RATIO AND PROPORTION

Ratio & Proportion is one of the trickiest chapters which leaves many candidates a bit confused and most of the candidates leave these questions untouched.

We will tell you here the basics of Ratio and Proportion along with some important short tricks and questions which will surely make the chapter easy for you all.

Ratio and Proportion

The number of times one quantity contains another quantity of the same kind is called the ratio of the two quantities.

Observe carefully that the two quantities must be of the same kind. There can be a ratio between Rs.20 and Rs 30, but there can be no ratio between Rs 20 and 30 mangoes.

The ratio 2 to 3 is written **as 2 : 3 or 2/3.** 2 and 3 are called the **terms of the ratio**. 2 is the first term and 3 is the second term.

Consequent

In the **ratio 2 :3** , 2 is the antecedent and 3 is the consequent.

Note:

(1) The word '**consequent** ' literally means 'that which goes after'.

(2) Since the quotient obtained on dividing one concrete quantity by another of the same kind is an abstract number, the ratio between two concrete quantities of the same kind is an abstract number. Thus, the ratio between Rs 5 and 7 is 5:7.

Compound Ratio

Ratios are compound by multiplying together the antecedents for a new antecedent, and the consequents for a new consequent.

Ex: Find the ratio compound of the ratio:

4:3, 9: 13, 26 : 5 and 2:15

Solution;

The required ratio = $\frac{4\times9\times26\times2}{3\times13\times5\times15}=\frac{16}{25}$

Inverse Ratio

- If 2:3 be the given ratio, then 1/2: 1/3 or 3 :2 is called its inverse or reciprocal ratio.
- If the antecedent = the consequent, the ratio is called the ratio of equality, such as 3:3.

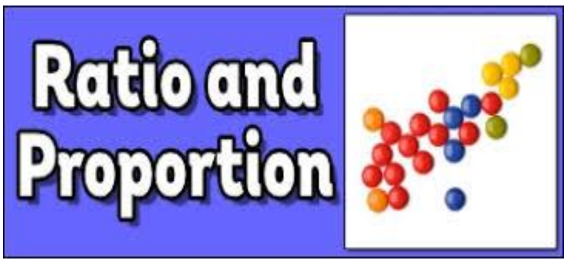
- If the antecedent >the consequent, the ratio is called the ratio of greater inequality, as 4 :3
- If the antecedent < the consequent, the ratio is called the ratio of less inequality, as 3 : 4.

Ex. **Divide 1458 into two parts such that one may be to the other as 2: 7.**

Solution:

1st part = $2\times\frac{1458}{2+7} = 2\times\frac{1458}{9} = 324$

2nd part = $7\times\frac{1458}{9} = 1134$



Proportion

Consider the two ratios:

1st ratio	2nd ratio
6 : 18	8 : 24

Since 6 is one-third of 18, and 8 is one – third of 24, the two ratios are equal. The equality of ratio is called **proportion.**

The number 6, 18, 8 and 24 are said to be in **proportion.**

The proportion may be written as

6 : 18 :: 8 : 24 (6 is to 18 as 8 is to 24)

Or, 6 :18 = 8:24 or $\frac{6}{18} = \frac{8}{24}$

The numbers 6, 18, 8 and 24 are called the **terms**. 6 is the **first terms**, 18 the **second,** 8 the **third,** and 24 **the fourth**. The first and fourth terms, i.e. 6 and 24 are called the **extremes** (end terms), and the second and the third terms, i.e., 18 and 8 are called the **means** (middle terms). 24 is called the **fourth** proportional.

1. If your quantities be in proportion, the product of the extremes is equal to the product of the means.

   Let the four quantities 3, 4, 9 and 12 be in proportion.

   We have , $\frac{3}{4} = \frac{9}{12}$

   $\frac{3}{4}\times 4\times 12 = \frac{9}{12}\times 4\times 12$

   $3\times 12 = 4\times 9$

2. Three quantities of the same kind are said to be in continued proportion when the ratio of the first to the second is equal to the ratio of the second the third.

   *The second quantity is called the **mean proportional** between the first and the third ; and the **third quantity** is called the third proportional to the first and second.*

   Thus, 9, 6 and 4 are in continued proportion for 9 :6 ::6 :4.

