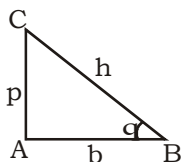


TRIGONOMETRY

1. Trigonometric Ratio:



To study different trigonometric ratio functions we will consider a right angled triangle. Suppose ABC is a right angled triangle with $\angle A = 90^\circ$.

We can obtain six different trigonometric ratio from the sides of these triangle. They are respectively

$\frac{AC}{BC}, \frac{AB}{BC}, \frac{AC}{AB}, \frac{AB}{AC}, \frac{BC}{AB}$ and

$\frac{BC}{AB}$. If $\angle B = q$ then these ratio are

respectively called $\sin q, \cos q, \tan q, \cot q, \sec q$ and $\operatorname{cosec} q$. Clearly for the given angle q , AC (p) is perpendicular, AB (b) is base and BC (h) is hypotenuse. Hence six different trigonometric ratios are follows (see the given figure)

Trigonometric Ratios:-

$$\sin \theta = \frac{AC}{BC} = \frac{p}{h} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{AB}{BC} = \frac{b}{h} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{AC}{AB} = \frac{p}{b} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\operatorname{cosec} \theta = \frac{BC}{AC} = \frac{h}{p} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$\sec \theta = \frac{BC}{AB} = \frac{h}{b} = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$\cot \theta = \frac{AB}{AC} = \frac{b}{p} = \frac{\text{Base}}{\text{Perpendicular}}$$

Clearly $\sin q$ and $\operatorname{cosec} q$ are reciprocals to each other. Similarly $\cos q$ and $\sec q$ are reciprocals to each other while $\tan q$ and $\cot q$ are re-

ciprocals to each other.

Relations between Trigonometric Ratios :-

$$(i) \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\text{or } \operatorname{cosec} \theta \times \sin \theta = 1$$

$$(ii) \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\text{or } \sec \theta \times \cos \theta = 1$$

$$(iii) \quad \cot \theta = \frac{1}{\tan \theta}$$

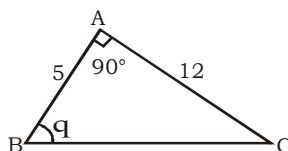
$$\text{or } \cot \theta \times \tan \theta = 1$$

$$(iv) \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

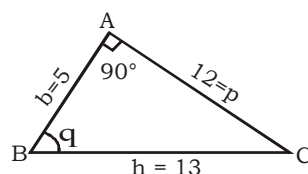
$$(v) \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

TYPE - 1

Ex.1 Write all the six t-ratios value in the given figure:



Sol. In $\triangle ABC$ is, a right angle triangle with $\angle A = 90^\circ$,



Let $AC = 12 = p$ and $AB = 5 = b$
Then from Pythagoras theorem,

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{5^2 + 12^2} \\ = \sqrt{25 + 144} = \sqrt{169} = 13$$

Here side opposite to q is AC which is p.

Side adjacent to q is AB, which is b.

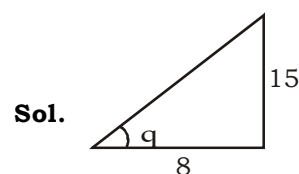
Side opposite to right angle is BC, which is hypotenuse h.

$$\sin q = \frac{p}{h} = \frac{12}{13}, \operatorname{cosec} q = \frac{h}{p} = \frac{13}{12}$$

$$\cos q = \frac{b}{h} = \frac{5}{13}, \sec q = \frac{h}{b} = \frac{13}{5}$$

$$\tan q = \frac{p}{b} = \frac{12}{5}, \cot q = \frac{b}{p} = \frac{5}{12}$$

Ex.2 If $15 \cot q = 8$ then calculate the remaining trigonometric ratio.



Sol.

$$\cot q = \frac{8}{15} = \frac{b}{p}$$

$$\text{Let } b = 8k \quad p = 15k$$

$$\text{From pythagoras theorem, } h^2 \\ = p^2 + b^2 = (15k)^2 + (8k)^2$$

$$\text{or, } h^2 = 225k^2 + 64k^2 = 289k^2$$

$$\text{or, } h = \sqrt{289k^2} = 17k$$

$$\text{Hence, } \sin q = \frac{p}{h} = \frac{15k}{17k} = \frac{15}{17}$$

$$\cos q = \frac{b}{h} = \frac{8k}{17k} = \frac{8}{17}$$

$$\tan q = \frac{p}{b} = \frac{15k}{8k} = \frac{15}{8}$$

$$\sec q = \frac{h}{b} = \frac{17k}{8k} = \frac{17}{8}$$

$$\operatorname{cosec} q = \frac{h}{p} = \frac{17k}{15k} = \frac{17}{15}$$

Ex.3 If $\tan \theta = \frac{4}{3}$, then $\cos \theta = ?$

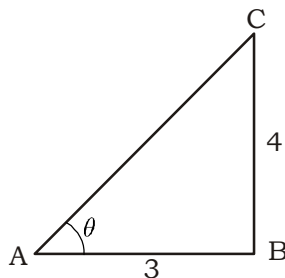
$$(a) \quad \frac{4}{5}$$

$$(b) \quad \frac{3}{5}$$

(c) $\frac{3}{4}$ (d) $\frac{1}{5}$

Sol.(b) $\tan \theta = \frac{BC}{AB} = \frac{4}{3}$

$\therefore AC = \sqrt{(4)^2 + (3)^2} = 5$



$\therefore \cos \theta = \frac{AB}{AC} = \frac{3}{5}$

Ex.4 If $\tan \theta = \frac{4}{3}$, the value of

$\frac{1 - \sin \theta}{1 + \sin \theta}$ is:-

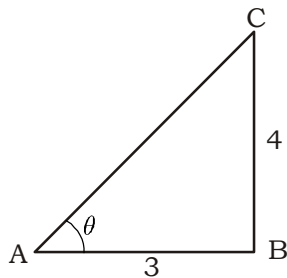
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{9}$ (d) $\frac{1}{13}$

Sol.(c) $\tan \theta = \frac{4}{3} = \frac{BC}{AB}$

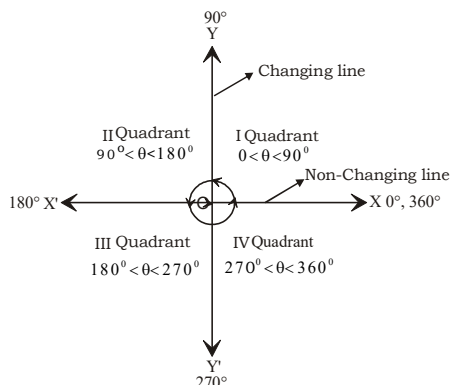
and $AC = \sqrt{(3)^2 + (4)^2} = 5$

$\therefore \sin \theta = \frac{BC}{AC} = \frac{4}{5}$



$\therefore \frac{1 - \sin \theta}{1 + \sin \theta} = \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{1}{9}$

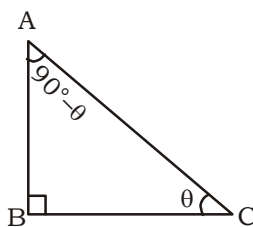
Quadrants:-



Let XOX' and YOY' be two mutually perpendicular lines. These lines divide the plane into four parts and each one of them is called a quadrant.

Complementary Angle.

For a given angle q its complementary angle is $(90^\circ - q)$.



