Pair of Linear Equations in Two Variables

Introduction and Graphical Method to Solve Linear Equation of Two Variable

INTRODUCTION

Equation:

A statement in which two algebraic expression are equal is known as equation.

Like: $2x + 3 = 0$, $\frac{2Y}{3} + 1 = \frac{Y}{3}$

Linear equation:

An equation involving linear polynomials is called a linear equation. For example

$$\frac{3}{2}x + 4 = 2x - 3$$

Remark: A linear equation in one variables has the standard form $ax + b = 0$, $a \neq 0$, $b \in R$

Solution (root) of a linear equation:

The value of the variable which makes the two sides of the equation equal and satisfies the equation is called the solution of the equation.

Rules for solving an equation:

(i) The same number is added or subtracted to both sides of an equation, the resulting equation is equivalent to the first.

(ii) If both sides of an equation are multiplied by the same non-zero number the resulting equation is equivalent to the first.

Remark:

Every linear equation in one variable has only one (unique) solution.

In this chapter we shall study about system of linear equation in two variables, solution of a system of linear equations in two variables.

LINEAR EQUATION IN TWO VARIABLE

Definition:

A linear equation is a rational and integral equation of the first degree.

Eg.: $3x + 2y = 7$; $2x - \sqrt{3}y = \sqrt{5}$

The equation is linear equation in two variables if

(i) neither x nor y is under a radical sign.

(ii) neither x nor y is in the denominator.

(iii) the exponent (power) of x and y in each term is one.

In General form:

$ax + by + c = 0$; a, b, c, $\in$ R; $a \neq 0$, $b \neq 0$ is a linear equation in two variables

(i) $ax + c = 0$; $a \neq 0$

(ii) $by + c = 0$; $b \neq 0$ are linear equation in one variable

Simultaneous equation:

A pair of linear equation in two variables is said to form a system of simultaneous equation.

Solution of a linear equation in two variables:

The pair of values of x and y which satisfies the given equation is called a solution of the equation.

Graphical method of solution of pair (Simultaneous) of line:

Let us consider a linear equation $ax + by + c = 0$ where $a \neq 0$, $b \neq 0$

Step-I: Write down $y = -\left(\frac{ax+c}{b}\right)$

Step-II: Substitute any arbitrary value of x in step-I and obtain the corresponding value of y.

Step-III: Plot these points on the graph paper

Step-IV: Join these two points. The line thus obtained is the required graph of $ax + by + c = 0$

Ex.1 Draw the graph of the equation y – x = 2.

Sol. We have,

$y - x = 2$

$\Rightarrow\ y = x + 2$

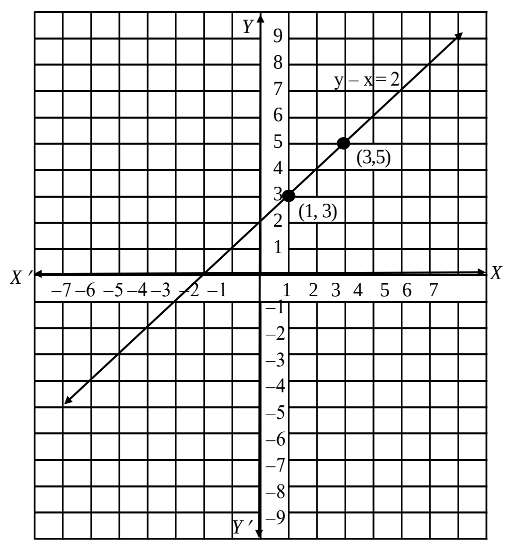
When x = 1, we have : $y = 1 + 2 = 3$

When x = 3, we have : $y = 3 + 2 = 5$

Thus, we have the following table exhibiting the abscissa and ordinates of points on the line represented by the given equation.

x	1	3
y	3	5

Plotting the points (1, 3) and (3, 5) on the graph paper and drawing a line joining them, we obtain the graph of the line represented by the given equation as shown in Fig.



Ex.2 Draw a graph of the line x – 2y = 3. From the graph, find the coordinates of the point when

(i) x = – 5 (ii) y = 0.

Sol. We have $x - 2y = 3 \Rightarrow y = \frac{x-3}{2}$

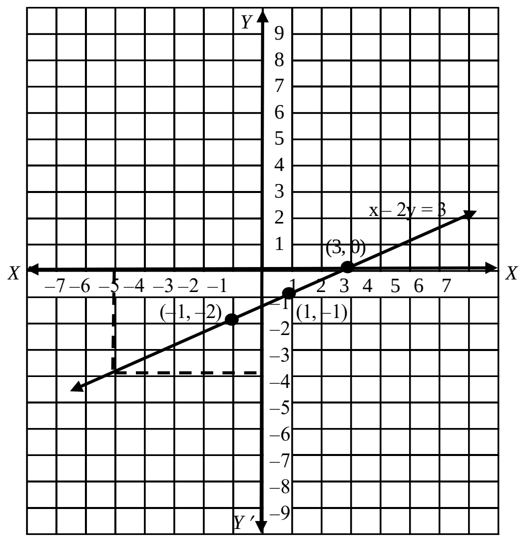
When x = 1, we have : $y = \frac{1-3}{2} = -1$

When x = –1, we have : $y = \frac{-1-3}{2} = -2$

Thus, we have the following table :

x	1	–1
y	–1	–2

Plotting points (1, –1) & (–1, –2) on graph paper & joining them, we get straight line as shown in fig. This line is required graph of equation x – 2y = 3.