   Hence, 6 is the mean proportional between 9 and 4, and 4 is the third proportional to 9 and 6.

Ex. Find the fourth proportional to the numbers 6, 8 and 15.

Solution:

If x be the fourth proportional, then 6 : 8 = 15:x

$$\therefore \frac{8\times 15}{6}=20$$

Ex. Find the third proportional to 15 and 20.

Solution:

Here, we have to find a fourth proportional to 15, 20 and 20. If x be the fourth proportional, we have 15 :20 = 20:x

$$\therefore x=\frac{20\times 20}{15}=\frac{80}{3}=26\frac{2}{3}$$

Direct Proportion: consider the following example .

Ex.: If 5 ball cost Rs 8, what do 15 balls cost?

Solution:

It will be seen at once that if the number of balls be increased 2, 3, 4,….times, the price will also be increased 2, 3, 4… times. Therefore, 5 balls is the same fraction of 15 balls that the cost of balls is of the cost of 15 balls.

5 balls : 15 balls :: Rs8 : required cost the

required cost = Rs $\frac{15\times 8}{5}$ =Rs 24

This, example is an illustration of what is called direct proportion. In this case, the two given quantities are so related to each other that if one of them is multiplied (or divided) by any number, the other is also multiplied (or divided) by the same number.

Inverse Proportion: Consider the following example

Ex. : If 15 men can reap a filed in 28 days, in how many days will 10 men reap it?

Solution:

Here, it will be seen that if the number of men be increased 2, 3,4,….times, the number of days will be decreased 2, 3, 4…times. Therefore, the inverse ratio of the number of men is equal to the ratio of the corresponding number of days.

$$\therefore \frac{1}{15}:\frac{1}{10}::28:\text{the required number of days}$$

Or , 10: 15 :: 28 : the required number of days the required number of days=

$$\frac{15\times 28}{10}=42$$

The above example is an illustration of what is called inverse proportion. In this case, the two quantities are so related that if one of them is multiplied by any number, the other is divided by the same number, and vice versa

Ex : The employer decreases the number of his employees in the ratio 10 :9 and increase their ways in the ratio 11:12. What is the ratio of his two expenditures?

Solution:

The required ratio = 10×11 : 9 ×12 = 55 : 54

Ex: A vessel contains liquid A and B in ratio 5 :3. If 16 liters of the mixture are removed and the same quantity of liquid B is added, the ratio becomes 3 :5. What quantity does the vessel hold?

Solution:

Quicker Method:

When the ratio is reversed (i.e., 5:3 becomes 3 :5), we can use the formula;

Total quantity = $\frac{(5+3)^2}{5^2-3^2}$ ×Quantity of A in the removed mixture

= $\frac{64}{16}\times10=40$ liters

Example: If A: B = 2:5 and B:C = 7:3 then find A:B:C

Solution:

A : B = 2 : 5

B : C = 7 : 3

In this, value of B has to be same so to equate the value of B, we can take LCM of both the values of B i.e. (5×7) = 35

So multiply (A : B) by 7 and (B : C) by 5

Hence, A : B : C = 14 : 35 : 15

Example : If ratio of A : B : C is given in reciprocals, then convert it.

Solution: Suppose, if A : B : C = (1/2): (1/3): (1/5)

take any number which is multiple of the product (2×3×5) = 30

Now multiply it in the numerator part

A:B:C

= (30/2) : (30/3) : (30/5) = 15:10:6

Example : The sum of the ages of Vinay and Arjun is 48 years. Vinay is 4 years older than Chirag. The ratio of the ages of Arjun and Chirag is 4 : 7. What was the age of Vinay 5 years back?

Solution:

Given Age of Vinay + Arjun = 48 and Vinay is 4 yrs older than Chirag So, Arjun + Chirag

= 44 and Arjun : Chirag = 4 : 7

Comparing B + C = 11

So, 11 = 44

1 = 4

Arjun = 4×4 = 16 yrs

Chirag = 7×4 = 28 yrs

Vinay = 32 yrs

Age of Vinay 5yrs back = 32-5 = 27 yrs

Example : The ratio of the ages of A and B is 6 : 5. The difference between the ages of C and A is more than 3 years. The age of D is a prime number between the ages of A and B. The ratio of the ages of B and C is 2 : 3. If the ages of all four are integers, what is the difference between the ages of C and D?

