PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

ALGEBRAIC METHOD TO SOLVE LINEAR EQUATION OF TWO VARIABLE

Algebraic Method to Solve Linear Equation of Two Variable

There are three methods for solving the linear equation

(i) Substitution method

(ii) Elimination method

(iii) Cross-multiplication method

(i) Substitution method

Step–I: Obtain the two equation: $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Step–II: Find the value of one variable, say y in terms of the other variable.

Step–III: Substitute this value of y in the other equation and reduce it to an equation in one variable.

Ex. 1 Solve the following pair of equations by substitution method:

$2x + 3y = 9$; $3x + 4y = 5$

Sol. **Step–I:** $2x + 3y = 9$ ....(i) $y = \left(\frac{9-2x}{3}\right)$ ...(ii)

Step–II: Substitute the value of x in equation $3x + 4y = 5$

$3x + 4\left(\frac{9-2x}{3}\right) = 5$; $9x + 36 - 8x = 15$

$\{x = -21\}$

$x = 15 - 36$

$x = -21$

Step–III: Putting the value of x in equation (ii),

$$y = \left(\frac{9-2(-21)}{3}\right)$$

$$y = \frac{9+42}{3}$$

= y = 51/3 [y = 17]

Therefore the solution of the given system of equation is $x = -21$, y = 17

Ex.2 Solve the following pair of equations by substitution method:

x + 4y = 14 .....(i)

7x - 3y = 5 ....(ii)

Sol. From equation (i) x = 14 - 4y ....(iii)

Substitute the value of x in equation (ii)

⇒ 7 (14 - 4y) - 3y = 5

⇒ 98 - 28y - 3y = 5

⇒ 98 - 31y = 5

⇒ 93 = 31y

⇒ $y = \frac{93}{31} \Rightarrow y = 3$

Ex.3 Solve each of the following system of equations by eliminating x (by substitution) :

(i) x + y = 7 2x – 3y = 11

(ii) x + y = 7 12x + 5y = 7

(iii) 2x – 7y = 1 4x + 3y = 15

(iv) 3x – 5y = 1 5x + 2y = 19

(v) 5x + 8y = 9 2x + 3y = 4

Sol. (i) We have ;

x + y = 7 ....(1)

2x – 3y = 11 ....(2)

We shall eliminate x by substituting its value from one equation into the other. from equaton (1), we get ;

$x + y = 7 \quad \Rightarrow \; x = 7 - y$

Substituting the value of x in equation (2), we get ;

$2 \times (7 - y) - 3y = 11$

$\Rightarrow \; 14 - 2y - 3y \; = 11$

$\Rightarrow \; -5y = -3$ or, $y = 3/5$.

Now, substituting the value of y in equation (1), we get;

$x + 3/5 = 7 \quad \Rightarrow \; x = 32/5$.

Hence, $x = 32/5$ and $y = 3/5$.

(ii) We have,

$x + y = 7$ ....(1)

$12x + 5y = 7$ ....(2)

From equation (1), we have;

$x + y = 7$

$\Rightarrow \quad x = 7 - y$

Substituting the value of y in equation (2), we get ;

$\Rightarrow \quad 12(7 - y) + 5y = 7$

$\Rightarrow \quad 84 - 12y + 5y = 7$

$\Rightarrow \quad -7y = -77$

$\Rightarrow \quad y = 11$

Now, Substituting the value of y in equation (1), we get ;

$x + 11 = 7 \quad \Rightarrow \; x = -4$

Hence, $x = -4$, $\quad y = 11$.

(iii) We have;

$2x - 7y = 1$ ....(1)

$4x + 3y = 15$ ....(2)

From equation (1), we get

$2x - 7y = 1 \quad \Rightarrow x = \frac{7y+1}{2}$

Substituting the value of x in equation (2), we get ;

$\Rightarrow 4 \times \frac{7y+1}{2} + 3y = 15$

$\Rightarrow \frac{28y+4}{2} + 3y = 15$

$\Rightarrow 28y + 4 + 6y = 30$

$\Rightarrow 34y = 26 \quad \Rightarrow y = \frac{26}{34} = \frac{13}{17}$

Now, substituting the value of y in equation (1), we get;

$2x - 7 \times \frac{13}{17} = 1$

$\Rightarrow 2x = 1 + \frac{91}{17} = \frac{108}{17} \quad \Rightarrow \quad x = \frac{108}{34} = \frac{54}{17}$

Hence, $x = \frac{54}{17}, y = \frac{13}{17}$

(iv) We have ;

$3x - 5y = 1$ .... (1)

$5x + 2y = 19$ .... (2)

From equation (1), we get;

$3x - 5y = 1 \quad \Rightarrow x = \frac{5y+1}{3}$

Substituing the value of x in equation (2), we get ;

$\Rightarrow 5 \times \frac{5y+1}{3} + 2y = 19$

$\Rightarrow 25y + 5 + 6y = 57 \quad \Rightarrow \quad 31y = 52$

Thus, $y = \frac{52}{31}$

Now, substituting the value of y in equation (1), we get ;

$3x - 5 \times \frac{52}{31} = 1$

$\Rightarrow\ 3x - \frac{260}{31} = 1 \Rightarrow 3x = \frac{291}{31}$

$\Rightarrow\ x = \frac{291}{31 \times 3} = \frac{97}{31}$

Hence, $x = \frac{97}{31}, \quad y = \frac{52}{31}$

(v) We have,

$5x + 8y = 9$ ....(1)

$2x + 3y = 4$ ....(2)

From equation (1), we get;

$5x + 8y = 9 \quad \Rightarrow \quad x = \frac{9-8y}{5}$

Substituting the value of x in equation (2), we get ;

$\Rightarrow\ 2 \times \frac{9-8y}{5} + 3y = 4$

$\Rightarrow\ 18 - 16y + 15y = 20$

$\Rightarrow\ -y = 2$ or $y = -2$

Now, substituting the value of y in equation (1), we get ;

$5x + 8(-2) = 9$

$\Rightarrow\ 5x = 25 \quad \Rightarrow \quad x = 5$

Hence, $x = 5$, $y = -2$.

Now substitute value of y in equation (iii)

$\Rightarrow \quad 7x - 3(3) = 5$

$\Rightarrow \quad 7x - 3(3) = 5$

$\Rightarrow$ $7x = 14$

$\Rightarrow$ $x = \frac{14}{7} = 2$

So, solution is $x = 2$ and $y = 3$

Elimination by Equating The Coefficient