CONSTRUCTIONS

CONTENTS

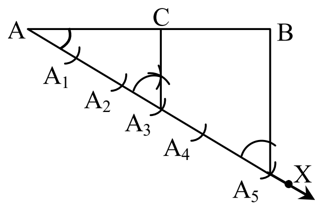
- To divide a line segment in a given ratio.
- To construct a triangle similar to a given triangle as per given scale factor.
- To construct a tangent to a circle at a given point on it (using the centre of the circle).
- To construct two tangents to a circle from a point outside the circle (using the center of the circle).

TO DIVIDE A LINE SEGMENT IN A GIVEN RATIO

Given a line segment AB, we want to divide it in the ratio m : n, where both m and n are positive integers. To help you to understand it, we shall take m = 3 and n = 2.

Steps of Construction :

1. Draw any ray AX, making an acute angle with AB.
2. Locate 5(= m + n) points A_1, A_2, A_3, A_4 and A_5 on AX so that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
3. Join BA_5.
4. Through the point A_3 (m = 3), draw a line parallel to A_5B (by making an angle equal to $\angle AA_5B$) at A_3 intersecting AB at the point C (see figure). Then, AC : CB = 3 : 2.



Let use see how this method gives us the required division.

Since A_3C is parallel to A_5B, therefore,

$$\frac{AA_3}{A_3A_5} = \frac{AC}{CB}$$

(By the Basic Proportionality Theorem)

By construction, $\frac{AA_3}{A_3A_5} = \frac{3}{2}$. Therefore, $\frac{AC}{CB} = \frac{3}{2}$.

This shows that C divides AB in the ratio 3 : 2.

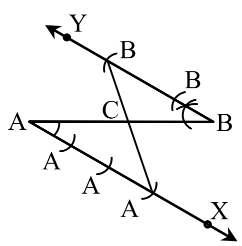
Alternative Method

Steps of Construction :

1. Draw any ray AX making an acute angle with AB.
2. Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$.
3. Locate the points A_1, A_2, A_3 (m = 3) on AX and B_1, B_2 (n = 2) on BY such that

   $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$.
4. Join A_3B_2.

Let it in intersect AB at a point C (see figure)



Then AC : CB = 3 : 2

Whey does this method work ? Let us see.

Here ΔAA_3C is similar to ΔAB_2C. (Why ?)

Then $\frac{AA_3}{BB_2} = \frac{AC}{BC}$

Since by construction, $\frac{AA_3}{BB_2} = \frac{3}{2}$,

therefore, $\frac{AC}{BC} = \frac{3}{2}$

In fact, the methods given above work for dividing the line segment in any ratio.

We now use the idea of the construction above for constructing a triangle similar to a given triangle whose sides are in a given ratio with the corresponding sides of the given triangle.

➢ TO CONSTRUCT A TRIANGLE SIMILAR TO A GIVEN TRIANGLE AS PER GIVEN SCALE FACTOR

Scale factor means the ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle.

This construction involves two different situations :

(i) The triangle to be constructed is smaller than the given triangle, here scale factor is less than 1.

(ii) The triangle to be constructed is bigger than the given triangle, here scale factor is greater than 1.

❖ EXAMPLES ❖

Ex.1 Construct a ΔABC in which AB = 4 cm, BC = 5 cm and AC = 6 cm. Now, construct a triangle similar to ΔABC such that each of its sides is two-third of the corresponding sides of ΔABC. Also, prove your assertion.

Sol. Steps of construction

Step I : Draw a line segment AB = 4 cm.

Step II : With A as centre and radius = AC = 6 cm, draw an arc.

Step III : With B as centre and radius = BC = 5 cm, draw another arc, intersecting the arc drawn in step II at C.

Step IV : Join AC and BC to obtain ΔABC.

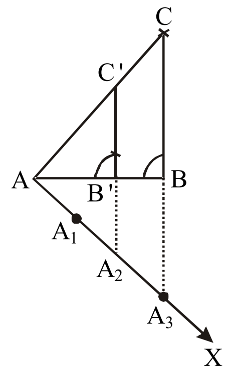
Step V : Below AB, make an acute angle $\angle BAX$.

