ARITHMETIC PROGRESSIONS

GENERAL TERM AND SUM OF N TERM IN A.P

General Term of an A.P :

a, a + d, a + 2d, a + 3d.....represents an arithmetic progression where a is the first term and d the common difference. This is called the general form of an A.P.

Let 'a' be the first term and 'd' be the common difference of an A.P. Then its n^{th} term is $a_n = a + (n - 1)d$

Sum of First n terms of an A.P.:

The sum S_n of n terms of an A.P. with first term 'a' and common difference 'd' is given by

(i) $S_n = \frac{n}{2}[2a + (n-1)d]$

(ii) $S_n = \frac{n}{2}[a + l]$ where $l = a + (n - 1)d$, l = last term

NOTE : (i) If sum of n terms Sn is given then general term Tn = Sn – Sn–1 where Sn–1 is sum of (n–1) terms of A.P.

(ii) Common difference of AP is given by d = S2 – 2S1 where S2 is sum of first two terms and S1 is sum of first term.

Ex.1 Find the sum of 20 terms of the A.P. 1,4,7,10.....

Sol. a = 1, d = 3

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{20} = \frac{20}{2}[2(1)+(20-1)3]$$

Ex.2 Find the sum of all three digit natural numbers. Which are divisible by 7.

Sol. 1st no. is 105 and last no. is 994.

Find n $994 = 105 + (n + 1)7$

$\therefore \quad n = 128$

$\therefore \quad$ Sum, $S_{128} = \frac{128}{2}[105 + 994]$

EX.3 The sum of three numbers in A.P. is 27, and their product is 504, find them

Sol. Let the three terms be (a – d), a and (a + d) where d is common different. It is given that $(a - d) + a + (a + d) = 27$

Or $3a = 27$

$\Rightarrow \quad a = 9$

and $(a - d)(a)(a + d) = 504$

or $a(a^2 - d^2) = 504$

$\Rightarrow \quad 9(81 - d^2) = 504$

$\Rightarrow \quad 81 - d^2 =$

$\Rightarrow \quad d^2 = 81 - 56 = 25$

$\Rightarrow \quad d^2 = 25$

or $d = +5$ or $d = -5$

Therefore, the terms are: $9 - 5, 9, 9 + 5 = 4, 9, 14$ **Ans.**

Selection of Terms in an A.P.:

No. of terms	Terms	Common difference
1	a-d, a , a+d	d
4	a-3d, a-d, a+d, a+3d	2d
5	a-2d, a-d, a, a+d, a+2d	d
6	a-5d, a-3d, a-d, a+d, a+3d, a+5d	2d

Ex.4 The sum of three number in A.P. is -3 and their product is 8. Find the numbers.

Sol. Three no. 's in A.P. be a - d, a, a + d

$\therefore$ $a - d + a + a + d = -3$

$3a = -3 \Rightarrow a = -1$

& $(a - d)\, a\, (a + d) = 8$

$a(a^2 - d^2) = 8$

$(-1)\,(1 - d^2) = 8$

$1 - d^2 = -8$

$\Rightarrow$ $d^2 = 9$

$\Rightarrow$ $d = \pm 3$

If $a = 8$ & $d = 3$ numbers are -4, -1, 2.

If $a = 8$ & $d = -$ numbers are 2, -1, -4.

Ex.5 The n^{th} term of a sequence is $3n - 2$. Is the sequence an A.P. ? If so, find its 10^{th} term.

Sol. We have $a_n = 3n - 2$

Clearly a_n is a linear expression in n. So, the given sequence is an A.P. with common difference 3.

Putting $n = 10$, we get

$$a_{10} = 3 \times 10 - 2 = 28$$

Ex.6 Determine the A.P. whose their term is 16 and the difference of 5^{th} term from 7^{th} term is 12.

Sol. Given : $a_3 = a + (3 - 1)\, d = a + 2d = 16$ .....(i)

$a_7 - a_5 = 12$ ....(ii)

$(a + 6d) - (a + 4d) = 12$

$a + 6d - a - 4d = 12$

$2d = 12$

$d = 6$

Put $d = 6$ in equation (i)

$a = 16 - 12$

$a = 4$

$\therefore$ A.P. is 4, 10, 16, 22, 28, ...

