LIMITS

1. INDETERMINATE FORM

Some times we come across with some functions which donot have definite value corresponding to some particular value of the variable.

For example for the function

$$f(x) = \frac{x^2-4}{x-2},\ f(2)$$

$$= \frac{4-4}{2-2} = \frac{0}{0}$$

which cannot be determined. Such a form is called an Indeterminate form. Some other indeterminate forms are

$0 \times \infty$, 0^0, 1^∞,

$\infty - \infty$, $\infty/\infty, \infty^\circ$, 0/0.

2. THEOREMS ON LIMITS

The following theorems are very helpful for evaluation of limits–

(i) $\lim_{x\to a}[k\ f(x)\ ] = k \lim_{x\to a} f(x)$,
where k is a constant

(ii) $\lim_{x\to a}[\ f(x) + g(x)\ ]$

$= \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(iii) $\lim_{x\to a}[f(x) - g(x)\ ] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

(iv) $\lim_{x\to a}[f(x).g(x)\ ] = \lim_{x\to a} f(x).\ \lim_{x\to a} g(x)$

(v) $\lim_{x\to a}[f(x)/g(x)]$

$= [\ \lim_{x\to a} f(x)\ ]/[\lim_{x\to a} g(x)]$

provided $g(x) \neq 0$

(vi) $\lim_{x\to a} f[g(x)\ ] = f[\lim_{x\to a} g(x)\ ]$

(vii) $\lim_{x\to a}[f(x) + k] = \lim_{x\to a} f(x) + k$ where k is a constant

(viii) $\lim_{x\to a} \log\{f(x)\} = \log\{\lim_{x\to a} f(x)\}$

(ix) If $f(x) \le g(x)$ for all x,

then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$

(x) $\lim_{x\to a}[f(x)]^{g(x)} = \{\lim_{x\to a} f(x)\}^{\lim_{x\to a} g(x)}$

(xi) $\lim_{x\to\pm\infty} f(x) = \lim_{x\to 0} f(1/x)$

(xii) $\lim_{x\to 0^+} f(-x) = \lim_{x\to 0^-} f(x)$

3. METHODS OF EVALUATION OF LIMITS

Factorisation method :

If f(x) is of the form $\frac{f(x)}{g(x)}$ and of indeterminate form then this form is removed by factorising g(x) and h(x) and cancel the common factors, then put the value of x.

Rationalisation Method:

In this method we rationalise the factor containing the square root and simplify and we put the value of x.

Expansion method

If $x \to 0$ and there is atleast one function in the given expression which can be expanded then we express numerator and Denominator in the ascending powers of x and remove the common factor there.

The following expansions of some standard functions are given-

(i) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

(ii) $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\ldots$

(iii) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots$

(iv) $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots\ldots$

(v) $a^x = 1 + (x \log a) + \frac{(x \log a)^2}{2!} + \frac{(x \log a)^3}{3!} +$

(vi) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots$

(vii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots$

(viii) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\ldots$

(ix) $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots$

(x) $\cosh x = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+.....$

(xi) $\tanh x = x-\frac{x^3}{3}+2x^5-.....$

(xii) $\sin^{-1}x = x+\frac{x^3}{3!}+\frac{9x^5}{5!}+.....$

(xiii) $\cos^{-1}x = \frac{\pi}{2}-\left(x+\frac{x^3}{3!}+\frac{9x^5}{5!}+....\right)$

(xiv) $\tan^{-1} x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.....$

(xv) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2+.....$

4. SOME STANDARD LIMITS

(i) $\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{x}{\sin x}= 1;$

$\lim_{x\to 0}\sin x = 0$

(ii) $\lim_{x\to 0}\cos x = \lim_{x\to 0}\left(\frac{1}{\cos x}\right)=1$

(iii) $\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{x}{\tan x}= 1;$

$\lim_{x\to 0}\tan x =0$

(iv) $\lim_{x\to 0}\frac{\sin^{-1}x}{x}=\lim_{x\to 0}\frac{x}{\sin^{-1}x}= 1$

(v) $\lim_{x\to 0}\frac{\tan^{-1}x}{x}=\lim_{x\to 0}\frac{x}{\tan^{-1}x}= 1$

(vi) $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = \lim_{x\to 0}(1+ax)^{1/x} = e^a$

