POLYNOMIALS

VALUE AND ZEROES OF POLYNOMIALS

Value of a polynomial :

The value of a polynomial f(x) at x = a is obtained by substituting x = a in the given polynomial and is denoted by f(a).

Consider the polynomial : $p(x) = 6x^2 + 7x - 2$

If we replace x by 1 everywhere in p(x), we get

$p(1) = 6(1)^2 + 7(1) - 2$

$= 6 + 7 - 2 = 11$

So, we say that the value of p(x) at x = 1 is 11.

Similarly, the value of polynomial

$f(x) = 3x^2 - 4x + 2$,

(i) at x = –2 is

$f(-2) = 3(-2)^2 - 4(-2) + 2$

$= 12 + 8 + 2 = 22$

(ii) at x = 0 is

$f(0) = 3(0)^2 - 4(0) + 2$

$= 0 - 0 + 2 = 2$

(iii) at $x = \frac{1}{2}$ is

$f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 2$

$= \frac{3}{4} - 2 + 2 = \frac{3}{4}$

Zero(es) / Root(s) of polynomials :

$x = r$ is a root or zero of a polynomial $p(x)$, if $p(r) = 0$.

In other words, $x = r$ is a root or zero of a polynomial $p(x)$, if it is a solution to the equation $p(x) = 0$.

The process of finding the zeros of $p(x)$ is nothing more than solving to the equation

$p(x) = 0$.

For example :

(i) For polynomial $p(x) = x - 2$; $p(2) = 2 - 2 = 0$

$x = 2$ or simply 2 is a zero of the polynomial

$p(x) = x - 2$.

(ii) For the polynomial $g(u) = u^2 - 5u + 6$;

$g(3) = (3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

3 is a zero of the polynomial $g(u) = u^2 - 5u + 6$.

Also, $g(2) = (2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$

2 is also a zero of the polynomial

$g(u) = u^2 - 5u + 6$

(a) Every linear polynomial has one and only one zero.

(b) A given polynomial may have more than one zeroes.

(c) If the degree of a polynomial is n; the largest number of zeroes it can have is also n.

For example:

If the degree of a polynomial is 5, the polynomial can have at the most 5 zeroes; if the degree of a polynomial is 8; largest number of zeroes it can have is 8.

(d) A zero of a polynomial need not be 0.

For example:

If $f(x) = x^2 - 4$,

then $f(2) = (2)^2 - 4 = 4 - 4 = 0$

Here, zero of the polynomial $f(x) = x^2 - 4$ is 2 which itself is not 0.

(e) 0 may be a zero of a polynomial.

For example:

If $f(x) = x^2 - x$,

then $f(0) = 0^2 - 0 = 0$

Here 0 is the zero of polynomial

$f(x) = x^2 - x$.

Ex.1 Show that $x = 2$ is a root of $2x^3 + x^2 - 7x - 6$

Sol. $p(x) = 2x^3 + x^2 - 7x - 6$.

Then, $p(2) = 2(2)^3 + (2)^2 - 7(2) - 6 = 16 + 4 - 14 = 0$

Hence $x = 2$ is a root of $p(x)$.

Ex.2 If $x=\frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. $f(x) = 6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right)=6\left(\frac{4}{3}\right)^3-11\left(\frac{4}{3}\right)^2+k\left(\frac{4}{3}\right)-20=0$$

$$6\left(\frac{64}{27}\right)-11\left(\frac{16}{9}\right)+\frac{4k}{3}-20=0\Rightarrow 6\left(\frac{64}{27}\right)-11\left(\frac{16}{9}\right)+\frac{4k}{3}-20=0$$

$128 - 176 + 12k - 180 = 0$

$12k + 128 - 356 = 0$

$12k = 228$

$k = 19$.

Ex.3 If $x = 2$ & $x = 0$ are roots of the polynomials $(f)x = 2x^3 - 5x^2 + ax + b$, then find the values of a and b.

Sol. $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow$ $16 - 20 + 2a + b = 0$

$\Rightarrow$ $2a + b = 4$ .....(i)

$\Rightarrow$ $f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\Rightarrow$ $b = 0$

$\Rightarrow$ $2a = 4$

$\Rightarrow$ $a = 2, b = 0.$