To find the coordinates of the point when

x = –5, we draw a line parallel to y-axis and passing through (–5, 0). This line meets the graph of x – 2y = 3 at a point from which we draw a line parallel to x-axis which crosses y-axis at y = –4. So, the coordinates of the required point are (–5, –4).

Since y = 0 on x-axis. So, the required point is the point where the line meets x-axis. From the graph the coordinates of such point are (3, 0).

Hence, required points are (–5, –4) and (3, 0).

GRAPHICAL REPRESENTATION OF PAIR OF LINEAR EQUATIONS

Let the system of pair of linear equations be

$a_1x + b_1y = c_1$ ....(1)

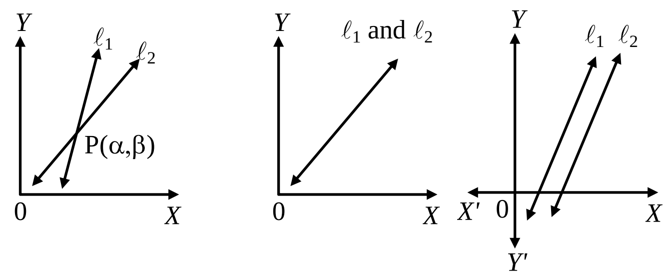
$a_2x + b_2y = c_2$ ....(2)

We know that given two lines in a plane, only one of the following three possibilities can happen -

(i) The two lines will intersect at one point.

(ii) The two lines will not intersect, however far they are extended, i.e., they are parallel.

(iii) The two lines are coincident lines.



Ex.3 The path of highway number 1 is given by the equation $x + y = 7$ and the highway number 2 is given by the equation $5x + 2y = 20$. Represent these equations geometrically.

Sol. We have, $x + y = 7$

$\Rightarrow y = 7 - x$ ....(1)

In tabular form

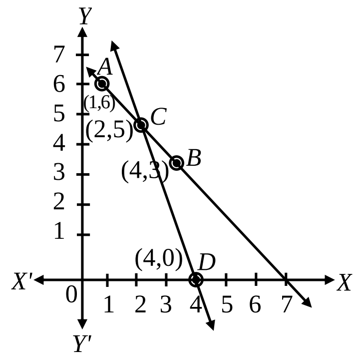
x	1	4
y	6	3
Points	A	B

and $5x + 2y = 20$

$\Rightarrow y = \frac{20-5x}{2}$ ....(2)

In tabular form

x	2	4
y	5	0
Points	C	D



Plot the points A (1, 6), B(4, 3) and join them to form a line AB.

Similarly, plot the points C(2, 5). D (4, 0) and join them to get a line CD. Clearly, the two lines intersect at the point C. Now, every point on the line AB gives us a solution of equation (1). Every point on CD gives us a solution of equation (2).

TYPES OF SOLUTION

There are three types of solutions :

1. Unique solution.
2. Infinitely many solutions
3. No solution.

(A) Consistent :

If a system of simultaneous linear equations has at least one solution then the system is said to be consistent.

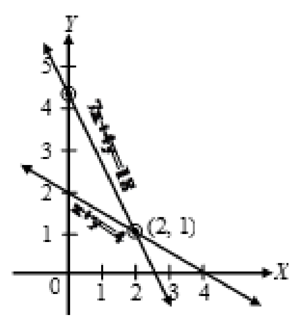
(i) Consistent equations with unique solution :

The graphs of two equations intersect at a unique point.

For example. Consider

$x + 2y = 4$

$7x + 4y = 18$

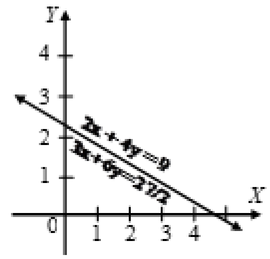


The graphs (lines) of these equations intersect each other at the point (2, 1) i.e., $x = 2, y = 1$.

Hence, the equations are consistent with unique solution.

(ii) Consistent equations with infinitely many solutions :

The graphs (lines) of the two equations will be coincident.



