COORDINATE GEOMETRY

APPLICATION OF SECTION FORMULA

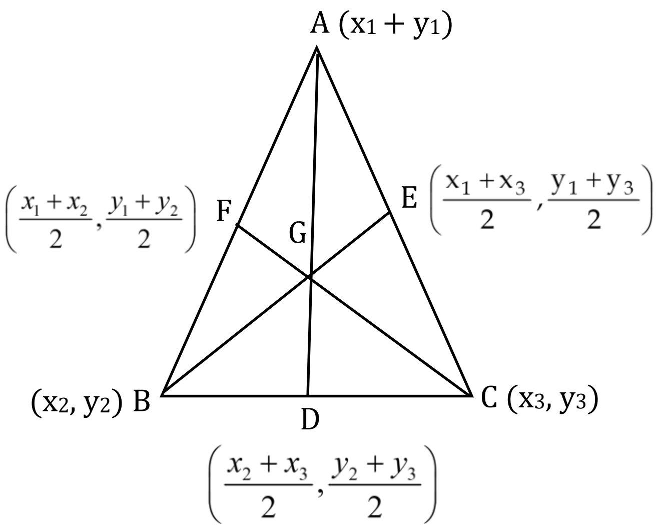
CENTROID OF A TRIANGLE :

Prove that the coordinates of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (y_3, y_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. Also, deduce that the medians of a triangle are concurrent.

Proof :

Let **A(x_1, y_1, B(x_2, y_2)** and **C(x_3, y_3)** be the vertices of **ΔABC** whose medians are AD, BE and CF respectively. So. D,E and F are respectively the mid-points of BC, CA and AB.

Coordinates of **D** are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$. Coordinates of a point dividing AD in the ratio **2 : 1** are



$$\left(\frac{1.x_1+2\left(\frac{x_2+x_3}{2}\right)}{1+2},\frac{1.y_1+\left(\frac{y_2+y_3}{2}\right)}{1+2}\right)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$. The coordinates of a point dividing BE in the ratio 2 : 1 are $\left(\frac{1.x_2+\frac{2(x_1+x_3)}{2}}{1+2},\frac{1.y_2+\frac{2(y_1+y_3)}{2}}{1+2}\right)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

Similarly the coordinates of a point dividing CF in the ratio 2 : 1 are
$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

Thus, the point having coordinates $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 1 : 2.

Hence, medians of a triangle are concurrent and the coordinates of the centroid are

$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$.

Ex.1 Find the coordinates of the centroid of a triangle whose vertices are (–1, 0), (5, –2) and (8, 2).

Sol. We know that the coordinates of the centroid of a triangle whose angular points are (x_1, y_1), (x_2, y_2), (x_3, y_3) are

$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

So, the coordiantes of the centroid of a triangle whose vertices are (–1, 0), (5, –2) and (8, 2) are

$\left(\frac{-1+5+8}{3},\frac{0-2+2}{3}\right)$ or, (4, 0)

Ex.2 If the coordinates of the mid points of the sides of a triangle are (1, 1), (2, – 3) and (3, 4) Find its centroid.

Sol. Let P (1, 1), Q(2, –3), R(3, 4) be the mid-points of sides AB, BC and CA respectively of triangle ABC. Let A (x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of triangle ABC. Then, P is the mid-point of BC

$\Rightarrow \frac{x_1+x_2}{2} = 1, \frac{y_1+y_2}{2} = 1$

$\Rightarrow x_1 + x_2 = 2$ and $y_1 + y_2 = 2$...(1)

Q is the mid-point of BC

$\Rightarrow \frac{x_2+x_3}{2} = 2, \frac{y_2+y_3}{2} = -3$

$\Rightarrow x_2 + x_3 = 4$ and $y_2 + y_3 = -6$...(2)

R is the mid-point of AC

$\Rightarrow \frac{x_1+x_3}{2} = 3$ and $\frac{y_1+y_3}{2} = 4$

$\Rightarrow x_1 + x_3 = 6$ and $y_1 + y_3 = 8$...(3)

From (1), (2) and (3), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$

and,$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$ ...(4)

The coordinates of the centroid of ΔABC are

$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right) = \left(\frac{6}{3},\frac{2}{3}\right)$

$= \left(2,\frac{2}{3}\right)$ [Using (4)]

Ex.3 Two vertices of a triangle are (3, –5) and (–7, 4). If its centroid is (2, –1). Find the third vertex.

Sol. Let the coordinates of the third vertex be (x, y). Then,

$\frac{x+3-7}{3} = 2$ and $\frac{y-5+4}{3} = -1$

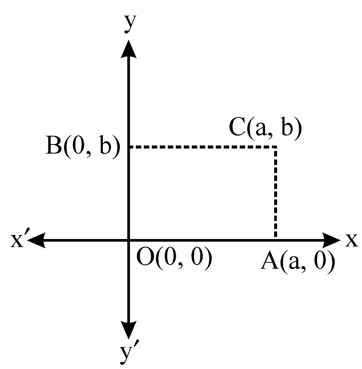
$\Rightarrow$ x – 4 = 6 and y – 1 = – 3

$\Rightarrow$ x = 10 and y = – 2

Thus, the coordinates of the third vertex are (10, –2).

Ex.4 Prove that the diagonals of a rectangle bisect each other and are equal.

Sol. Let OACB be a rectangle such that OA is along x-axis and OB is along y-axis. Let OA = a and OB = b.



Then, the coordinates of A and B are (a, 0) and (0, b) respectively.

Since, OACB is a rectangle. Therefore,

AC = Ob $\Rightarrow$ AC = b

Thus, we have

OA = a and AC = b

So, the coordiantes of C are (a, b).

The coordinates of the mid-point of OC are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Also, the coordinates of the mid-points of AB are $\left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Clearly, coordinates of the mid-point of OC and AB are same.

Hence, OC and AB bisect each other.

Also, $OC = \sqrt{a^2 + b^2}$ and

$AB = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$\therefore$ $OC = AB$