03

COMPOUND INTEREST

Money

I Said to be lent compound interest (C.I.), if the interest is not paid as soon as it falls due but it is added to the principal after a fixed period, so that the amount,s at the end of the period becomes the principal for the next period.

Note

(1) Unless there is a mention of CI, the problem should be treated as that of SI.

(2) The compound interest and the simple interest for one year are the same when the principal and the rate of interest are the same, provided that the interest is calculated annually.

(3) If the interest is payable half yearly, the time is doubled and the rate becomes half.

For example, if the rate of interest is 10% per annum and the money is kept for 1 year, then if the rate is calculated half yearly, then r = 5% and time is 2 years.

Important Facts and Formulae

If principal = Rs. P, Time = t years, Rate = R% p.a.

(i) When interest is compounded annually:

Amount after t years = A =

$$P\left(1+\frac{R}{100}\right)^{t}$$

(ii) When interest is compounded half-yearly

Principal = Rs. P, Time = t years = (2 t) half years,

Rate = R% p.a. = (R/2%) per half-yearly

Amount after t years =

$$P\left(1+\frac{R/2}{100}\right)^{2t}$$

(iii) When interest is compounded quarterly:

Principal = Rs. P, Rate = R% p.a. = (R/4)% per quarter,Time = t years = (4t)quarters.

Amount after t years =

$$A=P\left(1+\frac{R/4}{100}\right)^{4t}$$

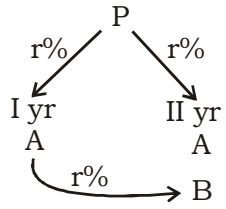
(iv) When rate of interest is R_1 %, R_2% & R_3% for 1st year, 2nd year and 3rd year respectively, then

Amount after 3 years

$$= \text{Rs. } P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

CI for two years

Let Principal = P and rate = r% per annum

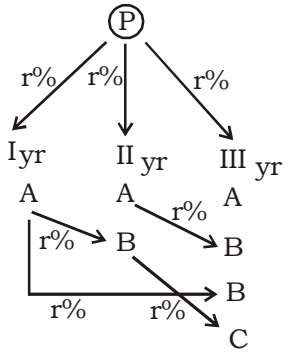


i.e. CI = 2A + B (for 2 years)

& CI for II year = A + B

CI for three years

Let principal = P and rate = r% p.a.



∴ CI for (3 years) = 3A + 3B + C

CI for II year = A + B and

CI for III year = A + 2B + C

e.g. P = Rs. 1000, r = 10%

∴ A = 10 % of 1000 = 100

B = 10% of 100 = 10

C = 10% of 10 = 1

∴ CI for 3 years = 3A + 3B + C

= 300 + 30 + 1 = 331

- When difference between the compound interest and simple interest on a certain sum of money for 2 years at R% rate is Rs. D then

$$\frac{R}{100}=\sqrt{\frac{D}{P}}$$

Where, P = Principal D = Difference

EXAMPLES

1. Raviraj invested a sum of ` 10,000 at compound interest rate of 10 percent per annum for a period of three years. What amount will Raviraj get after 3 years?

(a) ` 12340 (b) ` 13210

(c) ` 13320 (d) ` 13310

Sol. (d) P = Rs. 10,000

r = 10% p.a.

t = 3 years

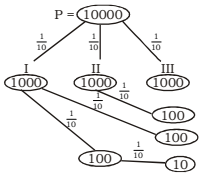
$$A = P\left(1+\frac{r}{100}\right)^{t}$$

$$A = 10000\left(1+\frac{10}{100}\right)^{3}$$

$$= 10000\left(\frac{11}{10}\right)^{3}$$

= Rs. 13310

Alternate



CI = 3310

Amount = 10000 + 3310

= Rs. 13310

2. Seema invested a sum of ` 16000 for two years at compound interest and received an amount of ` 17640 on maturity. What is the rate of interest?