Step VI : Along AX, mark off three points (greater of 2 and 3 in 2/3) A_1, A_2, A_3 such that $AA_1 = A_1A_2 = A_2A_3$.

Step VII : Join A_3B.

Step VIII : Since we have to construct a triangle each of whose sides is two-third of the corresponding sides of ΔABC. So, take two parts out of three equal parts on AX i.e. from point A_2, draw

$A_2B' \parallel A_3B$, meeting AB at B'.



Step IX : From B', draw $B'C' \parallel BC$, meeting AC at C'.

AB'C' is the required triangle, each of the whose sides is two-third of the corresponding sides of ΔABC.

Justification : Since $B'C' \parallel BC$.

So, $\Delta ABC \sim \Delta AB'C'$

$$\therefore \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{AB'}{AB} = \frac{2}{3} \qquad \left[\because \frac{AB'}{AB} = \frac{2}{3}\right]$$

Let ABC be the given triangle and we want to construct a triangle similar to ΔABC such that each of its sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding sides of ΔABC such that m < n. We follow the following steps to construct the same.

Steps of construction when m > n.

Step I : construct the given triangle by using the given data.

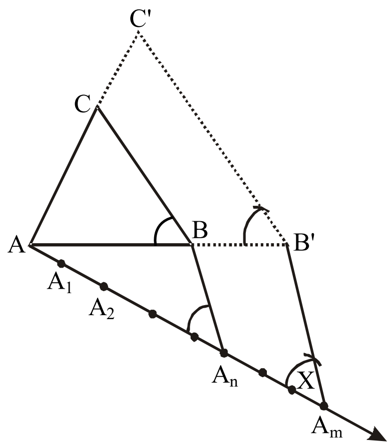
Step II : Take any of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.

Step III : At one end, say A, of base AB construct an acute angle ∠BAX below base AB i.e. on the opposite side of the vertex C.

Step IV : Along AX, mark-off m (large of m and n) points A_1, A_2,...,A_m on AX such that $AA_1 = A_1A_2 = ... = A_{m-1}A_m$.

Step V : Join A_n to B and draw a line through A_m Parallel to A_nB, intersecting the extended line segment AB at B'.

Step VI : Draw a line through B' parallel to BC intersecting the extended line segment AC at C'.



Step VII : ΔAB'C' so obtained is the required triangle.

Justification : For justification of the above construction consider triangles ABC and AB'C'. In these two triangles, we have

∠BAC = ∠B'AC'

∠ABC = ∠AB'C'   [∵ B'C' || BC]

So, by AA similarity criterion, we have

ΔABC ~ ΔAB'C'

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \quad ....(i)$$

In Δ A A_m B', A_nB || A_m B'.

$$\therefore \frac{AB}{BB'} = \frac{AA_n}{A_nA_m}$$

$$\Rightarrow \frac{BB'}{AB} = \frac{A_nA_m}{AA_n} \Rightarrow \frac{BB'}{AB} = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'-AB}{AB} = \frac{m-n}{n} \quad \Rightarrow \frac{AB'}{AB} - 1 = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n} \quad ....(ii)$$

From (i) and (ii), we have

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

Ex.2 Draw a triangle ABC with side BC = 7 cm,

∠B = 45°, ∠A = 105°. Then construct a triangle whose sides are (4/3) times the corresponding sides of ΔABC.

Sol. In order to construct ΔABC, we follow the following steps:

Step I : Draw BC = 7 cm.

Step II : At B construct ∠CBX = 45° and at C construct

∠BCY = 180° – (45° – 105°) = 30°

Suppose BX and CY intersect at A. ΔABC so obtained is the given triangle. To construct a triangle similar to ΔABC,

we follow the following steps.