Solution: Given ratio of A : B = 6 : 5 and B : C = 2 : 3

So, A : B : C = 12 : 10 : 15

Now let’s use conditions given i.e.

(i) All ages of A, B, C and D should be integers.

(ii) difference of age between A and C should be more than 3 yrs but from the above ratio difference is only 3 yrs.

So, we have to take A : B : C = 24 : 20 :

(iii) age of D lies between A and B and it should be prime number

Numbers between 24 and 20 are = 23, 22, 21 and 23 is the prime number. So, the age of D is 23

Difference of C and D = 30 – 23 = 7yrs

Example : Ten years ago, the ages of Alok and Sanjay were in the ratio 6 : 7. After six years, Alok's age would be 9.09% less than Sanjay's age. What would be age of Sanjay after 9 years?

Solution: In percentage topic we have learnt that 9.09% = 1/11, if age of Sanjay after 6 yrs is X then age of Alok will be X – (1/11)X = (10/11)X hence ratio of Alok and Sanjay = (10/11)X : X = 10 : 11

	Alok	:	Sanjay
10yrs ago	6	:	7
after 6yrs	10	:	11

Difference between both these are 16 yrs

So, 4 = 16

1 = 4

hence, age of Sanjay after 6 yrs = 11×4 = 44

age of Sanjay after 9 yrs = 44 + 3 = 47 yrs

Example : Five years ago, the ages of a father and son were in the ratio 7 : 2. After three years, their ages would be in the ratio 5 : 2. What was father's age 7 years back?

Solution: Father : Son

	Father	:	Son
5yrs ago	7	:	2
3yrs after	5	:	2

Example : The difference between age of father at 5yrs ago and 3 yrs after should be equal to the difference between age of son at 5yrs ago and 3 yrs. To achieve this condition, we manipulate the ratio.

	Father	:	Son
5yrs ago	21	:	6
3yrs after	25	:	10

now, we can see that 25-21 = 4 and 10-6 = 4

difference between 3yrs after and 5 yrs ago = 8yrs

So, 4 = 8 yrs

1 = 2 yrs

hence age of father's 5 yrs ago is 42 yrs and 7yrs ago = 40 yrs.

Example : A stick is broken up into two parts. The ratio of the lengths of the bigger part and the smaller part is equal to the ratio of the lengths of the full stick and the bigger part. What is this ratio?

Solution: Let the length of bigger part of the rod is 1 m and the length of smaller part of the rod is x m, then total length of the rod is (1+x) m

Given, ratio of bigger to smaller is equal to ratio of the length of full stick to bigger part

So, 1/x = (1+x) /1

$x^2 + x = 1$

$x^2 + x – 1 = 0$

using quadratics x = (-1 ± √ 5)/2

but x = (-1+√ 5)/2 is the answer, because another will give the negative answer.

Example : The contents of two vessels containing wine and water in the ratio 2 : 3 and 5 : 6 are mixed in the ratio 10 : 7. What would be the ratio of wine and water in the final mixture?

Solution: Wine Water

	Wine		Water
vessel 1	2	:	3
vessel 2	5	:	6

to make the total quantity same in both vessel we have to multiply ratio of vessel 1 by (5+6= 11) and vessel 2 by (2+3 = 5)

Now, Vessel 1 22 : 33

Vessel 2 25 : 30

But, vessel 1 and vessel 2 are mixed in the ratio of 10 : 7.

So, wine = 22×10+25×7 = 395

water = 33×10+30×7 = 540

It is given that two vessels containing wine and water in the ratio 2 : 3 and 5 : 6 but both vessels have same quantity of total mixture.

So, ratio of wine and water in final mixture = 395 : 540 = 79 : 108

How to express data in a smarter way.

Wine : water

vessel 1 (2 : 3) × 11 × 10

vessel2 (5 : 6) × 5 × 7

vessel 1 (2 : 3) × 110 = (2 : 3)× 22 = 44 : 66

vessel2 (5 : 6) × 35 = (5 : 6)× 7 = 35 : 42

Hence, ratio will be (44+35) : (66+42) = 79 : 108

Example : A mixture contains wine and water in the ratio 3 : 2 and another contains them in the ratio 4 : 5. How many litres of the former must be mixed with 15 litres of the latter so that the resultant mixture contains equal quantities of wine and water?