From definition,

$\sin q = \frac{\text{side opposite angle } \theta}{\text{hypotenuse}} = \frac{AB}{AC}$

and $\cos (90^\circ - q)$

$= \frac{\text{side along with angle } (90^\circ - \theta)}{\text{hypotenuse}} = \frac{AB}{AC}$

$\therefore \boxed{\sin q = \cos(90^\circ - q)}$

Similarly, we can prove that

$\therefore \boxed{\cos q = \sin(90^\circ - q)}$

$\Rightarrow 90^\circ, 270^\circ, \dots$ (odd multiple of 90°) will be changed

$\Rightarrow 0^\circ, 180^\circ, 360^\circ, \dots$ (multiple of 180°) will not be changed

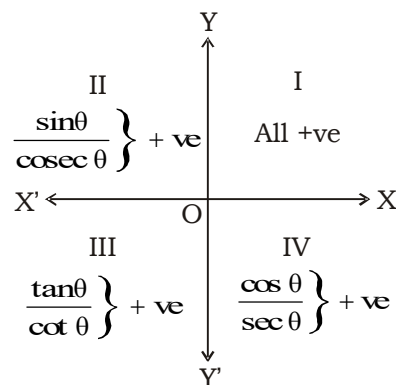
Change will be in following manner:

$\sin \theta \rightarrow \cos \theta$ & $\cos \theta \rightarrow \sin \theta$

$\tan \theta \rightarrow \cot \theta$ & $\cot \theta \rightarrow \tan \theta$

$\sec \theta \rightarrow \csc \theta$ & $\csc \theta \rightarrow \sec \theta$

Signs of Trigonometric Ratios:-



Trigonometric Ratios of Allied Angles

(A) T-ratios of $(-\theta)$ in terms of those of θ :-

1. $\sin(-\theta) = -\sin \theta$

2. $\cos(-\theta) = \cos \theta$

3. $\tan(-\theta) = -\tan \theta$

4. $\cot(-\theta) = -\cot \theta$

5. $\sec(-\theta) = \sec \theta$

6. $\csc(-\theta) = -\csc \theta$

(B) T-ratios of $(90^\circ - \theta)$ in terms of those of θ :-

1. $\sin(90^\circ - \theta) = \cos \theta$

2. $\cos(90^\circ - \theta) = \sin \theta$

3. $\tan(90^\circ - \theta) = \cot \theta$

4. $\cot(90^\circ - \theta) = \tan \theta$

5. $\sec(90^\circ - \theta) = \csc \theta$

6. $\csc(90^\circ - \theta) = \sec \theta$

(C) T-ratios of $(90^\circ + \theta)$ in terms of those of θ :-

1. $\sin(90^\circ + \theta) = \cos \theta$

2. $\cos(90^\circ + \theta) = -\sin \theta$

3. $\tan(90^\circ + \theta) = -\cot \theta$

4. $\cot(90^\circ + \theta) = -\tan \theta$

5. $\sec(90^\circ + \theta) = -\csc \theta$

6. $\csc(90^\circ + \theta) = \sec \theta$

(D) T-ratios of $(180^\circ - \theta)$ in terms of those of θ :-

1. $\sin(180^\circ - \theta) = \sin \theta$

2. $\cos(180^\circ - \theta) = -\cos \theta$

3. $\tan(180^\circ - \theta) = -\tan \theta$

$$4. \cot(180^\circ - \theta) = -\cot \theta$$

$$5. \sec(180^\circ - \theta) = -\sec \theta$$

$$6. \operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec} \theta$$

(E) T-ratios of $(180^\circ + \theta)$ in terms of those of θ :-

$$1. \sin(180^\circ + \theta) = -\sin \theta$$

$$2. \cos(180^\circ + \theta) = -\cos \theta$$

$$3. \tan(180^\circ + \theta) = \tan \theta$$

$$4. \cot(180^\circ + \theta) = \cot \theta$$

$$5. \sec(180^\circ + \theta) = -\sec \theta$$

$$6. \operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec} \theta$$

(F) T-ratios of $(270^\circ - \theta)$ in terms of those of θ :-

$$1. \sin(270^\circ - \theta) = -\cos \theta$$

$$2. \cos(270^\circ - \theta) = -\sin \theta$$

$$3. \tan(270^\circ - \theta) = \cot \theta$$

$$4. \cot(270^\circ - \theta) = \tan \theta$$

$$5. \sec(270^\circ - \theta) = -\operatorname{cosec} \theta$$

$$6. \operatorname{cosec}(270^\circ - \theta) = -\sec \theta$$

(G) T-ratios of $(270^\circ + \theta)$ in terms of those of θ :-

$$1. \sin(270^\circ + \theta) = -\cos \theta$$

$$2. \cos(270^\circ + \theta) = \sin \theta$$

$$3. \tan(270^\circ + \theta) = -\cot \theta$$

$$4. \cot(270^\circ + \theta) = -\tan \theta$$

$$5. \sec(270^\circ + \theta) = \operatorname{cosec} \theta$$

$$6. \operatorname{cosec}(270^\circ + \theta) = -\sec \theta$$

(H) T-ratios of $(360^\circ - \theta)$ in terms of those of θ :-

$$1. \sin(360^\circ - \theta) = -\sin \theta$$

$$2. \cos(360^\circ - \theta) = \cos \theta$$

$$3. \tan(360^\circ - \theta) = -\tan \theta$$

$$4. \cot(360^\circ - \theta) = -\cot \theta$$

$$5. \sec(360^\circ - \theta) = \sec \theta$$

$$6. \operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec} \theta$$

(I) T-ratios of $(360^\circ + \theta)$ in terms of those of θ :-

$$1. \sin(360^\circ + \theta) = \sin \theta$$

$$2. \cos(360^\circ + \theta) = \cos \theta$$

$$3. \tan(360^\circ + \theta) = \tan \theta$$

$$4. \cot(360^\circ + \theta) = \cot \theta$$

$$5. \sec(360^\circ + \theta) = \sec \theta$$

$$6. \operatorname{cosec}(360^\circ + \theta) = \operatorname{cosec} \theta$$

(J) T-ratios of $(n \times 360^\circ + \theta)$ in terms of those of θ :-

$$1. \sin(n \times 360^\circ + \theta) = \sin \theta$$

$$2. \cos(n \times 360^\circ + \theta) = \cos \theta$$

$$3. \tan(n \times 360^\circ + \theta) = \tan \theta$$

$$4. \cot(n \times 360^\circ + \theta) = \cot \theta$$

$$5. \sec(n \times 360^\circ + \theta) = \sec \theta$$

$$6. \operatorname{cosec}(n \times 360^\circ + \theta) = \operatorname{cosec} \theta$$

Value of some specific angle of trigonometrical (t)-ratio function.

We must learn the following table to solve the question based on trigonometrical (t)-ratio angle 0° , 30° , 45° , 60° , 90°

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

TYPE - II

Ex.5 find the value of following

$$(i) \sin 120^\circ \quad (ii) \cos 210^\circ$$

$$(iii) \tan 570^\circ \quad (iv) \cot 780^\circ$$

$$(v) \sin 960^\circ \quad (vi) \cos 1020^\circ$$

$$(vii) \sec 1500^\circ$$

$$\text{Sol. (i)} \sin 120^\circ = \sin(90^\circ + 30^\circ)$$

$$(\sin(90^\circ + q) = \cos q)$$

$$= \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Sol. (ii)} \cos 210^\circ$$

$$= \cos(180^\circ + 30^\circ)$$

$$(\cos(180^\circ + q) = -\cos q)$$

$$= -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\text{Sol. (iii)} \tan 570^\circ$$

$$= \tan(540^\circ + 30^\circ)$$

$(540^\circ \text{ multiple of } 180^\circ, \text{ Then no change})$

$$\tan(540^\circ + q) = \tan q$$

$$= \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{Sol. (iv)} \cot 780^\circ$$

$$= \cot(720^\circ + 60^\circ)$$

$$\cot(n \times 360^\circ + q) = \cot q$$

$$= \cot(2 \times 360^\circ + 60^\circ)$$

$$= \cot 60^\circ = \frac{1}{\sqrt{3}}$$

$$\text{Sol. (v)} \sin 960^\circ$$

$$= \sin(900^\circ + 60^\circ)$$

$(900^\circ \text{ multiple of } 180^\circ, \text{ so no change of Trigonometry function.})$

