# INTRODUCTION TO TRIGONOMETRY

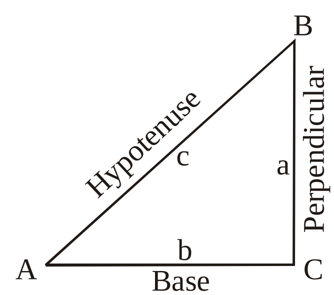
## TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES

**TRIGONOMETRIC RATIOS :**

With reference to angle A in a right angled D ABC, right angle at C.

a is opposite side of angle ∠A (Perpendicular), b is opposite side of angle ∠B and c is opposite side of angle ∠C.

The ratio of sides $\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{b}{a}, \frac{c}{b}, \frac{c}{a}$ have the following names




$\frac{a}{c}$ is called the sine of A, written as sin A

$\frac{b}{c}$ is called the co-sine of A, written as cos A

$\frac{a}{b}$ is called the tangent of A, written as tan A.

$\frac{b}{a}$ is called the co-tangent of A. written as cot A

$\frac{c}{b}$ is called the secant of A, written as sec A

$\frac{c}{a}$ is called the co-secant of A, written as cosec A.

**(i) Six Trigonometry Ratio are :**

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{P}{H} = \frac{AB}{AC}$  $\text{cosec}\,\theta = \frac{\text{Hypoteuse}}{\text{Perpendicular}} = \frac{H}{P} = \frac{AC}{AB}$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{B}{H} = \frac{BC}{AC}$  $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{H}{B} = \frac{AC}{BC}$

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{B} = \frac{AB}{BC}$  $\cot\theta = \frac{\text{Base}}{\text{Parpendicular}} = \frac{B}{P} = \frac{BC}{AB}$

**(ii) Interrelationship is Basic Trigonometric Ratio :**

$\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta}$

$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$

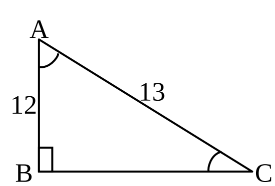
$\sin\theta = \frac{1}{\text{cosec}\,\theta} \Rightarrow \text{cosec}\,\theta = \frac{1}{\sin\theta}$

**We also observe that**

$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta}$

**Ex.1** If ABC is right angle triangle, $\angle B = 90º$, AB = 12 cm, AC = 13 cm then find sin A and cos C.

**Sol.** Using Pythagoras theorem



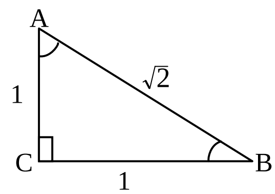
$BC = \sqrt{AC^2 - AB^2} = \sqrt{169 - 144} = 5$ cm

$\therefore \sin A = \frac{BC}{AC} = \frac{5}{13}$

$\cos C = \frac{AB}{AC} = \frac{12}{13}$ **Ans.**

**Ex.2** If $\sin A = \frac{1}{\sqrt{2}}$ in right triangle ABC, then find value of tan A, cosec A, tan B, cosec B.

**Sol.**




$\because \sin A = \frac{1}{\sqrt{2}} = \frac{BC}{AB}$

$\therefore AC = \sqrt{AB^2 - BC^2} = \sqrt{(\sqrt{2}k)^2 - (k)^2}$

$= \sqrt{2k^2 - k^2} = \sqrt{k^2} = k$

$\therefore \tan A = \frac{BC}{AC} = \frac{k}{k} = 1$

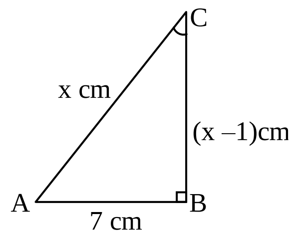
$\text{cosec } A = \frac{1}{\sin A} = \frac{\sqrt{2}k}{k} = \sqrt{2}$

$\tan B = \frac{AC}{BC} = \frac{k}{k} = 1$

$\text{cosec } B = \frac{AB}{AC} = \frac{\sqrt{2}k}{k} = \sqrt{2}$

**Ex.3** In $\Delta ABC$, right –angled at B, AB = 7 cm and (AC – BC) = 1 cm. Find the values of sin C and cos C.

**Sol.** Consider $\Delta ABC$ in which $\angle B = 90^\circ$, AB = 7 cm and (AC – BC) = 1 cm.




Let AC = x cm.

Then, BC = (x – 1) cm

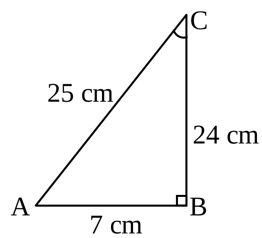
By Pythagoras theorem, we have :

$AB^2 + BC^2 = AC^2 \Rightarrow (7)^2 + (x - 1)^2 = x^2$

$\Rightarrow$ $49 + x^2 - 2x + 1 = x^2$

$\Rightarrow$ $2x = 50$

$\Rightarrow$ $x = 25$




$\therefore$ AC = 25 cm, BC = (25 – 1)cm = 24 cm and AB = 7 cm.