For example. Consider

$2x + 4y = 9 \qquad \Rightarrow 3x + 6y = \frac{27}{2}$

The graphs of the above equations coincide. Coordinates of every point on the lines are the solutions of the equations. Hence, the given equations are consistent with infinitely many solutions.

(B) Inconsistent Equation :

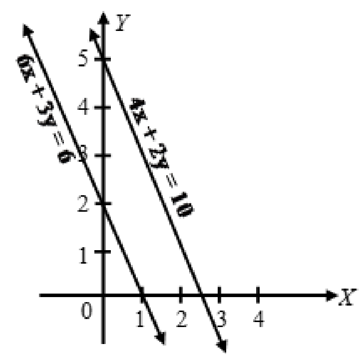
If a system of simultaneous linear equations has no solution, then the system is said to be inconsistent.

No Solution : The graph (lines) of the two equations are parallel.

For example. Consider

$4x + 2y = 10$

$6x + 3y = 6$



The graphs (lines) of the given equations are parallel. They will never meet at a point. So, there is no solution. Hence, the equations are inconsistent.

S.No	Graph of Two Equations	Types of Equations
1	Intersecting lines	Consistent, with unique solution
2	Coincident	Consistent with infinite solutions
3	Parallel lines	Inconsistent (No solution)

Ex.4 Show graphically that the system of equations

$x - 4y + 14 = 0$; $3x + 2y - 14 = 0$

is consistent with unique solution.

Sol. The given system of equations is

$x - 4y + 14 = 0$ ....(1)

$\Rightarrow y = \dfrac{x+14}{4}$

When $x = 6$, $y = \dfrac{6+14}{4} = 5$

When $x = -2$, $y = \dfrac{-2+14}{4} = 3$

In tabular form

x	6	–2
y	5	3
Points	A	B

$3x + 2y - 14 = 0$ ....(2)

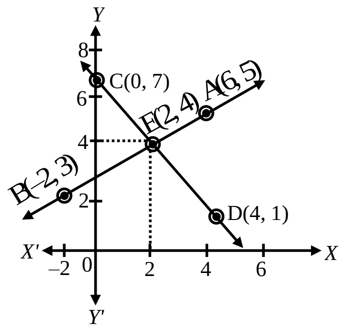
$\Rightarrow y = \dfrac{-3x+14}{2}$

When $x = 0$, $y = \dfrac{0+14}{2} = 7$

When $x = 4$, $y = \dfrac{-3\times4+14}{2} = 1$

In tabular form

x	0	4
y	7	1
Points	C	D



The given equations representing two lines, intersect each other at a unique point (2, 4). Hence, the equations are consistent with unique solution

Ex.5 Show graphically that the system of equations $2x + 5y = 16$; $3x + \frac{15}{2}y = 24$ has infinitely many solutions.

Sol. The given system of equations is

$2x + 5y = 16$ ....(1)

$\Rightarrow y = \frac{16-2x}{5}$

When $x = 3$, $y = \frac{16-6}{5} = 2$

When $x = -2$, $y = \frac{16-2\times(-2)}{5} = 4$

In tabular form

x	–2	3
y	4	2
Points	A	B

$3x + \frac{15}{2}y = 24$ ....(1)

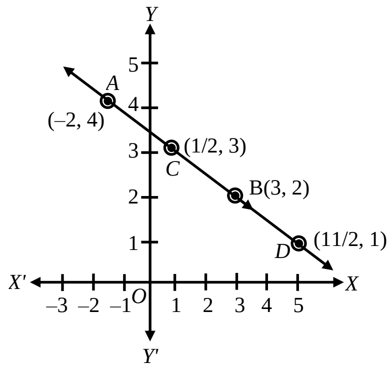
$\Rightarrow y = \frac{48-6x}{15}$ ....(2)

When $x = \frac{1}{2}$, $y = \frac{48-3}{15} = 3$

When $x = \frac{11}{2}$, $y = \frac{48-6\times\left(\frac{11}{2}\right)}{15} = 1$

In tabular form

x	$\frac{1}{2}$	$\frac{11}{2}$
y	3	1
Points	C	D



The lines of two equations are coincident. Coordinates of every point on this line are the solution.

Hence, the given equations are consistent with infinitely many solutions.

Pair of lines $a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	**Compare the ratio**
$2x+3y+4=0$ $5x+6y+9=0$	$\frac{2}{5}$	$\frac{3}{6}$	$\frac{4}{9}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$x+2y+5=0$ $3x+6y+15=0$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{5}{15}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$2x-3y+4=0$ $4x-6y+10=0$	$\frac{2}{4}$	$\frac{-3}{-6}$	$\frac{4}{10}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Graphical representation	Algebraic interpretation
Intersecting lines	Exactly one solution (unique)
Coincident lines	Infinitely many solutions
Parallel lines	No solution

From the table above you can observe that if the line $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i)	for the intersecting lines then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
(ii)	for the coincident lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(iii)	for the parallel lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$