce red face is opposite to white face, yellow face is opposite to green face, and brown face is opposite to blue face.
5. b
 6. b
 7. c; If yellow face is at the top then green will be at the bottom because yellow face and green face are opposite to

each other.

(8-13): Let 3 cm = 1 unit,

then 15 cm = 5 units

Number of total cubes = $5 \times 5 \times 5 = 125$

Number of cubes only one face painted comprises three types of cubes i.e. cubes having one face green, one face yellow and the cubes having one face blue.

Cubes having only one face green = $2(n - 2)^2 = 2(5 - 2)^2 = 18$.

Where n = side of cube

Cubes having one face blue = $2(n - 2)^2 = 2 \times 3 \times 3 = 18$

Cubes having one face yellow = $2(n - 2)^2 = 2 \times 3 \times 3 = 18$

Cubes having only one face painted are $18 + 18 + 18 = 54$

Cubes of only two faces painted comprises three types of cube

1st type: one face blue and one face yellow

$4(5 - 2) \Rightarrow 4(n - 2) = 4 \times 3 = 12$

2nd type: one face yellow and one face green

$4(n - 2) \Rightarrow 4(5 - 2) = 4 \times 3 = 12$

3rd type: one face green and one face blue

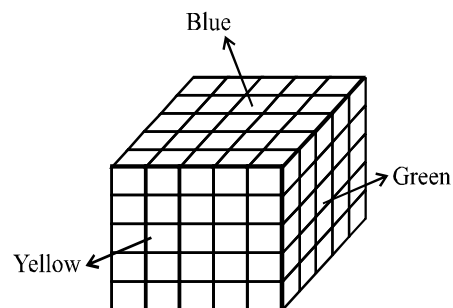
$4(n - 2) \Rightarrow 4(5 - 2) = 4 \times 3 = 12$

Cubes having only two faces painted are $12 + 12 + 12 = 36$

Cubes which have no any face painted are $(n - 2)(n - 2)(n - 2) = (5 - 2)(5 - 2)(5 - 2) = 27$

[where n is the side of a cube.]

8. a
9. b
10. c
11. a
12. c
13. e
14. b; At most one face painted means cubes which are painted. Only one side and cubes which are not painted.
15. c;



Edges which are not painted blue are 4

and one each edges 3 cubes have only two faces painted.

$$\text{So, } 4 \times 3 = 12$$

16. c; 1 cm of 3, 4 and 6 = 12

12 cm length is required

$$\frac{12}{3} = 4, \quad \frac{12}{4} = 3, \quad \frac{12}{6} = 2$$

$$4 \times 3 \times 2 = 24$$

17. d; LCM of 2, 3 and 4 = 12

Length of cube = 12 cm

$$\frac{12}{2} = 6, \quad \frac{12}{3} = 4, \quad \frac{12}{4} = 3$$

$$6 \times 4 \times 3 = 72.$$

72 cuboids are obtained from this cube.

18. d; In given 2 dice 1, 6, 4 and 2 are near to 3. So, 3 and 5 are opposite to each other.

19. a; After compiling both dice we get E and B are opposite to each other.

20. c; After compiling both dice we get the ★ is opposite to @.