POLYNOMIALS

GRAPH OF ZEROS OF POLYNOMIALS

Geometrical meaning of the zero(es) of a polynomial :

Zero(es) of a polynomial is/are the x-coordinate of the point(s) where the graph of $y = f(x)$ intersects the x-axis.

Graphs of polynomial :

In algebraic language the graph of a polynomial f(x) is the collection of all points (x,y), where y = f(x). In geometrical or in graphical language the graph of a polynomial f(x) is a smooth free hand curve passing through point (x_1, y_1), (x_2, y_2), (x_3, y_3)...........etc, where y_1, y_2, y_3 .... are the values of the polynomial f(x) at x_1, x_2, x_3 ....respectively. In order to draw the graph of a polynomial f(x), we may follow the following algorithm.

ALGORITHM

Step - 1: Find the values y_1, y_2...... y_n of polynomial f(x) at different points x_1, x_2......x_n and prepare a table that gives values of y or f(x) for various values of x.

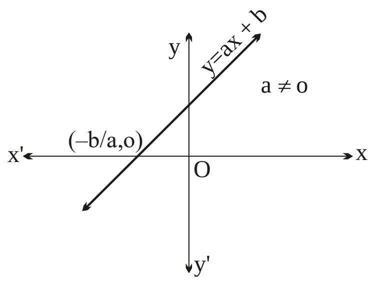
Step – 2: Plot the points $(x_1 y_1)$, (x_2,y_2), (x_3, y_3)........ (x_n, y_n) on rectangular coordinate system. In plotting these points we may use different scales on the x and y axes.

Step -3: Draw a free hand smooth curve passing through points plotted in step 2 to get the graph of the polynomial f(x).

x	x_1	x_2	....	x_n
$y = f(x)$	$y_1 = f(x_1)$	$y_2 = f(x_2)$	....	$y_n = f(x_n)$

Graph of a Linear Polynomial :-

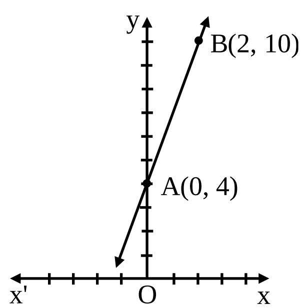
Consider a linear polynomial f(x) = ax + b, a0. We know that the graph of polynomial y = ax + b is a straight line, so it is called a linear polynomial. Since a straight line can be determined by two points, so only two points need to be poltted to draw the graph of y =ax +b. The graph of y = ax + b crosses the x axis at exactly one point namely $\left(\frac{-b}{a},0\right)$.



Let us consider linear polynomial ax + b. The graph of y = ax + b is a straight line.

For example : The graph of y = 3x + 4 is a straight line passing through (0, 4) and (2, 10).

x	0	2
y=3x+4	4	10
Points	A	B

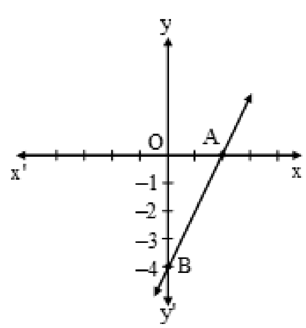


EXAMPLE:

Let us consider the graph of y = 2x – 4 intersects the x-axis at x = 2. The zero 2x – 4 is 2.

Thus, the zero of the polynomial 2x – 4 is the x-coordinate of the point where the graph y = 2x – 4 intersects the x-axis.

x	2	0
y=2x–4	0	–4
Points	A	B



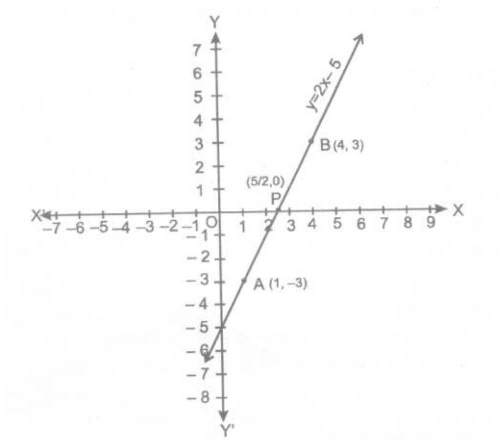
Ex.1 Draw the graph of the polynomial $f(x) = 2x - 5$. Also, find the coordinates of the point where it crosses X-axis.

