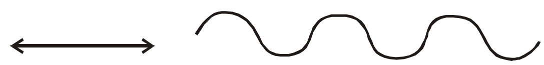
GEOMETRY

Importance : It is very important chapter for competitive exams and questions on different difficulty levels are asked.

Scope of questions : Questions are based on angles, ratio/measure of sides angles or bisectors, measure/ratio of trapesium/square/rectangle/parallelogram/pentagon sides/angles, centre, radius, diameter, angle, area and circumference of circle.

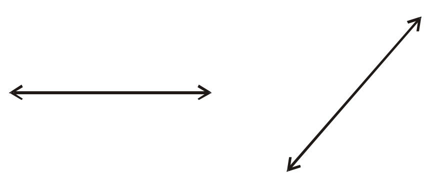
Way to success : The complete and thorough study of this chapter is a must.

- **Line :** A figure formed by joining collinear or non-collinear points is known as line. It has no width e.g.

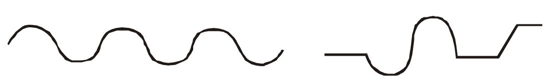


There are two types of line :

(i) **Straight line :** A line travels a distance without any diversion on straight path, is called straight line. It represents the shortest path between any two points lying on it.



(ii) **Curved line :** Line when travels on a diverted path, that is called curved line.

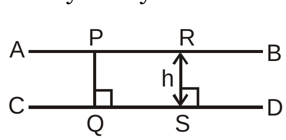


Line segment : A line segment has two end points. i.e. it can not be extended in any direction. Sometimes, a line and a line segment may be used in same sense as that of a line.

- **Ray :** A ray can be extended in one direction only, which is denoted by an arrow. On the other side we have an end point, called the initial point.

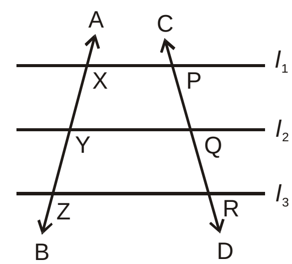
A ●⟶ B represents as $\overrightarrow{AB}$

- **Parallel lines :** Two lines are said to be parallel, if they do not intersect each other at any point and the distance (perpendicular distance h) between them is constant. They are denoted by the symbol ||.



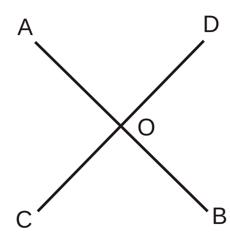
In the above figure AB || CD.

- **Transversal line :** A line that intersects two or more parallel lines at different points, is called a transversal.

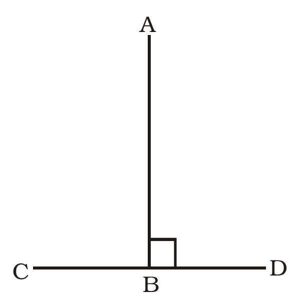


Here, $l_1 \parallel l_2 \parallel l_3$ and AB and CD are two transversal lines.

- **Intersecting lines :** Two lines that intersect each other or in other words, share a common point (Called point of intersection) are called intersecting lines.

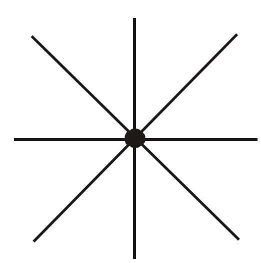


- **Perpendicular lines :** Two lines that intersect each other at right angle (90°) are called perpendicular lines. They are denoted by the symbol "⊥".



AB ⊥ CD

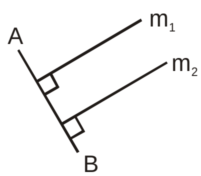
- **Concurrent lines :** Three or more lines are said to be concurrent if they all intersect at one common point as shown below.



- **Coplanar lines :** The lines that lie in the same plane are said to be coplanar lines, otherwise they are called non. coplanar lines. Same holds good for coplanar points (i.e., points that lie in the same plane) and non-coplanar points. (Points that not lie in the same plane.)

Points to remember :

- Three or more points lying on the same line are called collinear points.
- Only one line can be drawn through any two given points.
- Two line can intersect maximum at one point.
- If two different lines are perpendicular to a third line, then the former are parallel to each other, as shown below.



Here, $m_1 \perp AB$ and $m_2 \perp AB$. Hence, as per above rule $m_1 \parallel m_2$

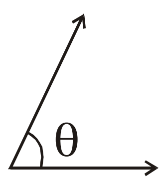
- There are infinite number of points on a straight line.
- Infinite number of lines can be made from a single point

ANGLES

- Angle : When two rays have same starting point or an ending point, then an angle is formed.

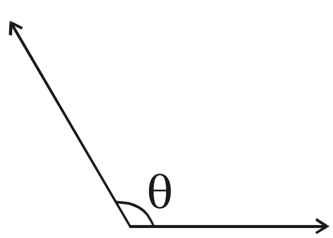
Types of Angles :

- **Acute angle :** An angle greater than 0° but less than 90° (i.e. $\frac{\pi}{2}$ radians) is an acute angle.



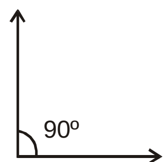
Here $0° < \theta < 90°$ or $0 < \frac{\pi}{2}$

- **Obtuse angle :** An angle which is greater than 90° and less than 180° (π radians) is an obtuse angle.



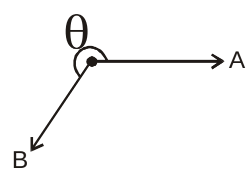
Here $90° < \theta < 180°$ or $\frac{\pi}{2} < \theta < \pi$

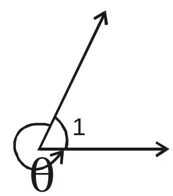
- **Right angle :** An angle equals to 90° (or $\frac{\pi}{2}$ radians) is right angle.



- **Straight angle :** An angle equals to 180° (or π radians) is straight angle.

- **Reflex angle :** It is an angle greater than 180° and (or π radians) but less than 360°(2π radians).

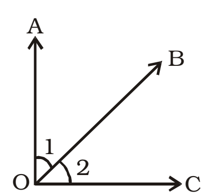




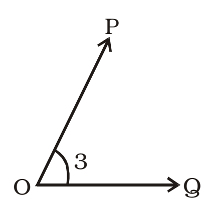
Here, $180° < \theta < 360°$ ($\angle\theta = 360° - \angle 1$)

or $\pi < \theta < 2\pi$

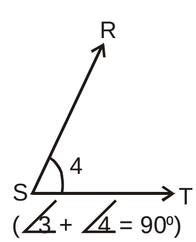
- **Complementary angle :** Two angles are said to be complementary if their sum is equal to 90° (or $\frac{\pi}{2}$ radians).



Case-I

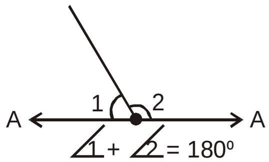


Case-II

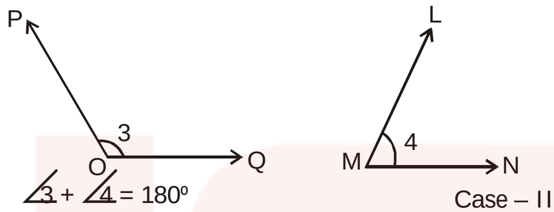


In the above figure, $\angle 1 + \angle 2 = 90°$: Hence, they are complementary angles. Also $\angle 3 + \angle 4 = 90°$, therefore, they are also complementary angles.

- **Supplementary angles :** If the sum of the angles is equal to 180°, then they are called supplementary angles. For example supplementary (or π radians) angle of 50° is = 180° – 50° = 130°

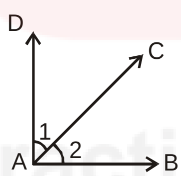


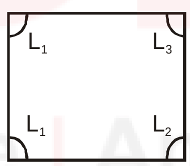
Case – I



Case – II

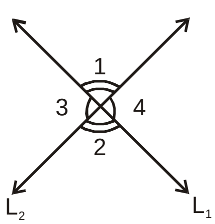
- **Linear pair :** In the case I above, $\angle 1$ and $\angle 2$ form a linear pair. Two angles form a linear pair, if they have one side common between them and the two angles are supplementary.
- **Adjacent angles :** $\angle 1$ and $\angle 2$ are called adjacent angles, since they have one side common between them.



