CONSTRUCTIONS

TO DIVIDE LINE SEGMENT IN A GIVEN RATIO

INTRODUCTION

In class IX, we have discussed a number of constructions with the help of ruler and compass e.g. bisecting a line segment, bisecting an angle, perpendicular bisector of line segment, some more constructions of triangles etc. with their justifications. In this chapter we will discuss more constructions by using the knowledge of the earlier construction.

DIVISION OF A LINE SEGENT :

In order to divide a line segment internally is a given ratio m: n, where both m and n are positive integers, we follow the following steps:

Step of construction :

(i) Draw a line segment AB of given length by using a ruler.

(ii) Draw and ray AX making an acute angle with AB.

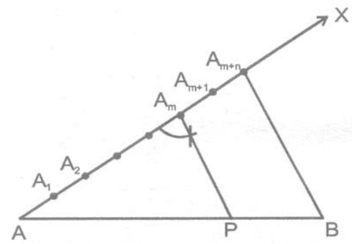
(iii) Along AX mark off (m + n) points A_1, A_2,..., A_{m+n} such that $AA_1 = A_1A_2 = ... = A_{m+n+}A_{m+n}$.

(iv) Join B A_{m+n}

(v) Through the point A_m draw a line parallel to A_{m+n} B by making an angle equal to $\angle AA_{m+n}B$ at A_m.

Suppose this line meets AB at a point P.

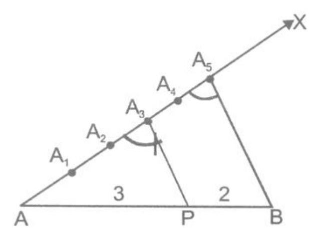
The point P so obtained is the required point which divides AB internally in the ratio m : n.



Ex.1 Divide a line segment of length 12 cm internally in the ratio 3 : 2.

Sol. Following are the steps of construction.

Step of construction :



(i) Draw a line segment AB = 12 cm by using a ruler.

(ii) Draw any ray making an acute angle $\angle BAX$ with AB.

(iii) Along AX, mark-off 5 (=3 + 2) points A_1, A_2, A_3, A_4 and A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

(iv) Join BA_5

(v) Through A_3 draw a line A_3P parallel to A_5B by making an angle equal to $\angle AA_5B$ at A_3 intersecting AB at a point P.

The point P so obtained is the required point, which divides AB internally in the ratio 3 : 2.

ALTERNATIVE METHOD FOR DIVISION OF A LINE SEGMENT INTERNALLY IN A GIVEN RATIO :

Use the following steps to divide a given line segment AB internally in a given ration m : n, where m and natural members.

Steps of Construction :

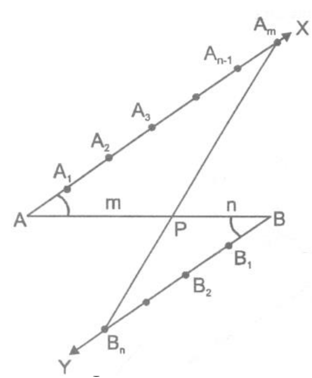
(i) Draw a line segment AB of given length.

(ii) Draw any ray AZ making an acute angle $\angle BAX$ with AB.

(iii) Draw a ray BY, on opposite side of AX, parallel to AX making an angle $\angle ABY$ equal to $\angle BAX$.

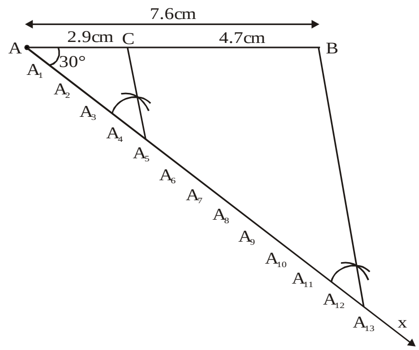
(iv) Mark off a points $A_1, A_2, \ldots A_{m'}$ on AX and n points $B_1, B_2, \ldots B_n$ on BY such that $AA_1 = A_1A_2 = \ldots\ldots = A_{m-1}A_m = B_1B_2 = \ldots B_{n-1}B_n$.

(v) Join A_mB_n. Suppose it intersect AB at P.



The point P is the required point dividing AB in the ratio m : n.

Ex. 2: Draw a segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.



Sol. STEPS OF CONSTRUCTION :

1 : Draw any ray AX making an angle of 30° with AB.

2 : Locate 13 points : A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12} and A_{13}

So that: $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = ... = A_{11}A_{12} = A_{12}A_{13}$

3 : Join B with A_{13}.

4 : Through the point A_5, draw a line $A_5C || A_{13}B$ such that $\angle AA_5C$ = corr.

$\angle AA_{13}B$ intersecting AB at a point C. Then AC : CB = 5 : 8.

Let us see how this method gives us the required division.

Since A_5C is parallel to $A_{13}B$.

Therefore $\frac{AA_5}{A_5A_{13}} = \frac{AC}{CB}$ (Basic Proportionality Theorem)

By construction, $\frac{AA_5}{A_5A_{13}} = \frac{5}{8}$

Therefore $\frac{AC}{CB} = \frac{5}{8}$

This given that C divides AB in the ratio 5 : 8.

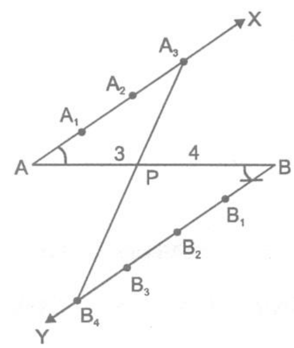
By measurement, we find, AC = 2.9 cm, CB = 4.7 cm.

By Calculation: AC = $\frac{7.6 \times 5}{13} = \frac{38}{13}$ = 2.9 cm

BC = $\frac{7.6 \times 8}{13} = \frac{60.8}{13}$ = 4.67 = 4.7 cm.

Ex.2 Decide a line segment of length 6 cm internally in the ratio 3:4.

Sol. Follow the following steps



Steps of Construction :

(i) Draw a line segment AB of length 6 cm.

(ii) Draw any ray AX making an acute angle $\angle BAX$ with AB.

(iii) Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$.

(iv) Mark of three point A_1,A_2,A_3 on AX and 4 points B_1, B_2m B_3, B_4 on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_2B_4$.

(v) Join A_3B_4. Suppose it intersects AB at a point P.

Then, P is the point dividing AB internally in the ratio 3:4.