Sol. Let $y = 2x - 5$.

The following table list the values of **y** corresponding to different values of **x.**

x	1	4
y	-3	3

The points A (1, - 3) and B (4, 3) are plotted on the graph paper on a suitable scale. A line is drawn passing through these points to obtain the graphs of the given polynomial.



Graph of a Quadratic Polynomial :

Consider a quadratic polynomial $f(x) = y = ax^2 + bx + c$. Where a, b and c be real numbers and a ^{1} 0. We know that the graph of quadratic polynomial is a cup shaped graph known as parabola.

In order to draw the graph of a quadratic polynomial f(x), we may follow the following algorithm.

Algorithm to draw the graph of quadratic polynomial:

Step 1: Write the given quadratic polynomial $f(x) = y = ax^2 + bx + c$.

Step 2: Determine the zero of the polynomial , if they exist. This can be done by putting $y = 0$ i.e. $ax^2 + bx + c = 0$.

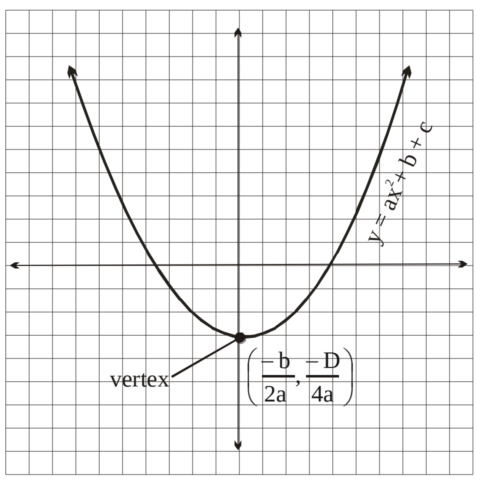
Step 3: Calculate the discriminant $D = b^2 - 4ac$

Step 4: Determine the point where the curve intersects y - axis. This can be done by putting $x = 0$ in the given function and calculating the value of y.

Step 5: Determine the vertex i.e., $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$.

Step 6: Prepare a table of selected values of x and corresponding values of y (generally two or three points on the left and two or three points on the right of the vertex.)

Step 7: Draw a smooth curve, through these points.



Graph of Quadratic Polynomial :

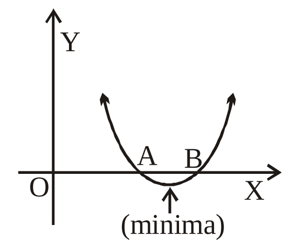
The graph of a quadratic polynomial $ax^2 + bx + c$, is a parabola which opens upward or downward as $a > 0$ or $a < 0$, we have the following possibilities :

Case-I: If $\mathbf{a > 0}$, then graph of a quadratic polynomial looks similar to one of the graphs in Figure (i), (ii) and (iii). In these figures parabola is opening upward.

(i) When $b^2 - 4ac > 0$

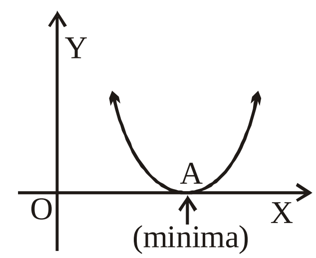
The graph $y = ax^2 + bx + c, a \neq 0$ cuts the x-axis at two distinct points A and B.

The x-coordinates of these points are the two zeroes of the polynomial $y = ax^2 + bx + c, a \neq 0$.



