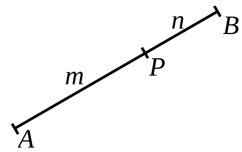
COORDINATE GEOMETRY

SECTION FORMULA

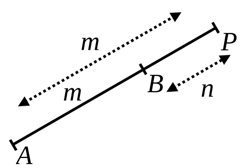
SECTION FORMULAE :

Let A and B be two points in the plane of the paper as shown in fig. and P be a point on the segment joining A and B such that AP : BP = m : n. Then, the point P divides segment AB internally in the ratio m : n.

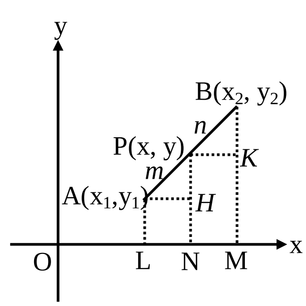


If P is a point on AB produced such that

AP : BP = m : n, then point P is said to divide AB externally in the ratio m : n.



The coordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m : n are given by $\left(x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}\right)$



The coordinates of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

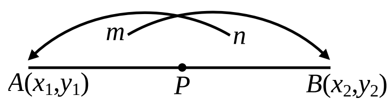
Note 1 :

If P is the mid-point of AB, then it divides AB in the ratio 1 : 1, so its coordinates are

$$\left(\frac{1.x_1+1.x_2}{1+1}, \frac{1.y_1+1.y_2}{1+1}\right) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Note 2 :

Fig. will help to remember the section formula.



Note 3 :

The ratio m : n can also be written as $\frac{m}{n}$: 1, or λ : 1, where $\lambda = \frac{m}{n}$.

So, the coordinates of point P dividing the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right) = \left(\frac{\frac{m}{n}x_2+x_1}{\frac{m}{n}+1}, \frac{\frac{m}{n}y_2+y_1}{\frac{m}{n}+1}\right)$$

$$= \left(\frac{\lambda x_2+x_1}{\lambda+1}, \frac{\lambda y_2+y_1}{\lambda+1}\right)$$

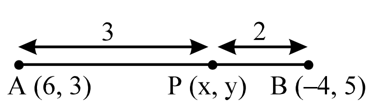
Type I : On finding the section point when the section ratio is given

Ex.1 Find the coordinates of the point which divides the line segment joining the points (6, 3) and (– 4, 5) in the ratio 3 : 2 internally.

Sol. Let P (x, y) be the required point. Then,

$x = \frac{3\times(-4)+2\times6}{3+2}$ and $y = \frac{3\times5+2\times3}{3+2}$

$\Rightarrow x = 0$ and $y = \frac{21}{5}$



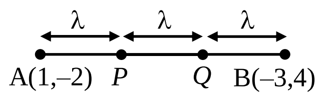
So, the coordinates of P are (0, 21/5).

Ex.2 Find the coordinates of points which trisect the line segment joining (1, –2) and (–3, 4).

Sol. Let A(1, –2) and B(–3, 4) be the given points.

Let the points of trisection be P and Q. Then,

AP = PQ = QB = λ (say).



$\therefore$ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = λ : 2λ = 1 : 2 and

AQ : QB = 2λ : λ = 2 : 1

So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1. Thus, the coordinates of P and Q are

$$P\left(\frac{1\times(-3)+2\times1}{1+2}, \frac{1\times4+2\times(-2)}{1+2}\right) = P\left(\frac{-1}{3}, 0\right)$$

$$Q\left(\frac{2\times(-3)+1\times1}{2+1}, \frac{2\times4+1\times(-2)}{2+1}\right) = Q\left(\frac{-5}{3}, 2\right) \text{ respectively}$$

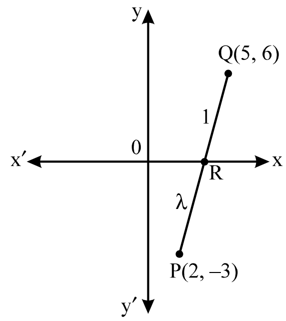
Hence, the two points of trisection are (–1/3, 0) and (–5/3, 2).

Type II : On Finding the section ratio or an end point of the segment when the section point is given

Ex.3 In what ratio does the x-axis divide the line segment joining the points (2, –3) and (5, 6) ? Also, find the coordinates of the point of intersection.

Sol. Let the required ratio be λ : 1. Then, the coordinates of the point of division are,

$$R\left(\frac{5\lambda+2}{\lambda+1}, \frac{6\lambda-3}{\lambda+1}\right)$$



But, it is a point on x-axis on which y-coordinates of every point is zero.