For T-ratios of $\angle C$, we have base = BC = 24 cm,

perpendicular = AB = 7 cm and

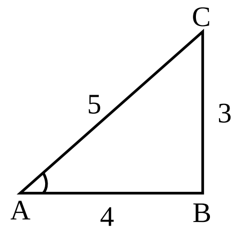
hypotenuse = AC = 25 cm.

$\therefore \sin C = \frac{AB}{AC} = \frac{7}{25}$ and $\cos C = \frac{BC}{AC} = \frac{24}{25}$.

**Ex.4** If $\sin A = \frac{3}{5}$, find cos A and tan A.

**Sol.** Since $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$, so

We draw a triangle ABC, right angled at B such that




Perpendicular = BC = 3 units,

and, Hypotenuse = AC = 5 units.

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow\ 5^2 = AB^2 + 3^2$

$\Rightarrow\ AB^2 = 5^2 - 3^2$

$\Rightarrow\ AB^2 = 16 \ \Rightarrow AB = 4$

When we consider the t-ratio of $\angle A$, we have

Base = AB = 4, Perpendicular = BC = 3, Hypotenuse = AC = 5.

$\therefore\ \cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

and, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

**TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES 0°, 30°, 45°, 60° AND 90°:**

| Ratio \ Angle | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| sin | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| tan | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not defined |
| cot | Not defined | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |
| sec | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | Not defined |
| cosec | Not defined | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |

- $\sin\theta \uparrow$ when $\theta \uparrow$, $0 \le \theta \le 90°$
- $\cos\theta \downarrow$ when $\theta \uparrow$, $0 \le \theta \le 90°$
- $\tan\theta$, $\cot\theta$ are not defined for $\theta = 90°$ & 0 respectively.
- $\operatorname{cosec}\theta$, $\sec\theta$ are not defined when $\theta = 0$ & $90°$ respectively.
- $\sin\theta = \cos\theta$ for only $\theta = 45°$
- $\because \; 180° = \pi^c$
- $\therefore \; 30° = \left(\frac{\pi}{6}\right)^c; \; 45° = \left(\frac{\pi}{4}\right)^c$

  $60° = \left(\frac{\pi}{3}\right)^c; \; 90° = \left(\frac{\pi}{2}\right)^c$

**Ex. 5** Find the value of the following: $4\cos^2 60º + 4\sin^2 45º - \sin^2 30º$

**Sol.** $4\cos^2 60º + 4\sin^2 45º - \sin^2 30º$

$$4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2 = 4\times\frac{1}{4} + 4\times\frac{1}{2} - \frac{1}{4} = \frac{11}{4}$$

**Ex.6** Evaluate each of the following in the simplest form :

(i) $\sin 60º \cos 30º + \cos 60º \sin 30º$

(ii) $\sin 60º \cos 45º + \cos 60º \sin 45º$

**Sol.** (i) $\sin 60º \cos 30º + \cos 60º \sin 30º$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

(ii) $\sin 60º \cos 45º + \cos 60º \sin 45º$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

**Ex.7** Evaluate the following expression :

(i) $\tan 60º \operatorname{cosec}^2 45º + \sec^2 60º \tan 45º$

(ii) $4\cot^2 45º - \sec^2 60º + \sin^2 60º + \cos^2 90º$.

**Sol.** (i) $\tan 60º \operatorname{cosec}^2 45º + \sec^2 60º \tan 45º$

$\tan 60º (\operatorname{cosec} 45º)^2 + (\sec 60º)^2 \tan 45º$

$= \sqrt{3} \times (\sqrt{2})^2 + (2)^2 \times 1$

$= \sqrt{3} \times 2 + 4 = 4 + 2\sqrt{3}$

(ii) $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + \cos^2 90^\circ$

$= 4(\cot 45^\circ)^2 - (\sec 60^\circ)^2 + (\sin 60^\circ)^2 + (\cos 90^\circ)^2$

$= 4 \times (1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 0$

$= 4 - 4 + \frac{3}{4} + 0 = \frac{3}{4}$

**Ex.8** Show that :

(i) $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ) = 6$

(ii) $2(\cos^4 60^\circ + \sin^4 30^\circ) - (\tan^2 60^\circ + \cot^2 45^\circ) + 3\sec^2 30^\circ = \frac{1}{4}$

**Sol.** (i) $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ)$

$= 2\left(\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2\right) - 6\left(\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right)$

$= 2\left(\frac{1}{2} + 3\right) - 6\left(\frac{1}{2} - \frac{1}{3}\right) = 2\left(\frac{1+6}{2}\right) - 6\left(\frac{3-2}{6}\right)$

$= 2 \times \frac{7}{2} - 6 \times \frac{1}{6} = 7 - 1 = 6$

(ii) $2(\cos^4 60^\circ + \sin^4 30^\circ) - (\tan^2 60^\circ + \cot^2 45^\circ) + 3\sec^2 30^\circ$

$= 2\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right) - \left((\sqrt{3})^2 + (1)^2\right) + 3\left(\frac{2}{\sqrt{3}}\right)^2$

$= 2\left(\frac{1}{16} + \frac{1}{16}\right) - (3 + 1) + 3 \times \frac{4}{3}$

$= 2 \times \frac{1}{8} - 4 + 4 = \frac{1}{4}$