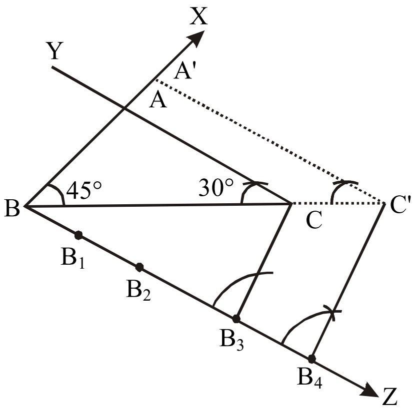
Step I : Construct an acute angle ∠CBZ at B on opposite side of vertex A of ΔABC.

Step II : Mark-off four (greater 4 and 3 in 4/3) points

B_1, B_2, B_3, B_4 on BZ such that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step III : Join B_3 (the third point) to C and draw a line through B_4 parallel to B_3C, intersecting the extended line segment BC at C'.



Step IV : Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

Triangle A'BC' so obtained is the required triangle such that

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{4}{3}$$

Ex.3 Construct a triangle similar to a given triangle ABC such that each of its sides is (6/7)th of the corresponding sides of ΔABC. It is given that

AB = 5 cm, AC = 6 cm and BC = 7 cm.

Sol. Steps of Construction

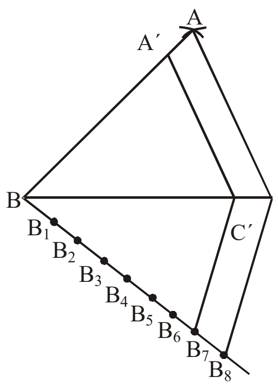
Step I : Draw a line segment BC = 7 cm.

Step II : With B as centre and

radius = AB = 5 cm, draw an arc.

Step III : With C as centre and

radius = AC = 6 cm, draw another arc, intersecting the arc drawn in step II at A.



Step IV : Join AB and AC to obtain the triangle ABC.

Step V : Below base BC, construct an acute angle ∠CBX.

Step VI : Along BX, mark off seven points B_1, B_2, B_3, B_4, B_5, B_6, B_7 such that $BB_1 = B_1B_2 = ... = B_6B_7$.

Step VII : Join B_7C.

Step VIII : Since we have to construct a triangle each of whose sides is (6/7)th of the corresponding sides of ΔABC. So take 6 parts out of 7 equal parts on BX i.e. from B_6, Draw $B_6C' \parallel B_7C$, intersecting BC at C′.

Step IX : From C′, draw C′A′ || CA, meeting BA at A′.

ΔA′BC′ is the required triangle each of whose sides is (6/7)th of the corresponding sides of ΔABC.

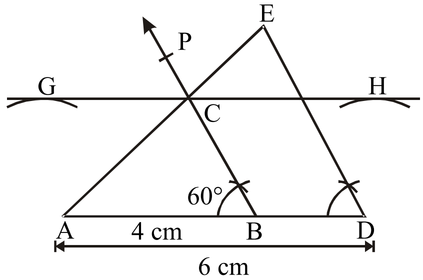
Ex.4 Construct a ΔABC in which AB = 4 cm,

∠B = 60º and altitude CL = 3 cm. Construct a ΔADE similar to ΔABC such that each side of ΔADE is 3/2 times that of the corresponding side of ΔABC.

Sol. Steps of construction

Step I : Draw a line segment AB = 4 cm.

Step II : Construct ∠ABP = 60º.



Step III : Draw a line GH || AB at a distance of 3 cm, intersecting BP at C.

Step IV : Join CA.

Thus, ΔABC is obtained.

Step V : Extend AB to D such that AD = 3/2

$AB = \left(\frac{3}{2} \times 4\right) cm = 6\ cm.$

Step VI : Draw DE || BC, cutting AC produced at E.