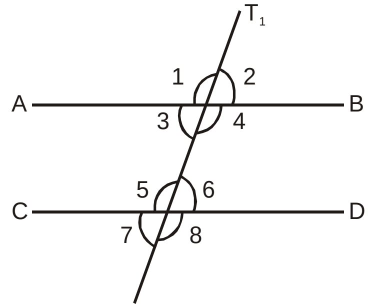


L_1 & L_2 are adjacent angles.

- **Vertically opposite angles :** Let L_1 and L_2 be two intersecting lines as shown below, then $\angle 1$ and $\angle 2$ are said to be vertically opposite angles. Also, $\angle 3$ and $\angle 4$ are vertically opposite angles.



- **Angles between parallel lines :** Let AB and CD be two parallel lines that are intercepted by a transversal T_1, then we have.



Corresponding angles : $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 4 = \angle 8$, and $\angle 3 = \angle 7$ are pair of corresponding angles will be equal.

Interior Alternate angles : $\angle 3 = \angle 6$, $\angle 4 = \angle 5$.

- **Vertically opposite angles :** $\angle 1 = \angle 4$, $\angle 2 = \angle 3$, $\angle 5 = \angle 8$, $\angle 6 = \angle 7$ are pair of vertically opposite angles.
- **Opposite interior angles :** $\angle 3 + \angle 5 = 180°$ and $\angle 4 + \angle 6 = 180°$.

TRIANGLES

Types of Triangle : According to Sides :

(i) **Equilateral Triangle :** A triangle whose all the three sides are equal, is called an equilateral Δ. If in ΔABC, if AB = BC = AC, then ΔABC is an equilateral triangle.

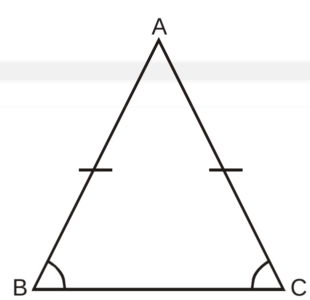
Also, all angles of an equilateral triangle are equal i.e., $\angle A = \angle B = \angle C = 60°$.

(ii) **Isosceles triangle :** A triangle with two equal sides is an isosceles triangle.

Also, angles opposite to equal sides are equal.

In isosceles ΔABC, if AB = AC

then $\angle ACB = \angle ABC$.



(iii) **Scalene triangle :** A triangle in which none of the three sides is equal is called a scalene triangle. In a scalene triangle $AB \neq BC \neq CA$ and $\angle A \neq \angle B \neq \angle C$

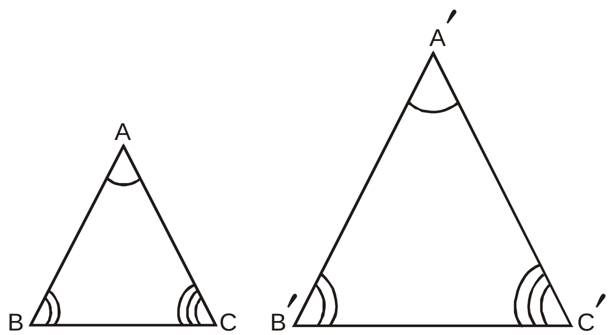
According to Angles :

(i) **Acute angled triangle :** It is one in which all the angles are less than 90° or in other words, all the angles are acute.

(ii) **Right angled triangle :** It is one with one angle equals to 90°. side opposite to 90° is called hypotenuse.

(iii) **Obtuse angled triangle :** A triangle with one of its angle greater than 90° or obtuse.

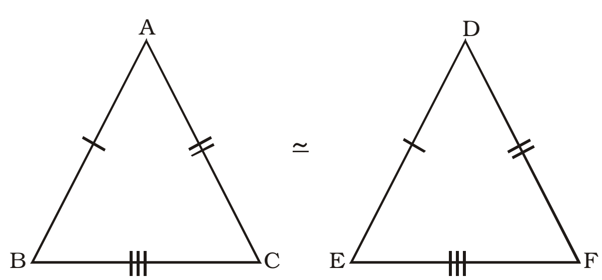
- **Similar Triangles :** If all the angles of a triangle are equal to the angles of another triangle, then both are called similar triangles [relation [represented as ~] to each other.



Here, $\angle A = \angle A'$, $\angle B = \angle B'$.$\angle C = \angle C'$, then ΔABC and $\Delta A'B'C$ will be similar. So. $\Delta ABC \sim \Delta A'B'C'$

$$\therefore \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$$

- **Congruent Triangles :** Any two triangles are called congruent triangles (relation represented as $\cong$), when a triangle covers totally the other triangle. In other words if both triangles are exactly same (identical) to each other in sides or angles.



Here, in ΔABC and ΔDEF

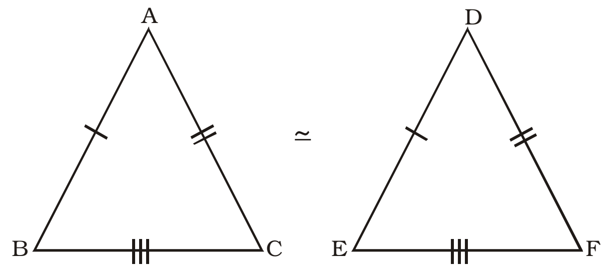
$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$, and AB = DE, BC = EF, CA = FD then

$\therefore \Delta ABC \simeq \Delta DEF$.

Congruency conditions :

- **S-S-S (Side-Side-Side) :** Here, AB = DE, BC = EF and AC = DF, then

  $\therefore \Delta ABC \simeq \Delta DEF$ by S-S-S congruency condition.

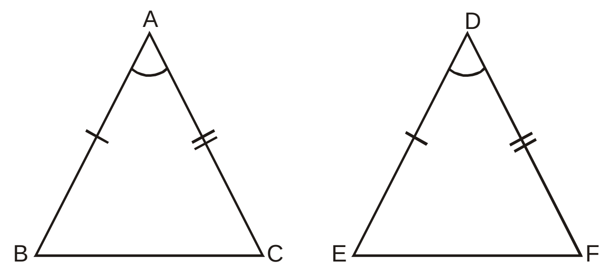


- **S-A-S (Side-Angle-Side) :**

  Here, AB = DE, AC = DF

  and $\angle A = \angle D$ then

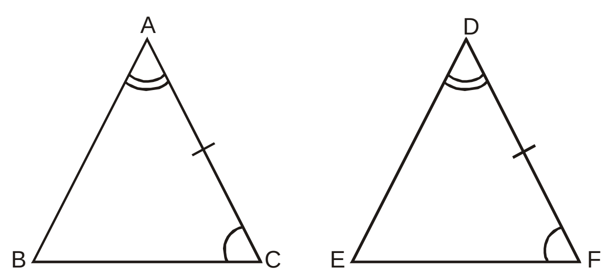
  $\therefore \Delta ABC \cong \Delta DEF$ by SAS congruency condition.



Note : The angle involved in SAS condition must lie between the sides.

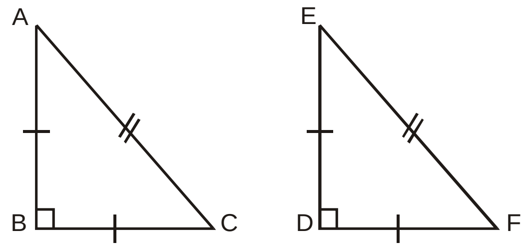
- **A-S-A (Angle-Side-Angle) :** Here, $\angle A = \angle D$, $\angle C = \angle F$ and AC = DF, then

  $\therefore \Delta ABC \cong \Delta DEF$ by ASA congruency condition.



Note : The side involved in ASA condition must lie between the angles.

R.H.S (Right-Hypotenuse-Side) :

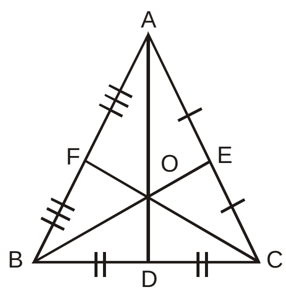


If any two sides of a right angled triangle are equal (separately) to any two corresponding sides of another right angled triangle then both triangles are congruent.