$$= \sin(2 \times 360^\circ + 180^\circ + 60^\circ) = \sin(180^\circ + 60^\circ) = -\sin 60^\circ$$

$$= -\frac{\sqrt{3}}{2} \quad \sin(180^\circ + q) = -\sin q$$

$$\text{Sol. (vi)} \cos(1020^\circ)$$

$$= \cos(1080^\circ - 60^\circ)$$

$1080^\circ \text{ multiple of } 180^\circ, \text{ so no change in Trigonometry function.}$

$$= \cos(3 \times 360^\circ - 60^\circ)$$

$$= \cos 60^\circ = \frac{1}{2}$$

$$\text{Sol. (vii)} \sec(1500^\circ)$$

$$= \sec(1440^\circ + 60^\circ)$$

$$= \sec(4 \times 360^\circ + 60^\circ)$$

$$\sec(n \times 360^\circ + q) = \sec q$$

$$= \sec 60^\circ = 2$$

Ex.6

$$\frac{\cos(90^\circ + A) \cdot \sec(360^\circ - A) \cdot \tan(180^\circ - A)}{\sec(A - 720^\circ) \cdot \sin(A + 540^\circ) \cdot \cot(A - 90^\circ)} = ?$$

$$(a) 0 \quad (b) 1 \quad (c) -1$$

(d) None of these

Sol. (b)

$$\frac{\cos(90^\circ + A) \cdot \sec(360^\circ - A) \cdot \tan(180^\circ - A)}{\sec(A - 720^\circ) \cdot \sin(A + 540^\circ) \cdot \cot(A - 90^\circ)}$$

$$= \frac{(-\sin A) \cdot (\sec A) \cdot (-\tan A)}{\sec(2 \times 360^\circ - A) \cdot \sin(2 \times 180^\circ + A) \cdot \cot(2 \times 90^\circ - A)}$$

$$(Q \sec(-\theta) = \sec \theta)$$

$$\text{and } \cot(-\theta) = -\cot \theta]$$

$$= \frac{\sin A \cdot \sec A \cdot \tan A}{\sec A (-\sin A) (-\tan A)}$$

$$= \frac{\sin A \cdot \sec A \cdot \tan A}{\sin A \cdot \sec A \cdot \tan A} = 1$$

Ex. 7 $\sin 720^\circ - \cot 270^\circ - \sin 150^\circ$
 $\cos 120^\circ$ is equal to:-

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{5}$ (d) $\frac{1}{4}$

Sol. (d). $\sin 720^\circ - \cot 270^\circ - \sin 150^\circ$
 $\cos 120^\circ$

$$= \sin(2 \times 360^\circ + 0^\circ) - \cot(360^\circ - 90^\circ)$$

$$= \sin(90^\circ + 60^\circ) \cdot \cos(90^\circ + 30^\circ)$$

$$= \sin 0^\circ + \cot 90^\circ + \cos 60^\circ \cdot \sin 30^\circ$$

$$= 0 + 0 + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Ex. 8. Find the value of :- $\frac{\sin 37^\circ}{\cos 53^\circ}$

(a) 1 (b) -1 (c) 0 (d) 0

Sol. (a)

$$\frac{\sin 37^\circ}{\cos 53^\circ} = \frac{\sin 37^\circ}{\cos(90^\circ - 37^\circ)} = \frac{\sin 37^\circ}{\sin 37^\circ} = 1$$

Ex. 9. Evaluate :- $\sin^2 60^\circ + \cos^2 30^\circ + \cot^2 45^\circ$
 $+ \sec^2 60^\circ - \operatorname{cosec}^2 30^\circ + \cos^2 0^\circ$:-

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) 2

Sol. (c) We know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \cot 45^\circ = 1$$

$$\sec 60^\circ = 2$$

$$\therefore \sin^2 60^\circ + \cos^2 30^\circ + \cot^2 45^\circ + \sec^2 60^\circ$$

$$- \operatorname{cosec}^2 30^\circ + \cos^2 0^\circ$$

$$= \left[\left(\frac{\sqrt{3}}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2 + 1^2 + 2^2 - 2^2 + 1^2 \right] = \frac{7}{2}$$

Ex. 10 If $\frac{x \cos \operatorname{cosec}^2 30^\circ \cdot \sec^2 45^\circ}{8 \cos^2 45^\circ \cdot \sin^2 60^\circ}$

$= \tan^2 60^\circ - \tan^2 30^\circ$, then the value of x is :-

(a) -1 (b) 0 (c) 1 (d) 2

Sol. (c)

$$\frac{x \times (2)^2 \times (\sqrt{2})^2}{8 \times \left(\frac{1}{\sqrt{2}} \right)^2 \times \left(\frac{\sqrt{3}}{2} \right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}} \right)^2$$

$$\Rightarrow 8x = \left(\frac{8}{3} \times 8 \times \frac{1}{2} \times \frac{3}{4} \right) \Rightarrow x = 1$$

Some Useful formula

(i) $\sin^2 q + \cos^2 q = 1$

or $\sin^2 q = 1 - \cos^2 q$

or $\cos^2 q = 1 - \sin^2 q$

(ii) $1 + \tan^2 = \sec^2 q$

or $\sec^2 q - 1 = \tan^2 q$

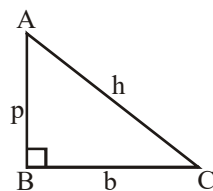
or $\sec^2 q - \tan^2 q = 1$

(iii) $1 + \cot^2 q = \operatorname{cosec}^2 q$

or $\operatorname{cosec}^2 q - 1 = \cot^2 q$

or $\operatorname{cosec}^2 q - \cot^2 q = 1$

Proof we know,



$$\sin q = \frac{p}{h} \quad \cos q = \frac{b}{h}$$

$$\frac{b}{h}$$

Now,

$$\sin^2 q + \cos^2 q$$

$$= \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{p^2 + b^2}{h^2}$$

$$\frac{p^2 + b^2}{h^2}$$

Q In right angle DABC

$$p^2 + b^2 = h^2$$

then $\sin^2 q + \cos^2 q$

$$= \frac{h^2}{h^2} = 1$$

★ Same as we can proof all remaining results same this process

TYPE - III

If $A + B = 90^\circ$,

Results

- (i) $\sin A \cdot \sec B = 1$
or $\sin A = \cos B$
- (ii) $\cos A \cdot \operatorname{cosec} B = 1$
or $\sec A = \operatorname{cosec} B$
- (iii) $\tan A \cdot \tan B = 1$
or $\tan A = \cot B$
- (iv) $\cot A \cdot \cot B = 1$
- (v) $\sin^2 A + \sin^2 B = 1$
- (vi) $\cos^2 A + \cos^2 B = 1$

Proof

(i) **$\sin A \cdot \sec B = 1$**

$A + B = 90^\circ$ (given)

Then, $B = 90^\circ - A$

Now, $\sin A \cdot \sec(90^\circ - A)$

$\sin A \cdot \operatorname{cosec} A$

$$\sin A \times \frac{1}{\sin A} = 1$$

★ Same as we can proof all remaining results same this process

★ **And their vice-versa are also true.**

when $\sin A \cdot \sec B = 1$,

then we can say $A + B = 90^\circ$

Ex. 11 The value of $(\sin 25^\circ \cdot \sec 65^\circ)$ is equal to:-

Sol. $25^\circ + 65^\circ = 90^\circ$

∴ $A + B = 90^\circ$ ∴

∴ $\sin A \cdot \sec B = 1$

So, $\sin 25^\circ \cdot \sec 65^\circ = 1$

Ex. 12 The value of $(\tan 23^\circ \cdot \tan 67^\circ)$

is equal to :-

Sol. $23^\circ + 67^\circ = 90^\circ$

∴ $A + B = 90^\circ$ ∴

∴ $\tan A \cdot \tan B = 1$

So, $\tan 23^\circ \cdot \tan 67^\circ = 1$

Ex. 13 The value of $\tan 10^\circ \cdot \tan 25^\circ \cdot \tan 65^\circ \cdot \tan 80^\circ$ is

Sol.

$$\tan 10^\circ \tan 25^\circ \tan 65^\circ \tan 80^\circ$$

$$= 1$$

Ex. 14 If $\sin(3x - 6) = \cos(6x - 3)$ find the value of x .