Solution: In this, after mixing of both different mixture , quantity of wine and water becomes equal.

Let X litre of mixture 1 is mixed with 15 litres of mixture 2, then

(3/5)X + (4/9)15 = (2/5)X + (5/9)15

X/5 = 15/9

X = 75/9 = 25/3

How to do it by using ratio concept and above discussed methodology?

Ultimately we have to equal the ratio of wine and water in the final mixture.

	Wine	:	Water
Mix 1	3	:	2
Mix 2	4	:	5

We have to make the sum of quantity of wine in mix 1 and mix 2 equal to sum of quantity of water in mix 1 and mix2.

But in the above question it is already given so, we will mix it in the same quantity.

If we mix 5 litres of mixture1 and 9 litres of mixture2, then the ratio of quantity will be same.

So, if it is 15 litres of mixture2, then mixture1 = (15×5)/9 = 25/3 litres.

Example : A mixture contains wine and water in the ratio 3 : 2 and another contains them in the ratio 7 : 3. In what ratio should the two be mixed to get a resultant mixture with wine and water in the ratio 17 : 8?

Solution: Let these are mixed in ratio of 1:X , we will not assume a : b because there will be two variables and will make the calculation harder.

Now, wine : water

mixture1 3 : 2

mixture2 7 : 3

First we will solve it as done in 8.

So,(3:2)×(10)×1 = (3:2)×2

(7:3)×(5)× X = (7:3)×X

ratio of wine and water in final mixture is

(3×5+7X):(2×5+3X)

(6+7X) / (4+3X) = 17/8

solving it, X = 4

So, it will me mixed in the ratio of 1:4

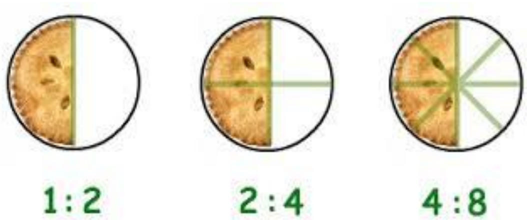
Wrong Approach

Some of you will mark answer (1:2), why?

3 : 2 × 1 = 3 : 2

7 : 3 × 2 = 14: 6

So, (3+14): (2+6)= 17 : 8. this will not be applicable here because the quantity of mixtures 1 & 2 are not equal.



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Alligation Approach: we will discuss it in detail in the next article.

In this approach, alligation will be applied only on one object throughout the solution.

So, let's apply it on wine

in mixture1, % of wine is 60% in solution

In mixture2, % of wine is 70% in solution

In final mixture, % of wine is 68%

So, 60--------68-------70

60 8 68 2 70, you can see difference of 68 and 60 is 8 and 70 and 68 is 2.

For calculating the ratio of mixing, answer will be the reciprocal of ratio of these differences.

ratio in which it is mixed = 2 : 8 = 1 : 4

SOME IMPORTANT RULES

1. If a : b = m : n and b : c = q. Then a : c =

$\frac{m}{n} \times \frac{p}{q}$

Note : This rule can be extended for any number of ratios.

Example : If a : b = 2, b : c = 4 : 3 and c : d = 6 : 7, then a : d = ?

(A) 2 : 7 (B) 16 : 35

(C) 12 : 21 (D) None of these

Sol.

$\frac{a}{d} = \frac{2}{3} \times \frac{4}{3} \times \frac{6}{7} = \frac{16}{35} \Rightarrow a : d = 16 : 35$

2. If x times of a number is equal to y times of other number

Then ratio between two numbers

$= \frac{1}{x} : \frac{1}{y}$ = y : x

Example : If 5 times a number is equal to 3 times another number. Find ratio between two number.

(A) 2 : 3 (B) 4 : 5

(C) 3 : 5 (D) 5 : 3

Sol.

Ratio between two numbers = $\frac{1}{5} : \frac{1}{3}$ = 3 : 5

3. If the sum of two numbers is x and their difference is y, ratio of two numbers is (x + y) : (x-y).

Note: The two numbers are $\frac{x+y}{2}$ and $\frac{x-y}{2}$ respectively.

