QUADRATIC EQUATION

Introduction of Quadratic Equation

INTRODUCTION

A polynomial of degree one is called linear polynomial and that of degree two is called Quadratic polynomial. For example $x^2 + 4$, $2x^2 + 3x + 4$, are quadratic polynomials.

A quadratic polynomial can have at most three terms namely the terms containing x^2 , x and the constant term.

The general form of a quadratic polynomial in x is $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$. When we equate this polynomial to zero we get a quadratic equation.

QUADRATIC EQUATION

(i) Let $p(x) = ax^2 + bx + c$, where a, b, c Î R be the quadratic polynomial, then $p(x) = 0$,

i.e. $ax^2 + bx + c = 0$ is called a **quadratic equation** where a, b, c are real numbers and a 0.

Since a 0, quadratic equations in general are of the following types

(a) $b = 0$, c 0 i.e. $ax^2 + c = 0$

(b) $b = 0, c = 0$ i.e. $ax^2 + bx = 0$

(c) $b = 0$, $c = 0$ i.e. $ax^2 = 0$

(d) $b = 0, c = 0$ i.e. $ax^2 + bx + c = 0$

(ii) An equation involving the square of unknown quantity (variable) and no other higher power, is called a quadratic equation or a second degree equation

Ex. 1 Which of the following are quadratic equations.

(i) $3x^2 - 8x = 0$ (ii) $x^2 + \frac{1}{x^2} = 8$ (iii) $x^2 - 6x + 5\sqrt{x} - 7 = 0$

Sol. (i) $3x^2 - 8x = 0$ is a quadratic equation.

VALUE OF A QUADRATIC POLYNOMIAL

The value of a quadratic polynomial $ax^2 + bx + c$

(i) at $x = \alpha$ is $a(\alpha)^2 + b(\alpha) + c = a\alpha^2 + b\alpha + c$

(ii) at $x = \beta$ is $a\beta^2 + b\beta + c$

(iii) at $x = 5$ is a $(5)^2 + b(5) + c = 25a^2 + 5a + c$

In the same way :

(i) Value of $5x^2 - 3x + 4$ at $x = 2$ is

$$= 5(2)^2 - 3(2) + 4$$

$$= 20 - 6 + 4 = 18$$

(ii) Value of $x^2 - 8x - 15$ at $x = -1$ is

$$= (-1)^2 - 8(-1) - 15$$

$$= 1 + 8 - 15 = -6$$

(iii) Value of $7x^2 - 4$ at $x = \frac{2}{3}$ is $= 7\left(\frac{2}{3}\right)^2 - 4$

$$= 7 \times \frac{4}{9} - 4 = \frac{28-36}{9} = \frac{8}{9}$$

ZEROS OF A QUADRATIC POLYNOMIAL

The value of the polynomial $x^2 - 7x + 10$ at :

(i) $x = 1$ is $(1)^2 - 7 \times 1 + 10 = 1 - 7 + 10 = 4$

(ii) $x = 2$ is $(2)^2 - 7 \times 2 + 10 = 4 - 14 + 10 = 0$

(iii) $x = 3$ is $(3)^2 - 7 \times 3 + 10 = 9 - 21 + 10 = -2$

(iv) $x = 5$ is $(5)^2 - 7 \times 5 + 10 = 25 - 35 + 10 = 0$

It is observed here that for $x = 2$ and $x = 5$; the value of polynomial $x^2 - 7x + 10$ is zero. These two values of x are called zeros of the polynomial.

Thus, if for $x = \alpha$, where α is a real number, the value of given quadratic polynomial is zero; the real number α is called zero of the quadratic polynomial.

Ex.2 Show that :

(i) $x = 3$ is a zero of quadratic polynomial $x^2 - 2x - 3$.

(ii) $x = -2$ is a zero of quadratic polynomial $3x^2 + 7x + 2$.

(iii) $x = 4$ is not a zero of quadratic polynomial $2x^2 - 7x - 5$.

Sol.(i) The value of $x^2 - 2x - 3$ at $x = 3$ is

$(3)^2 - 2 \times 3 - 3 = 9 - 6 - 3 = 0$

$\Rightarrow x = 3$ is a zero of quadratic polynomial $x^2 - 2x - 3$.

