POLYNOMIALS

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(A) BASED ON DEGREE: If degree of polynomial is

			Examples
1	One	Linear	$x + 3$, $y - x + 2$, $\sqrt{3}x - 3$
2	Two	Quadratic	$2x^2 - 7$, $\frac{1}{3}x^2 + y^2 - 2xy$, $x^2 + 1 + 3y$
3	Three	Cubic	$x^3 + 3x^2 - 7x + 8$, $2x^2 + 5x^3 + 7$,
4	Four	bi-quadratic	$x^4 + y^4 + 2x^2y^2$, $x^4 + 3$,…

(B) BASED ON TERMS: If number of terms in polynomial is

			Examples
1.	One	Monomial	$7x$, $5x^9$, $\frac{7}{3}x^{16}$, xy, ……
2.	Two	Binomial	$2 + 7y^6$, $y^3 + x^{14}$, $7 + 5x^9$,…
3.	Three	Trinomial	$x^3 - 2x + y$, $x^{31} + y^{32} + z^{33}$,…..

Note : (1) Degree of constant polynomials

(**Ex.**5, 7, –3, 8/5, ...) is zero.

(2) Degree of zero polynomial (zero = 0

= zero polynomial) is not defined.

POLYNOMIAL IN ONE VARIABLE

If a polynomial has only one variable then it is called polynomial in one variable.

Ex. $P(x) = 2x^3 + 5x - 3$	Cubic trinomial
$Q(x) = 7x^7 - 5x^5 - 3x^3 + x + 3$	polynomial of degree 7
$R(y) = y$	Linear, monomial
$S(t) = t^2 + 3$	Quadratic Binomial

Note: General form of a polynomial in one variable x of degree 'n'

is $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$, $a_n \neq 0$, where $a_n, a_{n-1}, \ldots a_2, a_1, a_0$ all are constants.

for linear	$ax + b$,	$a \neq 0$
for quadratic	$ax^2 + bx + c$,	$a \neq 0$
for cubic	$ax^3 + bx^2 + cx + d$,	$a \neq 0$

REMAINDER THEOREM

(i) Remainder obtained on dividing polynomial p(x) by x – a is equal to p(a) .

(ii) If a polynomial p(x) is divided by (x + a) the remainder is the value of p(x) at x = –a.

(iii) (x – a) is a factor of polynomial p(x) if p(a) = 0

(iv) (x + a) is a factor of polynomial p(x) if p(–a) = 0

(v) (x – a) (x – b) is a factor of polynomial p(x), if p(a) = 0 and p(b) = 0.

Ex.1 Find the remainder when $4x^3 - 3x^2 + 2x - 4$ is divided by

(a) $x - 1$ (b) $x + 2$ (c) $x + \frac{1}{2}$

Sol. Let $p(x) = 4x^3 - 3x^2 + 2x - 4$

(a) When p(x) is divided by (x – 1), then by remainder theorem, the required remainder will be p(1)

$$p(1) = 4(1)^3 - 3(1)^2 + 2(1) - 4$$
$$= 4 \times 1 - 3 \times 1 + 2 \times 1 - 4$$
$$= 4 - 3 + 2 - 4 = -1$$

(b) When p(x) is divided by (x + 2), then by remainder theorem, the required remainder will be p (–2).

$$p(-2) = 4(-2)^3 - 3(-2)^2 + 2(-2) - 4$$
$$= 4 \times (-8) - 3 \times 4 - 4 - 4$$
$$= -32 - 12 - 8 = -52$$

(c) When p(x) is divided by, $\left(x+\frac{1}{2}\right)$ then by remainder theorem, the required remainder will be

$$p\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 4$$
$$= 4 \times \left(-\frac{1}{8}\right) - 3 \times \frac{1}{4} - 2 \times \frac{1}{2} - 4$$
$$= -\frac{1}{2} - \frac{3}{4} - 1 - 4 = \frac{1}{2} - \frac{3}{4} - 5$$
$$= \frac{-2-3-20}{4} = \frac{-25}{4}$$