(ii) When $b^2 - 4ac = 0$

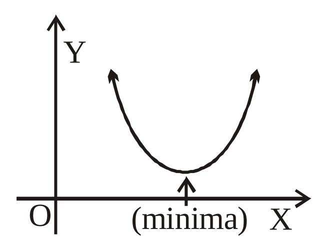
In this case, the graph of polynomial $y = ax^2 + bx + c, a \neq 0$, touches the x-axis at exactly one point A and whose coordinates are $\left(\frac{-b}{2a}, 0\right)$. So, in this case the x-coordinates of point A gives two equal zeros of the polynomial $y = ax^2 + bx + c, a \neq 0$.



(iii) When $b^2 - 4ac < 0$

(In this case polynomial $ax^2 + bx + c$ is not factorizable.)

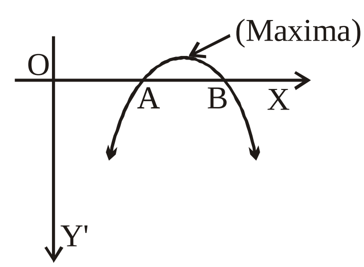
The graph of polynomial $y = ax^2 + bx + c, a \neq 0$ does not cut or touch x-axis. The curve of parabola remains completely above the x-axis.



Case-II : If $\mathbf{a < 0}$, then graph of the quadratic polynomial looks similar to one of the graphs in Figure (iv), (v) and (vi). In these figures parabola is opening downwards.

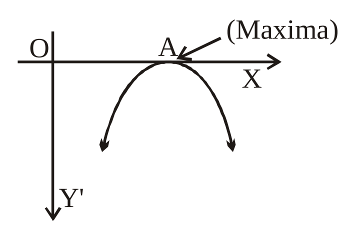
(iv) When $b^2 - 4ac > 0$

The graph $y = ax^2 + bx + c, a \neq 0$ cuts the x-axis at two distinct points A and B. The x-coordinates of these points are the two zeroes of the polynomial $y = ax^2 + bx + c, a \neq 0$.



(v) When $b^2 - 4ac = 0$

In this case, the graph of polynomial $y = ax^2 + bx + c, a \neq 0$, touches the x-axis at exactly one point A and whose coordinates

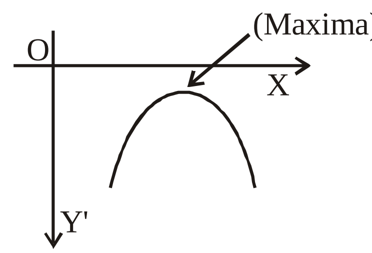


are $\left(\frac{-b}{2a}, 0\right)$. So, in this case the x-coordinates of point A gives two equal zeros of the polynomials $y = ax^2 + bx + c$, a ^{1} 0.

(vi) When $b^2 - 4ac < 0$

(In this case polynomial $ax^2 + bx + c$ is not factorizable.)

The graph of polynomial $y = ax^2 + bx + c$, a ^{1} 0 does not cut or touch x-axis. The curve of parabola remains completely below the x-axis.



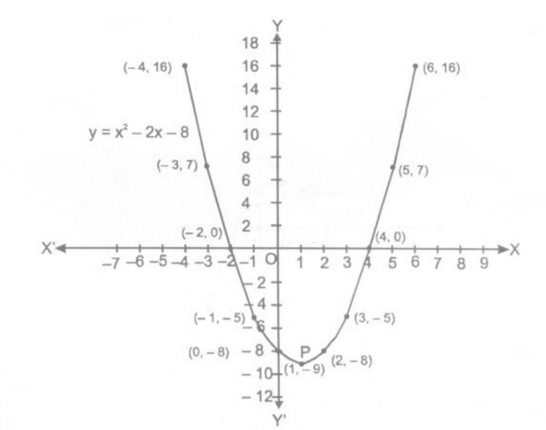
Ex.2 Draw the graph of the polynomial $f(x) = x^2 - 2x - 8$

Sol. Let $y = x^2 - 2x - 8$.

