QUADRATIC EQUATION

Solution of Quadratic Equation by Completing the Square

ALGORITAM

Step-(i) Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$.

Step-(ii) Make the coefficient of x^2 unity, if it is not unity. i.e., obtained $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Step-(iii) Shift the constant term $\frac{c}{a}$ on R.H.S. to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step-(iv) Add square of half of the coefficient of x i.e. $\left(\frac{b}{2a}\right)^2$ on both sides to obtain

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Step-(v) Write L.H.S. as the perfect square of a binomial expression and simplify R.H.S. to Get $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$.

Step-(vi) Take square root of both sides to get $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Step (vii) Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on RHS.

Ex. 1 Solve :- $x^2 + 3x + 1 = 0$

Sol. We have $x^2 + 3x + 1 = 0$

Add and subtract $(\frac{1}{2}\text{coefficient of } x)^2$ in L.H.S. and get

$$x^2+3x+1+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2=0$$

$$\Rightarrow \quad x^2+2\left(\frac{3}{2}\right)x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+1=0$$

$$\Rightarrow \quad \left(x+\frac{3}{2}\right)^2-\frac{5}{4}=0$$

$$\Rightarrow \quad \left(x+\frac{3}{2}\right)^2=\left(\frac{\sqrt{5}}{2}\right)^2 \quad \Rightarrow \quad x+\frac{3}{2}=\pm\frac{\sqrt{5}}{2}$$

This gives $x=\frac{-(3+\sqrt{5})}{2}$ or $x=\frac{-3+\sqrt{5}}{2}$

Therefore $x=-\frac{3+\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}$ are the solutions of the given equation.

Ex. 2 By using the method of completing the square, show that the equation $4a^2 + 3x + 5 = 0$ has no real roots.

Sol. We have, $4x^2 + 3x + 5 = 0$

$$\Rightarrow \quad x^2+\frac{3}{4}x+\frac{5}{4}=0$$

$$\Rightarrow \quad x^2+2\left(\frac{3}{8}x\right)=-\frac{5}{4}$$

$$\Rightarrow \quad x^2+2\left(\frac{3}{8}\right)x+\left(\frac{3}{8}\right)^2=\left(\frac{3}{8}\right)^2-\frac{5}{4}$$

$$\Rightarrow \quad \left(x+\frac{3}{8}\right)^2=-\frac{71}{64}$$

Clearly, RHS is negative

But, $\left(x+\frac{3}{8}\right)^2$ cannot be negative for any real value of x.

Hence, the given equation has no real roots.

Ex. 3 Illustration showing solution of a quadratic equation using completion of square:

Find the roots of $2x^2 + 6x + 1 = 0$

Sol. Here the coefficient of x^2 is not equal to 1 so first we make the coefficient of x^2 equal to 1.

Hence the given equation can be written as : $2(x^2 + 3x + \frac{1}{2}) = 0$

(Dividing both sides of the equation by 2)

Now, we have $x^2 + 3x + \frac{1}{2} = 0$

Here coefficient of x is 3, so adding and subtracting $(\frac{1}{2})^2$ to the given equation:

$$\Rightarrow x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{1}{2}=0$$

$$\Rightarrow \left(x+\frac{3}{2}\right)^2-\frac{9}{4}+\frac{1}{2}=0$$

$$\Rightarrow \left(x+\frac{3}{2}\right)^2-\frac{7}{4}=0$$

$$\Rightarrow \left(x+\frac{3}{2}\right)^2=\frac{7}{4}$$

$$\Rightarrow x+\frac{3}{2}=\pm\frac{\sqrt{7}}{2}$$

$$\Rightarrow x=-\frac{3}{2}\pm\frac{\sqrt{7}}{2}$$

$$\Rightarrow x=\frac{-3+\sqrt{7}}{2}, x=\frac{-3-\sqrt{7}}{2}$$

Hence, , $x=\frac{-3+\sqrt{7}}{2}, x=\frac{-3-\sqrt{7}}{2}$

Ex. 4 Find the roots of the following quadratic equations (if they exist) by the method of completing the square.

(i) $2x^2 - 7x + 3 = 0$

(ii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(ii) $2x^2 + x + 4 = 0$

Sol.(i) $2x^2 - 7x + 3 = 0$

$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

[Dividing each term by 2]

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} + \frac{3}{2} = 0$

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{49-24}{16}\right) = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$

i.e., $\left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow x - \frac{7}{4} = \pm \frac{5}{4}$

i.e., $x - \frac{7}{4} = \frac{5}{4}$ or $x - \frac{7}{4} = -\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = 3$ or $x = \frac{1}{2}$

(ii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = 0$

i.e., $x^2 + 2 \times x \times \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0$

i.e., $\left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$

$\Rightarrow x + \frac{\sqrt{3}}{2} = 0$ and $x = \frac{-\sqrt{3}}{2}$

$\therefore$ Roots are : $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$

(iii) $2x^2 + x + 4 = 0 \quad \Rightarrow x^2 + \frac{x}{2} + 2 = 0$

i.e., $x^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} + 2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 + \frac{31}{16} = 0$

$\left\lfloor -\frac{1}{16} + 2 = \frac{-1+32}{16} = \frac{31}{16} \right\rfloor$

i.e., $\left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

This is not possible as the square of a real number can not be negative