(ii) The value of $3x^2 + 7x + 2$ at $x = -2$ is

$3(-2)^2 + 7(-2) + 2 = 12 - 14 + 2 = 0$

$\Rightarrow$ $x = -2$ is a zero of quadratic polynomial $3x^2 + 7x + 2$

(iii) The value of $2x^2 - 7x - 5$ at $x = 4$ is

$2(4)^2 - 7(4) - 5 = 32 - 28 - 5 = -1 \neq 0$

$\Rightarrow$ $x = 4$ is not a zero of quadratic polynomial $2x^2 - 7x - 5$.

Ex. 3 Find the value of m, if $x = 2$ is a zero of quadratic polynomial $3x^2 - mx + 4$.

Sol. Since, $x = 2$ is a zero of $3x^2 - mx + 4$

$\Rightarrow 3(2)^2 - m \times 2 + 4 = 0$

$\Rightarrow 12 - 2m + 4 = 0,$

$\Rightarrow 12 + 4 = 2m$

$\Rightarrow m = \frac{16}{2}$ i.e., $m = 8$.

QUADRATIC EQUATION AND IT'S ROOTS

Since, $ax^2 + bx + c, a \neq 0$ is a quadratic polynomial, $ax^2 + bx + c = 0, a \neq 0$ is called a quadratic equation.

(i) $-x^2 - 7x + 2 = 0$ is a quadratic equation, as $-x^2 - 7x + 2$ is a quadratic polynomial.

(ii) $5x^2 - 7x = 0$ is a quadratic equation.

(iii) $5x^2 + 2 = 0$ is a quadratic equation, but

(iv) $-7x + 2 = 0$ is not a quadratic equation.

Ex. 4 Which of the following are quadratic equations, give reason :

(i) $x^2 - 8x + 6 = 0$

(ii) $3x^2 - 4 = 0$

(iii) $2x + \frac{5}{x} = x^2$

(iv) $x^2 + \frac{2}{x^2} = 3$

Sol. (i) Since, $x^2 - 8x + 6$ is a quadratic polynomial

$\Rightarrow$ $x^2 - 8x + 6 = 0$ is a quadratic equation.

(ii) $3x^2 - 4 = 0$ is a quadratic equation.

(iii) $2x + \frac{5}{x} = x^3$

$\Rightarrow$ $2x^2 + 5 = x^3$

$\Rightarrow$ $x^3 - 2x^2 - 5 = 0$; which is cubic and not a quadratic equation.

(iv) $x^2 + \frac{2}{x^2} = 3$

$\Rightarrow$ $x^4 + 2 = 2x^2$

$\Rightarrow$ $x^4 - 2x^2 + 2 = 0$; which is biquadratic and not a quadratic equation.

Ex. 5 In each of the following, determine whether the given values are solutions (roots) of the equation or not :

(i) $3x^2 - 2x - 1 = 0$; $x = 1$

(ii) $x^2 + 6x + 5 = 0$; $x = -1, x = -5$

(iii) $x^2 + \sqrt{2}x - 4 = 0$; $x = \sqrt{2}, x = -2\sqrt{2}$

Sol. (i) Value of $3x^2 - 2x - 1$ at $x = 1$ is

$3(1)^2 - 2(1) - 1 = 3 - 2 - 1 = 0 = \text{RHS}$

$\therefore$ $x = 1$ is a solution of the given equation.

(ii) For $x = -1$, L.H.S. $= (-1)^2 + 6(-1) + 5$

$= 1 - 6 + 5 = 0 = \text{R.H.S.}$

$\Rightarrow$ $x = -1$ is a solution of the given equation

For $x = -5$, L.H.S. $= (-5)^2 + 6(-5) + 5$

$= 25 - 30 + 5 = 0 = \text{R.H.S.}$

$\Rightarrow$ $x = -5$ is a solution of the given equation.

(iii) For $x = \sqrt{2}$, L.H.S. $= x^2 + \sqrt{2}x - 4$

$= (\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4$

$= 2 + 2 - 4 = 0$

$= \text{R.H.S.}$

$\therefore$ $x = \sqrt{2}$ is a solution of the given equation For $x = -2\sqrt{2}$,

L.H.S. $= (-2\sqrt{2})^2 + \sqrt{2} \times -2\sqrt{2} - 4$

$= 4 \times 2 - 2 \times 2 - 4 = 0$ R.H.S.

$\therefore$ $x = -2\sqrt{2}$ is a solution of the given equation.