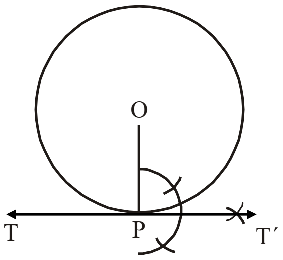
Then ΔADE is the required triangle similar to ΔABC such that each side of ΔADE is 3/2 times the corresponding side of ΔABC.

Proof : Since DE || BC, we have ΔADE ~ ΔABC.

$$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} = \frac{3}{2}$$

> **TO CONSTRUCT A TANGENT TO A CIRCLE AT A GIVEN POINT ON IT (USING THE CENTRE OF THE CIRCLE)**

Steps of Construction



Step I : Take a point O on the plane of the paper and draw a circle of given radius.

Step II : Take a point P on the circle.

Step III : Join OP.

Step IV : Construct ∠OPT = 90º.

Step V : Produce TP to T′ to get TPT′ as the required tangent.

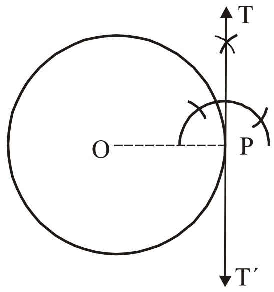
❖ EXAMPLES ❖

Ex.5 Take a point O on the plane of the paper. With O as centre draw a circle of radius 3cm. Take a point P on this circle and draw a tangent at P.

Sol. Steps of Construction

Step I : Take a point O on the plane of the paper and draw a circle of radius 3 cm.

Step II : Take a point P on the circle and join OP.



Step III : Construct ∠OPT = 90º

Step IV : Produce TP to T′ to obtain the required tangent TPT′.

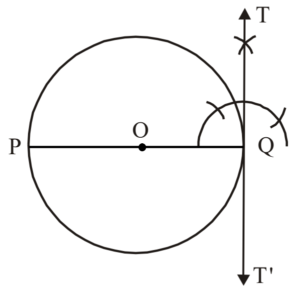
Ex.6 Draw a circle of radius 4 cm with centre O. Draw a diameter POQ. Through P or Q draw tangent to the circle.

Sol. Steps of Construction

Step I : Taking O as centre and radius equal to 4 cm draw a circle.

Step II : Draw diameter POQ.

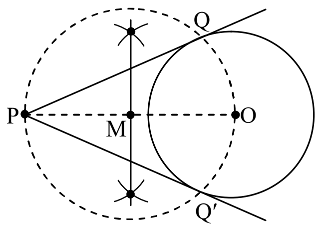
Step III : Construct ∠PQT = 90º



Step IV : Produce TQ to T′ to obtain the required tangent TQT′.

> **TO CONSTRUCT TWO TANGENTS TO A CIRCLE FROM A POINT OUTSIDE THE CIRCLE (USING CENTRE OF CIRCLE)**

Steps of construction



1. Take given circle and a point P outside the circle. O is centre of the circle
2. Joint OP

3. Bisect OP and get its mid-point M
4. Draw circle with centre M and

   radius = PM = MO
5. Circle drawn meets the given circle at Q above PO and at Q′ below PO.
6. Join PQ and PQ′
7. PQ and PQ′ are the required tangents drawn to the circle from the point P.

   We observe that PQ = PQ′.

❖ EXAMPLES ❖

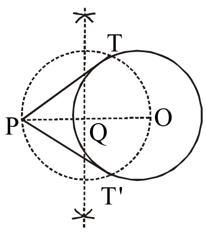
Ex.7 Draw a circle of radius 3 cm. Take a point at a distance of 5.5 cm from the centre of the circle. From point P, draw two tangents to the circle.

Sol. Steps of Construction

Step I : Take a point O in the plane of the paper and draw a circle of radius 3 cm.

Step II : Mark a point P at a distance of 5.5 cm from the centre O and join OP.

Step III : Draw the right bisector of OP, intersecting OP at Q.



Step IV : Taking Q as centre and OQ = PQ as radius, draw a circle to intersect the given circle at T and T′.

Step V : Join PT and PT′ to get the required tangents.