Sol. $3x - 6 + 6x - 3 = 90^\circ$

$$9x = 99^\circ$$

$$x = 11$$

∴ $A + B = 90^\circ$ ∴

∴ then $\sin A = \cos B$

Ex. 15 The value of $\cos 40^\circ \cdot \operatorname{cosec} 50^\circ$

Sol. $40^\circ + 50^\circ = 90^\circ$

$$\text{If } A + B = 90^\circ \quad \text{ö} \\ \text{cos} A \cdot \text{cosec} B = 1 \quad \text{ö}$$

$$\text{So, } \cos 40^\circ \cdot \text{cosec} 50^\circ = 1$$

TYPE-IV

Sum and Difference Formula

- (i) $\sin(A+B) = \sin A \cdot \cos B + \cos A \sin B$
(ii) $\sin(A-B) = \sin A \cdot \cos B - \cos A \sin B$
(iii) $\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$
(iv) $\cos(A-B) = \cos A \cdot \cos B + \sin A \sin B$
(v) $2 \sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$
(vi) $2 \cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$
(vii) $2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$
(viii) $2 \cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$
(ix) $\sin^2 A - \sin^2 B = \sin(A+B) \cdot \sin(A-B)$
(x) $\cos^2 A - \cos^2 B = \cos(A+B) \cdot \cos(A-B)$

Tangent Formulae

- (i) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
(ii) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$
(iii) $\cot(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$
(iv) $\cot(A-B) = \frac{\cot B \cdot \cot A + 1}{\cot B - \cot A}$
(v) $\tan(45^\circ + \theta) = \frac{1 + \tan \theta}{1 - \tan \theta} = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$
(vi) $\tan(45^\circ - \theta) = \frac{1 - \tan \theta}{1 + \tan \theta} = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$

Ex.26 Find the value of the following

- (i) $\sin 75^\circ$ (ii) $\cos 75^\circ$
(iii) $\tan 15^\circ$ (iv) $\tan 75^\circ$

Sol.(i) $\sin 75^\circ$
 $\sin(45^\circ + 30^\circ)$
 $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15^\circ$$

(ii) $\cos 75^\circ$
 $\cos(45^\circ + 30^\circ)$
 $= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

★ $\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} = \sin 15^\circ$

(iii) $\tan 15^\circ$
 $\tan(45^\circ - 30^\circ)$
 $= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

★ $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \cot 75^\circ$

(iv) $\tan 75^\circ$
 $\tan(45^\circ + 30^\circ)$
 $= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$
 $= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

★ $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \cot 15^\circ$

Trigonometric Ratios of Specific Angles

(i) $\sin 18^\circ = \left(\frac{\sqrt{5}-1}{4} \right)$

(ii) $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

(iii) $\cos 36^\circ = \left(\frac{\sqrt{5}+1}{4} \right)$

(iv) $\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$

(vii) $\sin 22 \frac{1}{2}^\circ = \frac{\sqrt{2}-\sqrt{2}}{2}$

(viii) $\cos 22 \frac{1}{2}^\circ = \frac{\sqrt{2}+\sqrt{2}}{2}$

Ex.16 The value of

$$\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ} \text{ is}$$

Sol. $\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ}$
 $= \tan(45^\circ - 15^\circ)$
 $= \tan 30^\circ = \frac{1}{\sqrt{3}}$

Ex.17 The value of $\tan 40^\circ + 2 \tan 10^\circ$ is equal to
(a) $\tan 40^\circ$ (b) $\cot 40^\circ$
(c) $\sin 40^\circ$ (d) $\cos 40^\circ$

Sol. We know,
 $40^\circ + 10^\circ = 50^\circ$
both sides take tan
 $\tan(40^\circ + 10^\circ) = \tan 50^\circ$

Þ $\frac{\tan 40^\circ + \tan 10^\circ}{1 - \tan 40^\circ \cdot \tan 10^\circ} = \tan 50^\circ$

Þ $\tan 40^\circ + \tan 10^\circ = \tan 50^\circ -$

$$\tan 50^\circ \cdot \tan 40^\circ \cdot \tan 10^\circ$$

$$\hat{u} \left(\begin{array}{l} \tan A \cdot \tan B = 1 \\ \text{if } A + B = 90^\circ \end{array} \right)$$

Þ $\tan 40^\circ + \tan 10^\circ = \tan 50^\circ - \tan 10^\circ$

Þ $\tan 40^\circ + 2 \tan 10^\circ = \tan 50^\circ$
Now, $\tan 50^\circ = \tan(90^\circ - 40^\circ) = \cot 40^\circ$

Ex.18 The value of $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$ is equal to
(a) $\tan 33^\circ \cdot \cot 53^\circ$
(b) $\tan 53^\circ \cdot \cot 37^\circ$
(c) $\tan 33^\circ \cdot \cot 57^\circ$
(d) $\tan 57^\circ \cdot \cot 37^\circ$

Sol. $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$

Þ $\frac{\tan 57^\circ + \cot 37^\circ}{\tan(90^\circ - 57^\circ) + \cot 53^\circ}$

Þ $\frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\cot 57^\circ + \cot(90^\circ - 37^\circ)}$

Þ $\frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\frac{1}{\tan 57^\circ} + \tan 37^\circ}$
 $= \frac{(\tan 57^\circ \tan 37^\circ + 1) \tan 37^\circ}{(\tan 57^\circ \tan 37^\circ + 1)}$

Þ $\tan 57^\circ$

$$\text{p } \frac{1}{\tan 37^\circ} \times \tan 57^\circ$$

$$\text{p } \tan 57^\circ \cdot \cot 37^\circ$$

TYPE-V

Use of componendo and dividendo-

$$\text{If } \frac{x}{y} = \frac{a}{b}, \text{ Then } \frac{x}{y} = \frac{a}{b}$$

$$\frac{x+y}{x-y} = \frac{a+b}{a-b}$$

Proof $\frac{x}{y} = \frac{a}{b}$

Add 1 in both side.

$$\frac{x}{y} + 1 = \frac{a}{b} + 1$$

$$\frac{x+y}{y} = \frac{a+b}{b} \quad \dots\dots\dots(i)$$

subtract 1 in both side.

$$\frac{x}{y} - 1 = \frac{a}{b} - 1$$

$$\frac{x-y}{y} = \frac{a-b}{b} \quad \dots\dots\dots(ii)$$

(i) / (ii)

$$\frac{x+y}{x-y} = \frac{a+b}{a-b}$$

Ex.19 If $\frac{\sin q + \cos q}{\sin q - \cos q} = 9$ find the value $\tan q$ and $\cos q$

Sol. $\frac{\sin q + \cos q}{\sin q - \cos q} = \frac{9}{1}$

Apply C & D

$$\text{p } \frac{(\sin q + \cos q) + (\sin q - \cos q)}{(\sin q + \cos q) - (\sin q - \cos q)}$$

$$= \frac{9+1}{9-1}$$

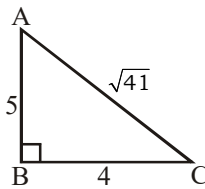
$$\text{p } \frac{2\sin q}{2\cos q} = \frac{10}{8}$$

$$\tan = \frac{5}{4}$$

Now, $\tan q = \frac{\text{Perpendicular}}{\text{Base}}$

$$\text{Hypotenuse} = \sqrt{(5)^2 + (4)^2}$$

$$= \sqrt{41}$$



$$\text{Then, } \cos q = \frac{b}{h} = \frac{4}{\sqrt{41}}$$

Ex.20 If $\frac{\sec q + \tan q}{\sec q - \tan q} = \frac{5}{3}$, then find

The value of $\sin q$

Sol. $\frac{\sec q + \tan q}{\sec q - \tan q} = \frac{5}{3}$

Apply C & D

$$\text{p } \frac{(\sec q + \tan q) + (\sec q - \tan q)}{(\sec q + \tan q) - (\sec q - \tan q)} = \frac{5+3}{5-3}$$

$$\text{p } \frac{2\sec q}{2\tan q} = \frac{8}{2}$$

$$\text{p } \frac{1}{\frac{\cos q}{\sin q}} = 4 \quad \text{p } \frac{1}{\sin q} = 4$$

$$\text{So, } \sin q = \frac{1}{4}$$

TYPE - VI

Some pythagorean natural number will help in solving the problem on trigonometric ratio angle.