Here, $\angle B = \angle D = 90°$ and AB = DE and AC = EF, then

$\therefore$ $\Delta ABC = \cong$ DEF.

Median :



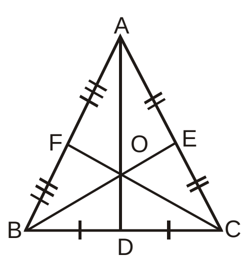
A line drawn from a vertex to the opposite side of a triangle, which divides the side into 2 equal parts is called a median.

Here, AD, BE and CF are medians and

BD = DC, CE = AE and

AF = BF

Centroid (Centre of gravity) :



A Centroid (point in figure) is the point of intersection of three medians.

Rule 1. The centroid divides a median in the ratio of 2 : 1 with the larger part towards the vertex, i.e., G divides BE, CF and AD in the ratio of 2 : 1.

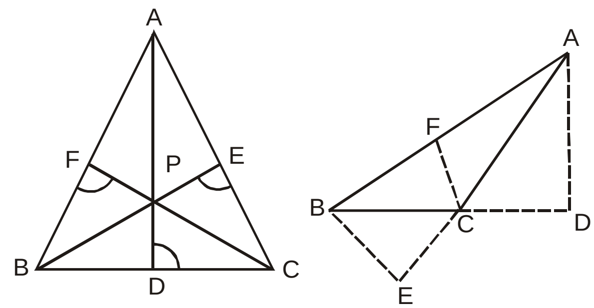
$$\therefore \frac{AO}{OD} = \frac{BO}{OE} = \frac{CO}{OF} = \frac{2}{1}$$

The medians make 6 triangles of equal areas. as–

ar ΔAFO = ar ΔFOB = ar ΔOBD = ar ΔODC = ar ΔCOE

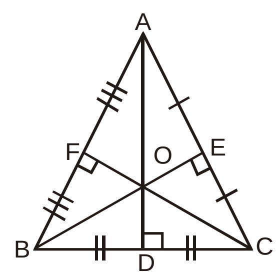
= ar ΔAOE.= $\frac{1}{6}$ ar ΔABC

- **Altitude :** An altitude is nothing but the height of a triangle. It is a perpendicular drawn from a vertex to the opposite side.

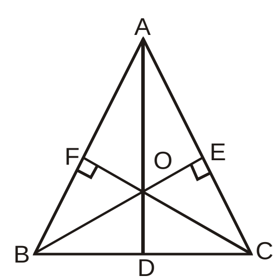


A triangle can have three altitudes. In case of an obtuse triangle atleast one altitude lies out side the triangle. AD, BE and CF are altitudes.

- **Perpendicular Bisector :** A line that bisect a side of the triangle at right angle is called the perpendicular Bisector. OD is the perpendicular bisector of BC if BD = DC and $\angle ODC = \angle ODB = 90°$.



- **Ortho centre :** It is the point of intersection of three Altitudes of a triangle. In ΔABC, O is the Orthocentre.



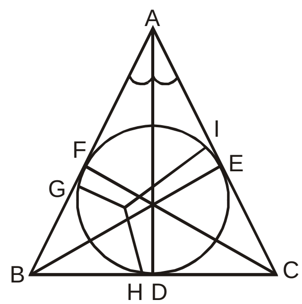
Here, AD, BE and CF are altitudes of ΔABC.

$\therefore$ $\angle BOC + \angle A = 180°$

$\angle AOB + \angle C = 180°$

$\angle COA + \angle B = 180°$

- **Incentre :** The point of intersection of the Angle Bisectors of a triangle is called the Incentre.



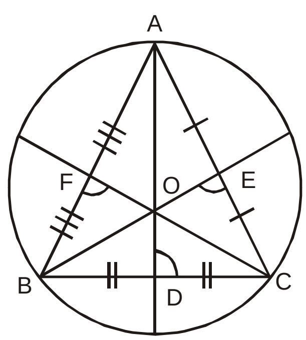
In ΔABC, given above AD, BE and CF are the angle bisectors of A, B and C respectively. Therefore O is the incentre, and OH, OI and OG are in-radii.

Circumcentre :

The point of intersection of the Perpendicular Bisectors of the sides of a triangle is called the circumcentre.

Here, O is the circum centre and OA, OB and OC are circum radii.

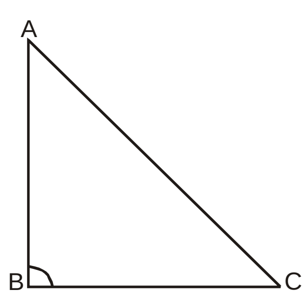
Here, $\angle BOC = 2\angle A$, $\angle COA = 2\angle B$, $\angle AOB = 2\angle C$.



Properties of Triangles :

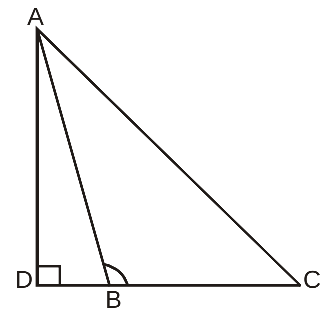
(1) **Pythagoras theorem :** In any right angled triangle $AB^2 + BC^2 = AC^2$, where

AB is Perpendicular, BC is Base, AC is Hypoteneuse

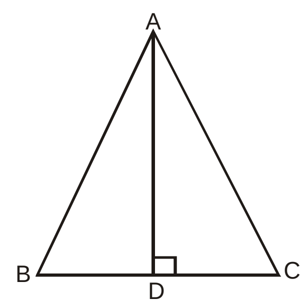


(2) If in a certain triangle ABC, $\angle B$ is obtuse angle, and $AD \perp BC$, then

$AC^2 = AB^2 + BC^2 + 2BC.AD$

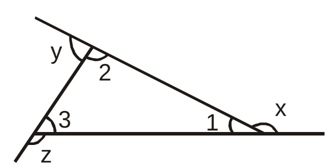


(3) If in a certain ΔABC,



$\angle C$ is acute angle, and $AD \perp BC$, then $AB^2 = BC^2 + AC^2 - 2BC.DC$.

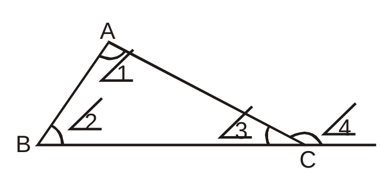
(4) Sum of interior angles of a triangle is 180° and sum of exterior angles is 360°



$\therefore$ $\angle 1 + \angle 2 + \angle 3 = 180°$. $\angle x + \angle y + \angle z = 360°$.

(5) In a triangle, sum of two sides is always greater than third side.

(6) In the given ΔABC

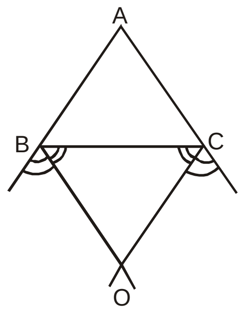


$\angle 4 = \angle 1 + \angle 2$ and $\angle 3 + \angle 4 = 180°$.

(7) In the given ΔABC If OB and OC are the bisectors of angles of triangle $\angle B$ and $\angle C$, then $\angle BOC$

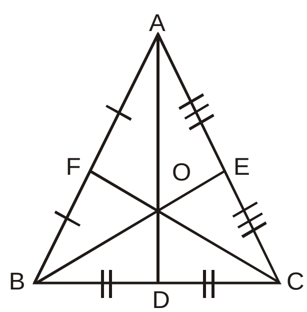
$= 90^\circ + \frac{\angle A}{2}$

(8) If in the ΔABC, the sides AB and AC are extended and the bisectors of exterior angles of $\angle B$ and $\angle C$ meet at O then $\angle BOC = 90 - \frac{\angle A}{2}$

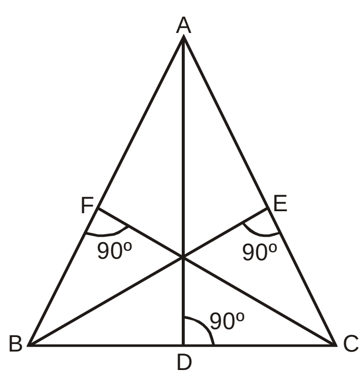


(9) In the given ΔABC, AD, BE and CF are the medians, then

$$AD + BE + CF > \frac{AB + BC + CA}{2}$$

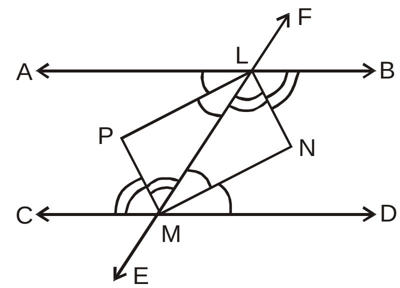


(10) In the given ΔABC, if AD, BE and CF are the Perpendiculars, then, AB + BC + CA > AD + BE + CF.