(vii) $\lim_{x\to 0}\frac{a^x-1}{x}= \log_e a \quad (a>0)$

(viii) $\lim_{x\to 0}\frac{e^x-1}{x}= 1$

(ix) $\lim_{x\to a}\frac{x^n-a^n}{x-a}= n\ a^{n-1}$

(x) $\lim_{x\to 0}\frac{\log(1+x)}{x}= 1$

(xi) $\lim_{x\to 0}\frac{(1+x)^n-1}{x}= n$

(xii) $\lim_{x\to\infty}\frac{\sin x}{x}= \lim_{x\to\infty}\frac{\cos x}{x}= 0$

(xiii) $\lim_{x\to\infty}\frac{\sin 1/x}{1/x}= 1$

(xiv) $\lim_{x\to\infty} 1/x = 0$

(xv) $\lim_{x\to 0}\frac{1}{|x|}= \infty$

(xvi) $\lim_{x\to\infty} a^x = \begin{cases} 0, \text{ if } |a|<1 \\ 1, \text{ if } a=1 \\ \infty, \text{ if } a>1 \\ \text{does not exist, if } a\le -1 \end{cases}$

(xvii) $\lim_{x\to a}[f(x)]^{g(x)} = \lim_{x\to a} g(x)\ \{f(x) - 1\}$

5. SOME LIMITS WHICH DO NOT EXIST

(i) $\lim_{x\to 0}\left(\frac{1}{x}\right)$ (ii) $\lim_{x\to 0} x^{1/x}$

(iii) $\lim_{x\to 0}\frac{|x|}{x}$ (iv) $\lim_{x\to a}\frac{|x-a|}{x-a}$

(v) $\lim_{x\to 0}\sin 1/x$ (vi) $\lim_{x\to 0}\cos 1/x$

(vii) $\lim_{x\to 0} e^{1/x}$ (viii) $\lim_{x\to\infty}\sin x$

(ix) $\lim_{x\to\infty}\cos x$

SOLVED PROBLEMS

Ex.1 If $f(x) = \begin{cases} x^2+2, & x \geq 1 \\ 2x+1, & x<1 \end{cases}$, then find $\underset{x\to 1}{\text{Lim}}\ f(x)$

Sol. $\lim\limits_{x\to 1-0} f(x) = \lim\limits_{h\to 0} [2(1-h)+1] = 3$

$\lim\limits_{x\to 1+0} f(x) = \lim\limits_{h\to 0} [(1+h)^2 + 2] = 3$

$\because$ LHL = RHL , so $\lim\limits_{x\to 1} f(x) = 3$.

Ex.2 Evaluate $\underset{x\to 0}{\text{Lim}} \dfrac{1+e^{-1/x}}{1-e^{-1/x}}$

Sol. LHL $= \underset{h\to 0}{\text{Lim}} \dfrac{1+e^{1/h}}{1-e^{1/h}}$

$= \underset{h\to 0}{\text{Lim}} \dfrac{e^{-1/h}+1}{e^{-1/h}-1} = -1$

RHL $= \underset{h\to 0}{\text{Lim}} \dfrac{1+e^{-1/h}}{1-e^{-1/h}} = \dfrac{1+0}{1-0} = 1.$

LHL $\neq$ RHL, so given limit does not exist.

Ex.3 Evaluate $\lim\limits_{x\to -1}\left(\dfrac{x^2-1}{x^2+3x+2}\right)$

Sol. Limit $= \lim\limits_{x\to -1} \dfrac{(x-1)(x+1)}{(x+2)(x+1)} = \dfrac{-1-1}{-1+2} = -2$

Ex.4 Evaluate $\lim\limits_{x\to a}\left[\dfrac{x^2-(a+1)x+a}{x^3-a^3}\right]$

Sol. $\lim\limits_{x\to a}\left[\dfrac{x^2-(a+1)x+a}{x^3-a^3}\right]\left(\dfrac{0}{0}\text{ form}\right)$

$= \lim\limits_{x\to a} \dfrac{2x-a-1}{3x^2} = \dfrac{a-1}{3a^2}$

Ex.5 Evaluate $\lim\limits_{x\to 3} \dfrac{x-3}{|x-3|}$

Sol. LHL $= \lim\limits_{h\to 0} \dfrac{(3-h)-3}{|(3-h)-3|}$

$= \lim\limits_{h\to 0} \dfrac{-h}{|-h|} = -1$

RHL $= \lim\limits_{h\to 0} \dfrac{(3+h)-3}{|(3+h)-3|}$

$= \lim\limits_{h\to 0} \dfrac{h}{|h|} = 1$

LHL $\neq$ RHL , so limit does not exist.