pythagorean theorem

$$(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$$

$$\begin{aligned} 3^2 + 4^2 &= 5^2, & 6^2 + 8^2 &= 10^2, \\ 5^2 + 12^2 &= 13^2, & 10^2 + 24^2 &= 26^2, \\ 8^2 + 15^2 &= 17^2, & 7^2 + 24^2 &= 25^2, \\ 20^2 + 21^2 &= 29^2, & 9^2 + 40^2 &= 41^2, \text{ etc.} \end{aligned}$$

Ex.21 If $\sin q + \cos q = \frac{17}{13}$ find the value of $\sin q \cdot \cos q$

Sol. $\sin q + \cos q = \frac{17}{13}$
squaring of both side

$$\text{p } (\sin q + \cos q)^2 = \frac{17^2}{13^2}$$

$$\text{p } \sin^2 q + \cos^2 q + 2\sin q \cos q = \frac{289}{169}$$

$$\text{p } 1 + 2\sin q \cos q = \frac{289}{169}$$

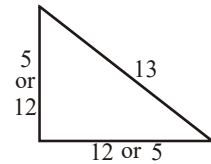
$$\text{p } 2\sin q \cos q = \frac{289}{169} - 1$$

$$\text{p } 2\sin q \cos q = \frac{289-169}{169}$$

$$\text{p } 2\sin q \cos q = \frac{120}{169}$$

$$\text{p } \sin q \cos q = \frac{60}{169}$$

Alternate:-



$$\begin{aligned} \sin q + \cos q &= \frac{17}{13} \rightarrow \frac{p+b}{h} \\ \downarrow \quad \downarrow & \\ \frac{p}{h} \quad \frac{b}{h} & \end{aligned}$$

Apply pythagorean here hypotenuse is 13, Then other sides of right angle triangle will be 5 and 12.

Now,

Check

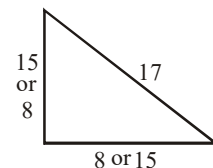
$$\frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

But we cannot find exact value of base and perpendicular, here no affect of value of $\sin q$ and $\cos q$. This question because both are product.

$$\text{Hence, } \sin q \cos q = \frac{5}{13} \cdot \frac{12}{13} = \frac{60}{169}$$

Ex.22 If $\sin q + \cos q = \frac{23}{17}$, find the value of $\sin q \cdot \cos q$

Sol.



$$\sin q + \cos q = \frac{23}{17}$$

- -

$$\frac{p}{h} + \frac{b}{h} = \frac{p+b}{h}$$

so, h = 17

Apply pythagorean sides here hypotenuse is 17, then other sides 8 and 15

Now, Check $\frac{8}{17} + \frac{15}{17} = \frac{23}{17}$

Hence, $\sin q \cdot \cos q = \frac{8}{17} \cdot \frac{15}{17} = \frac{120}{289}$

TYPE -VII

Function and Inverse function ®

(a) If $\sin \theta + \operatorname{cosec} \theta = 2$
then $\sin \theta = \operatorname{cosec} \theta = 1$

Q $\sin^n \theta + \operatorname{cosec}^n \theta = 2$
n ® natural no.

Ex.23 If $\sin \theta + \operatorname{cosec} \theta = 2$ find the value of $\sin^{100} \theta + \operatorname{cosec}^{100} \theta$

Sol. $\sin \theta + \operatorname{cosec} \theta = 2$

Then, $\sin \theta = \operatorname{cosec} \theta = 1$

so, $\sin^{100} \theta + \operatorname{cosec}^{100} \theta = 1$
 $= (1)^{100} + (1)^{100} = 2$

(b) If $\cos \theta + \sec \theta = 2$
then $\cos \theta = \sec \theta = 1$

Q $\cos^n \theta + \sec^n \theta = 2$

Ex.24 If $\cos \theta + \sec \theta = 2$, find the value of $\cos^{10} \theta + \sec^{10} \theta = ?$

Sol. $\cos \theta + \sec \theta = 2$
 $\cos \theta = \sec \theta = 1$
Then, $\cos^{10} \theta + \sec^{10} \theta$
 $= (1)^{10} + (1)^{10} = 1 + 1 = 2$

(c) If $\tan \theta + \cot \theta = 2$
so $\tan \theta = \cot \theta = 1$
 $\tan^n \theta + \cot^n \theta = 2$

Ex.25 If $\tan \theta + \cot \theta = 2$ find the value of $\tan^{50} \theta + \cot^{60} \theta$

Sol. $\tan \theta + \cot \theta = 2$
 $\tan \theta = \cot \theta = 1$

$\tan^{50} \theta + \cot^{60} \theta = (1)^{50} + (1)^{60} = 1 + 1 = 2$

(d) If $\sin A + \cos B = 2$
Then $A = 90^\circ$
 $B = 0^\circ$

TYPE - VIII

Series Base →

Ex. 26 The value of $\cos 1^\circ$, $\cos 2^\circ, \cos 3^\circ, \dots, \cos 179^\circ$ is:-
(a) 1 (b) -1 (c) 2 (d) 0

Sol. $Q \cos 90^\circ = 0$

$\therefore \cos 1^\circ \cdot \cos 2^\circ \dots \cos 179^\circ = 0$

Ex. 27 The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ is :

(a) 1 (b) 0 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

Sol. $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 45^\circ \dots \tan 88^\circ \tan 89^\circ$
 $= (\tan 1^\circ \cdot \tan 89^\circ)$
 $(\tan 2^\circ \cdot \tan 88^\circ) \dots \tan 45^\circ$

$= (\tan 1^\circ \cdot \cot 1^\circ) \cdot (\tan 2^\circ \cdot \cot 2^\circ)$
 $\dots \tan 45^\circ = 1$
[$\tan (90^\circ - \theta) = \cot \theta$, $\tan \theta \cdot \cot \theta = 1$]

Ex.28 The value of : $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \dots + \cos 160^\circ + \cos 180^\circ$ is:-

(a) 1 (b) -1
(c) 0 (d) $\frac{1}{2}$

Sol. $\cos (180^\circ - \theta) = -\cos \theta$

$\therefore \cos 160^\circ = \cos (180^\circ - 20^\circ) = -\cos 20^\circ$

similarly

$\Rightarrow \cos 140^\circ = -\cos 40^\circ$,

$\cos 120^\circ = -\cos 60^\circ$

$\cos 100^\circ = -\cos 80^\circ$

Now,

$\Rightarrow (\cos 20^\circ + \cos 160^\circ) +$

$(\cos 40^\circ + \cos 140^\circ)$

$+ (\cos 60^\circ + \cos 120^\circ)$

$+ (\cos 80^\circ + \cos 100^\circ) + \cos 180^\circ$

$= (\cos 20^\circ - \cos 20^\circ) + (\cos 40^\circ - \cos 40^\circ) +$

$(\cos 60^\circ - \cos 60^\circ) + (\cos 80^\circ - \cos 80^\circ)$

$+ \cos 180^\circ$

$\Rightarrow \cos 180^\circ = -1$

Ex. 29 $\sin^2 5^\circ + \sin^2 6^\circ + \dots + \sin^2 84^\circ + \sin^2 85^\circ = ?$