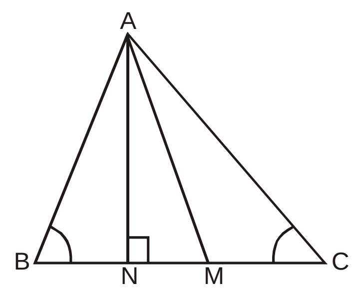


(11) If a line intersects two parallel lines, then bisectors of the interior angles make a rectangle.

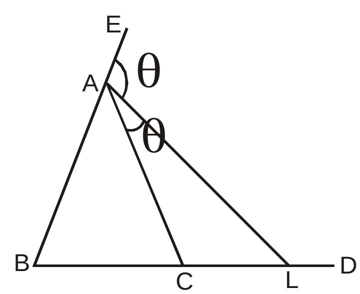
$\therefore$ LNMP is a rectangle.



(12) In the given ΔABC, AM is the bisector of angle $\angle BAC$ and $AN \perp BC$ then $\angle MAN = \frac{1}{2}(\angle B - \angle C)$

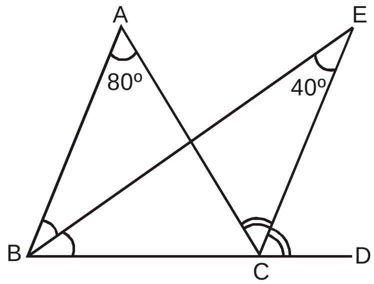


(13) In ΔABC, If BC is extended to D, and AL is the bisector of exterior angle $\angle A$ then $\angle ABC + \angle ACD = 2\angle ALC$.

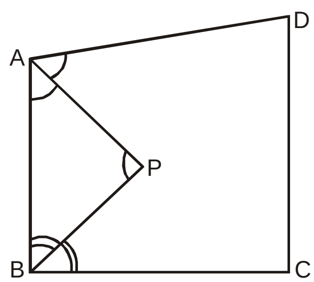


Here, BA is extended to BE.

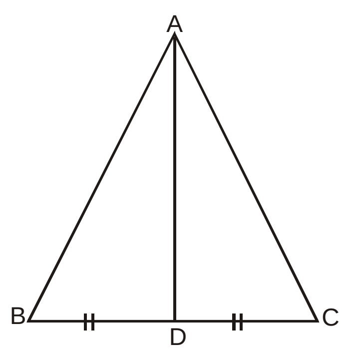
(14) In a ΔABC, if BC is extended to D and BE and CE are the bisectors of $\angle ABC$ and $\angle ACD$ which meet at E, then $\angle BEC = \frac{1}{2}\angle A$



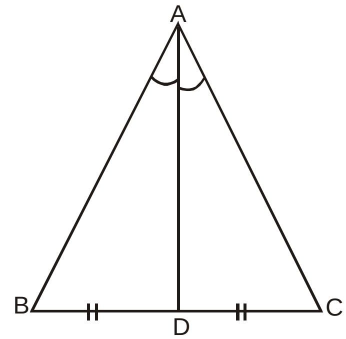
(15) In the given quadrilateral, the bisectors of adjacent angles meet at P, then, $\angle APB = -\frac{1}{2}(\angle C - \angle D)$ where < C > < D



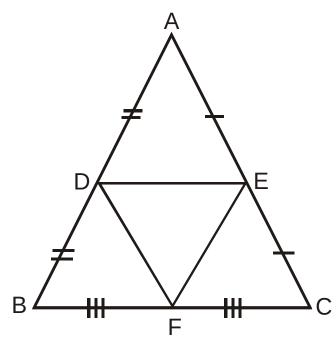
(16) In any ΔABC, if AD is the median then $\frac{AB}{AC} = \frac{BD}{DC}$.



(17) In any ΔABC, if AD is the bisector of angle ∠A then, $\frac{AB}{AC} = \frac{BD}{DC}$

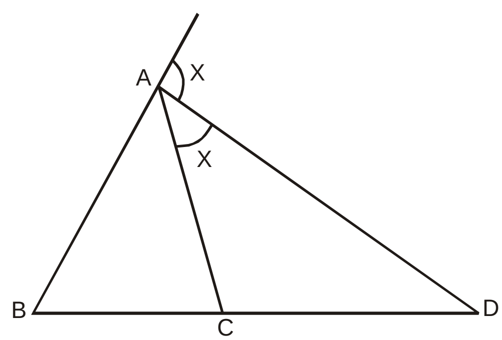


(18) In any ΔABC, D and E are the mid-point of sides AB and AC respectively, then DE| | BC and DE = $\frac{1}{2}$ BC, area ΔADE = $\frac{1}{4}$ area (ΔABC) and $\frac{AD}{BD} = \frac{AE}{EC}$



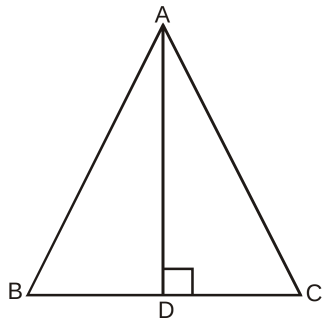
According to figure, F is mid-point of BC then area ΔDEF = $\frac{1}{4}$ area (ΔABC)

(19) In ΔABC, AD is the bisector of exterior angle ∠A, then, $\frac{BD}{DC} = \frac{AB}{AC}$



(20) In an equilateral ΔABC, if AD ⊥ BC, then, $\frac{(AB)^2}{(AD)^2} = \frac{4}{3}$

$\therefore\ 3(AB)^2 = 4(AD)^2$



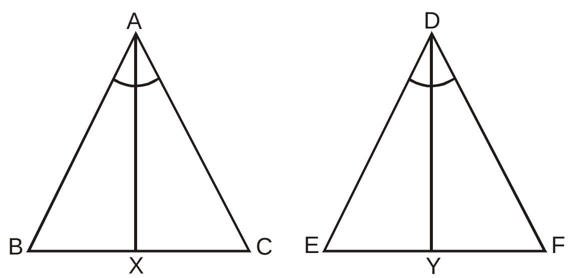
(21) Here, ΔABC and ΔDEF are similar, then

$$\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \left(\frac{AX}{DY}\right)^2$$

$$= \left(\frac{P_1}{P_2}\right)^2$$

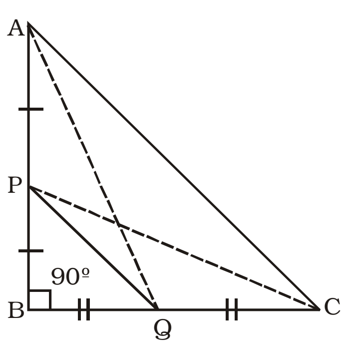
P_1 = Perimeter of ΔABC

P_2 = Perimeter of ΔDEF

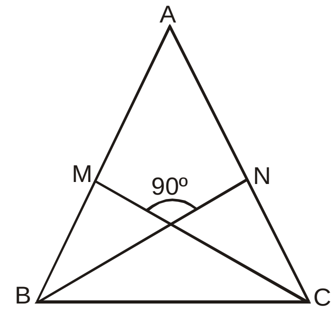


(22) In the given figure, ΔABC is a right angled triangle, in which P and Q are the mid-point on the side AB and BC.

then $4\left((AQ)^2 + (CP)^2\right) = 5\,(AC)^2 = 20\,(PQ)^2$

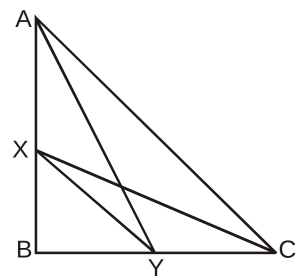


(23) If in ΔABC, BN and CM are the medians of the triangle intersecting at 90°, then $(AB)^2 + (AC)^2 = 5\,(BC)^2$.