Ex.6 If $f(x) = \dfrac{x+|x|}{x}$, then find $\lim\limits_{x\to 0} f(x)$

Sol. LHL $= \lim\limits_{h\to 0} \dfrac{-h+|h|}{-h} = \lim\limits_{h\to 0} (0) = 0$

RHL $= \lim\limits_{h\to 0} \dfrac{h+|h|}{h} = 2$

LHL $\neq$ RHL $\Rightarrow$ does not exist.

Ex.7 Evaluate $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{1+x}-\sqrt{1-x}}$

Sol. Limit $= \lim\limits_{x\to 0} \dfrac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{(1+x)-(1-x)}$

$= \lim\limits_{x\to 0} \dfrac{\sqrt{1+x}+\sqrt{1-x}}{2} = 1.$

Ex.8 Evaluate $\lim\limits_{x\to 0} \dfrac{xe^x - \log(1+x)}{x^2}$

Sol. $= \lim\limits_{x\to 0} \dfrac{x\left(1+x+\dfrac{x^2}{2!}+\ldots\right)-\left(x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\ldots\right)}{x^2}$

$= \lim\limits_{x\to 0}\left(\dfrac{3}{2}+\dfrac{1}{6}x+\ldots.\right) = \dfrac{3}{2}$

Ex.9 Evaluate $\lim\limits_{x\to 0}\left[\dfrac{1}{x^2}-\dfrac{1}{\sin^2 x}\right]$

Sol. Limit $= \lim\limits_{x\to 0} \dfrac{\sin^2 x - x^2}{x^2.\sin^2 x}$

$$= \lim_{x\to 0} \frac{\left(x-\frac{x^3}{3!}+\ldots\right)^2 - x^2}{x^2\left(x-\frac{x^3}{3!}+\ldots\right)^2}$$

$$= \lim_{x\to 0} \frac{x^2 - \frac{1}{3}x^4+\ldots - x^2}{x^4\left(1-\frac{x^2}{3!}+\ldots\right)^2} = -1/3$$

Ex.10 Evaluate $\lim_{x\to 0} \frac{\sin x^\circ}{x}$

Sol. Limit = $\lim_{x\to 0} \frac{\sin(\pi/180)x}{x}$ $\left(\frac{0}{0}\text{form}\right)$

$$= \lim_{x\to 0} \frac{(\pi/180)\cos(\pi/180)x}{1}$$

$$= \frac{\pi}{180}$$

Ex.11 If $f(x) = \begin{cases} x-1, & x<0 \\ 1/4, & x=0 \\ x^2, & x>0 \end{cases}$ then find $\lim_{x\to 0} f(x)$

Sol. Here $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$

and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x-1) = -1$

$\because \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\therefore \lim_{x\to 0} f(x)$ does not exist.

Ex.12 Evaluate $\lim_{x\to 0} \frac{2^x-1}{\sqrt{(1+x)}-1}$

Sol. Given Limit

$$= \lim_{x\to 0} \frac{2^x-1}{\sqrt{(1+x)}-1} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0} \frac{2^x-1}{x} \lim_{x\to 0} \left(\sqrt{1+x}+1\right)$$

$$= 2.\lim_{x\to 0} \frac{2^x \log 2}{1} = 2.\log 2$$

Ex.13 Evaluate $\lim_{x\to 0} \frac{x(2^x-1)}{1-\cos x}$

Sol. The given limit

$$= \lim_{x\to 0} \frac{2^x-1}{x} \cdot \frac{x^2}{1-\cos x}$$

$$= \lim_{x\to 0} \frac{2^x-1}{x} \lim_{x\to 0} \frac{x^2}{2\sin^2\frac{x}{2}}$$

$$= \log x \,.\, 2 \lim_{x\to 0} \left(\frac{x/2}{\sin(x/2)}\right)^2$$

$$= 2 \log 2.$$

Ex.14 Evaluate $\lim_{x\to 0} \left[\frac{a}{x} - \cot\frac{x}{a}\right]$

Sol. Given Limit = $\lim_{x\to 0} \left[\frac{a}{x} - \frac{\cos(x/a)}{\sin(x/a)}\right]$