(a) $39\frac{1}{2}$ (b) $40\frac{1}{2}$

(c) 40 (d) $39\frac{1}{\sqrt{2}}$

Sol. Let the number of terms be n,
then By $t_n = a + (n-1)d$

Here,

$\Rightarrow a = 5, d = 1$

$\Rightarrow 85 = 5 + (n-1)1$

$\Rightarrow n-1 = 85-5 = 80$

$\Rightarrow n = 81$

$\therefore \sin^2 5^\circ + \sin^2 6^\circ + \dots + \sin^2 45^\circ + \dots + \sin^2 84^\circ + \sin^2 85^\circ$

$= (\sin^2 5^\circ + \sin^2 85^\circ) + (\sin^2 6^\circ + \sin^2 84^\circ) + \dots + \text{to 40 terms} + \sin^2 45^\circ$

$= (\sin^2 5^\circ + \cos^2 5^\circ) + (\sin^2 6^\circ + \cos^2 6^\circ) + \dots + \text{to 40 terms} + \sin^2 45^\circ$

$= 40 + \frac{1}{2} = 40\frac{1}{2}$

$\sin(90^\circ - \theta) = \cos \theta$
 $\therefore \sin^2 \theta + \cos^2 \theta = 1$

Ex.30 The value of $\sin 10^\circ + \sin 20^\circ + \dots + \sin 340^\circ + \sin 350^\circ$

Sol. $\sin 10^\circ + \sin 20^\circ + \dots + \sin 340^\circ + \sin 350^\circ$

$\sin(360^\circ - 350^\circ) + \sin(360^\circ - 340^\circ) + \dots + \sin 180^\circ \dots \sin 340^\circ + \sin 350^\circ$

$-\sin 350^\circ - \sin 340^\circ \dots + \sin 180^\circ + \dots \sin 340^\circ + \sin 350^\circ = 0$

[$\sin(360^\circ - \theta) = -\sin \theta$, $\sin 180^\circ = 0$]

TYPE- IX

(A)

$$\boxed{\begin{array}{c} \sin^2 \theta + \cos^2 \theta = 1 \\ \text{or} \\ \sin^2 \theta = 1 - \cos^2 \theta \\ \text{or} \\ \cos^2 \theta = 1 - \sin^2 \theta \end{array}}$$

Ex. 31 What is the value of $\sin^2 1000^\circ + \cos^2 1000^\circ$?

(a) 1000 (b) 100 (c) 10 (d) 1

Sol. (d) $\sin^2 1000^\circ + \cos^2 1000^\circ = 1$

for every value of q in $\sin^2 q + \cos^2 q$ will be 1

Ex. 32 If $\sin^2 60^\circ + \cos^2 (3x - 9^\circ) = 1$
Then value of x is

Sol. $\sin^2 60^\circ + \cos^2 (3x - 9^\circ) = 1$

This is similar to $\sin^2 q + \cos^2 q$

P

So,

$60^\circ = 3x - 9$

$69^\circ = 3x$

$x = 23^\circ$

Ex.33 If $3\sin^2 \alpha + 7\cos^2 \alpha = 4$, then the value of $\tan \alpha$ is (where $0 < \alpha < 90^\circ$) :

(a) $\sqrt{2}$ (b) $\sqrt{5}$

(c) $\sqrt{3}$ (d) $\sqrt{6}$

Sol. (a) $3\sin^2 \alpha + 7(1 - \sin^2 \alpha) = 4$

$\Rightarrow 3\sin^2 \alpha + 7 - 7\sin^2 \alpha = 4$

$\Rightarrow 7 - 4\sin^2 \alpha = 4$

$\Rightarrow 4\sin^2 \alpha = 3 \Rightarrow \sin \alpha = \frac{\sqrt{3}}{2}$

So, $\alpha = 60^\circ$

$\tan \alpha = \tan 60^\circ = \sqrt{3}$

Ex.34 If $\cos^4 \theta - \sin^4 \theta = \frac{2}{3}$, then

the value of $2\cos^2 \theta - 1$ is :

(a) 0 (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Sol.(c) $\cos^4 \theta - \sin^4 \theta = \frac{2}{3}$

$$\Rightarrow (\cos^2 \theta + \sin^2 \theta) (\cos^2 \theta - \sin^2 \theta) = \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta = \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - (1 - \cos^2 \theta) = \frac{2}{3}$$

$$\Rightarrow 2 \cos^2 \theta - 1 = \frac{2}{3}$$

Ex.35 $\sin q + \sin^2 q = 1$

Find the value of $\cos^2 q + \cos^4 q$

Sol. $\sin q + \sin^2 q = 1$

$$\sin q = 1 - \sin^2 q$$

$$\sin q = \cos^2 q$$

Now, $\cos^2 q + \cos^4 q$

Put the value $\cos^2 q$

$$\cos^2 q + (\cos^2 q)^2$$

$$\sin q + \sin^2 q = 1 \text{ (given)}$$

TYPE - X

(A) $1 + \tan^2 \theta = \sec^2 \theta$

$$\sec^2 \theta - 1 = \tan^2 \theta$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

Ex.36 If $\sec^2 \theta + \tan^2 \theta = 9$ find the value of $\sin \theta$ ($0^\circ < \theta < 90^\circ$)

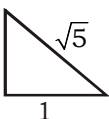
Sol. $\sec^2 \theta + \tan^2 \theta = 9$

$$1 + \tan^2 \theta + \tan^2 \theta = 9$$

$$2 \tan^2 \theta = 8$$

$$\tan^2 \theta = 4$$

$$\tan \theta = 2, = \frac{p}{b}$$

Now,  $\sin \theta = \frac{p}{h} = \frac{2}{\sqrt{5}}$

Ex.37 If $\sec^2 \theta + \tan^2 \theta = 11$, find the value of $\operatorname{cosec} \theta$ ($0^\circ < \theta < 90^\circ$)

Sol. $\sec^2 \theta + \tan^2 \theta = 11$

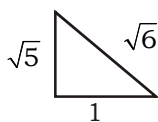
$$1 + \tan^2 \theta + \tan^2 \theta = 11$$

$$2 \tan^2 \theta = 10$$

$$\tan^2 \theta = 5$$

$$\tan \theta = \sqrt{5}$$

Now,



$$\operatorname{cosec} \theta = \frac{\sqrt{6}}{\sqrt{5}}$$

Ex.38 If $\sec^2 \theta + \tan^2 \theta = \frac{5}{3}$ find the value of θ . while ($0^\circ \leq \theta \leq 90^\circ$)

Sol. $\sec^2 \theta + \tan^2 \theta = \frac{5}{3}$

$$1 + \tan^2 \theta + \tan^2 \theta = \frac{5}{3}$$

$$2 \tan^2 \theta = \frac{5}{3} - 1$$

$$2 \tan^2 \theta = \frac{2}{3}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\text{so, } \theta = 30^\circ$$

Ex.39 If $\tan^2 \alpha = 1 + 2 \tan^2 \beta$, find the value of $\sqrt{2} \cos \alpha - \cos \beta$