Proof : Let the numbers be a and b. Then a + b = x, and a- b = y

Now, (a+b) + (a-b) = x+y $\Rightarrow$ a = $\frac{x+y}{2}$ and

(a+b) – (a-b) = x – y $\Rightarrow$ b = $\frac{x-y}{2}$

$\therefore$ a : b = $\frac{x+y}{2} : \frac{x-y}{2}$ = (x+y) : (x-y)

Example :

The ratio of the sum and difference of two numbers is 5 : 2. What is the ratio of two numbers?

(A) 7 : 3 (B) 3 : 7

(C) 1 : 10 (D) 5 : 2

Sol.

Ratio of the numbers = $\frac{5+2}{5-2}$ = 7 : 3

4. **If $\frac{am}{bn} = \frac{x}{y} \Rightarrow \frac{a}{b} = \frac{x}{y} : \frac{m}{n} = \frac{x}{m} : \frac{y}{m}$**

Example :

Product of two sets of two numbers are in the ratio of 3 : 5. If the ratio of two numbers one from each set is 2 : 3, find the ratio between the two other numbers.

Sol.

Required ratio = $\frac{3}{2} : \frac{5}{3}$ = 9 : 10

5. **What must be added to each of the four numbers a, b, c and d so that the resultant numbers are in proportion?**

Sol.

Let the number to be added is x where

$$x = \frac{bc - ad}{(a+d) - (b+c)} = \frac{\text{Product of means-Product of extremes}}{\text{Sum of extremes - Sum of means}}$$

Example :

What must be added to each of the numbers 7, 10, 19 and 25 so that the resultant numbers are in proportion?

(A) 2 (B) 3

(C) 4 (D) 5

Sol.

Required number

$$x = \frac{10 \times 19 - 7 \times 25}{(7+25) - (10+19)} = \frac{15}{3} = 5$$

6. If $\frac{a}{b} = \frac{c}{d}$ then

(i) $\frac{b}{a} = \frac{d}{c}$ (Invertendo Rule: Obtained by dividing 1 by the given ratios)

(ii) $\frac{a}{c} = \frac{b}{d}$ (Alternendo Rule)

(iii) $\frac{a+b}{b}=\frac{c+d}{d}$ (Rule of Componendo. Arrived at by adding 1 to each side)

(iv) $\frac{a-b}{b}=\frac{c-d}{d}$ (Rule of Dividendo. Arrived at by subtracting 1 from each side)

(v) $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (Rule of Componendo and Dividendo : Arrived at by dividing (iii) by (iv)

(vi) $\frac{a}{b}=\frac{c}{d}=\frac{a-c}{b-d}$ (Rule of Subtranendo)

(viii) $\frac{a}{a-b}=\frac{c}{c-d}$ (Rule of Convertendo)

7. **If two numbers are in the ratio x : y and on adding a constant number 'a' to both the numbers, the new ratio becomes (x+1) : (y+1), Or on subtracting 'a' from both the numbers, the ratio becomes (x-1) : (y-1). then the original numbers are ax and ay respectively.**

Example. If two numbers are in the ratio 2 : 3 and the ratio becomes 3 : 4 when 8 is added to both the numbers, then the sum of the two numbers is:

(a) 80 (b) 40

(c) 10 (d) 100

Sol.

On adding 8 to both the numbers, the new ratio is increased by same ratio i.e. 1 in each case.

The original numbers are 8 × 2 and 8 × 3 i.e. 16 and 24.

Example. Two numbers are in the ratio 8 : 11. If we subtract 6 from each number, then new ratio becomes 7 : 10. The original numbers are:

Sol.

On subtracting 6 from both the numbers, the new ratio is decreased by same ratio i.e. 1 in each case.

The original numbers are 8 × 6 and 11 × 6 i.e. 48 and 66.

8. **If two numbers are in the ratio x : y and on adding a constant number a to both the numbers, the new ratio becomes (x+k) : (y+k), Or On subtracting a from both the numbers, the ratio becomes (x-k) : (y-k). then the original number are $\frac{a}{k}$ x and $\frac{a}{k}$ y respectively.**

Example . The ratio of monthly income of A, B is 6 : 5 and their monthly expenditures are in the ratio 4 : 3. If each of them saves Rs. 400 per month, find the sum of their monthly incomes.

