Application of Trigonometry

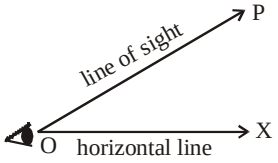
Angle of Elevation

INTRODUCTION

In this chapter, we shall be applying trigonometric results to discuss simple problems based on height and distance. One of the object of trigonometry is to find the distance between two points or the height of tower, building and the height of definite objects without actually measuring these distances or these heights. We begin by some definitions which will be used in this chapter.

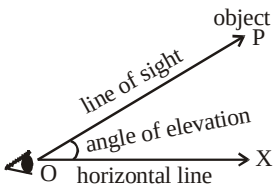
LINE OF SIGHT

It is the line from the eyes of the observer to a point where the person is viewing.
In figure, the eye is at point O and the object is at P therefore OP is the line of sight.



ANGLE OF ELEVATION

It is the angle formed by the line of sight with horizontal line through the eyes of observer, when the object is above the horizontal level.
In figure, the eye is at point O and the position of the object is 'P'. Therefore OP is the line of sight which makes an angle XOP from horizontal line OX.
Hence, the angle of elevation $= \angle XOP$.

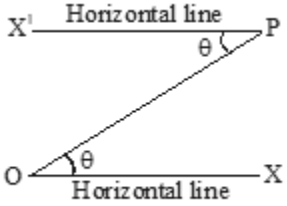


Note :

(i) First of all read the question carefully and draw the figure.

(ii) In right triangle, trigonometric ratio of known angles (sine, cosine, tangent etc.) are express in the term of known side.

(iii) From given figure, it is clear that the angle of elevation of 'O' with respect to 'P' is equal to the angle of depression of 'P' with respect to O'. i.e. the angle of elevation of one object is equal to the angle of depression of the other object with respect to the first object.



More important results

Let $\angle BAC = q$ be an acute angle of a right-angled D ABC.

We define the following ratios, known as trigonometric ratios for q

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\tan\theta = \frac{\text{prependicular}}{\text{base}} = \frac{BC}{AB}$$

Reciprocal Relation

(i) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

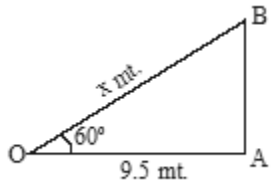
(iii) $\cot\theta = \frac{1}{\tan\theta}$

Ex. 1 The angle of elevation of a ladder leaning against a wall is 60º and the foot of the ladder is 9.5 mt. away from the wall. Find the length of the ladder.

Sol. Let OB be the ladder leaning against the wall AB, then ĐAOB = 60º and OA = 9.5 mt.

Let length of the ladder = OB = x mt.

In right angled triangle OAB.



$\therefore \quad \frac{OB}{OA} = \sec 60º$

Or $\frac{X}{9.5} = 2 \quad \therefore x = (9.5 \times 2)$ mt. = 19 mt.

Hence length of ladder is 19 mt.

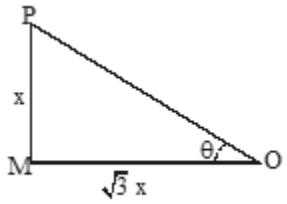
Ex. 2 If the length of a shadow cast by a pole be times the length of the pole, find the angle of elevation of the sun.

Sol. Let MP be the pole, then shadow

OM = $\sqrt{3}$ MP (given)

Let $\angle MOP = \theta$ (the elevation of the sun)

From right angled D OMP, we get



$\tan \theta = \frac{MP}{OM} = \frac{X}{X\sqrt{3}} = \frac{1}{\sqrt{3}} \tan 30º$

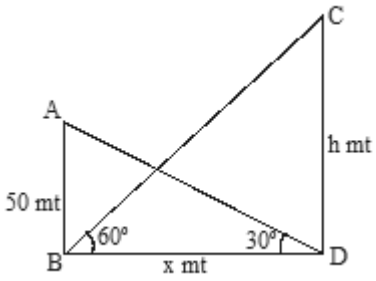
$\Rightarrow \quad \theta = 30º$

Ex. 3 The angle of elevation of the top of a hill from the foot of a tower is 60º and the angle of elevation of the top of the tower from the foot of hill is 30º. If the tower is 50 mt. high, what is the height of the hill.

Sol. Let AB be the tower and CD be the hill

Let CD = h mt and BD = x mt

in D ABD, we have



$\frac{50}{X} = \tan 30º$

$\Rightarrow \frac{50}{x} = \frac{1}{\sqrt{3}}$ mt.

$\Rightarrow x = 50\sqrt{3}$ mt. .....(i)

in D BCD, we have

$h/x = \tan 60^\circ$

$\Rightarrow h = x \tan 60^\circ$ .....(ii)

Substitute value of x in (ii)

$h = 50\sqrt{3} \times \sqrt{3}$

$h = 150$ mt.

Hence, Height of hill is 150 mt.

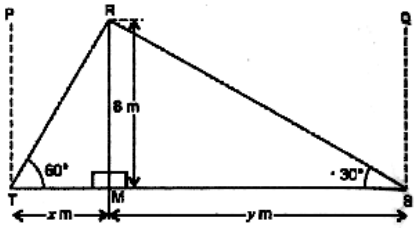
Ex. 4 Ranjan is sitting at a height of 8 m on a tall tree in the middle of a river. He observes two poles directly opposite each other on the two banks of the river and in line with the foot of the tree. If the angles of depression of the feet of the poles from the point at which Ranjan is sitting on either side of the river are 60° and 30° respectively, find the width of the river.

Sol. Let Ranjan is sitting on the tree RM which is 8 m high. PT and QS are the two poles On the opposite banks of the river.

Let the distance TM and MS be x and y respectively.

From right triangle TMR, $\tan 60^\circ = \frac{RM}{TM}$

$$\sqrt{3} = \frac{8}{X} \quad X = \frac{8}{\sqrt{3}} \quad \text{.... (i)}$$



From right triangle SMR, $\tan 30^\circ = \frac{RM}{SM}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{y} \qquad \Rightarrow \quad y = 8\sqrt{3} \qquad \text{.... (ii)}$$

The width of the river is given by

$$= \quad x + y = \frac{8}{\sqrt{3}} + 8\sqrt{3}$$

$$\Rightarrow \qquad \frac{8+24}{\sqrt{3}} = \frac{32}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow$$

$\Rightarrow \quad \frac{32\sqrt{3}}{3} = \frac{32 \times 1.732}{3} = \frac{55.424}{3} =$ 18.475 metres. (Taking $\sqrt{3} = 1.732$.)

The width of the river is 18.475 metres.