The following table gives the values of y or f(x) for various values of x.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
$y = x^2 - 2x - 8$	16	7	0	-5	-8	-9	-8	-5	0	7	16

Let us plot the points (-4, 16), (-3, 7), (-2, 0), (-1, -5), (0, - 8), (1, - 9), (2, - 8), (3, - 5), (4, 0), (5, 7) and (6, 16) on a graphs paper and draw a smooth free hand curve passing through these points. The curve thus obtained represents the graphs of the polynomial $f(x) = x^2 - 2x - 8$. This is called a parabola. The lowest point P, called a minimum points, is the vertex of the parabola. Vertical line passing through P is called the axis of the parabola. Parabola is symmetric about the axis. So, it is also called the line of symmetry.



Observations :

Fro the graphs of the polynomial $f(x) = x^2 - 2x - 8$, following observations can be drawn :

(i) The coefficient of x^2 in $f(x) = x^2 - 2x - 8$ is 1 (a positive real number) and so the parabola opens upwards.

(ii) $D = b^2 - 4ac = 4 + 32 = 36 > 0$. So, the parabola cuts X-axis at two distinct points.

(iii) On comparing the polynomial $x^2 - 2x - 8$ with $ax^2 + bx + c$, we get $a = 1$, $b = -2$ and $c = -8$.

The vertex of the parabola has coordinates (1, -9) i.e. $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$, where $D \equiv b^2 - 4ac$.

(iv) The polynomial $f(x) = x^2 - 2x - 8 = (x - 4)(x + 2)$ is factorizable into two distinct linear factors (x - 4) and (x + 2). So, the parabola cuts X-axis at two distinct points (4, 0) and (-2, 0). the x-coordinates of these points are zeros of f(x).

Ex.3 Draw the graphs of the quadratic polynomial $f(x) = 3 - 2x - x^2$.

Sol. Let $y = f(x)$ or, $y = 3 - 2x - x^2$.

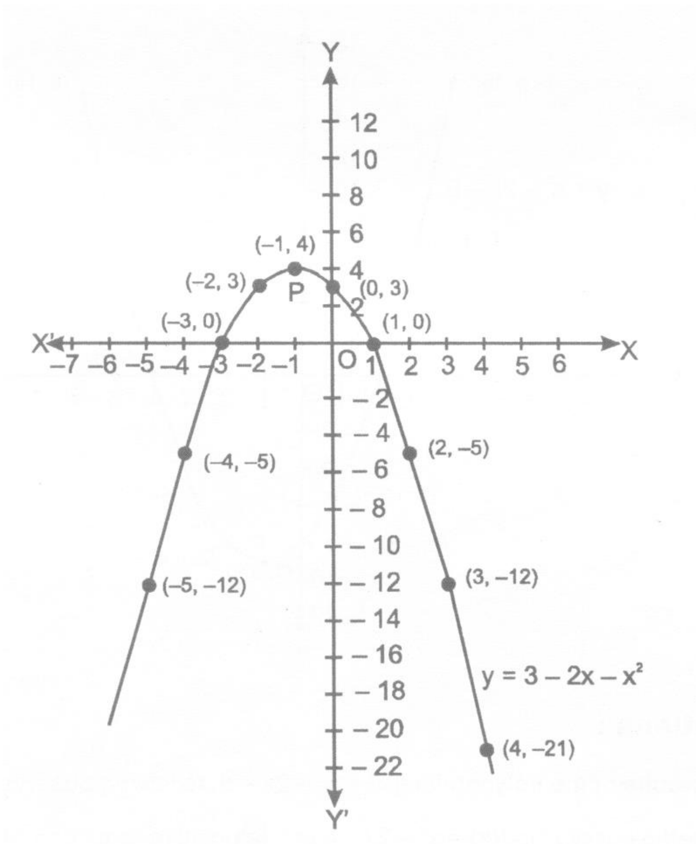
Let us list a few values of $y = 3 - 2x - x^2$ corresponding to a few values of x as follows:

x	-5	-4	-3	-2	-1	0	1	2	3	4
$y = 3 - 2x - x^2$	-12	-5	0	3	4	3	0	-5	-12	-21

Thus, the following points lie on the graph of the polynomial $y = 2 - 2x - x^2$:

(-5, -12), (-4, -5), (-3, 0), (-2, 4), (-1, 4), (0, 3), (1, 0), (2, - 5), (3, -12) and (4, - 21).