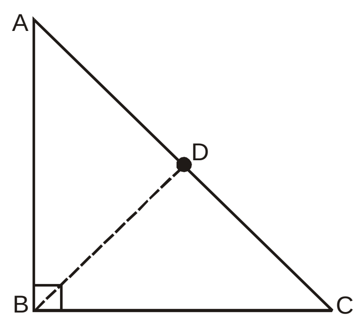


(24) In ΔABC, $\angle B = 90°$ and X and Y are the points on sides AB and BC respectively,

then $(AY)^2 + (XC)^2 = (AC)^2 + (XY)^2$



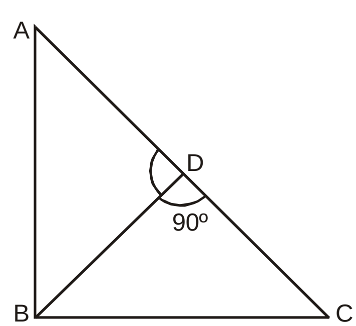
(26) Circumcentre of a right angled triangle lies on the mid-point of the hypotenuse, therefore, AD = BD = DC = radius of circumcircle



(27) If ΔABC is a right angled Δ, such that $\angle B = 90°$.

And $BD \perp AC$.

then, $BD = \sqrt{AD \times DC}$. and $BD = \dfrac{AB \times BC}{AC}$

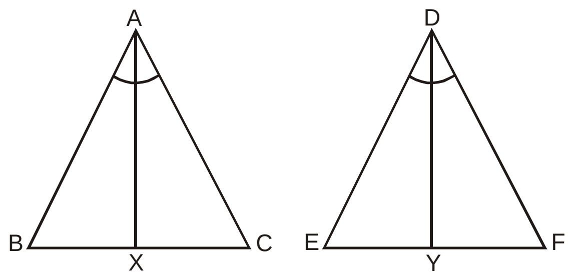


(28) In the given figure $\Delta ABC \sim \Delta DEF$

P_1 = Perimeter of ΔABC

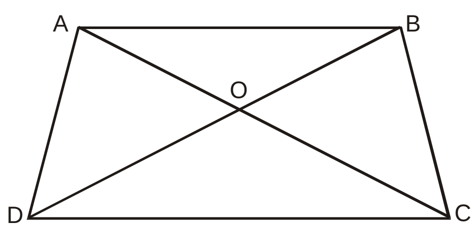
P_2 = Perimeter of ΔDEF

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AX}{DY} = \frac{P_1}{P_2}$$



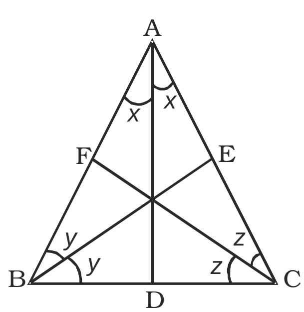
(29) In the given figure, $\square$ ABCD is a trapezium with AB | | DC

then, $\dfrac{OA}{OC} = \dfrac{OB}{OD}$.

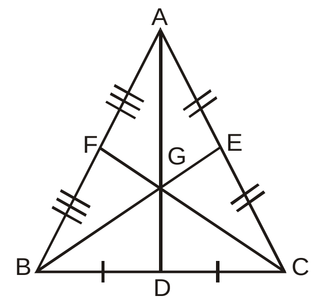


(30) If all of these medians bisect the angles from where it start, then Δ will be equilateral. In the given figure, AD and BE are medians of ΔABC, then BD = DC, AE = CE, $\angle BAD = \angle CAD$, $\angle EBC = \angle EBA$ then AB = BC = AC

$\therefore$ ΔABC is an equilateral triangle.

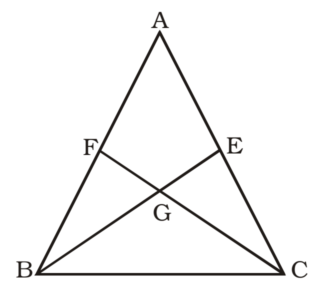


(31) In the given ΔABC, AD, BE and CF are the medians. If AD = BE = CF, then AB = BC = AC So, triangle ΔABC will be an equilateral triangle.

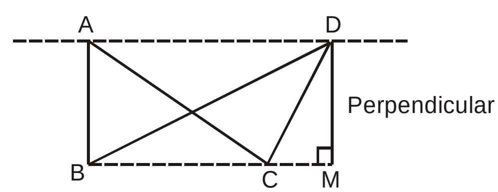


(32) In the triangle ABC If medians BE and CF are equal i.e., BE = CF then AB = AC

$\therefore$ ΔABC is an isosceles triangle.



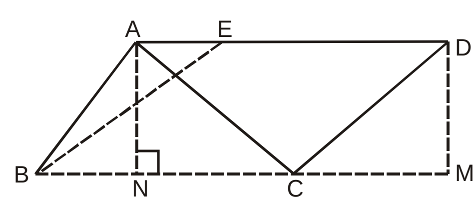
(33) Triangles on the same base and between two parallel lines are equal in area.



$\therefore$ Area ΔABC = Area ΔBDC. (as both triangles lie on base BC)

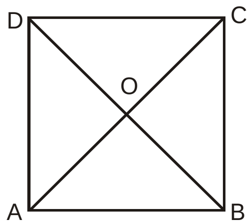
(34) If a parallelogram and a triangle lie on same base and between two parallel lines, then area of ΔABC

$= \dfrac{1}{2}$ (Area parallelogram EBCD)

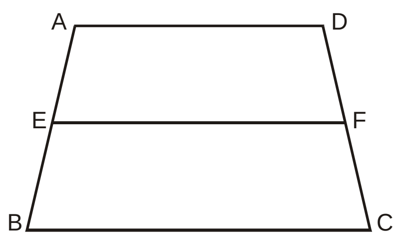


(35) The perimeter of a quadrilateral is greater than the sum of its diagonals.

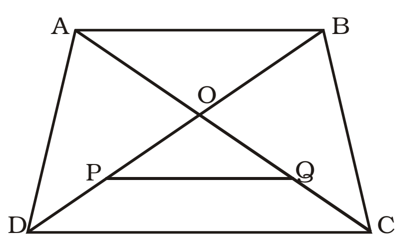
$\therefore$ AB + BC + CD + DA > AC + BD.



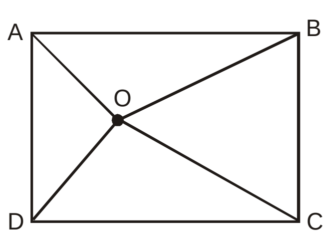
(36) $\square$ABCD is a trapezium, AD | | BC and E and F are the mid-points of AB and DC respectively, then EF $= \frac{1}{2}$ (AB + CD)



(37) $\square$ABCD is a trapezium, then AB | | DC, P and Q are the mid-points of diagonals BD and AC respectively, then $PQ = \frac{(DC - AB)}{2}$

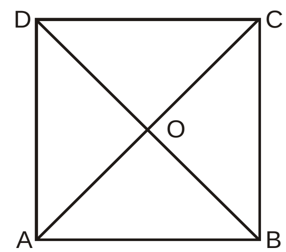


(38) $\square$ ABCD is a rectangle, O is any point in $\square$ABCD. then. $OA^2 + OC^2 = OB^2 + OD^2$.



QUADRILATERALS

- **Quadrilaterals :** Quadrilateral is a figure which is bounded by four straight lines. Here, ABCD is a quadrilateral in which BD and AC are two diagonals, which cut each other at O. $\angle A + \angle B + \angle C + \angle D = 360°$



- **Square :** The quadrilateral whose all sides are equal is called square. Every angle is right angle (90°). Diagonals AC and BD are equal and cut each other at 90°.