$$= \lim_{x\to 0} \left[\frac{a\sin(x/a) - x\cos(x/a)}{x\sin(x/a)}\right]$$

$$= a \lim_{x\to 0} \left[\frac{a\sin(x/a) - x\cos(x/a)}{x^2}\right] \times \frac{(x/a)}{\sin(x/a)}$$

$$= a \lim_{x\to 0} \left[\frac{a\sin(x/a) - x\cos(x/a)}{x^2}\right] \left(\frac{0}{0}\text{form}\right)$$

$$= a \lim_{x\to 0} \left[\frac{\cos(x/a) - \cos(x/a) + (x/a)\sin(x/a)}{2x}\right]$$

EXERCISE – I

UNSOLVED PROBLEMS

Evaluate the following limits :

Q.1 $\lim_{x\to 2}(5-x)$

Q.2 $\lim_{x\to 1}(6x^2-4x+3)$

Q.3 $\lim_{x\to 3}\left(\frac{x^2+9}{x+3}\right)$

Q.4 $\lim_{x\to 3}\left(\frac{x^2-4x}{x-2}\right)$

Q.5 $\lim_{x\to 5}\left(\frac{x^2-25}{x-5}\right)$

Q.6 $\lim_{x\to 1}\left(\frac{x^3-1}{x-1}\right)$

Q.7 $\lim_{x\to -2}\left(\frac{x^3+8}{x+2}\right)$

Q.8 $\lim_{x\to 3}\left(\frac{x^4-81}{x-3}\right)$

Q.9 $\lim_{x\to 3}\left(\frac{x^2-4x+3}{x^2-2x-3}\right)$

Q.10 $\lim_{x\to \frac{1}{2}}\left(\frac{4x^2-1}{2x-1}\right)$

Q.11 $\lim_{x\to 4}\left(\frac{x^3-64}{x^2-16}\right)$

Q.12 $\lim_{x\to 2}\left(\frac{x^5-32}{x^3-8}\right)$

Q.13 $\lim_{x\to a}\left(\frac{x^{5/2}-a^{5/2}}{x-a}\right)$

Q.14 $\lim_{x\to a}\left\{\frac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}\right\}$

Q.15 $\lim_{x\to 1}\left(\frac{x^n-1}{x-1}\right)$

Q.16 $\lim_{x\to a}\left(\frac{\sqrt{x}-\sqrt{a}}{x-a}\right)$

Q.17 $\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)$

Q.18 $\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$

Q.19 $\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}\right)$

Q.20 $\lim_{x\to 0}\left(\frac{\sqrt{2-x}-\sqrt{2+x}}{x}\right)$

Q.21 $\lim_{x\to 0}\left(\frac{\sqrt{1+x+x^2}-1}{x}\right)$

Q.22 $\lim_{x\to 0}\left(\frac{\sqrt{3-x}-1}{2-x}\right)$

Q.23 $\lim_{x\to 0}\left(\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}\right)$

Q.24 $\lim_{x\to 1}\left(\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}\right)$

Q.25 $\lim_{x\to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right)$

Q.26 $\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$

Q.27 $\lim_{x\to 0}\left(\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}\right)$

Q.28 $\lim_{x\to 0}\left(\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right)$

Q.29 $\lim_{x\to 1}\left(\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right)$