(a) 9 p.c.p.a. (b) 5 p.c.p.a.

(c) 4 p.c.p.a. (d) 3 p.c.p.a.

Sol. (b) P = Rs. 16000

A = Rs. 17640

t = 2 years

$$A = P\left[1+\frac{r}{100}\right]^t$$

$$17640 = 16000\left(1+\frac{r}{100}\right)^2$$

$$\frac{1764}{1600} = \left(1+\frac{r}{100}\right)^2$$

$$\frac{441}{400} = \left(1+\frac{r}{100}\right)^2$$

$$\left(\frac{21}{20}\right)^2 = \left(1+\frac{r}{100}\right)^2$$

Rooting both sides

$$\frac{r}{100} = \frac{1}{20}$$

r = 5%

Alternate

$$\sqrt{1600} : \sqrt{1764}$$

40 : 42

20 : 21

(1)

Þ Rate $\frac{1}{20} \times 100 = 5\%$

3. Find the amount of ` 1000 in 1 year at 5 percent per annum compound interest payable half-yearly.

(a) ` 1050 (Approx)

(b) ` 950 (Approx)

(c) ` 1125 (Approx)

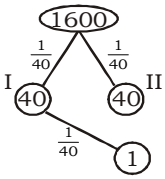
(d) ` 1025 (Approx)

Sol. (a) P = Rs. 1000

$$r = \frac{5}{2}\% \text{ (half yearly)} = \frac{1}{40}$$

t = 1 × 2 = 2 half years

Let principal = 1600 units



C.I = 40 + 40 + 1= 81units

Amount = 1600 + 81 = 1681 units

Now, 1600 units = Rs. 1000

$$1 \text{ unit} = \frac{1000}{1600} = \text{Rs.}\frac{5}{8}$$

then, Amount

$$= 1681 \times \frac{5}{8}$$

= Rs. 1050.625

4. Find the amount on ` 6400 in 1 year 6 months at 5 p.c.p.a. compound interest, interest being calculated half yearly.

(a) ` 6882.10 (b) ` 6892.10

(c) ` 6982.10 (d) ` 7282.05

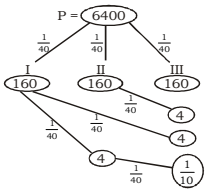
Sol. (b) P = Rs. 6400

Q Compounded half yearly

$$t = \frac{3}{2} \times 2$$

= 3 half years

$$r = \frac{5}{2}\% = \frac{1}{40}$$



$$CI = (3 \times 160) + (3 \times 4) + \frac{1}{10}$$

= 492.10

A = 6400 + 492.10

= Rs. 6892.10

5. Find the compound interest on ` 10000 in 9 months at 4 p.c.p.a interest payable quarterly.

(a) ` 303 (Approx)

(b) ` 313 (Approx)

(c) ` 203 (Approx)

(d) ` 204 (Approx)