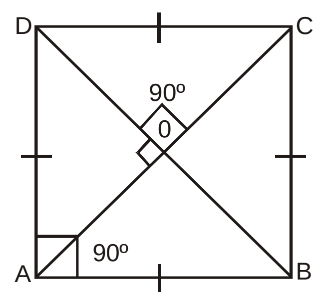
  $\therefore$ (i) AB = BC = CA = AD = side = a

  (ii) diagonal (AC) = diagonal (BD) = $\sqrt{2}$ side = a$\sqrt{2}$

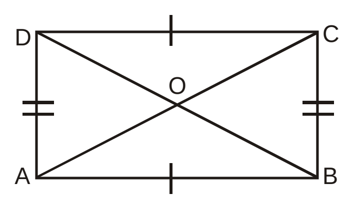
  (iii) $\angle A = \angle B = \angle C = \angle D = 90°$ (every angle)

  (iv) OD = OB = OA = OC

  (v) Note that square is a special kind of rectangle as well as rhombus. Hence, all properties of Rectangle and Rhombus will be satisfied for a square.



- **Rectangle :** $\square$ABCD is a rectangle whose properties are –



(i) AB = CD and AB | | CD, BC = DA and BC | | AD.

(ii) Diagonal AC = Diagonal BD.

(iii) $\angle A = \angle B = \angle C = \angle D = 90°$

(iv) $AC^2 = AB^2 + BC^2 = BD^2 = BC^2 + CD^2$

(v) AC bisects BD and vice versa

Parallelogram :

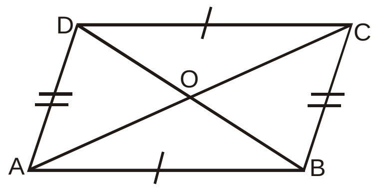
$\square$ABCD is a parallelogram whose properties are–

(i) AB = CD and AB | | CD and BC = DA and BC | | DA.

(ii) diagonals AC and BD bisect each other means OA = OC and OB = OD, but AC $\neq$ BD. [Note]

(iii) $\angle A = \angle C$, $\angle B = \angle D$ (opposite angles are equal)

(iv) $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle D + \angle A = 180°$ (Sum of adjacent angles is 180°)



Rhombus :

$\square$ ABCD is a Rhombus whose properties are–

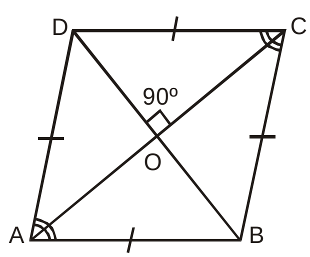
(i) AB = BC = CD = DA (all sides are equal)

(ii) AD | | BC, and AB | | CD.

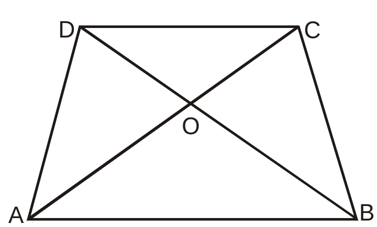
(iii) $\angle A = \angle C$ and $\angle B = \angle D$ (but not equal to 90°)

(iv) $\angle A + \angle B = 180°$, $\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$, $\angle D + \angle A = 180°$ [i.e. sum of Adjacent angles is 180°]

(v) The diagonals AC and BD bisect each other at 90°. It means $AC \perp BD$ and OA = OC and OD = OB but $AC \neq BD$. [Note]



- **Trapezium :** $\square$ABCD is a quadrilateral in which two sides (AB | | DC) are parallel to each other but they are not equal ($AB \neq DC$), that is called trapezium.



Here, AB | | DC But $AD \neq BC$

- **Rhomboid :** The quadrilateral in which two adjacent sides are equal to each other.

Some Properties of Quadrilaterals

- **Polygon :** Polygon is a sector (2D-shape) which is bounded by three or more than three straight lines. On the basis of number of sides, there are different names of polygon. In Regular Polygons all sides are equal.

Polygon	No. of sides
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

Properties of Regular Polygons :

(i) Sum of interior angles of a polygon

$= (n - 2) \times 180°$ where n is no. of sides.

(ii) Each exterior angle of a polygon = 180° – (every interior angle).

(iii) Each interior angle of a polygon $= \dfrac{(n-2) \times 180°}{n}$

(iv) Sum of all exterior angles of a polygon is 360°

(v) Every exterior angle of a polygon $= \dfrac{360°}{n}$.

(vi) Measurement of each angle at the centr e made by any side of a polygon $= \dfrac{360°}{n}$

(vii) Number of diagonals of a polygon $= \dfrac{n(n-3)}{2}$

Area of polygon :

(i) The area of a polygon of n sides

$= \dfrac{na^2}{4} \cot\left(\dfrac{\pi}{n}\right)$, where n = no. of sides, a

= length of side.

(ii) Radius of outer circle of a polygon having n sides

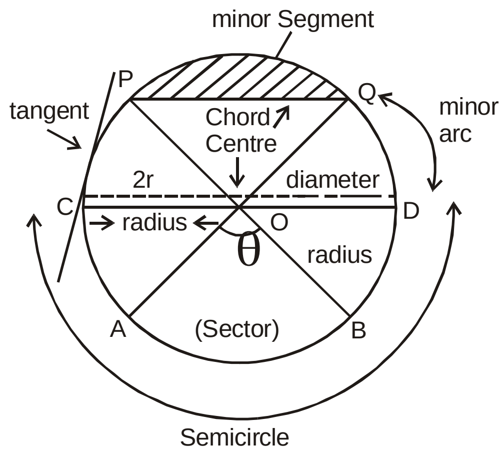
$(R) = \dfrac{a}{2} \operatorname{cosec} \dfrac{180°}{n}$.

(iii) Radius of inner circle of a polygon having n sides (r)

$= \dfrac{a}{2} \cot \dfrac{180°}{n}$.

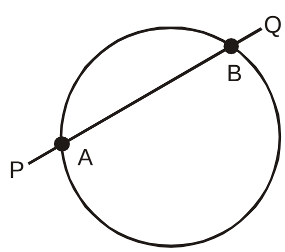
CIRCLE

- **Circle :** A circle is a set of points, lying at a constant distance from a fixed point. That constant distance is called radius (r) and the fixed point is called its centre.

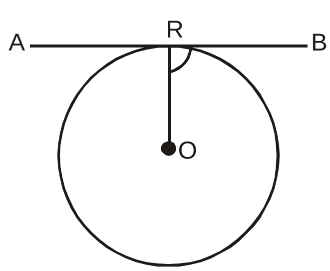


- **Centre :** The fixed point is called the centre of the circle. In the above figure, O is the centre of the circle.
- **Radius :** OA, OB. OC are the radii of circle in the above figure. A radius is the distance from centre of a circle to any point on it's circumference.
- **Chord :** Any line segment whose end points lie on the circle is called a chord. PQ is the chord of circle with centre O as shown above.
- **Diameter :** Diameter is the longest chord of the circle. It is that chord, which passes through the centre.

  A diameter is twice the radius of a circle.
- **Secant :** A line segment that intersects a circle at two points is called a secant. Here, PQ is the secant that intersects the circle at points A and B.



- **Tangent :** A line that touches the circle at one and only one point is called a tangent.

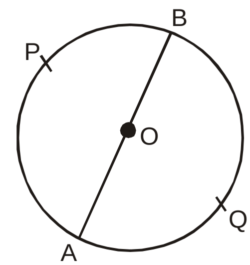


Note : Radius is always perpendicular to the tangent.

$\therefore$ $\angle ORB = \angle ORA = 90°$, So, $OR \perp AB$.

- **Semi-circle :** As the name suggest, semicircle is half the circle.

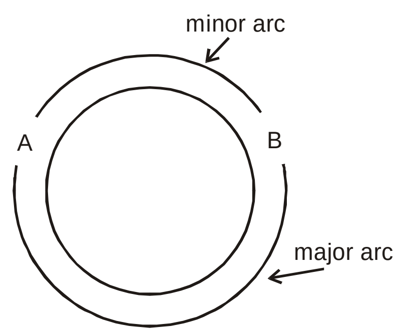
  A diameter divides a circle into two semi-circles. APB and AQB are two semicircles made by diameter AB. Measure of a semicircle = 180°



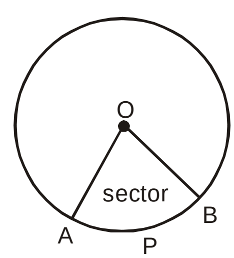
- **Arc :** In the given circle, let A and B be any two points on the circle.