Sol. $\tan^2 \alpha = 1 + 2 \tan^2 \beta$ (Using identity)

$$\sec^2 \alpha - 1 = 1 + 2 (\sec^2 \beta - 1)$$

$$\sec^2 \alpha - 1 = 1 + 2 \sec^2 \beta - 2$$

$$\sec^2 \alpha - 1 = 2 \sec^2 \beta - 1$$

$$\sec^2 \alpha = 2 \sec^2 \beta$$

$$\sec \alpha = \sqrt{2} \sec \beta$$

$$\frac{1}{\cos \alpha} = \sqrt{2} \frac{1}{\cos \beta}$$

$$\cos \beta = \sqrt{2} \cos \alpha$$

$$\sqrt{2} \cos \alpha - \cos \beta = 0$$

Alternative:-

$$\alpha = 45^\circ \text{ \& } \beta = 0^\circ \text{ satisfies}$$

$$\tan^2 \alpha = 1 + 2 \tan^2 \beta$$

$$\text{put } \alpha = 45^\circ \text{ \& } \beta = 0^\circ \text{ in } \sqrt{2}$$

$$\cos \alpha - \cos \beta$$

$$= \sqrt{2} \cos 45^\circ - \cos 0^\circ = 1 - 1 = 0$$

(B). $(\sec \theta + \tan \theta) (\sec \theta - \tan \theta) = 1$

$$\sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta}$$

$$\sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$$

if $\sec \theta + \tan \theta = x$

then, $\sec \theta - \tan \theta = \frac{1}{x}$

Ex.40 If $\sec \theta + \tan \theta = 3$, find the value of $\cos \theta$

Sol. $\sec \theta + \tan \theta = 3$ (i)

then $\sec \theta - \tan \theta = \frac{1}{3}$ (ii)

adding (i) + (ii)

$$2 \sec \theta = 3 + \frac{1}{3}$$

$$2 \sec \theta = \frac{10}{3}$$

$$\sec \theta = \frac{5}{3}$$

$$\text{Hence, } \cos \theta = \frac{3}{5}$$

TYPE - XI

(A) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$\operatorname{cosec}^2 \theta - 1 = \cot^2 \theta$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

Ex.41 $\operatorname{cosec}^2 \theta + 2 \cot^2 \theta = 10$, then find the value of $\sin \theta + \cos \theta$ when $0^\circ < \theta < 90^\circ$

Sol. $\operatorname{cosec}^2 \theta + 2 \cot^2 \theta = 10$

$$1 + \cot^2 \theta + 2 \cot^2 \theta = 10$$

$$3 \cot^2 \theta = 9$$

$$\cot \theta = \sqrt{3}$$

So, $\theta = 30^\circ$

Now, $\sin \theta + \cos \theta = \sin 30^\circ + \cos 30^\circ$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

Ex.42 If $\operatorname{cosec}^2 \theta + \cot^2 \theta = 3$, find the value of $\cos \theta$. when ($0^\circ < \theta < 90^\circ$)

Sol. $\operatorname{cosec}^2 \theta + \cot^2 \theta = 3$

$$1 + \cot^2 \theta + \cot^2 \theta = 3$$

$$2 \cot^2 \theta = 2$$

$$\cot^2 \theta = 1$$

$$\cot \theta = 1$$

So, $\theta = 45^\circ$

Now, $\cos \theta$

$$= \cos 45^\circ = \frac{1}{\sqrt{2}}$$

(B) $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$
 $(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) = 1$

$$\frac{\operatorname{cosec} \theta - \cot \theta}{\operatorname{cosec} \theta + \cot \theta} = 1$$

If $\operatorname{cosec} \theta - \cot \theta = x$,

then, $\operatorname{cosec} \theta + \cot \theta = \frac{1}{x}$

Ex.43 If $\operatorname{cosec} \theta - \cot \theta = 4$, find the value of $\cos \theta$

Sol. $\operatorname{cosec} \theta - \cot \theta = 4$ (i)

then, $\operatorname{cosec} \theta + \cot \theta = \frac{1}{4}$

.....(ii)
 (i) and (ii)

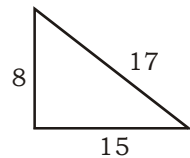
$$2 \operatorname{cosec} \theta = 4 + \frac{1}{4}$$

$$2 \operatorname{cosec} \theta = \frac{17}{4}$$

$$\operatorname{cosec} \theta = \frac{17}{8} = \frac{h}{p}$$

$$b = \sqrt{(17)^2 - (8)^2} = 15$$

Now,



$$\cos \theta = \frac{b}{h} = \frac{15}{17}$$

Ex.44 If $\operatorname{cosec} \theta + \cot \theta = \sqrt{5} + 2$, then find the value of $\sin \theta$.

Sol. $\operatorname{cosec} \theta + \cot \theta = \sqrt{5} + 2$ (i)

Then, $\operatorname{cosec} \theta - \cot \theta = \frac{1}{\sqrt{5} + 2} = \sqrt{5} - 2$ (ii)

(i) + (ii)

$$2 \operatorname{cosec} \theta = 2\sqrt{5}$$

$$\operatorname{cosec} \theta = \sqrt{5}$$

So, $\sin \theta = \frac{1}{\sqrt{5}}$

(c) $a \operatorname{cosec} \theta - b \cot \theta = c$
 $b \operatorname{cosec} \theta - a \cot \theta = d$

or

$$a \cot \theta - b \operatorname{cosec} \theta = d$$

$$(i)^2 - (ii)^2$$

$$\text{Then, } (a^2 - b^2 = c^2 - d^2)$$

or

$$a \operatorname{cosec} \theta + b \cot \theta = c \quad \dots\dots (i)$$

$$b \operatorname{cosec} \theta + a \cot \theta = d \quad \dots\dots (ii)$$

$$(i)^2 - (ii)^2$$

$$(a^2 - b^2 = c^2 - d^2)$$

Ex.45 If $4 \operatorname{cosec} \theta + 5 \cot \theta = 7$, then find the value of 5

$$\operatorname{cosec} \theta + 4 \cot \theta = ?$$

Sol. $4 \operatorname{cosec} \theta + 5 \cot \theta = 7$ (given)

$$5 \operatorname{cosec} \theta + 4 \cot \theta = m \text{ (let)}$$

Using identity

$$(4)^2 - (5)^2 = (7)^2 - (m)^2$$

$$16 - 25 = 49 - m^2$$

$$m^2 = 49 - 9$$

$$m = \pm \sqrt{58}$$

TYPE - XII

(A) If $A+B = 45^\circ$ or 225°

then,

(i) $(1+\tan A)(1+\tan B) = 2$

and

(ii) $(1-\cot A)(1-\cot B) = 2$

Proof

(i) $A+B = 45^\circ$

Both side take tan.

$$\tan(A+B) = \tan 45^\circ$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$$

$$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$$

Adding 1 both side.

$$\Rightarrow 1 + \tan A + \tan B + \tan A \tan B = 1 + 1$$

$$\Rightarrow 1(1+\tan A) + \tan B(1+\tan A) = 2$$

$$\text{Hence, } (1+\tan B)(1+\tan A) = 2$$

(ii) $A+B = 45^\circ$

Both side take cot.

$$\cot(A+B) = \cot 45^\circ$$

$$\frac{\cot A \cdot \cot B - 1}{\cot A + \cot B} = 1$$

$$\cot A \cot B - 1 = \cot A + \cot B$$

$$\cot A [\cot B - 1] - 1 - \cot B + 1 = 0$$

$$\cot A [\cot B - 1] - 1 [\cot B - 1] = 2$$

$$(\cot A - 1) (\cot B - 1) = 2$$

Ex.46 Find the value of $(1+\tan 5^\circ)(1+\tan 40^\circ)$

Sol. $A+B = 5^\circ + 40^\circ = 45^\circ$

then, $(1+\tan 5^\circ)(1+\tan 40^\circ) = 2$

Ex.47 Find the value of $(1+\tan 1^\circ)(1+\tan 2^\circ)(1+\tan 3^\circ)(1+\tan 4^\circ)(1+\tan 3^\circ)(1+\tan 42^\circ)$

Sol. $(1+\tan 1^\circ)(1+\tan 44^\circ)$

$$(1+\tan 2^\circ)(1+\tan 43^\circ)$$

$$(1+\tan 3^\circ)(1+\tan 42^\circ)$$

$$1^\circ + 44^\circ = 2^\circ + 43^\circ$$

$$= 3^\circ + 42^\circ = 45^\circ$$

so, 3 pair of such term

$$= 2 \times 2 \times 2 = 8$$

TYPE - XIII

Morri's law

If $4\theta < 60^\circ$

(i) $\sin \theta \cdot \sin 2\theta \cdot \sin 4\theta = \frac{1}{4} \sin 3\theta$