Sol. (a) P = Rs. 10,000

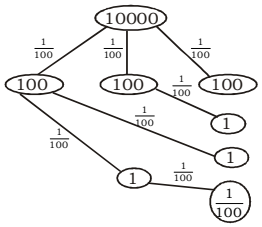
t = 9 months

$$= \frac{9}{12} \times 4 \text{ (quarterly)}$$

= 3 quarters

$$r = \frac{4}{4} \text{ (quarterly)}$$

$$= 1\% \text{ per. quarter} = \frac{1}{100}$$



$$CI = 100+100+100+1+1+1+\frac{1}{100}$$

= 303.01

6. Find the compound interest on ` 8000 in 3 months at 5 p.c.p.a interest payable quarterly

(a) ` 250 (b) ` 200

(c) ` 150 (d) ` 100

Sol. (d) P = Rs. 8000

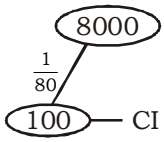
t = 3 months

$$= \frac{3}{12} \times 4 \text{ (quarterly)}$$

= 1 quarter

$$r = \frac{5}{4} \times \frac{1}{100} \text{ (quarterly)}$$

$$= \frac{1}{80} \text{ per. quarter}$$



\ CI = ` 100

7. What principal will amount to ` 1352 in 2 years at 4 p.c.p.a compound interest?

(a) ` 1520 (b) ` 1260

(c) ` 1250 (d) ` 1220

Sol. (c) A = Rs. 1352

t = 2 years

r = 4% p.a

$$1352 = P\left(1+\frac{4}{100}\right)^2$$

$$1352 = P\left(\frac{26}{25}\right)^2$$

$$p = \frac{1352' 25' 25}{26' 26}$$

p = Rs. 1250

Alternate:

$$4\% = \frac{1}{25}$$

Principal	Amount
25	26
25	26
625	676
×2 ↓	×2 ↓
1250	1352

XC

\ P = Rs. 1250

8. On what principal will the compound interest for 3 years at 5 p.c.p.a amount to ` 63.05?

(a) ` 400 (b) ` 500

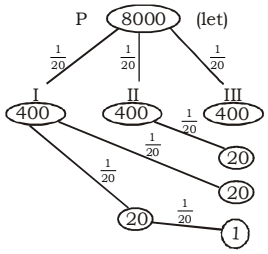
(c) ` 450 (d) ` 550

Sol. (a) t = 3 years

$$r = 5\% = \frac{1}{20}$$

CI = Rs. 63.05

Let P = $(20)^3$ = 8000



1261 units ——— 63.05

1 unit ——— $\frac{63.05}{1261}$

$$\text{P(8000 units)} = \frac{6305}{100} \times \frac{8000}{1261}$$

= Rs. 400

Alternate:

$$5\% = \frac{1}{20}$$

Principal	Amount
20	21
20	21
20	21
8000	9261

1261

1261 units ® 63.05

Principal (8000 units) ®

$$\frac{63.05}{1261} \times 8000 = \text{Rs.}400$$

9. ` 50000 is borrowed at CI at the rate of 1% for the first year, 2% for the second year and 3% for the third year. Find the amount to be paid after 3 years.

(a) ` 50355.3 (b) ` 53055.3

(c) ` 53505.3 (d) ` 53053.5

Sol. (b) P = Rs. 50,000

t = 3 years

r = 1%, 2%, 3%

$$1\%, 2\% = 1 + 2 + \frac{1' 2}{100} = 3.02\%$$

Now, 3.02%, 3% = 3.02 + 3 + $\frac{3.02' 3}{100}$ = 6.1106%

OR

1% 2% 3%

3.02%

6.1106%

CI = 6.1106% of 50000

$$= \frac{61106}{10000}, \frac{50000}{100}$$

= Rs. 3055.3

A = 50000 + 3055.3

= Rs. 53055.3

Alternate

$$\text{Amount} = 50000 \times \frac{101}{100} \times \frac{51}{50} \times \frac{103}{100}$$

= Rs. 53055.3

10. ` 125000 is borrowed at CI at the rate of 2% for the first year, 3% for the second year and 4% for the third year. Find the amount to be paid after 3 years.

(a) ` 135678 (b) ` 136587

(c) ` 163578 (d) ` 136578

Sol. (d) P = Rs. 125000

r = 2%, 3%, 4%

t = 3 years

Amount =

$$125000 \times \frac{51}{50} \times \frac{103}{100} \times \frac{26}{25}$$

= Rs. 136578

11. At what rate percent compound interest, will ` 400 amount to ` 441 in 2 years?

(a) 4% (b) 5%

(c) 6% (d) 3%

Sol. (b) P = Rs. 400

A = Rs. 441

t = 2 years

$$A = P\left(1+\frac{r}{100}\right)^t$$

$$441 = 400\left(1+\frac{r}{100}\right)^2$$

$$\left(\frac{21}{20}\right)^2 = \left(1+\frac{r}{100}\right)^2$$

$$\frac{21}{20} = 1 + \frac{r}{100}$$

$$\frac{r}{100} = \frac{1}{20}$$

r = 5% p.a.