Let plot these points on a graph paper and draw a smooth free hand curve passing through these points to obtain the graphs of $y = 3 - 2x - x^2$. The curve thus obtained represents a parabola, as shown in figure. The highest point P(-1, 4), is called a maximum points, is the vertex of the parabola. Vertical line through P is the axis of the parabola. Clearly, parabola is symmetric about the axis.



Observations :

Following observations from the graph of the polynomial f(x) = 3 - 2x - x^2 is as follows :

(i) The coefficient of x^2 in f(x) = 3 - 2x - x^2 is - 1 i.e. a negative real number and so the parabola opens downwards.

(ii) D ≡ b^2 - 4ax = 4 + 12 = 16 > 0. So, the parabola cuts x-axis two distinct points.

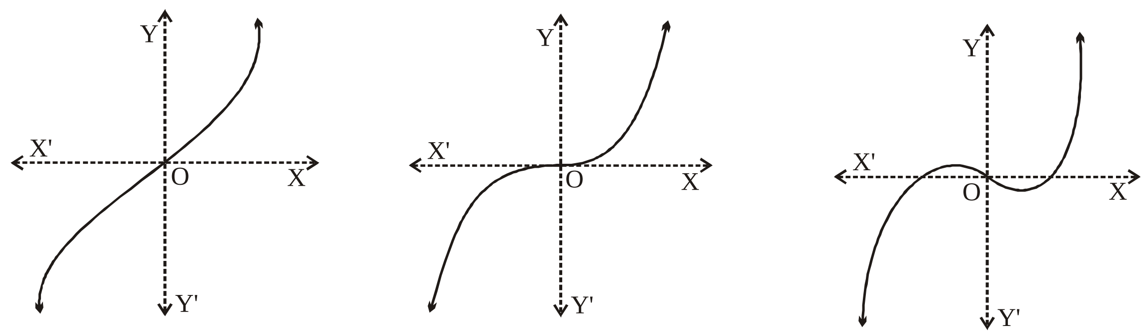
(iii) On comparing the polynomial 3 - 2x - x^2 with ax^2 + bc + c, we have a = - 1, b = - 2 and c = 3. The vertex of the parabola is at the point (-1, 4) i.e. at $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$, where D = b^2 - 4ac.

(iv) The polynomial $f(x) = 3 - 2x - x^2 = (1 - x)(x + 3)$ is factorizable into two distinct linear factors $(1 - x)$ and $(x + 3)$. So, the parabola cuts X-axis at two distinct points (1, 0) and (-3, 0). The co-ordinates of these points are zeros of f(x).

Graph of a cubic polynomial :

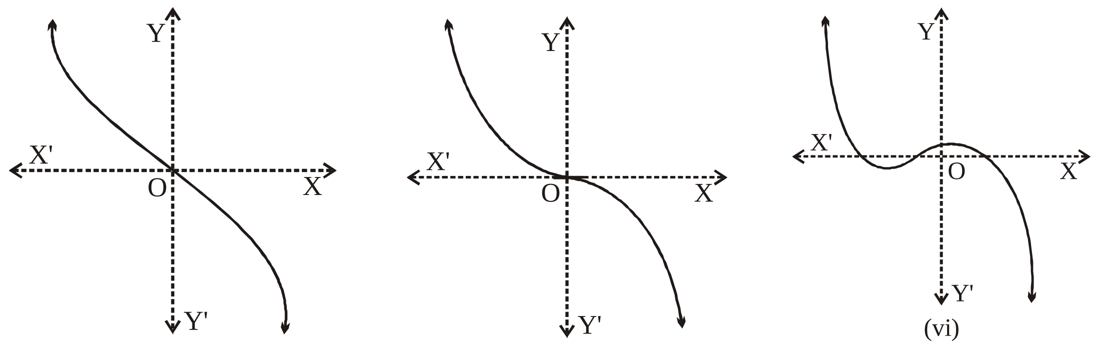
A cubic polynomial is a function of the form $y = ax^3 + bx^2 + cx + d$.
where $a \neq 0$, and a, b, c and d are real constants.