  We get two arcs here (by two points) i.e. minor arc AB and major arc AB. An arc is denoted by the symbol $\frown$

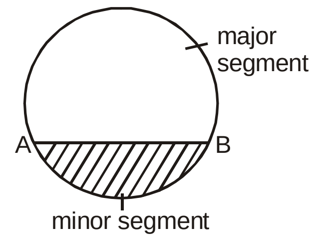
  e.g. arc AB or ($\overline{AB}$)



- **Sector :** The part of the circle which is bounded by an arc and two radius is called sector.



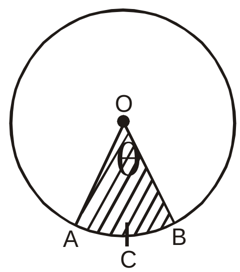
  Here, OAPB is a sector.
- **Segment :** A circle is divided into two parts by a chord, which are called segments. In the given figure, chord AB divides circle into two segments, minor segment and major segment.



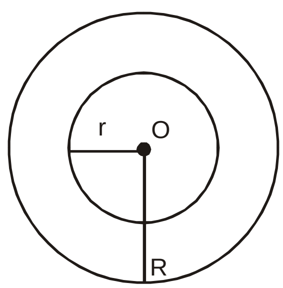
- **Circumference :** The perimeter of a circle is called its circumference (C) and it is equal to $2\pi r$. i.e. $C = 2\pi r$

- **Area of Sector :** Area of sector OACB = $\frac{\pi r^2 \theta}{360°}$, where

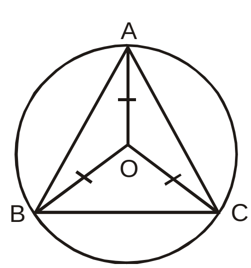
  θ is the angle sub-tended at centre by $\overset{\frown}{ACB}$.



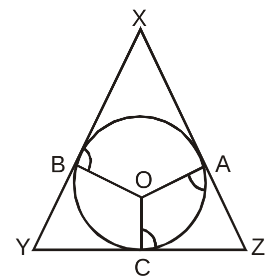
- **Concentric circles :** Two circles are said to be concentric if they have the same centre.



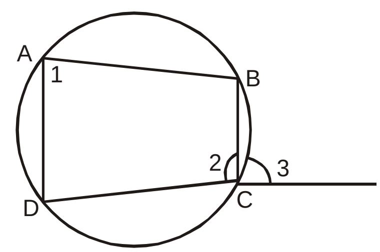
  In the given figure, we have two concentric circles with radius r and R, but with same centre O.
- **Congruent Circles :** Two circles having equal radii, are called congruent circles.
- **Central angle :** Angle subtended at the centre is called the central angle.
- **Circumcircle :** It is the circle drawn around a triangle, in such a way that the vertices of a triangle lie on the circle, as shown here.



- **Incircle :** It is the circle drawn inside a triangle such that all the three sides of triangle are tangents to the circle.
  Since radius is perpendicular to the tangent $OA \perp XZ$, $OC \perp YZ$ and $OB \perp XY$. Also $OA = OB = OC = r$, O is the incentre.

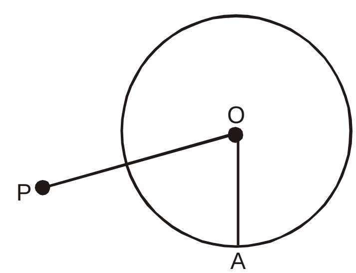


- **Cyclic Quadrilateral :** It is a quadrilateral whose all four vertices lie on the circle. Also the sum of opposite angles is equal to 180°.
  $\therefore\ \angle 1 + \angle 2 = 180°$, also $\angle 2 + \angle 3 = 180°$
  $\Rightarrow \angle 1 = \angle 3$.
  i.e. In a cyclic quadrilateral, exterior angle is equal to interior opposite angle.

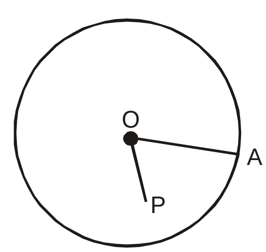


Properties of Circles :

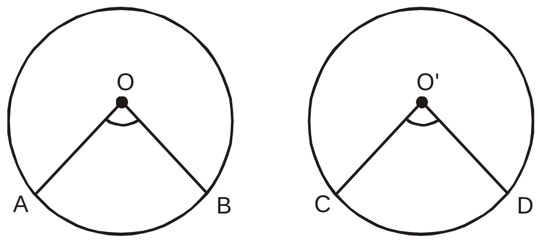
- If a point lies outside the circle, then distance from that point to centre is greater than radius i.e. $OP > OA$



- If a point lies inside the circle, then distance from that i.e. $OP < OA$ point to centre is less than radius.



- In the given figure. In two circles of same radii.



If $m\,\overset{\frown}{AB} = m\,\overset{\frown}{CD}$ then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

It means, $\angle AOB = \angle CO'D$ then arc AB $\simeq$ arc CD

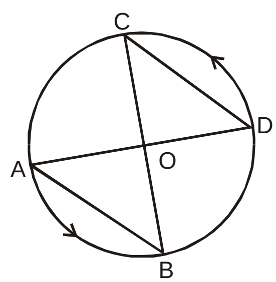
Again, if $\overset{\frown}{AB} \simeq \overset{\frown}{CD}$ then $m\overset{\frown}{AB} = m\overset{\frown}{CD}$

i.e. if arc AB $\simeq$ arc CD then $\angle AOB = \angle CO'D$.

* In the given circle,

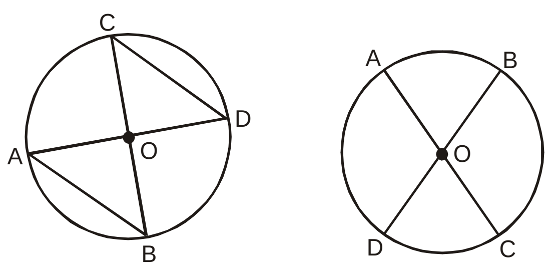
If $\overset{\frown}{AB} = \overset{\frown}{CD}$

$\therefore$ AB = CD (Chords)



- In the given figure chord AB = Chord CD, then minor arc AB $\simeq$ minor arc CD as chord AC = Chord BD

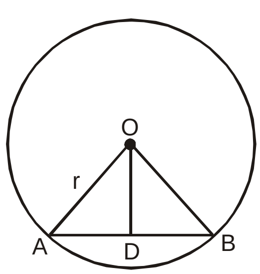
  $\therefore$ $\overset{\frown}{AC} \simeq \overset{\frown}{BD}$



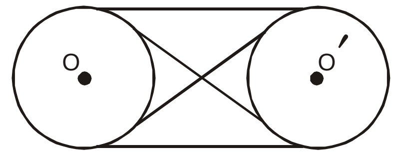
- In the given figure, if $OD \perp AB$

  $\therefore$ $AD = BD = \dfrac{AB}{2}$

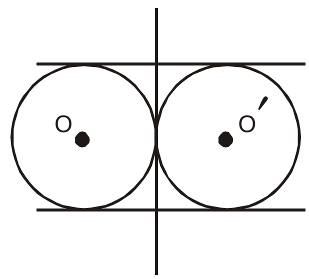
  $\therefore$ $OD = \sqrt{OA^2 - (AD)^2}$



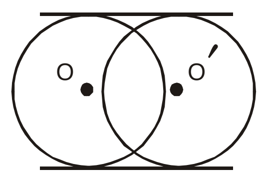
- If two circles do not touch each other then 4 tangents can be drawn.



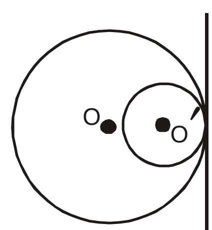
- If two circles touch each other externally, then 3 tangents can be drawn.



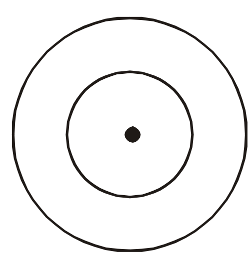
- If two circles cut each other, then two tangents can be drawn.



