Applications of Trigonometry

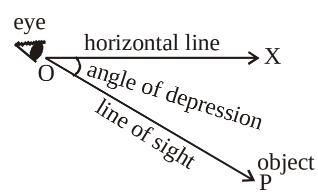
Angle of Depression

Angle of Depression

It is the angle formed by the line of sight with the horizontal line when the object is below the horizontal level.

In figure, the eye is at point O and the position of the object is P therefore OP is the line of sight which makes an angle XOP from Horizontal line.

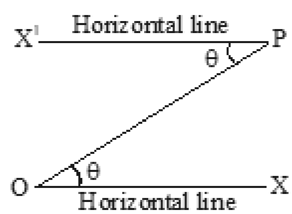
Hence the angle the angle of depression = $\angle XOP$.



Note :

(i) First of all read the question carefully and draw the figure.

(ii) In right triangle, trigonometric ratio of known angles (sine, cosine, tangent etc.) are express in the term of known side.



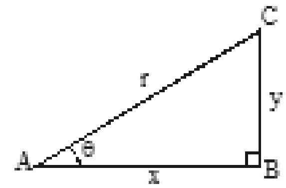
(iii) From given figure, it is clear that the angle of elevation of 'O' with respect to 'P' is equal to the angle of depression of 'P' with respect to O'. i.e. the angle of elevation of one object is equal to the angle of depression of the other object with respect to the first object.

More important results

Let $\angle BAC = \theta$ be an acute angle of a right-angled D ABC.

We define the following ratios, known as trigonometric ratios for θ

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC}$$



$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\tan\theta = \frac{\text{prependicular}}{\text{base}} = \frac{BC}{AB}$$

Reciprocal Relation

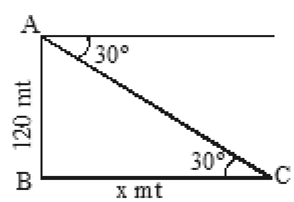
(i) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ (ii) $\sec\theta = \frac{1}{\cos\theta}$ (iii) $\cot\theta = \frac{1}{\tan\theta}$

Ex. 1 From the top of a tower 120 mt. high, the angle of depression of a car is 30º, find how far is the car from the tower.

Sol. Let AB be the tower and C be position of a car, then AB = 120 mt. and

let BC = x mt.

then $\frac{BC}{AB}$ = cot 30º



⇒ $\frac{X}{120} = \sqrt{3}$

⇒ x = 120 $\sqrt{3}$ = 120 × 1.723 mt.

⇒ x = 207.840 mt.

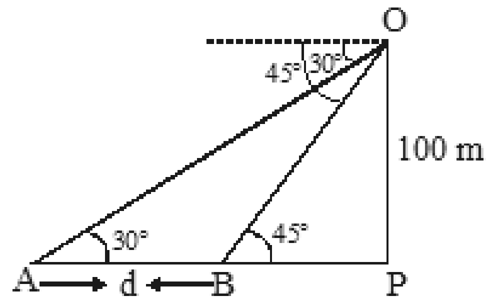
Hence, distance of the car from the tower = 207.84 mt.

Ex. 2 As observed from the top of a light house, 100m above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 45°. Determine the distance travelled by the ship during the period of observation.

Sol. Let A and B be the two positions of the ship. Let d be the distance travelled by the ship during the period of observation i.e. AB = d metres.

Let the observer be at O, the top of the light house PO.

It is given that PO = 100 m and the angles of depression from O of A and B are 30° and 45° respectively.



$\angle OAP = 30°$ and $\angle OBP = 45°$,

In ΔOPB, we have

$$\tan 45° = \frac{OP}{BP}$$

$$\Rightarrow 1 = \frac{100}{BP} \qquad \Rightarrow BP = 100 \text{ m}$$

In DOPA, we have

$$\Rightarrow \tan 30° = \frac{OP}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{d+BP}$$

$$\Rightarrow d + BP = 100\sqrt{3}$$

$$\Rightarrow d + 100 = 100\sqrt{3}$$

$$\Rightarrow d = 100\sqrt{3} - 100$$

$$\Rightarrow d = 100\left(\sqrt{3} - 1\right)$$

$$= 100(1.732 - 1) = 73.2m$$

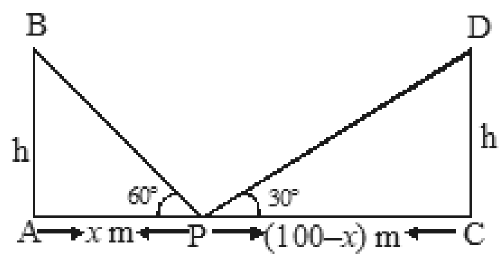
Hence, the distance travelled by the ship from A to B is 73.2 m.

Ex. 3 Two pillars of equal height are on either side of a road, which is 100m wide. The angles of elevation of the top of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of the point between the pillars and the height of each pillar.

Sol. Let AB and CD be two pillars, each of height h metres. Let P be a point on the road such that AP = x metres. Then, CP = (100–x) metres.

It is given that ∠APB = 60° and ∠CPD = 30°

In PAB, we have tan 60° = $\frac{AB}{AP}$



$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$

In ΔPCD, we have tan 30° = $\frac{CD}{PC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$

$\Rightarrow h\sqrt{3} = 100 - x$ ....(ii)

Eliminating h from equation (i) and (ii), we get $3x = 100 - x = 4x = 100$ = x =25

Substituting x = 25 in equation (i), we get

h = $25\sqrt{3} = 25 \times 1.732 = 43.3$

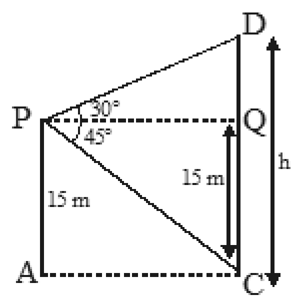
Thus, the required point is at a distance of 25 metres from the first pillar and 75 Metres from the second pillar. The height of the pillars is 43.3 metres.

Ex. 4 From a window 15 metres high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of the opposite house is 23.66 metres (Take)

Sol. Let the window be P at a height of 15 metres above the ground and CD be the house On the opposite side of the street such that the angles of elevation of the top D of house CD as seen from P is of 30° and the angle of depression of the foot C of house CD as seen from P is of 45°.

Let h metres be the height of the house CD. we have,

QD = CD – CQ = CD – AP = (h – 15) metres.



In DPQC, we have tan 45° = $\frac{QC}{PQ}$

$\Rightarrow 1 = \frac{15}{PQ}$ Þ PQ = 15 metres.

In D PQD, we have tan 30° = $\frac{DQ}{PQ}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-15}{15} \Rightarrow h-15 = \frac{15}{\sqrt{3}} \Rightarrow h-15 = 5\sqrt{3}$$

h = 15 + 5 × 1.732 = 23.66 metres,

Hence, the height of the opposite house is 23.66 metres.