Alternate

$$\sqrt{400} : \sqrt{441}$$

20 : 21

1

Rate = $\frac{1}{20} \times 100 = 5\%$ p.a.

12. At what rate percent compound interest will ` 625 amount to ` 676 in 2 years?
(a) 3% (b) 2%
(c) 4% (d) 5%

Sol. (c) P = Rs. 625
A = Rs. 676
t = 2 years

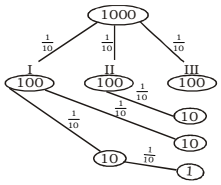
$\sqrt{625} : \sqrt{676}$
25 : 26
1

Rate = $\frac{1}{25} \times 100 = 4\%$ p.a.

13. On what sum will the amount for 2.5 years at 10 p.c.p.a becomes ` 6352.50?
(a) ` 4900 (b) ` 5500
(c) ` 5000 (d) ` 5800

Sol. (c) R = 10% = $\frac{1}{10}$

Let P = $(10)^3$ = 1000



CI = 100 + 100 + 10 + $\frac{121}{2}$

= 270.5 (for 2.5 years)

\ A = 1270.50

1270.50 units —— 6352.50

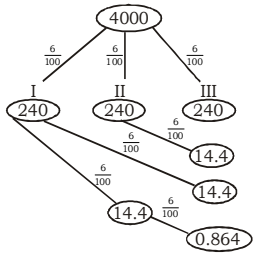
1 unit —— $\frac{635250}{127050}$

Principal (1000 units)

—— Rs. 5000

14. Find the amount on ` 4000 for 2.5 years at 6 p.c.p.a compound interest.
(a) ` 4629.23 (b) ` 4692.32
(c) ` 4639.32 (d) ` 4682.32

Sol. (a)



CI for IIIrd year = 269.664

CI for six months of 3rd year

= $\frac{269.664}{2}$ = 134.832

A = P + CI

= 4000 + 240 + 254.4 + 134.832

= Rs. 4629.232

15. A sum of money placed at compound interest doubles itself in 6 years. In how many years will it amount to 16 times itself?
(a) 24 years (b) 26 years
(c) 22 years (d) 20 years

Sol. (a) ① 6 yr ②

2^1 ——→ 6 years

2^4 ——→ 4×6

= 24 years

16. A sum of money placed at compound interest thrice itself in 4 years. In how many years will it amount to 27 times itself?
(a) 12 years (b) 15 years
(c) 14 years (d) 10 years

Sol. (a) ① 4 yr ③

3^1 ———— 4 years

3^3 ———— 4 × 3

= 12 years

17. If a sum of money at compound interest amount to thrice itself in 3 years, then in how many years will it be 9 times itself?
(a) 12 years (b) 6 years
(c) 9 years (d) 15 years

Sol. (b) ① 3 yr ③

3^1 ———— 3 years

3^2 ———— 3 × 2

= 6 years

18. At what rate in the compound interest, does a sum of money becomes four folds in 2 years?
(a) 150% (b) 100%
(c) 200% (d) 75%

Sol. (b) $4 = 1\left(1+\frac{r}{100}\right)^2$

$2 = 1 + \frac{r}{100}$

r = 100% p.a.

19. At what rate p.c.p.a in the compound interest, does a sum of money becomes 27 times in 3 years?
(a) 100% (b) 150%
(c) 75% (d) 200%

Sol. (d) $27 = 1\left(1+\frac{r}{100}\right)^3$

$3 = 1+\frac{r}{100}$

$\frac{r}{100} = 2$

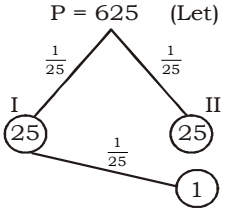
r = 200% p.a.

