PROBABILITY

BASIC CONCEPT OF PROBABILITY

INTRODUCTION

In our day-to-day conversation, we generally use the phrases like :

(i) **Probably,** Satya will visit my house today.

(ii) **Most probably,** Megha is preparing for CAT.

(iii) Khusboo is **quite sure** to be on the top.

(iv) **Chances** are high that Regi will head the organisation.

The words' probably', 'most probably', 'quite sure', 'chances' etc involve an element of uncertainty.

Probability - Probability is the mathematical measurement of uncertainty.

Probability Theory - It is that branch of mathematics in which the degree of uncertainty (or certainty of occurrence of event) is measured numerically.

SOME BASIC CONCEPTS/TERMS

1. **Experiment :**

   An action or operation which can produce some well defined result is known as **experiment.**

2. **Deterministic experiment :**

If we perform an experiment and repeat it under identical conditions, we get almost the same result every time, such an experiment is called a **deterministic experiment.**

3. **Random experiment :**

An experiment is said to be a random experiment if it satisfies the following two conditions :

(i) It has more than one possible outcomes.

(ii) It is not possible to predict the outcome (result) in advance.

Ex (i) Tossing a pair of fair coins. (ii) Rolling an unbiased die.

4. **Outcomes :** The possible results of a random experiment are called **outcomes.**

5. **Trial :**

When an experiment is repeated under similar conditions and it does not give the same result each time but may result in any one of the several possible outcomes, the result is called a **trial.**

Ex If a coin is tossed 100 times, then one toss of the coin is called a trial.

6. **Event :** The collection of all or some outcomes of a random experiment is called an **event.**

Ex. Suppose we toss a pair of coins simultaneously and let E be the event of getting exactly one head. Then, the even E contains HT and TH.

Ex. Suppose we roll a die and let E be the event of getting an even number. Then the event E contains 2, 4 and 6.

7. **Elementary or Simple Event :** An outcome of a trial is called an **elementary event.**

NOTE : An elementary event has only one element.

Ex. Let a pair of coins is tossed simultaneously. Then, possible outcomes of this experiment are.

HH : Getting H on first and H on second (= E_1) [H = Head, T = Tail and E = event]

HT : Getting H on first and T on second (= E_2)

TH : Getting T on first and H on second (= E_3)

TT : Getting T on first and T on second (= E_4)

Here, E_1, E_2, E_3 and E_4 are the elementary events associated with the random experiment of tossing of two coins.

8. **Compound event or composite event or mixed event :**

An event associated to a random experiment and obtained by combining two or more simple events associated to the same random experiment, is called a **compound event.**

OR

A compound event is an aggregate of some simple (elementary) event and is decomposable into simple events.

Ex. If we throw a die, then the event E of getting an odd number is a compound event because the event E contains three elements 1, 3 and 5, which is a compound of three simple events E_1, E_2 and E_3 containing 1, 3 and 5 respectively.

9. **Equally likely events :**

The out comes of an experiment are said to be equally likely events if the chances of their happenings are neither less nor greater than other.

In other words, a given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

Ex. In tossing a coin, getting head (H) and tail (T) are equally likely events.

MATHEMATICAL DEFINITION OF PROBABILITY

Let there are n exhaustive, mutually exclusive and equally likely cases for an event A and m of those are favourable to it, then probability of happening of the event A is defined by the ratio m/n which is denoted by P(A). Thus

$$P(A) = \frac{m}{n}$$

$$= \frac{\text{No of favourable cases to A}}{\text{No of exhaustive cases to A}}$$

Note : It is obvious that $0 \le m \le n$. If an event A is certain to happen, then m = n thus

P (A) = 1.

If A is impossible to happen then m = 0 and so

P (A) = 0. Hence we conclude that

$0 \le P(A) \le 1$

Further, if $\bar{A}$ denotes negative of A i.e. event that A doesn't happen, then for above cases m, n ; we shall have

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$\therefore$ P (A) + P ($\overline{A}$) = 1

Ex.1 If the probability of winning a game is 0.3, what is the probability of lossing it ?

Sol. Probability of winning a game = 0.3.

Probability of losing it = q (say).

$\Rightarrow$ 0.3 + q = 1

$\Rightarrow$ q = 1 – 0.3

$\Rightarrow$ q = 0.7

Ex.2 A bag contains 12 balls out of which x are white,

(i) If one ball is drawn at random, what is the probability that it will be a white ball?

(ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will double than that in (i). Find x.

Sol. Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

$\therefore$ Total number of possible outcomes = 12

Number of white balls = x

(i) Out of total 12 outcomes, favourable outcomes = x

$$P(\text{White ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x}{12}$$

(ii) If 6 more white balls are put in the bag, then

Total number of white balls = x + 6

Total number of balls in the bag = 12 + 6 = 18

P(White ball) $= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x+6}{12+6}$

According to the question,

Probability of drawing white ball in second case

= 2 × probability drawing of white ball in first case

$\Rightarrow \frac{x+6}{18} = 2\left(\frac{x}{12}\right) \quad \Rightarrow \frac{x+6}{18} = \frac{x}{6}$

$\Rightarrow$ 6x + 36 – 18x

$\Rightarrow$ 12x = 36

$\Rightarrow$ x = 3

Hence, number of white balls = 3

Ex.3 What is the probability that a leap year, selected at random will contain 53 Sundays?

Sol. Number of days in a leap year = 366 days

Now, 366 days = 52 weeks and 2 days

The remaining two days can be

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

For the leap year to contain 53 Sundays, last two days are either Sunday and Monday or Saturday and Sunday.

$\therefore$ Number of such favourable outcomes = 2

Total number of possible outcomes = 7

$\therefore$ P(a leap year contains 53 sundays) = $\frac{2}{7}$

Ex.4 A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at random, find the probability that it is

(i) Black (ii) Red (iii) Not green.

Sol. Total number of balls in the bag = 5 + 8 + 4 + 7 = 24

$\therefore$ Total number of elementary events = 24

(i) There are 7 black balls in the bag.

$\therefore$ Favourable number of elementary events = 7

Hence, P (Getting a black ball) = $\frac{7}{24}$.

(ii) There are 5 red balls in the bag.

$\therefore$ Favourable number of elementary events = 5

Hence, P (Getting a red ball) = $\frac{5}{24}$

(iii) There are 5 + 8 + 7 = 20 balls which are not green.

$\therefore$ Favourable number of elementary events = 20

Hence, P (No getting a green ball) = $\frac{20}{24}=\frac{5}{6}$