AREAS RELATED TO CIRCLE

SEGMENT OF CIRCLE AND ITS AREA

Segment of Circle and its Area

Segment of a circle :

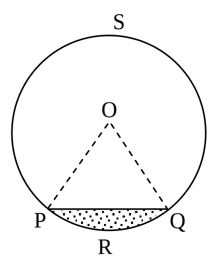
The region enclosed by an arc and a chord is called the segment of the circle.

Minor segment :

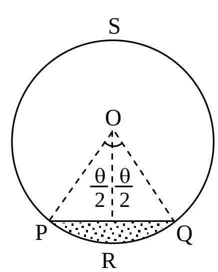
If the boundary of a segment is a minor arc of a circle, then the corresponding segment is called a minor segment.

Major segment :

A segment corresponding a major arc of a circle is known as the major segment.



Area of the sector OPRQ = Area of the segment PRQ + Area of ΔOPQ



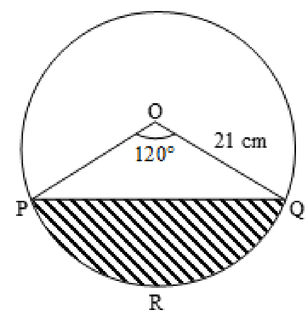
$\Rightarrow$ Area of segment PRQ = $\left\{\frac{\pi}{360}\times\theta-\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$

Ex.1 Find the area of the segment of a circle,given that the angle of the sector is 120º and the radius of the circle is 21 cm. (Take $\pi = 22/7$)

Sol. Here, r = 21 cm and $\pi = 120$

$\therefore$ Area of the segment

$$= \left\{\frac{\pi}{360}\times\theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2$$



$$= \left\{\frac{22}{7}\times\frac{120}{360} - \sin 60^\circ \cos 60^\circ\right\}(21)^2 \text{ cm}^2$$

$$= \left\{\frac{22}{21} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right\}(21)^2 \text{ cm}^2$$

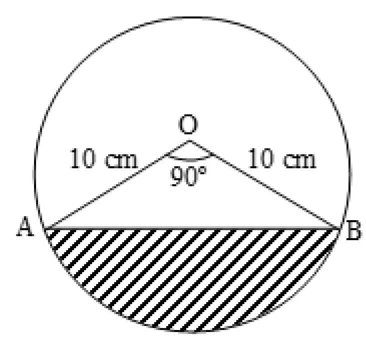
$$= \left\{\frac{22}{21}\times(21)^2 - (21)^2\times\frac{\sqrt{3}}{4}\right\}\text{cm}^2$$

$$= \left(462 - \frac{441}{4}\sqrt{3}\right)\text{cm}^2 = \frac{21}{4}(88 - 21\sqrt{3})\ \text{cm}^2$$

Ex.2 A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of major and minor segments (Take $\pi = 3.14$)

Sol. We know that the area of a minor segment of angle θº in a circle of radius r is given by

$$A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2$$



Here, r = 10 and θ = 90º

$$\therefore\ A = \left\{\frac{3.14\times 90}{4} - \sin 45^\circ \cos 45^\circ\right\}(10)^2 \text{ cm}^2$$

$$\Rightarrow\ A = \left\{\frac{3.14}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right\} 10)^2 \text{ cm}^2$$

$$\Rightarrow\ A = \{3.14\times 25 - 50\}\text{cm}^2 = (78.5 - 50)\ \text{cm}^2$$

$$= 28.5 \text{ cm}^2$$

Area of the major segment = Area of the circle – Area of the minor segment

$$= \{3.14\times 10^2 - 28.5\}\ \text{cm}^2$$

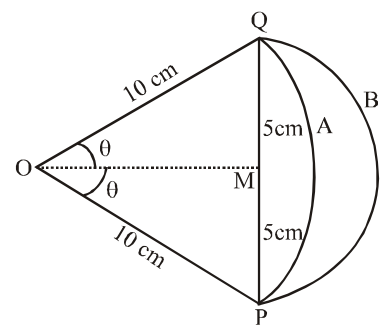
$= (314 - 28.5)\ cm^2 = 285.5\ cm^2$

Ex.3 The diagram shows two arcs, A and B. Arc A is part of the circle with centre O and radius of PQ. Arc B is part of the circle with centre M and radius PM, where M is the mid-point of PQ. Show that the area enclosed by the two arcs is equal to $25\left(\sqrt{3}-\frac{\pi}{6}\right)$ cm^2.

Sol. We have, Area enclosed by arc B and chord

PQ = Area of semi-circle of radius 5 cm

$= 1/2 \times \pi \times 5^2\ cm^2 = \frac{25\pi}{2}\ cm^2$



Let $\angle MOQ = \angle MOP = \theta$

In ΔOMP, we have

$\sin\theta = \frac{PM}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ \quad \Rightarrow \angle POQ = 2\theta = 60^\circ$

$\therefore$ Area enclosed by arc A and chord PQ.

= Area of segment of circle of radius 10 cm and sector containing angle 60º

$= \left\{\frac{\pi\times 60}{360} - \sin 30^\circ \times \cos 30^\circ\right\} \times 10^2\ cm^2$

$\left[\because A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2\right]$

$= \left\{\frac{50\pi}{3} - 25\sqrt{3}\right\} cm^2$

Hence,

Required area $= \left\{\frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)\right\} cm^2$

$\Rightarrow$ Required area $= \left\{25\sqrt{3} - \frac{25\pi}{6}\right\} cm^2$

$= 25\left\{\sqrt{3} - \frac{\pi}{6}\right\} cm^2$