Ex.7 Find the 12^{th}, 24^{th} and nth term of the A.P. given by 9, 13, 17, 21, 25, ...

Sol. We have,

a = First term = 9 and,

d = Common difference = 4

[$\because$ 13 – 9 = 4, 17 – 13 = 4, 21 – 7 = 4 etc.]

We know that the nth term of an A.P. with first term a and common difference d is given by

$a_n = a + (n - 1)\, d$

Therefore,

$a_{12} = a + (12 - 1)\, d$

$= a + 11d = 9 + 11 \times 4 = 53$

$a_{24} = a + (24 - 1)\, d$

$= a + 23\, d = 9 + 23 \times 4 = 101$

and, $a_n = a + (n - 1)\, d$

$= 9 + (n - 1) \times 4 = 4n + 5$

$a_{12} = 53$, $a_{24} = 101$ and $a_n = 4n + 5$

Ex.8 Which term of the sequence –1, 3, 7, 11, ... , is 95 ?

Sol. Clearly, the given sequence is an A.P.

We have,

a = first term = –1 and,

d = Common difference = 4.

Let 95 be the n^{th} term of the given A.P. then,

$a_n = 95$

$\Rightarrow$ $a + (n - 1)\, d = 95$

$\Rightarrow$ $-1 + (n - 1) \times 4 = 95$

$\Rightarrow$ $-1 + 4n - 4 = 95 \Rightarrow 4n - 5 = 95$

$\Rightarrow$ $4n = 100 \quad \Rightarrow n = 25$

Thus, 95 is 25^{th} term of the given sequence.

Ex.9 Which term of the sequence 4, 9 , 14, 19, ... is 124 ?

Sol. Clearly, the given sequence is an A.P. with first term a = 4 and common difference d = 5.

Let 124 be the n^{th} term of the given sequence. Then, $a_n = 124$

$a + (n - 1)\, d = 124$

$\Rightarrow 4 + (n - 1) \times 5 = 124$

$\Rightarrow$ $n = 25$

Hence, 25^{th} term of the given sequence is 124

Ex.10 Which term of the sequence 72, 70, 68, 66, ... is 40 ?

Sol. Here 1^{st} term x = 72 and common difference d = 70 - 72 = - 2

$\therefore$ For finding the value of n

$a_n = a + (n - 1)d$

$\Rightarrow$ $40 = 72 + (n - 1)(-2)$

$\Rightarrow$ $40 - 72 = -2n + 2$

$\Rightarrow$ $-32 = -2n + 2$

$\Rightarrow$ $-34 = -2n$

$\Rightarrow$ $n = 17$

$\therefore$ 17^{th} term is 40.

n^{th} term of an A.P. from the end:

If 'a' be the first term and 'd' be the common difference of an A.P. having 'm' terms. Then the n^{th} term from the end is $(m - n + 1)^{th}$ term from the beginning.

[n^{th} term from the end: $a_{m-n+1} = [a + (m - n + 1 - 1)d] = [a + (m - n)d]$

Note :

(i) General term is also denoted by λ (last term)

(ii) n (No. of terms) always belongs to set of natural numbers.

(iii) Common difference can be zero, + ve or – ve.

$d = 0 \Rightarrow$ then all terms of AP are same

Ex.11 Find 20^{th} term from the end of an A.P. 3,7,11..... 407.

Sol. $407 = 3 + (n - 1)4 \Rightarrow n = 102$

$\therefore 20^{th}$ term from end $\Rightarrow m = 20$

$a_{102-(20-1)} = a_{102-19} = a_{83}$ from the beginning.

$a_{83} = 3 + (83 + 1)4 = 331.$

Ex.12 Determine the 10^{th} term from the end of the A.P. 4, 9, 14, ..., 254.

Sol. We have,

l = Last term = 254 and,

d = Common difference = 5,

10^{th} term from the end $= l - (10 - 1)\, d$

$= l - 9d = 254 - 9 \times 5 = 209.$