STATISTICS

CONCEPT OF ARITHMETIC MEAN

MEAN (ARITHMETIC MEAN OF INDIVIDUAL OBSERVATIONS) OR UNGROUPED DATA

Mean of 'n' numbers $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n = \frac{sum\ of\ observation}{number\ of\ observation}$

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots\ldots\ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Ex. 1 If the heights of 5 persons are 144 cm, 152 cm, 151 cm, 158 cm and 155 cm respectivly find the mean height.

Sol. Mean height $= \frac{144+152+151+158+155}{5} = \frac{760}{5} cm = 152 cm$

Ex.2 Neeta and her four friends secured 65, 78, 82, 94 and 71 marks in a test of mathematics. Find the average (arithmetic mean) of their marks.

Sol. Arithmetic mean or average

$$= \frac{65+78+82+94+71}{5} = \frac{390}{5} = 78$$

Hence, arithmetic mean = 78

Ex.3 The marks obtained by 10 students in physics out of 40 are 24, 27, 29, 34, 32, 19, 26, 35, 18, 21. Compute the mean of the marks.

Sol. Mean of the marks is given by

$$\overline{x} = \frac{24+27+29+34+32+19+26+35+18+21}{10}$$

$$= \frac{265}{10} = 26.50$$

Ex.4 The mean of 20 observations was found to be 47. But later it was discovered that one observation 66 was wrongly taken as 86. Find the correct mean.

Sol. Here, $n = 20$, $\bar{X} = 47$

We have, $\bar{X} = \dfrac{\sum_{i=1}^{n} x_i}{n}$ $\therefore\ 47 = \dfrac{\sum_{i=1}^{n} x_i}{20}$

$\sum_{i=1}^{n} x_i = 47 \times 20 = 940.$

But the score 66 was wrongly taken as 86.

$\therefore$ Correct value of $\sum_{i=1}^{n} x_i = 940 + 66 - 86 = 920$

$\therefore$ Correct mean $= \dfrac{920}{20} = 46$

Ex.5 If $\bar{X}$ denote the mean of $x_1, x_2, ..., x_n$, show that $\sum_{i=1}^{n} = (x_i - \bar{x})$

Sol. $\bar{x} = \dfrac{x_1 + x_2 + ... + x_n}{n}$

$= x_1 + x_2 + ... + x_n = n\bar{X}$ (i)

$= \Sigma(x_1 - \bar{X}) = (x_1 - \bar{X}) + (x_2 - \bar{X}) + ... + (x_n - x_1)$

$= (x_1 + x_2 + ... + x_n) - n\bar{X} = n\bar{X} - n\bar{X}$

$= 0$ (from (i))

MEAN OF UNGROUPED DATA OR DISCRETE FREQUENCY DISTRIBUTION

(i) Direct method

(ii) Assumed mean method

(iii) Step deviation method or shortcut method

1 **Direct method (**For the discrete frequency distribution)

x_i	x_1	x_2	x_3	....	x_n
f_i	f_1	f_2	f_3	....	f_n

Mean of 'n' observations x_1, x_2, x_3, x_4 ....x_n with frequencies f_1, f_2, f_3......f_n is given by

Mean = $X = \dfrac{f_1x_1 + f_2x_2 + f_3x_3 .... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n} = \sum_{i=1}^{n} f_i$

Ex. 6 Find the mean of the following distribution:

X:	**4**	**6**	**9**	**10**	**15**
F:	**5**	**10**	**10**	**7**	**8**

Sol.

x_i	f_i	f_ix_i
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
	$N = \sum f_i$ $= 40$	$\sum f_ix_i = 360$

Mean

$$\overline{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{360}{40} = 9$$

$$\overline{X} = 9$$

Ex. 7 Find the value of K if mean of the following data is 14.

x_i	5	10	15	20	25
f_i	7	k	8	4	5

Sol.

x_i	f_i	$f_i x_i$
5	7	35
10	k	10k
15	8	120
20	4	80
25	5	125
Total	$\sum_{i=1}^{n} f_i = 24 + k$	$\sum_{i=1}^{n} f_i x_i = 360 + 10k$

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$\Rightarrow \quad 14 = \dfrac{360 + 10k}{24 + k}$

$\Rightarrow \quad 336 + 14\text{ k} = 360 + 10\text{ k}$

$\Rightarrow \quad 14\text{ k} - 10\text{ k} = 360 - 336$

$\Rightarrow \quad 4\text{ k} = 24$

$\Rightarrow \quad \text{k} = 6$

Ex.8 Find the mean of the following distribution :

x :	4	6	9	10	15
F:	5	10	10	7	8

Sol. Calculation of Arithmetic Mean

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
$N = \Sigma f_i = 40$		$\Sigma f_i x_i = 360$

$\therefore$ Mean = $\bar{X} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{360}{40} = 9$

ASUUMED MEAN METHOD

$$\text{Arithmetic mean} = a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

Note : The assumed mean is chosen, in such a manner, that

1. It should be one of the central values.
2. The deviation are small.
3. One deviation is zero.

Working Rule :

Step 1 : Choose a number 'a' from the central values of x of the first column, that will be our assumed mean.

Step 2 : Obtain deviations d_i by subtracting 'a' from x_i. Write down hese deviations against the corresponding frequencies in the third column.

Step 3 : Multiply the frequencies of second column with corresponding deviations d_i in the third column to prepare a fourth column of $f_i d_i$.

Step 4 : Find the sum of all the entries of fourth column to obtain $\sum f_i d_i$ and also, find the sum of all the frequencies in the second column to obtain $\sum f_i$.