Case-I : If **$a > 0$**, then graph of a cubic function looks similar to one of the graphs in Figure (i), (ii) and (iii).



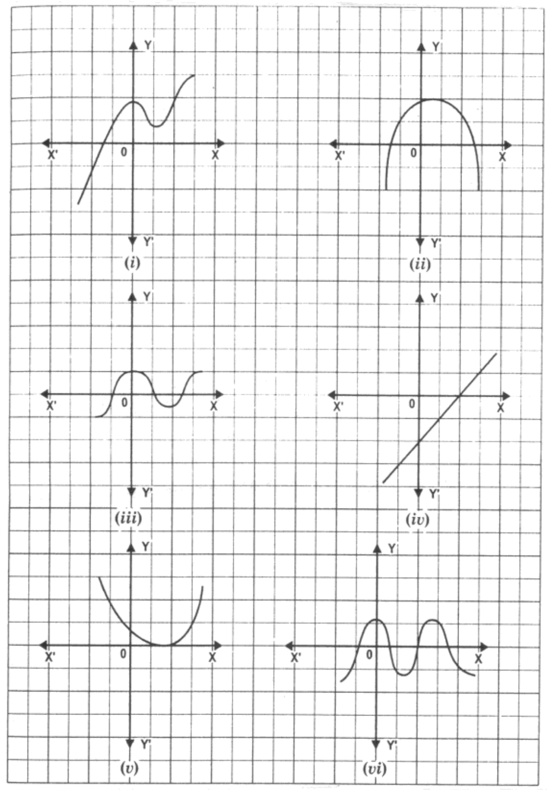
(i) (ii) (iii)

Case-II : If **$a < 0$**, then graph of the cubic function looks similar to one of the graphs in Figure (iv), (v) and (vi).



(iv) (v) (vi)

Ex.4: Look at the graphs given below. Each is the graph of $y = p(x)$ where p(x) is a polynomial. For each of the following graphs, find the number of zeroes of p(x).



Sol: (i) Since the graph intersects the x-axis at one point only, the polynomial p(x) has only one zero.

(ii) Since the graph intersects the x-axis at two points, the polynomial p(x) has two zeroes.

Ex.4 Draw the graphs of the polynomial $f(x) = x^3 - 4x$.

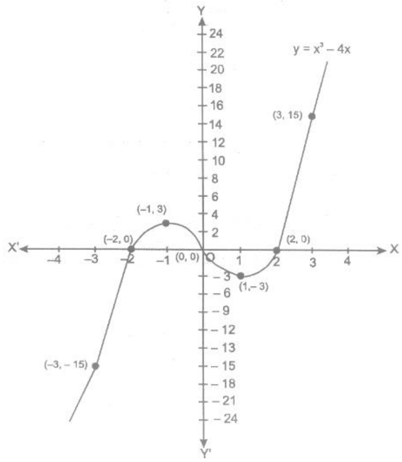
Sol. Let $y = f(x)$ or, $y = x^2 - 4x$.

The values of y for variable value of x are listed in the following table :

x	-3	-2	-1	0	1	2	3
$y = x^3 - 4x$	-15	0	3	0	-3	0	15

Thus, the curve $y = x^3 - 4x$ passes through the points (-3, -15), (-2, 0), (-1, 3), (0 ,0), (1, -3), (2, 0), (3, 15), (4, 48) etc.

Plotting these points on a graph paper and drawing a free hand smooth curve through these points, we obtain the graph of the given polynomial as shown figure.



Observations :

For the graphs of the polynomial $f(x) = x^3 - 4x$, following observations are as follows :-

(i) The polynomial $f(x) = x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2)$ is factorizable into three distinct linear factors. The curve $y = f(x)$ also cuts X-axis at three distinct points.

(ii) We have, $f(x) = x(x - 2)(x + 2)$

Therefore 0, 2 and -2 are three zeros of f(x). The curve y = f(x) cuts X-axis at three points O (0, 0), P(2, 0) and Q (-2, 0).