Q.30 $\lim_{x\to 2}\left(\frac{3^x+3^{3-x}-12}{3^{3-x}-3^{x/2}}\right)$

Q.31 $\lim_{x\to 0}\left(\frac{e^{4x}-1}{x}\right)$

Q.32 $\lim_{x\to 0}\left(\frac{e^{2+x}-e^2}{x}\right)$

Q.33 $\lim_{x\to 4}\left(\frac{e^x-e^4}{x-4}\right)$

Q.34 $\lim_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$

Q.35 $\lim_{x\to 0}\left(\frac{e^x-x-1}{x}\right)$

Q.36 $\lim_{x\to 0}\left(\frac{e^{bx}-e^{ax}}{x}\right)$

Q.37 $\lim_{x\to 0}\left(\frac{a^x-b^x}{x}\right)$

Q.38 $\lim_{x\to 0}\left(\frac{a^x - a^{-x}}{x}\right)$

Q.39 $\lim_{x\to 0}\left(\frac{2^x - 1}{x}\right)$

Q.40 $\lim_{x\to 0}\left(\frac{3^{2+x} - 9}{x}\right)$

Q.41 $\lim_{x\to 0}\frac{\sin 4x}{6x}$

Q.42 $\lim_{x\to 0}\frac{\sin 5x}{\sin 8x}$

Q.43 $\lim_{x\to 0}\frac{\tan 3x}{\tan 5x}$

Q.44 $\lim_{x\to 0}\frac{\tan \alpha x}{\tan \beta x}$

Q.45 $\lim_{x\to 0}\frac{\sin 4x}{\tan 7x}$

Q.46 $\lim_{x\to 0}\frac{\tan 3x}{\sin 4x}$

Q.47 $\lim_{x\to 0}\frac{\sin mx}{\tan nx}$

Q.48 $\lim_{x\to 0}\frac{\sin x - 2\sin 3x + \sin 5x)}{x}$

Q.49 $\lim_{x\to \pi/6}\frac{(2\sin^2 x + \sin x - 1)}{(2\sin^2 x - 3\sin x + 1)}$

Q.50 $\lim_{x\to 0}\frac{(\sin 2x + 3x)}{(2x + \sin 3x)}$

Q.51 $\lim_{x\to 0}\frac{(\tan 2x - x)}{(3x - \tan x)}$

Q.52 $\lim_{x\to 0}\frac{x^2 - \tan 2x}{\tan x}$

Q.53 $\lim_{x\to 0}\frac{(x\cos x + \sin x)}{(x^2 + \tan x)}$

Q.54 $\lim_{x\to 0}\frac{(\tan x - \sin x)}{\sin^3 x}$

Q.55 $\lim_{x\to 0}$ (xcosec x)

Q.56 $\lim_{x\to 0}$ (xcot 2x)

Q.57 $\lim_{x\to 0}\frac{(\sin x\cos x)}{3x}$

Q.58 $\lim_{x\to 0}\frac{\sin(x/4)}{x}$

Q.59 $\lim_{x\to 0}\frac{\tan(x/2)}{3x}$

Q.60 $\lim_{x\to 0}\frac{(1-\cos x)}{\sin^2 x}$

Q.61 $\lim_{x\to 0}\frac{(1-\cos 3x)}{x^2}$

Q.62 $\lim_{x\to 0}\frac{(1-\cos 2x)}{\sin^2 2x}$

Q.63 $\lim_{x\to 0}\frac{(1-\cos 2x)}{3\tan^2 x}$

Q.64 $\lim_{x\to 0}\frac{(1-\cos 4x)}{(1-\cos 6x)}$

Q.65 $\lim_{x\to 0}\frac{(1-\cos mx)}{(1-\cos nx)}$

Q.66 $\lim_{x\to 0}\frac{(2\sin x - \sin 2x)}{x^3}$

Q.67 $\lim_{x\to 0}\frac{(\tan x - \sin x)}{x^3}$

Q.68 $\lim_{x\to 0}\frac{(\tan 2x - \sin 2x)}{x^3}$

Q.69 $\lim_{x\to 0}\frac{(\cos ecx - \cot x)}{x}$

Q.70 $\lim_{x\to 0}\frac{(\cot 2x - \cos ec2x)}{x}$

Q.71 $\lim_{x\to 0}\frac{\sin 2x(1-\cos 2x)}{x^3}$

Q.72 $\lim_{x\to \frac{\pi}{4}}\frac{(\sec^2 x - 2)}{(\tan x - 1)}$

Q.73 $\lim_{x\to \frac{\pi}{4}}\frac{(co\sec^2 x - 2)}{(\cot x - 1)}$

Q.74 $\lim_{x\to \frac{\pi}{4}}\frac{(1-\tan x)}{\left(x - \frac{\pi}{4}\right)}$