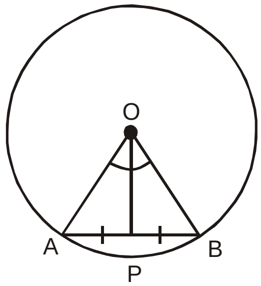
- If one circle touches another circle internally, then only one tangent can be drawn.



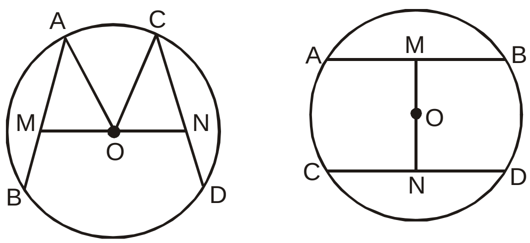
- There is no common tangent of two concentric circles.



- In the given figure if OP bisect AB, such that AP = BP, then $OP \perp AB$.

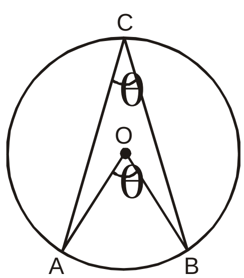


- Only one circle can be drawn through 3 non-collinear points.
- In the given figure, if AB = CD, then OM = ON where OM $\perp$ AB and ON $\perp$ CD

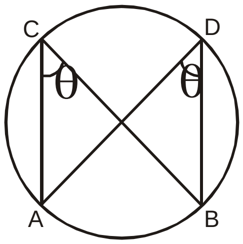


Converse : If OM = ON then,
AB = CD.

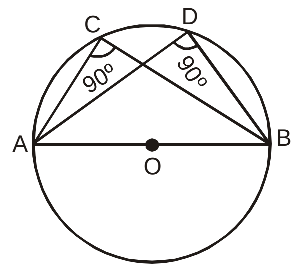
- In the given figure, $\angle AOB = 2\ \angle ACB$. i.e. Angle made by an Arc on centre = 2 × Angle made at circumference by same arc.



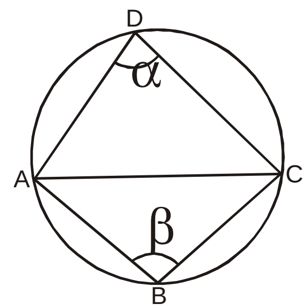
- Two angles subtended by the same arc AB on two different points C&D at circumference are equal i.e. $\angle ACB = \angle ADB$.



- Angles subtended by semicircle is right angle i.e. (90°), . $\angle ACB = \angle ADB = 90°$

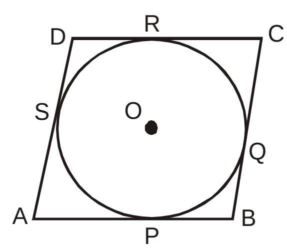


- Angles lying on both sides of segments subtended by a chord are supplementary to each other. Here AC is a chord, then, $\angle\alpha + \angle\beta = 180°$

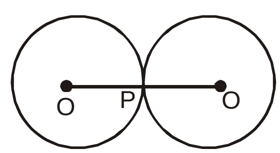


- If a circle drawn interior to a parallelogram touches all its sides, then the parallelogram is a Rhombus.

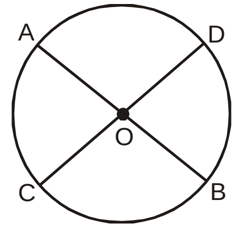
$\therefore$ $\square$ABCD is a rhombus.



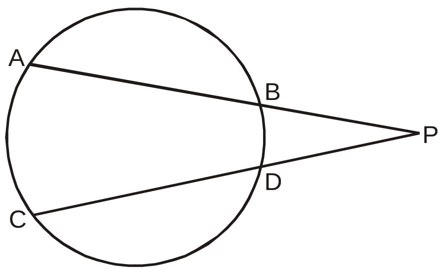
- Only one tangent can be drawn through a point on circumference of circle.
- Two circles touch each other at point P then their centres O and O' and P will be collinear.



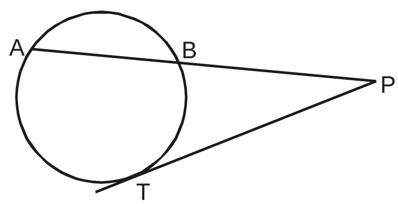
- AB and CD are chords of circle which cut each other at O. then OA × OB = OC × OD.



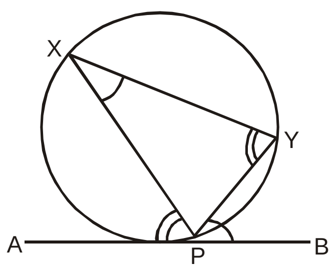
- According to given figure, chords AB and CD cut each other at P externally, then, PA × PB = PC × PD.



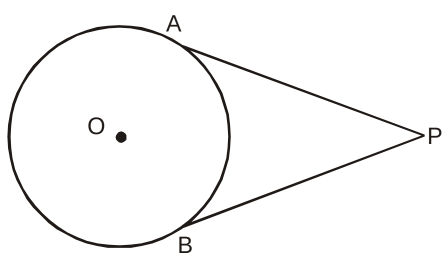
- According to figure, PT is tangent at point T and AB is a chord, then, PA × PB = $(PT)^2$.



- According to figure, AB is tangent at point P, then, ∠APX = ∠PYX, ∠BPY = ∠PXY.

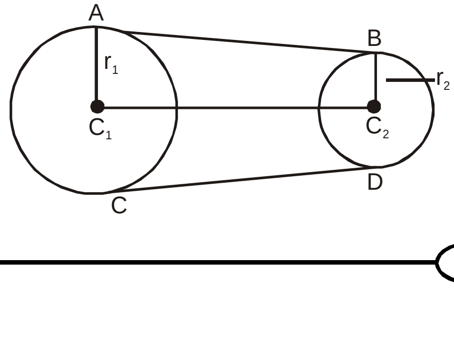


- In the given figure, PA and PB are two tangents, from a point P, then PA = PB.



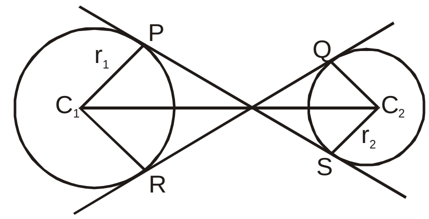
- Here, C_1C_2 is the distance between centre of circles. ∴ length of each of the common tangent

$$AB = CD = \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$$

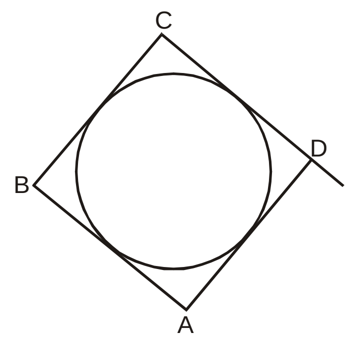


and length of the transverse tangent

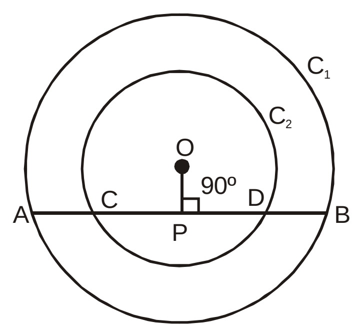
$$= PS = QR = \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$$



- If a circle touches all the four sides of a quadrilateral then sum of the opposite sides is equal.
∴ AB + DC = AD + BC

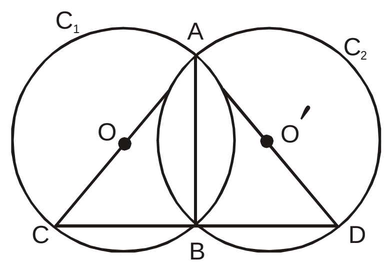


- If C_1 and C_2 are two concentric circles and AB is common chord and point P is such that OP ⊥ AB, then AC = BD



- C_1 and C_2 are two circles having centres O and O' and AC and AD diameters respectively. Both circles cut each other at A and B. Then, C, B and D will be collinear.

$\boxed{\text{B lies on CD.}}$ ∴ ∠CBD = 180°



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