PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

HOMOGENOUS EQUATION AND WORLD PROBLEMS

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The system of equations

$a_1x + b_1y = 0$

$a_2x + b_2y = 0$

called homogeneous equations has only solution $x = 0, y = 0$, when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

The system of equations has only one solution, and the system is consistent.

When $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

The system of equations has infinitely many solutions and the system is consistent.

Ex.1 Find the value of k for which the system of equations

$4x + 5y = 0; kx + 10y = 0$

has infinitely many solutions.

Sol. The given system is of the form

$a_1x + b_1y = 0$

$a_2x + b_2y = 0$

$a_1 = 4, b_1 = 5$ and $a_2 = k, \ b_2 = 10$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the system has infinitely many solutions.

$\Rightarrow \ \frac{4}{k} = \frac{5}{10}$

$\Rightarrow \ k = 8$

PROBLEMS BASED ON ARTICLES

Ex.2 The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, he buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball.

Sol. Let the cost of one bat be ₹ x and cost of one ball be ₹ y. Then

$7x + 6y = 3800$ ....(1)

$3x + 5y = 1750$ ....(2)

From (1) $y = \frac{3800 - 7x}{6}$

Putting $y = \frac{3800 - 7x}{6}$ in (2), we get

$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$ ....(3)

Multiplying (3) by 6, we get

$18x + 5(3800 - 7x) = 10500$

$\Rightarrow 18x + 19000 - 35x = 10500$

$\Rightarrow -17x = 10500 - 19000$

$\Rightarrow -17x = -8500 \quad \Rightarrow x = 500$

Putting $x = 500$ in (1), we get

$7(500) + 6y = 3800$

$\Rightarrow 3500 + 6y = 3800$

$\Rightarrow 6y = 3800 - 3500$

$\Rightarrow 6y = 300 \Rightarrow y = 50$

Hence, the cost of one bat = ₹ 500

and the cost of one ball = ₹ 50

PROBLEMS BASED ON NUMBERS

Ex.3 What number must be added to each of the numbers, 5, 9, 17, 27 to make the numbers in proportion ?

Sol. Four numbers are in proportion if

First × Fourth = Second × Third.

Let x be added to each of the given numbers to make the numbers in proportion. Then,

$(5 + x)(27 + x) = (9 + x)(17 + x)$

$\Rightarrow 135 + 32x + x^2 = 153 + 26x + x^2$

$\Rightarrow 32x - 26x = 153 - 135$

$\Rightarrow 6x = 18 \quad \Rightarrow x = 3$

PROBLEMS BASED ON AGES

Ex.4 Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Sol. Let the age of father = x years.

And the sum of the ages of his two children = y years

According to the question

Father's age = 3 × (sum of the ages of his two children)

$\Rightarrow x = 3y$ ....(1)

After 5 years

Father's age = (x + 5) years

sum of the ages of his two childrens = y + 5 + 5 = y + 10

[Age of his each children increases by 5 years]

According to the question,

After 5 years

Father's age = 2 × (sum of ages of his two children)

$\Rightarrow$ x + 5 = 2 × (y + 10)

$\Rightarrow$ x + 5 = 2y + 20

$\Rightarrow$ x – 2y = 15 ....(2)

Putting x = 3y from (1) in (2), we get

3y – 2y = 15

$\Rightarrow$ y = 15 years

And x = 3y $\Rightarrow$ x = 3 × 15 = 45

$\Rightarrow$ x = 45 years.

PROBLEMS BASED ON TWO DIGIT NUMBERS

Ex.5 The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

Sol. Let the unit's place digit be x and the ten's place digit be y.

$\therefore$ Original number = x + 10 y

The number obtained by reversing the digits = 10x + y

According to the question,

Original number + Reversed number = 99

$\Rightarrow$ (x + 10y) + (10x + y) = 99

$\Rightarrow$ 11x + 11y = 99

$\Rightarrow$ x + y = 9

$\Rightarrow$ x = 9 – y ....(1)

Given the difference of the digit = 3

$\Rightarrow$ x – y = 3 ....(2)

On putting the value of x = 9 – y from equation (1) in equation (2), we get

(9 – y) – y = 3 $\Rightarrow$ 9 – 2y = 3

$\Rightarrow$ 2y = 6 $\Rightarrow$ y = 3

Substituting the the value of y = 3 in equation (1), we get

x = 9 – y = 9 – 3 = 6

PROBLEMS BASED ON FRACTION

Ex.6 The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Sol. Let Numerator = x and Denominator = y

$\therefore$ Fraction = $\frac{x}{y}$

According to the first condition,

Numerator + denominator = 2 × numerator + 4

$\Rightarrow$ x + y = 2x + 4

$\Rightarrow$ y = x + 4 ....(1)

According to the second condition,

$$\frac{\text{Increased numerator by 3}}{\text{Increased denominator by 3}} = \frac{2}{3}$$

$\Rightarrow$ $\frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow$ 3x + 9 = 2y + 6

$\Rightarrow$ 3x – 2y + 3 = 0 ....(2)

Substituting the value of y form equation (1) into equation (2), we get

$3x - 2(x + 4) + 3 = 0$

$\Rightarrow\ 3x - 2x - 8 + 3 = 0$

$\Rightarrow\ x = 5$

On putting $x = 5$ in equation (1), we get

$y = 5 + 4$

$\Rightarrow\ y = 9$

Hence, the fraction $= \frac{x}{y} = \frac{5}{9}$

PROBLEM ON FIXED CHARGES & RUNNING CHARGES

Ex.7 A Taxi charges consist of fixed charges and the remaining depending upon the distance travelled in kilometers. If a persons travels 10 km, he pays ₹ 68 and for travelling 15 km, he pays ₹ 98. Express the above statements with the help of simultaneous equations and hence, find the fixed charges and the rate per km.