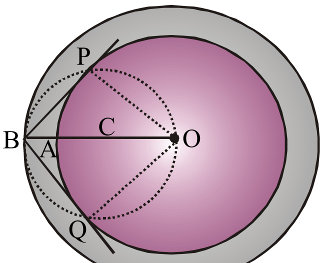
Ex.8 Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Sol. In order to do the desired construction,

we follow the following steps:

Step I : Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm. Also, draw a concentric circle of radius OB = 6 cm.

Step II : Find the mid-point C of OB and draw a circle of radius OC = BC. Suppose this circle intersects the circle of radius 4 cm at P and Q.



Step III : Join BP and BQ to get the desired tangents from a point B on the circle of radius 6 cm.

By actual measurement, we find the

BP = BQ = 4.5 cm

Justification: In ΔBPO, we have

OB = 6 cm and OP = 4 cm

$\therefore$ $OB^2 = BP^2 + OP^2$ [Using Pythagoras theorem]

$\Rightarrow$ $BP = \sqrt{OB^2 - OP^2}$

$\Rightarrow$ $BP = \sqrt{36 - 16} = \sqrt{20}$ cm = 4.47 cm $\simeq$ 4.5 cm

Similarly, BQ = 4.47cm $\simeq$ 4.5 cm

Ex.9 Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other an angle of 60º.

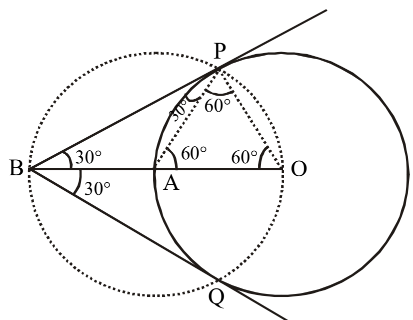
Sol. In order to draw the pair of tangents, we follow the following steps.

Step I : Take a point O on the plane of the paper and draw a circle of radius OA = 5 cm.

Step II : Produce OA to B such that
OA = AB = 5 cm.

Step III : Taking A as the centre draw a circle of radius AO = AB = 5 cm. Suppose it cuts the circle drawn in step I at P and Q.

Step IV : Join BP and BQ to get the desired tangents.



Justification: In OAP, we have

OA = OP = 5 cm (= Radius) Also,

AP = 5 cm (= Radius of circle with centre A)

$\therefore$ ΔOAP is equilateral.

$\Rightarrow$ $\angle PAO = 60^o \Rightarrow \angle BAP = 120^o$

In ΔBAP, we have

BA = AP and $\angle BAP = 120^o$

$\therefore$ $\angle ABP = \angle APB = 30^o \Rightarrow \angle PBQ = 60^o$

Ex.10 Draw a circle of radius 3 cm. Draw a pair of tangents to this circle, which are inclined to each other at an angle of 60º.

Sol. Steps of construction

Step I : Draw a circle with O as centre and radius = 3 cm.

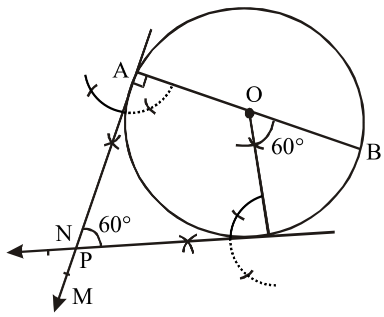
Step II : Draw any diameter AOB of this circle.

Step III : Construct $\angle BOC = 60^o$ such that radius OC meets the circle at C.

Step IV : Draw AM $\perp$ AB and CN $\perp$ OC.

Let AM and CN intersect each other at P.

Then, PA and PC are the desired tangents to the given circle, inclined at an angle of 60º



Proof : $\angle AOC = (180^o - 60^o) = 120^o$

In quad. OAPC, we have

$\angle OAP = 90^o, \angle AOC = 120^o, \angle OCP = 90^o.$

$\therefore \angle APC = [360^o - (90^o + 120^o + 90^o)] = 60^o.$