AREAS RELATED TO CIRCLE

PERIMETER AND AREA OF CIRCLE

INTRODUCTION :

We already know about the perimeter and areas of some simple figures like rectangle, square, rhombus, parallelogram etc. In this chapter, we shall learn how to find the perimeter and areas related to circular figures and we will apply this knowledge to find the areas of some special parts of a circular region like sector, segment and combinations of plane figures.

CIRCLE AND ITS RELATED TERMS :

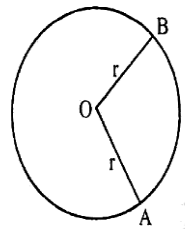
Circle:

A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.

Radius:

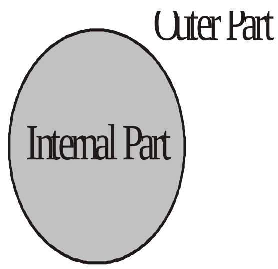
A line segment joining the centre of the circle to a point on the circle is called its radius.

In Figure, there is a circle with centre O and its radius is OA. The length of the radius of a circle is generally denoted by the letter 'r'.



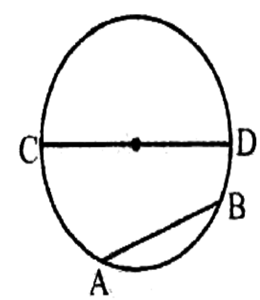
Internal and Outer Part :

A closed geometric figure in the plane divides the plane into three parts namely, the inner part of the figure, the figure and the outer part. In Figure the shaded portion is the inner part of the circle, the boundary is the circle and the unshaded portion is the outer part of the circle.



Chord:

A line segment joining any two points of a circle is called a chord. A chord passing though the centre of circle is called its diameter. AB and CD both are chords but the chord CD passes through the centre.



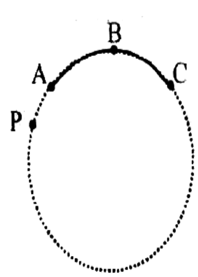
Hence CD is the diameter also.

Diameter of a circle = twice the radius of the circle.

Note: Diameter is the longest chord of a circle.

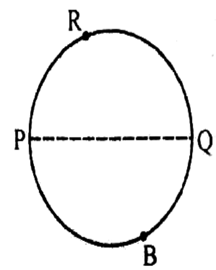
Arc: A part of a circle is called an arc.

In Figure, AC is an arc and is denoted by arc ABC or . $\overset{\frown}{ABC}$



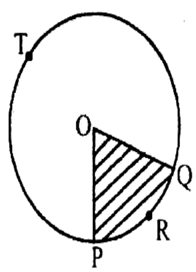
Semicircle:

A diameter of a circle divides a circle into two equal arcs, each known as a semicircle. In Figure, PQ is a diameter and arc PRQ is a semicircle and so is arc PBQ.



Sector:

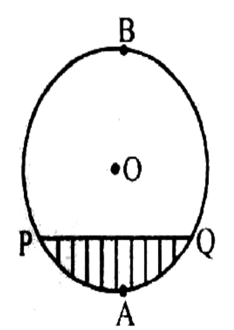
The region bounded by an arc of a circle and two radii at its end points is called a sector. In the figure, the shaded portion is a sector formed by the arc PRQ and the unshaded portion is a sector formed by the arc PTQ.



Segment:

A chord divides the interior of a circle into two parts, each called a segment. The segment in which the centre of the circle does not lie is called minor segment and the segment in which the centre of the circle lies is called major segment.

In the figure, the shaded region PAQP and the unshaded region PBQP are both segments of the circle. PAQP is called a minor segment and PBQP is called a major segment.



PERIMETER AND AREA OF A CIRCLE

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains same.

The fixed point is called the centre and the given constant distance is known as the radius of the circle.

The perimeter of a circle is known as its circumference.

If r is the radius of a circle, then

(i) Circumference = $2\pi r$ or πd, where $d = 2r$ is the diameter of the circle.

(ii) Area = πr^2 or $\pi d^2/4$

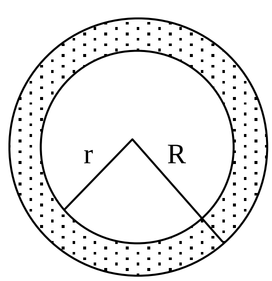
(iii) Area of semi-circle = $\frac{\pi r^2}{2}$

(iv) Area of a quadrant of a circle = $\frac{\pi r^2}{4}$

AREA ENCLOSED BY TWO CONCENTRIC CIRCLE

If R and r are radii of two con-centric circles, then area enclosed by the two circles

$= \pi R^2 - \pi r^2 = \pi (R^2 - r^2) = \pi (R + r)(R - r)$



Some useful results :

(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(ii) If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) The number of revolutions completed by a rotating wheel in one minute

$$= \frac{\text{Distance moved in one minute}}{\text{Circumference}}$$

Ex.1 Find the area of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle. Then,

Circumference = 22 cm

$\Rightarrow 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = 7/2$ cm

$\therefore$ Area of the circle $= \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= 38.5 \text{ cm}^2$

Ex.2 Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle. Then,

Circumference = 22 cm

$\Rightarrow 2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2}$ cm

$\therefore$ Area of a quadrant $= \frac{1}{4}\pi r^2$

$= \left\{\frac{1}{4}\times\frac{22}{7}\left(\frac{7}{2}\right)^2\right\} cm^2$

$= \left\{\frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right\} cm^2$

$= \frac{77}{4}\ cm^2 = 9.625\ cm^2$

Ex.3 If the perimeter of a semi-circular protractor is 66 cm, find the diameter of the protractor (Take $\pi = 22/7$).

Sol. Let the radius of the protractor be r cm. Then,

Perimeter = 66 cm

$\Rightarrow 1/2(2\pi r) = 66$

$\left[\because \text{Perimeter of semi-circle} = \frac{1}{2}(2\pi r)\right]$

$\Rightarrow \pi r = 66 \Rightarrow 22/7 \times r = 66 \Rightarrow r = 21$ cm

$\therefore$ Diameter of the protractor = 2r = (2 × 21) cm

= 42 cm

Ex.4 The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

Sol. Let the radius of the circle be r cm. Then,

Diameter = 2r cm and Circumference = $2\pi r$ cm

It is given that the circumference exceeds the diameter by 16.8 cm

$\therefore$ Circumference = Diameter + 16.8

$\Rightarrow 2\pi r = 2r + 16.8$

$\Rightarrow 2 \times \frac{22}{7} \times r = 2r + 16.8$ $\left[\because \pi = \frac{22}{7}\right]$

$\Rightarrow 44r = 14r + 16.8 \times 7$

$\Rightarrow 44r - 14r = 117.6 \Rightarrow 30\,r = 117.6$

$\Rightarrow r = \frac{117.6}{30} = 3.92$

Hence, radius = 3.92 cm