(ii) $\cos \theta \cdot \cos 2\theta \cdot \cos 4\theta = \frac{1}{4} \cos 3\theta$

(iii) $\tan \theta \cdot \tan 2\theta \cdot \tan 4\theta = \tan 3\theta$

For all value of θ

(i) $\sin(60-\theta) \sin \theta \cdot \sin(60+\theta)$

$$= \frac{1}{4} \sin 3\theta$$

(ii) $\cos(60-\theta) \cos \theta \cdot \cos(60+\theta)$

$$= \frac{1}{4} \cos 3\theta$$

(iii) $\tan(60-\theta) \tan \theta \cdot \tan(60+\theta) = \tan 3\theta$

Ex.48 The value of $\tan 10^\circ \tan 20^\circ \tan 40^\circ = ?$

Sol. Here $\theta = 10^\circ$

$$\tan(10^\circ) \tan(2 \times 10^\circ) \tan(4 \times 10^\circ) = \tan(3 \times 10^\circ)$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

Ex.49 The value of $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ = ?$

Sol. $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ$

$$\theta \quad 60-\theta \quad 60+\theta$$

Here, $\theta = 20^\circ$

$$= \frac{1}{4} \sin 3\theta$$

$$= \frac{1}{4} \sin(3 \times 20^\circ)$$

$$= \frac{1}{4} \sin 60^\circ$$

$$\frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

Ex.50 The value of

$$\sin \frac{p}{9} \cdot \sin \frac{5p}{9} \cdot \sin \frac{7p}{9} \cdot \sin \frac{3p}{9} \text{ is equal to}$$

Sol. $\sin \frac{p}{9} \cdot \sin \frac{5p}{9} \cdot \sin \frac{7p}{9} \cdot \sin \frac{3p}{9}$

Put value of $p = 180^\circ$

$\sin 20^\circ \cdot \sin 100^\circ \cdot \sin 140^\circ \cdot \sin 60^\circ$

$\sin 20^\circ \cdot \sin(180^\circ - 80^\circ)$

$$\sin(180^\circ - 40^\circ) \cdot \frac{\sqrt{3}}{2}$$

$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ \cdot \frac{\sqrt{3}}{2}$
[$\sin(180^\circ - \theta) = \sin \theta$]

$\frac{1}{4} \sin 60^\circ \times \frac{\sqrt{3}}{2}$

$\frac{1}{4} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

TYPE - XIV

T-radius of Multiple Angles :-

(i) $\sin 2A = 2 \sin A \cdot \cos A = \frac{2 \tan A}{1 + \tan^2 A}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1$
 $= 1 - 2 \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

(iii) $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

(iv) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(v) $\cos 3A = 4 \cos^3 A - 3 \cos A$

(vi) $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

(vii) $\sin C + \sin D = 2$

$$\sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$$

(viii) $\sin C - \sin D$

$$= 2 \cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$$

(ix) $\cos C + \cos D =$

$$2 \cos \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$$

(x) $\cos C - \cos D =$

$$2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$$

Ex.51. If $\sin 2x = \frac{1}{5}$, the value of

$(\sin x + \cos x)$ is :-

(a) $\sqrt{\frac{7}{5}}$ (b) $\sqrt{\frac{4}{5}}$

(c) $\sqrt{\frac{6}{5}}$ (d) $\sqrt{\frac{2}{5}}$

Sol. $\sin 2x = \frac{1}{5}$

add 1 both side

$$1 + \sin 2x = 1 + \frac{1}{5} = \frac{6}{5}$$

$$\therefore \sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x = \frac{6}{5}$$

$$[Q \sin^2 x + \cos^2 x = 1 \text{ and}$$

$$\sin 2x = 2 \sin x \cdot \cos x]$$

$$\Rightarrow (\sin x + \cos x)^2 = \frac{6}{5}$$

$$\Rightarrow \sin x + \cos x = \sqrt{\frac{6}{5}}$$

Ex.52 The value of

$$\cos 15^\circ \cdot \cos 7 \frac{1}{2}^\circ \cdot \sin 7 \frac{1}{2}^\circ = ?$$

Sol. $\cos 15^\circ \cdot \cos 7 \frac{1}{2}^\circ \cdot \sin 7 \frac{1}{2}^\circ$

Multiply and divide by 2

$\cos 15^\circ \cdot \frac{1}{2} \cdot 2 \cos 7 \frac{1}{2}^\circ \cdot \sin 7 \frac{1}{2}^\circ$

$\frac{1}{2} \cos 15^\circ \times \sin 2 \cdot \frac{15}{2}$

$\frac{1}{2} \cos 15^\circ \cdot \sin 15^\circ$

Again multiply and divide by 2

$\frac{1}{2} \times 2 \sin 15^\circ \cdot \cos 15^\circ$

$\frac{1}{4} \cdot \sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

TYPE- XV

Trigonometry expression is in-

dependent of angle so we can put any value of θ except result should not indeterminate

$$\frac{\frac{1}{5}, \frac{1}{5}, 0, \frac{1}{5}}{\frac{1}{5}, \frac{1}{5}, 0, \frac{1}{5}}$$

Note:- $\frac{1}{5} = 0, \frac{1}{5+1} = \frac{1}{5} = 0$

It is better to put $\theta = 0^\circ$ if expression does not contain cosec θ or cot θ otherwise $\theta = 45^\circ$

(i) If $\sin \theta, \cos \theta$ in equation, Try to put $\theta = 0^\circ$ or 90°

(ii) If $\sin \theta, \cos \theta, \tan \theta, \sec \theta, \operatorname{cosec} \theta, \cot \theta$ try to put $\theta = 45^\circ$

Ex.53 If $\tan \theta + \sin \theta = m$ and $\tan \theta - \sin \theta = n$, then find the value of $m^2 - n^2 = ?$

(a) $4\sqrt{mn}$ (b) mn

(c) $m^2 n^2$ (d) $m^3 n^3$

Sol. $m^2 - n^2 = (\tan \theta + \sin \theta)^2 - (\tan \theta - \sin \theta)^2$

$$= 4 \tan \theta \cdot \sin \theta$$

$$[Q (a+b)^2 - (a-b)^2 = 4ab]$$

$$= 4 \tan q \cdot \sin q$$

$$= 4 \sqrt{\tan^2 q \cdot \sin^2 q}$$

$$= 4 \sqrt{\tan^2 q (1 - \cos^2 q)}$$

$$= 4 \sqrt{\tan^2 q - \tan^2 q \cdot \cos^2 q}$$

$$= 4 \sqrt{\tan^2 q - \frac{\sin^2 q}{\cos^2 q} \cdot \cos^2 q}$$

$$= 4 \sqrt{\tan^2 q - \sin^2 q}$$

$$\text{Now, } mn = (\tan q + \sin q)$$

$$(\tan q - \sin q)$$

$$mn = \tan^2 q - \sin^2 q \quad \dots (i)$$

From equation (i)

$$= 4\sqrt{mn}$$

Alternative:-

Let $q = 45^\circ$

$$m = \tan q + \sin q = 1 + \frac{1}{\sqrt{2}}$$

$$n = \tan q - \sin q = 1 - \frac{1}{\sqrt{2}}$$

Now, $m^2 - n^2 = (m + n)(m - n)$

$$= (2) \times \frac{1}{\sqrt{2}} = 4 \times \frac{1}{\sqrt{2}}$$

take option (a) $4\sqrt{mn}$

$$= mn = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$= (1)^2 - \frac{1}{2} = 1 - \frac{1}{2} = \frac{1}{2}$$

So, $4\sqrt{mn} = 4 \times \sqrt{\frac{1}{2}}$

So option (a) is correct.