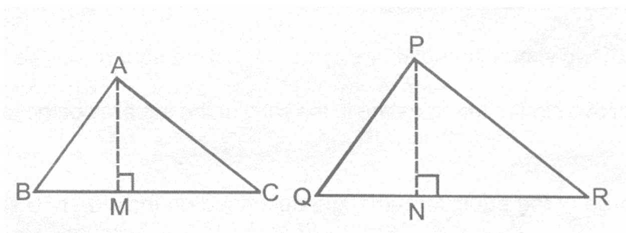
TRIANGLES

AREA OF TWO SIMILAR TRIANGLE

AREAS OF SIMILAR TRIANGLS :

Statement : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given : Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$ [Shown in the figure]



To Prove : $\dfrac{ar(ABC)}{ar(PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2$

Construction : Draw altitudes AM and PN of the triangle ABC an PQR.

Proof : $ar(ABC) = \dfrac{1}{2} BC \times AM$

And $ar(PQT) = \dfrac{1}{2} QR \times PN$

So, $\dfrac{ar(ABC)}{ar(PQR)} = \dfrac{\frac{1}{2} BC \times AM}{\frac{1}{2} QR \times PN} = \dfrac{BC \times AM}{QR \times PN}$ ....(i)

Now, in Δ ABM and Δ PQN,

And $\angle B = \angle Q$ [As Δ ABC $\sim$ ΔPQR]

$\angle M = \angle N$ [90^0 each]

So, $\Delta ABM \sim \Delta PQN$ [AA similarity criterion]

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ ....(ii)

Also, $\Delta ABC \sim \Delta PQR$ [Given]

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ .....(iii)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{AB}{PQ}$ [From (i) and (ii)]

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (iii)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (iii), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

(a) Properties of Areas of Similar Triangles :

(i) The areas of two similar triangles are in the ratio of the squares of corresponding altitudes.

(ii) The areas of two similar triangles are in the ratio of the squares of the corresponding medians.

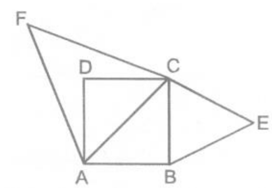
(iii) The area of two similar triangles are in the ratio of the squares of the corresponding angle bisector segments.

Ex.1 Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on this diagonals. **[CBSE - 2001]**

Sol. **Given :** A square ABCD. Equilateral triangles ΔBCE and ΔACF have been described on side BC and diagonals AC respectively.

To prove : Area (ΔBCE) $= \frac{1}{2}$. Area (ΔACF)

Proof : Since ΔBCE and ΔACF are equilateral. Therefore, they are equiangular (each angle being equal to 60^0) and hence ΔBCE ~ ΔACF.



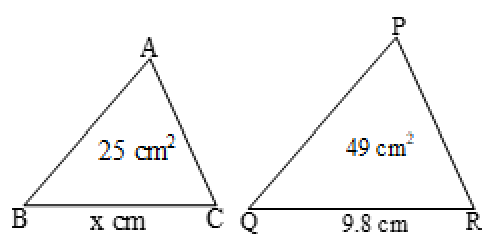
$\Rightarrow \quad \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{AC^2}$

$\Rightarrow \quad \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{(\sqrt{2}BC)^2} = \frac{1}{2}$ $\left[\begin{array}{l} \because ABCD \text{ is a square} \\ \because \text{Diagonal} = \sqrt{2}(\text{side}) \\ \Rightarrow AC = \sqrt{2}BC \end{array}\right]$

$\Rightarrow \quad \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{1}{2}$ **Hence Proved.**

Ex.2 The areas of two similar triangles ΔABC and ΔPQR are 25 cm^2 and 49 cm^2 respectively. If QR = 9.8 cm, find BC.

Sol. It is being given that ΔABC ~ ΔPQR, ar (ΔABC) = 25 cm^2 and ar (ΔPQR) = 49 cm^2. We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$

$\Rightarrow \frac{25}{49} = \frac{x^2}{(9.8)^2}$, where BC = x cm

$\Rightarrow x^2 = \left(\frac{25}{49} \times 9.8 \times 9.8\right)$

$\Rightarrow x = \sqrt{\frac{25}{49} \times 9.8 \times 9.8} = \left(\frac{5}{7} \times 9.8\right) = (5 \times 1.4) = 7.$

Hence BC = 7 cm.

Ex.3 In two similar triangles ABC and PQR, if their corresponding altitudes AD and PS are in the ratio 4 : 9, find the ratio of the areas of ABC and PQR.

Sol. Since the areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.

$$\therefore \quad \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

[$\because$ AD : PS = 4 : 9]

Hence, Area (ΔABC) : Area (ΔPQR) = 16 : 81

Ex.4 If ΔABC is similar to ΔDEF such that ΔDEF = 64 cm^2, DE = 5.1 cm and area of ΔABC = 9 cm^2. Determine the area of AB.

Sol. Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$$\therefore \quad \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{9}{64} = \frac{AB^2}{(5.1)^2}$$

$$\Rightarrow AB = \sqrt{3.65}$$

$\Rightarrow$ AB = 1.912 cm