Sol. Let fixed charges of taxi = ₹ x.

And running charges of taxi = ₹ y per km.

According to the question,

Expenses of travelling 10 km = ₹ 68.

$\therefore\ x + 10y = 68$ ....(1)

Again expenses of travelling 15 km = ₹ 98.

$\therefore\ x + 15y = 98$ ....(2)

Subtracting equation (1) from equation (2), we get

$5y = 30 \quad \Rightarrow\ y = 6$

On putting $y = 6$ in equation (1), we have

$x + 10 \times 6 = 68$

$\Rightarrow$ x = 68 – 60

$\Rightarrow$ x = 8

Hence, fixed charges of taxi = x = ₹ 8 and running charges per km = y = ₹ 6.

PROBLEMS BASED ON SPEED & TIME

Ex.8 Places A and B are 100 km apart on the highway. One car stars from A and another from B at the same time. If the cars travel in the same direction at a different speed, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speed of the two cars ?

Sol. Let the speed of the first car, starting from

A = x km/hr.

And the speed of second car, starting from

B = y km/hr.

Distance travelled by first car in 5 hours = AC = 5x

Distance travelled by second car in 5 hours = BC = 5y

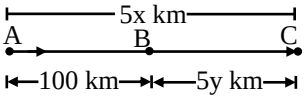
According to the question,

Let they meet at C, when moving in the same direction.

AC = AB + BC

5x = 100 + 5y

$\Rightarrow$ x = 20 + y ....(1)

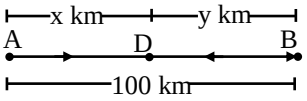


Distance = Speed × Time

When moving in the opposite direction, let they meet at D

Distance travelled by first car in 1 hour = AD = x. Distance travelled by second car in 1 hour

= BD = y



AD + BD = AB

$\Rightarrow$ x + y = 100 ....(2)

Substituting x = 20 + y from equation (1) in equation (2), we have

(20 + y) + y = 100

$\Rightarrow$ 20 + 2y = 100

$\Rightarrow$ 2y = 100 – 20 = 80

$\Rightarrow$ y = 40 km/hour

On putting y = 40 in equation (1), we get

x = 20 + 40 = 60 km/hour

Hence, the speed of first car = 60 km/hour

PROBLEMS BASED ON BOAT & STREAM

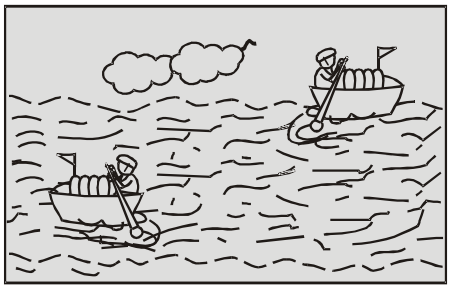
Ex.9 A boat goes 16 km upstream and 24 km downstream in 6 hours. It can go 12 km upstream and 36 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

Sol. Let the speed of stream = y km/hr ;

speed of boat in still water = x km/hr.

And the speed of boat in upstream = (x – y) km/hr.

The speed of boat in downstream = (x + y) km/hr.



According to the question,

Time taken in going 16 km upstream + time taken in going 24 km downstream = 6 hours.

$\Rightarrow \frac{16}{x-y} + \frac{24}{x+y} = 6$ ....(1)

$\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

Again, according to the question,

Time taken in going 12 km upstream + time taken in going 36 km downstream = 6 hours.

$\Rightarrow \frac{12}{x-y} + \frac{36}{x+y} = 6$ ....(2)

Let $\frac{1}{x-y} = p$, $\frac{1}{x+y} = q$

Equation (1) becomes $16p + 24q = 6$ ....(3)

Equation (2) becomes $12p + 36q = 6$ ....(4)

Multiplying equation (3) by 3 and equation (4) by 4, we get

$48p + 72q = 18$ ....(5)

$48p + 144q = 24$ ....(6)

Subtracting equation (5) from equation (6), we get

$72q = 6 \Rightarrow q = \frac{6}{72} = \frac{1}{12}$

Putting the value of q in equation (3), we get

$16p + 24\left(\frac{1}{12}\right) = 6$

$\Rightarrow 16p + 2 = 6$

$\Rightarrow 16p = 6 - 2 = 4$

$\Rightarrow p = 1/4$

$\therefore \quad \frac{1}{x-y} = \frac{1}{4}$ and $\frac{1}{x+y} = \frac{1}{12}$

$\Rightarrow$ x – y = 4 ....(7)

And, x + y = 12 ....(8)

By adding 2x = 16

$\Rightarrow$ x = 8

Putting x = 8 in equation (7), we get

8 – y = 4

$\Rightarrow$ y = 8 – 4 = 4

Hence, speed of boat in still water = 8 km/hr.

and speed of stream = 4 km/hr.