Ex. 9 The following table shows the weights of 12 students:

Weight (kg)	67	70	72	73	75
Number of students	4	3	2	2	1

Find the mean weight

Sol. Let the assumed mean be $a = 72$

Weight (in kg) x_i	Number of students f_i	$d_i = x_i - a$ $= x_i - 72$	$f_i d_i$
67	4	-5	-20
70	3	-2	-6
72	2	0	0
73	2	1	2
75	1	3	3
	$N = \Sigma_{f_i} = 12$		$\sum f_i d_i = -21$

We have $N = 12$ $f_i d_i = -21$ and $a = 72$

$\therefore$ Mean = $a + \frac{1}{N}\sum f_i d_i$

$$72 + \left(\frac{-21}{12}\right) = 72 - \frac{7}{4} = \frac{288-7}{4} = \frac{281}{4}$$

$= 70.25$ kg

Ex.10 The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure (in rupees)	100-150	150-200	200-250	250-300	300-350	350-400	400-450	450-500
Frequency	24	40	33	28	30	22	16	7

Sol. Let assumed mean = 275

Expenditure (in rupees)	Frequency (f_i)	Mid value (x_i)	$d_i = x - 275$	$f_i d_i$
100–150	24	125	–150	–3600
150–200	40	175	–100	–4000
200–250	33	225	–50	–1650
250– 300	28	275	0	0
300– 350	30	325	50	1500
350– 400	22	375	100	2200
400– 450	16	425	150	2400
450– 500	7	475	200	1400
	$\sum f_i = 200$			$\sum f_i d_i = -1750$

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 275 + \frac{-1750}{200} = \text{Rs } 266.25$$

Step Deviation Method

Deviation method can be further simplified on dividing the deviation by width of the class interval h. In such a case the arithmetic mean is reduced to a great extent.

$\Rightarrow$ Mean $(\bar{x}) = a + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times h$

Working Rule :

Step-1 : Choose a number 'a' from the central values of x(mid-values)

Step-2 : Obtain $u_i = \dfrac{x_i - a}{h}$

Step-3 : Multiply the frequency f_i with the corresponding u_i to get $f_i u_i$.

Step-4 : Find the sum of all $f_i u_i$ i.e., $\Sigma f_i u_i$

Step-5 : Use the formula $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} . h$ to get the required mean.

Ex.11 Apply step- deviation method to find the AM of the distribution

Variate(x):	5	10	15	20	25	30	35	40	45	50
Frequency(f):	20	43	75	67	72	45	39	9	8	6

Sol. Let the assumed mean be a = 25 & h = 5

Variate x_i	Frequency f_i	Deviations $d_i = x_i - 25$	$u_i = \frac{x_i - 25}{5}$	$f_i u_i$
5	20	-20	-4	-80
10	43	-15	-3	-129
15	75	-10	-2	-150
20	67	-5	-1	-67
25	72	0	0	0
30	45	5	1	45
35	39	10	2	78
40	9	15	3	27
45	8	20	4	32
50	6	25	5	30
	N = f_i = 384			$f_i u_i$ = - 214

We have N = 384, a = 25, h = 5, and $f_i u_i$ = 214

∴ Mean = = a + h

$$\therefore \text{Mean} = \bar{X} = A + h\left(\frac{1}{N}\sum f_i u_i\right)$$

$$\text{Mean} = 25 + 5 \times \left(\frac{-214}{384}\right)$$

25 – 2 : 786

$$\left[\bar{X} \quad = 22 : 214\right]$$

Ex.12 To find out the concentration of SO_2 in the air (in parts per million, i.e.ppm), the data was collected for 30 localities in a certain city and is presented below :

Concentration of SO_2 (in ppm)	Frequency
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2

Find the mean concentration of SO_2 in the air.

Sol. Let the assumed mean a = 0.10.

Concentration of SO_2 (in ppm)	Frequency f_i	Mid value x_i	$u_i = \frac{x_i - 0.10}{0.04}$	$f_i u_i$
0.00 – 0.04	4	0.02	–2	–8
0.04 – 0.08	9	0.06	–1	–9
0.08 – 0.12	9	0.10	0	0
0.12 – 0.16	2	0.14	1	2
0.16 – 0.20	4	0.18	2	8
0.20 – 0.24	2	0.22	3	6
	$\sum f_i = 30$			$\sum f_i u_i = -1$

By step deviation method

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 0.10 + \frac{-1}{30} \times 0.04$$

$$= 0.10 - 0.0013$$

$$= 0.0987$$

$$= 0.099 \text{ ppm}$$

SOME IMPORTANT RESULTS ABOUT MEAN :

1. The algebraic sum of deviations taken about the mean is zero i.e., $\sum_{i=1}^{n}\left(x_i - \bar{x}\right) = 0$

2. The value of the mean depends on all the observations.

3. The A.M. of two numbers a and b is $\frac{a+b}{2}$

4. **Combined mean :** If and are the arithmetic means of two series with n_1 and n_2 observations respectively, then the combined mean is :

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Note: The above result can be extended to any number of groups of data.

5. If is the mean of $x_1, x_2, ...x_n$, then the mean of $x_1 + a, x_2 + a, x_3 + a, ...x_n$ +ais + a, for all values of a.

6. If is the mean of $x_1, x_2, ...x_n$, then the mean of $ax_1, ax_2, ... ax_n$ is a and that of $\frac{x_1}{a}, \frac{x_2}{a}, ... \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$.

7. The mean of the first n natural numbers is $\left(\frac{n+1}{2}\right)$

8. The mean of the square of the first n natural numbers = $\frac{(n+1)(2n+1)}{6}$

9. The mean of the cubes of the first n natural numbers = $\frac{n(n+1)^2}{4}$

10. The mean cannot be calculated graphically.