20. If the CI on a certain sum for 2 years at 4 p.c.p.a be ` 510, what would be the SI?
(a) ` 500 (b) ` 505
(c) ` 400 (d) ` 475

Sol. (a) r = 4% = $\frac{1}{25}$

t = 2 years

P = 625 (Let)



51 units —— 510

□ 1 unit —— 10

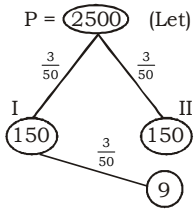
□ SI = 50 × 10

= Rs. 500

21. If the CI on a certain sum for 2 years at 6 p.c.p.a be ` 25.75, what would be the SI?
(a) ` 25 (b) ` 24
(c) ` 20 (d) ` 15

Sol. (a) t = 2 years

r = 6% = $\frac{6}{100} = \frac{3}{50}$



309 units —— 25.75

1 unit —— $\frac{25.75}{309}$

S.I. = $\frac{25.75}{309}$ × 300 = Rs. 25

22. The simple interest on a certain sum of money for 2 years at 5 p.c.p.a is ` 100. Find the compound interest at the same rate and for the same time.

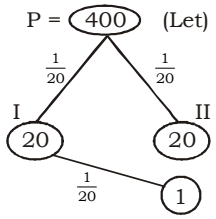
(a) ` 102.50 (b) ` 103

(c) ` 103.50 (d) ` 102.25

Sol. (a) t = 2 years

r = 5% = $\frac{5}{100}$ = $\frac{1}{20}$

SI = Rs. 100 (Given)



40 units —— 100

1 unit —— $\frac{100}{40}$

Q CI (41units) = $\frac{100}{40}$ × 41

= Rs. 102.50

23. The simple interest on a certain sum of money for 2 years at 6 p.c.p.a is ` 300. Find the compound interest at the same rate and for the same time.

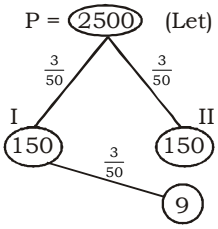
(a) ` 310 (b) ` 308

(c) ` 307 (d) ` 309

Sol. (d) t = 2 years

r = 6% = $\frac{6}{100}$ = $\frac{3}{50}$

SI = Rs. 300



300 units —— Rs. 300

1 unit —— Rs. 1

∴ CI (30 units) = $\frac{300}{300}$ × 309

= Rs. 309

24. The compound interest on a certain sum for 2 years is ` 105 and simple interest is ` 100. Find the rate of interest per annum and the sum.

(a) 10%, ` 500

(b) 10%, ` 1000

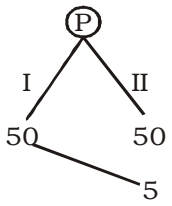
(c) 20%, ` 1000

(d) 4%, ` 1500

Sol. (a) SI = Rs. 100

CI = Rs. 105

t = 2 years



Rate = $\frac{5}{50}$ × 100 = 10% p.a.

and, P × $\frac{10}{100}$ = 50

P = Rs. 500

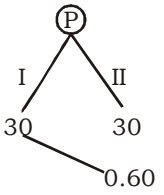
25. The compound interest on a certain sum for 2 years is ` 60.60 and simple interest is ` 60. Find the rate of interest per annum and the sum.

(a) 2%, ` 1600 (b) 2%, ` 1400

(c) 3%, ` 1500 (d) 2%, ` 1500

Sol. (d) SI = Rs.60

CI = Rs. 60.60



Rate = $\frac{0.60}{30}$ × 100

= $\frac{60}{30}$ × $\frac{100}{100}$ = 2%

and, P × $\frac{2}{100}$ = 30

P = Rs. 1500

= 8 + 8 + $\frac{8 \times 8}{100}$

= 16.64%

CI – SI = 16.64 – 16 = 0.64%

0.64 = 56

According to the question,

(P)100% = $\frac{56}{0.64}$ ×100

= Rs. 8750