(A) Rs. 2300 (B) Rs. 2400

(C) Rs. 2200 (D) Rs. 2500

Sol.

	A		B	
Income	6	:	5	
Expenditure	4	:	3	
Saving	2	:	2	

Hence saving of each = 2 ratio = Rs. 400

⇒ 1 ratio = Rs. 2200

9. **If two numbers are in the ratio x : y. On adding a constant number 'a' to the first number, the new ratio becomes (x+1) : (y), Or on subtracting a from the first number, the ratio becomes (9x-1) : (y). then the original numbers are ax and ay respectively.**

Example: Ratio of milk and water in a mixture is 4 : 1. If 5 litre of milk is added in the mixture, new ratio becomes 5 : 1. The quantity of milk in the mixture originally was :

(A) 10 litres (B) 15 litres

(C) 20 litres (D) 25 litres

Sol.

On adding 5 litres of milk, its ratio in the mixture is increased by 1.

Original quantity of milk in the mixture was 4 × 5 = 20 litres.

10. **If two numbers are in the ratio a : b and on adding k, a constant number to one of the numbers, new ratio becomes m : n, i.e. both the ratio change instead of one ratio only.**

1. **Multiply the ratio (old, New or both) so that the old and new ratios become the same for the number, to which k is not added to or subtracted from.**
2. **Find increase or decrease in the revised new ratio from revised old ratio.**
3. **Number added to get the desired change in the ratio is:**

$$\frac{\text{Difference in revised old an new ratios}}{\text{Sum of the revised old ratio}}$$

Example: 132 litres of a mixture contains milk and water in the ratio 8 : 3. How much water must be added to the mixture so as to make the ratio of milk and water 2 : 1?

(A) 11 liters (B) 12 litres

(C) 13 litres (D) 15 litres

Sol.

LCM of ratios of milk i.e. LCM of 8 and 2 is 8.

We make ratio of milk equal to 8 in the two ratios.

	Milk	:	Water
New ratio = 2 : 1 =	8	:	4
Old ratio =	8	:	3
Difference =	0	:	1

Water to be added = $\frac{1}{8+3} \times 132$ = 12 litres

11. **Two numbers are in the ratio a : b. If k, a constant number is added to both the numbers, new ratio of the numbers becomes m : n.**

then the original numbers are ax and bx, where x = $\frac{(m-n)k}{an-bm}$

$$\frac{\text{Difference between new ratio × Number added to both numbers}}{\text{Difference between cross multiplication of the two ratios}}$$

Note:

Constant number k added to both the numbers is k = $\frac{an-bm}{m-n} \times x$

proof: Let the original numbers are 'ax' and 'bx'. Then $\frac{ax+k}{bx+k} = \frac{m}{n}$

anx + nk = bmx + mk ⇒ (an – bm

(x = (m-n)k

$\Rightarrow x = \frac{(m-n)k}{an-bm}$

Example

The ratio between two numbers is 5 : 3. If 3 is added to both the numbers , the ratio becomes 14 : 9. find the smaller number.

(A) 15 (B) 18

(C) 25 (D) 28

Sol.

$x = \frac{(14-9)\times 3}{5\times 9 - 3\times 14} = \frac{5\times 3}{45-42} = \frac{5\times 3}{3} = 5$

Smaller number = 3x = 3 × 5 = 15

12. **The incomes of two persons are in the ratio a : b and their expenditures are in the ratio m :n. If both save Rs. S, then their incomes are ax and bx, where x =**

$\frac{S(n-m)}{an-bm}$

Example.

The monthly incomes of two persons are in the ratio 2 : 3 and their monthly expenses are in the ratio 5 : 9. If each of the save Rs. 600 per month, then their monthly incomes are:

(A) Rs. 1500; Rs. 2250

(B) Rs. 1200; Rs. 1800

(C) Rs. 1600; Rs. 2400

(D) Rs. 1400; Rs. 2100

Sol:

$x = \frac{S(n-m)}{an-bm} = \frac{600\times(9-5)}{2\times 9 - 3\times 5} = \frac{600\times 4}{3}$ = Rs. 800

A's income = 2 × Rs. 800 = Rs. 1600.

B's income = 3 × Rs. 800 = Rs. 2400