COORDINATE GEOMETRY

DISTANCE BETWEEN TWO POINT

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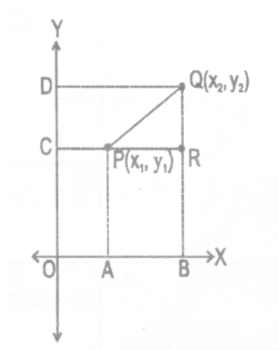
Let two points be **P** (x_1, y_1) and **Q**(x_2, y_2)

Take two mutually perpendicular lines as the coordinate axis with **O** as origin. Mark the points **P**(x_1, y_1) and **Q** (x_2, y_2). Draw lines **PA, QB** perpendicular to **X-axis** from the points **P** and **Q,** which meet the **X-axis** in points A and B, respectively.



Draw lines PC and QD perpendicular to **Y-axis,** which meet the **Y-axis** in C and D, respectively. Produce CP to meet BQ in R. Now

OA = abscissa of P = x_1

Similarly, OB = x_2, OC = y_1 and OD = y_2

Therefore, we have

PR = AB = OB - OA = $x_2 - x_1$

Similarly, QR = QB - RB = QB - PA = $y_2 - y_1$

Now, using Pythagoras Theorem, in right angled triangle PRQ, we have

$PQ^2 = Pr^2 + RQ^2$

or $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Since the distance or length of the line-segment PQ is always non-negative, on taking the positive square root, we get the distance as

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This result is known as **distance formula.**

Corollary : The distance of a point $\mathbf{P(x_1, y_1)}$ from the origin **(0,0)** is given by

$$OP = \sqrt{x_1^2 + y_1^2}$$

Some useful points :

1. In questions relating to geometrical figures, take the given vertices in the given order and proceed as indicated.

(i) For an **isosceles triangle -** We have to prove that at least two sides are equal.

(ii) For an **equilateral triangle -** We have to prove that three sides are equal.

(iii) For a **right -angled triangle** - We have to prove that the sum of the squares of two sides is equal to the square of the third side.

(iv) for a **square** - We have to prove that the four sides are equal, two diagonals are equal.

(v) For a **rhombus** - We have to prove that four sides are equal (and there is no need to establish that two diagonals are unequal as the square is also a rhombus).

(vi) For a **rectangle** - We have to prove that the opposite sides are equal and two diagonals are equal.

(vii) For a **Parallelogram** - We have to prove that the opposite sides are equal (and there is no need to establish that two diagonals are unequal sat the rectangle is also a parallelogram).

2. for three points to be **collinear -** We have to prove that the sum of the distances between two pairs of points is equal to the third pair of points.

Ex.1 Find the distance between the points (8 , -2) and (3, -6).

Sol. Let the points (8, -2) and (3, -6) be denoted by P and Q, respectively.

Then, by distance formula, we obtain the distance PQ as

$$PQ=\sqrt{(3-8)^2+(-6+2)^2}$$

$$=\sqrt{(-5)^2+(-4)^2}=\sqrt{41}\text{ uni}$$

Ex.2 Prove that the points $(1,-1),\left(-\frac{1}{2},\frac{1}{2}\right)$ and (1, 2) are the vertices of an isosceles triangle.

Sol. Let the point (1, -1), $\left(-\frac{1}{2},\frac{1}{2}\right)$and (1, 2) be denoted by P, Q and R, respectively. Now

$$PQ=\sqrt{\left(-\frac{1}{2}-\right)^2+\left(\frac{1}{2}+1\right)^2}=\sqrt{\frac{18}{4}}=\frac{3}{2}\sqrt{2}$$

$$QR=\sqrt{\left(1+\frac{1}{2}\right)^2+\left(2-\frac{1}{2}\right)^2}=\sqrt{\frac{18}{4}}=\frac{3}{2}\sqrt{2}$$

$$PR=\sqrt{(1-1)^2+(2+1)^2}=\sqrt{9}=3$$

From the above, we see that PQ = QR

$\therefore$ The triangle is isosceles.

Ex.3 Using distance formula, show that the points (-3, 2), (1, -2) and (9, -10) are collinear.

Sol. Let the given points (-3, 2), (1, -2) and (9, -10) be denoted by A, B and C, respectively. Points A, B and C will be collinear, if the sum of the lengths of two line-segments is equal to the third.

Now, $AB = \sqrt{(1+3)^2 + (-2-2)^2} = \sqrt{16+16} = 4\sqrt{2}$

$BC = \sqrt{(9-1)^2 + (-10+2)^2} = \sqrt{64+64} = 8\sqrt{2}$

$AC = \sqrt{(9+3)^2 + (-10-2)^2} = \sqrt{144+144} = 12\sqrt{2}$

Since, AB + BC = $4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2} = AC$, the, points A, B and C are collinear.

Ex.4 Find a point on the X-axis which is equidistant from the points (5, 4) and (-2, 3).

Sol. Since the required point (say P) is on the X-axis, its ordinate will be zero. Let the abscissa of the point be x.

Therefore, coordinates of the point P are (x, 0).

Let A and B denote the points (5, 4) and (-2, 3), respectively.

Since we are given that AP = BP, we have

$AP^2 = BP^2$

i.e., $(x - 5)^2 + (0 - 4)^2 = (x + 2)^2 + (0 - 3)^2$

or $x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$

or $-14x = -28$

or $x = 2$

Thus, the required point is (2, 0).