$\therefore \quad \frac{6\lambda-3}{\lambda+1} = 0$

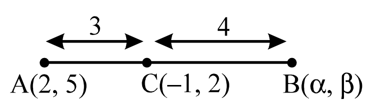
$\Rightarrow \lambda = \frac{1}{2}$

Thus, the required ratio is $\frac{1}{2}$: 1 or 1 : 2.

Putting λ = 1/2 in the coordinates of R, we find that its coordinates are (3, 0).

Ex.4 If the point C (–1, 2) divides internally the line segment joining A (2, 5) and B in ratio 3 : 4, find the coordinates of B.

Sol. Let the coordinates of B be (α, β). It is given that AC : BC = 3 : 4. So, the coordinates of C are



$$\left(\frac{3\alpha+4\times2}{3+4}, \frac{3\beta+4\times5}{3+4}\right) = \left(\frac{3\alpha+8}{7}, \frac{3\beta+20}{7}\right)$$

But, the coordinates of C are (–1, 2)

$\therefore \quad \frac{3\alpha+8}{7} = -1$ and $\frac{3\beta+20}{7} = 2$

$\Rightarrow \alpha = -5$ and $\beta = -2$

Thus, the coordinates of B are (–5, –2).

Type III : On determination of the type of a given quadrilateral

Ex.5 Prove that the points (–2, –1), (1, 0), (4, 3) and (1, 2) are the vertices of a parallelogram. Is it a rectangle ?

Sol. Let the given point be A, B, C and D respectively. Then,

Coordinates of the mid-point of AC are

$$\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = (1, 1)$$

Coordinates of the mid-point of BD are

$$\left(\frac{1+1}{2}, \frac{0+2}{2}\right) = (1, 1)$$

Thus, AC and BD have the same mid-point. Hence, ABCD is a parallelogram.

Now, we shall see whether ABCD is a rectangle or not.

We have, AC = $\sqrt{(4-(-2))^2+(3-(-1))^2} = 2$

and, BD = $\sqrt{(1-1)^2+(0-2)^2} = 2$

Clearly, AC $\neq$ BD. So, ABCD is not a rectangle.

Ex.6 Prove that (4, – 1), (6, 0), (7, 2) and (5, 1) are the vertices of a rhombus. Is it a square ?

Sol. Let the given points be A, B, C and D respectively. Then,

Coordinates of the mid-point of AC are

$$\left(\frac{4+7}{2}, \frac{-1+2}{2}\right) = \left(\frac{11}{2}, \frac{1}{2}\right)$$

Coordinates of the mid-point of BD are

$$\left(\frac{6+5}{2}, \frac{0+1}{2}\right) = \left(\frac{11}{2}, \frac{1}{2}\right)$$

Thus, AC and BD have the same mid-point.

Hence, ABCD is a parallelogram.

Now, AB = $\sqrt{(6-4)^2+(0+1)^2} = \sqrt{5}$,

BC = $\sqrt{(7-6)^2+(2-0)^2} = \sqrt{5}$

$\therefore$ AB = BC

So, ABCD is a parallelogram whose adjacent sides are equal.

Hence, ABCD is a rhombus.

We have,

AC = $\sqrt{(7-4)^2+(2+1)^2}$ = 3 $\sqrt{2}$ and

BD = $\sqrt{(6-5)^2+(0-1)^2}$ = $\sqrt{2}$

Clearly, AC $\neq$ BD.

So, ABCD is not a square.

Type IV : On finding the unknown vertex from given points

Ex.7 The three vertices of a parallelogram taken in order are (−1, 0), (3, 1) and (2, 2) respectively. Find the coordinates of the fourth vertex.

Sol. Let A(−1, 0), B(3, 1), C(2, 2) and D(x, y) be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

$\therefore$ Coordiantes of the mid-point of AC = Coordinates of the mid-point of BD

$$\Rightarrow \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$$

$$\Rightarrow \left(\frac{1}{2}, 1\right) = \left(\frac{3+x}{2}, \frac{y+1}{2}\right)$$

$$\Rightarrow \frac{3+x}{2} = \frac{1}{2} \text{ and } \frac{y+1}{2} = 1$$

$\Rightarrow$ x = – 2 and y = 1

Hence, the fourth vertex of the parallelogram is (−2, 1).

Ex.8 If the points A (6, 1), B (8, 2), C(9, 4) and D (p, 3) are vertices of a parallelogram, taken in order, find the value of p.

Sol. We know that the diagonals of a parallelogram bisect each other. So, coordinates of the mid-point of diagonal AC are same as the coordinates of the mid-point of diagonal BD.

$$\therefore \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$$

$$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\Rightarrow \frac{15}{2} = \frac{8+p}{2}$$

$$\Rightarrow 15 = 8 + p$$

$$\Rightarrow p = 7$$