Q.75 $\lim_{x\to \pi}\frac{(\sin 3x - 3\sin x)}{(\pi - x)^3}$

Q.76 $\lim_{x\to \frac{\pi}{2}}\frac{(1+\cos 2x)}{(\pi - 2x)^2}$

Q.77 $\lim_{x\to a}\frac{(\cos x - \cos a)}{(x-a)}$

Q.78 $\lim_{x\to a}\frac{(\sin x - \sin a)}{(x-a)}$

Q.79 $\lim_{x\to a}\frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})}$

Q.80 $\lim_{x\to 0}\frac{(\sin 5x - \sin 3x)}{\sin x}$

Q.81 $\lim_{x\to 0}\frac{(\cos 3x - \cos 5x)}{x^2}$

Q.82 $\lim_{x\to 0}\frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)}$

Q.83 $\lim_{x\to 0}\frac{[\sin(2+x) - \sin(2-x)]}{x}$

Q.84 $\lim_{x\to 0}\frac{(1-\cos 2x)}{(\cos 2x - \cos 8x)}$

Q.85 $\lim_{x\to \frac{\pi}{2}}\left(\frac{\pi}{2} - x\right)\tan x$

Q.86 $\lim_{x\to 0}\frac{(\sqrt{1+2x} - \sqrt{1-2x})}{\sin x}$

Q.87 $\lim_{x\to 0}\frac{(a+x)^2\sin(a+x) - a^2\sin a}{x}$

Q.88 $\lim_{x\to 0}\frac{(e^{3+x} - \sin x - e^3)}{x}$

Q.89 $\lim_{x\to 0}\frac{(e^{\tan x} - 1)}{\tan x}$

Q.90 $\lim_{x\to 0}\frac{(e^{\tan x} - 1)}{x}$

Answers

1.	3	**2.**	5	**3.**	3	**4.**	–3	**5.**	10	**6.**	3
7.	12	**8.**	108	**9.**	$\frac{1}{2}$	**10.**	2	**11.**	6	**12.**	$\frac{20}{3}$
13.	$\frac{5}{2}a^{\frac{3}{2}}$	**14.**	$\frac{5}{3}(a+2)^{2/3}$	**15.**	n	**16.**	$\frac{1}{2\sqrt{a}}$	**17.**	$\frac{1}{2\sqrt{x}}$	**18.**	$\frac{-1}{2x^{3/2}}$
19.	$\frac{1}{2}$	**20.**	$-\frac{1}{\sqrt{2}}$	**21.**	$\frac{1}{2}$	**22.**	$\frac{1}{\sqrt{3}+1}$	**23.**	$2\sqrt{a}$	**24.**	$\frac{1}{4}$
25.	–8	**26.**	$\frac{-1}{3}$	**27.**	$\frac{1}{2a^{3/2}}$	**28.**	1	**29.**	$\frac{1}{2}$	**30.**	$\frac{-4}{3}$
31.	4	**32.**	e^2	**33.**	e^4	**34.**	1	**35.**	0	**36.**	(b – a)
37.	log a – log b	**38.**	2log a	**39.**	log 2	**40.**	9 log 3	**41.**	$\frac{2}{3}$	**42.**	$\frac{5}{8}$
43.	$\frac{3}{5}$	**44.**	$\frac{\alpha}{\beta}$	**45.**	$\frac{4}{7}$	**46.**	$\frac{3}{4}$	**47.**	$\frac{m}{n}$	**48.**	0
49.	–3	**50.**	1	**51.**	$\frac{1}{2}$	**52.**	–2	**53.**	2	**54.**	$\frac{1}{2}$
55.	1	**56.**	$\frac{1}{2}$	**57.**	$\frac{1}{3}$	**58.**	$\frac{1}{4}$	**59.**	$\frac{1}{6}$	**60.**	$\frac{1}{2}$
61.	$\frac{9}{2}$	**62.**	$\frac{1}{2}$	**63.**	$\frac{2}{3}$	**64.**	$\frac{4}{9}$	**65.**	$\frac{m^2}{n^2}$	**66.**	1
67.	$\frac{1}{2}$	**68.**	4	**69.**	$\frac{1}{2}$	**70.**	–1	**71.**	4	**72.**	2
73.	2	**74.**	–2	**75.**	–1	**76.**	$\frac{1}{2}$	**77.**	–sin a	**78.**	cosa
79.	$2\sqrt{a}$ cos a	**80.**	2	**81.**	8	**82.**	4	**83.**	2cos 2	**84.**	$\frac{1}{15}$
85.	1	**86.**	2	**87.**	(2asina + a^2cosa)	**88.**	(e^3 – 1)	**89.**	1	**90.**	1