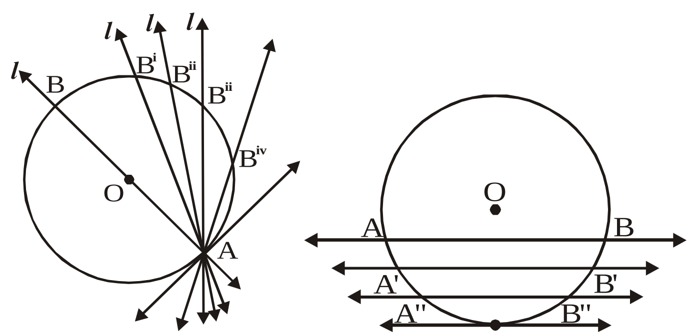
CIRCLE

PROPERTIES OF TANGENT TO A CIRCLE (THEOREM 1 AND 2)

Tangent

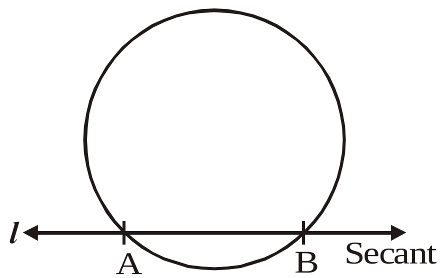
A tangent is a straight line that meets the circle at one and only one point. This point 'A' is called point of contact or point of tangency in fig. (c).

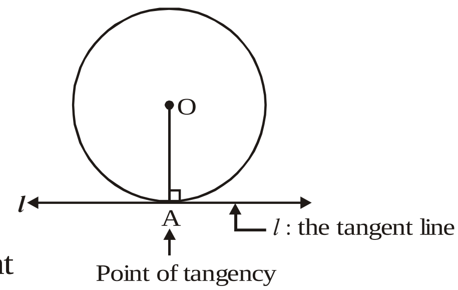
Tangent as a limiting case of a secant



In the fig. the secant l cuts the circle at A and B. If this secant l is turned around the point A, keeping A fixed then B moves on the circumference closer to A. In the limiting position, B coincides with A. The secant l becomes the tangent at A. Tangent to a circle is a secant when the two end points of its corresponding chord coincide.

In the fig. l is a secant which cuts the circle at A and B. If the secant is moved parallel to itself away from the centre, then the points A and B come closer and closer to each other. In the limiting position, they coincide into a single point at A, the secant l becomes the tangent at A. Thus a tangent line is the limiting case of a secant when the two points of intersection of the secant and a circle coincide with the point A. The point A is called the point of contact of the tangent. The line l touches the circle at the point A. i.e., the common point of the tangent and the circle is called the point of contact and the tangent is said to touch the circle at the common point.

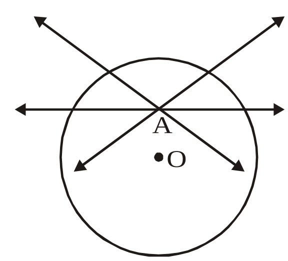




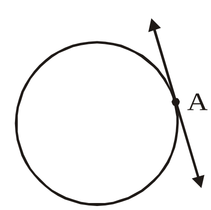
Note : The line containing the radius through the point of contact is called normal to the circle at the point.

NUMBER OF TANGENTS TO A CIRCLE FROM A POINT

1. If a point A lies inside a circle, no line passing through 'A' can be a tangent to the circle. i.e., No tangent can be drawn from the point A.

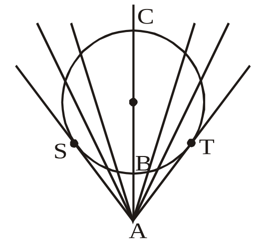


2. If A lies on the circle, then one and only one tangent can be drawn to pass through 'A'. i.e. Exactly one tangent can be drawn through A.



3. If A lies outside the circle then exactly two tangents can be drawn through 'A'.
In the fig., a secant ABC is drawn from a point 'A' outside the circle, if the secant is turned around A in the clockwise direction, in the limiting position, it becomes a tangent at T. Similarly if the secant

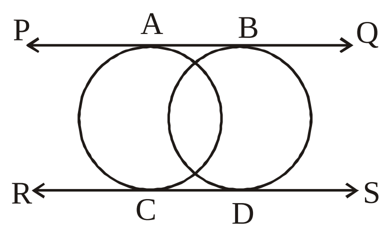
is turned in the anti-clockwise direction, in the limiting position, it becomes a tangent at S. Thus from a point A outside a circle only two tangents can be drawn. The points S and T where the lines touch the circle are called the points of contact.



Two circles and their common tangents

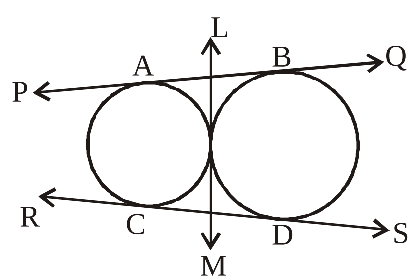
Case-I: When circles intersect in two points:

In this case, there will be two common tangents PQ and RS to the two circles as shown in figure.

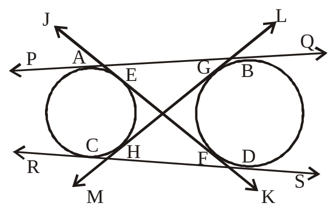


Case-II: When circles touch externally:

Here, the two circles will have three common tangents LM, PQ and RS as shown in figure.



Case-III: When one circle lies entirely outside the other circle without having a common point.

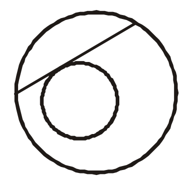


In this case, there will be four common tangents, PQ, RS, JK and LM as shown in figure.

(i) PQ and RS are two direct common tangents.

(ii) KJ and LM are two indirect common tangents i.e. transversals.

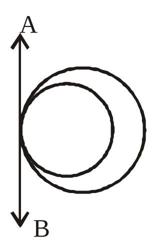
Case-IV: When one circle lies entirely inside the other circle without having a common point.



If any tangent is drawn at any point of the inner circle, it will intersect The outer circle in two distinct points and, therefore, cannot be a tangent to the outer circle. Thus, no common tangent can be drawn in this case.

Case-V: When circles touch internally

There is only one common tangent AB to the two circles.



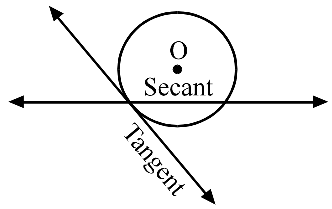
Secant :

A line which intersects a circle in two distinct points is called a secant.

Tangent :

A line meeting a circle only in one point is called a tangent to the circle at that point.

The point at which the tangent line meets the circle is called the point of contact.



Number of Tangents to a Circle

(i) There is no tangent passing through a point lying inside the circle.

(ii) There is one and only one tangent passing through a point lying on a circle.

(iii) There are exactly two tangents through a point lying outside a circle.

Length of Tangent

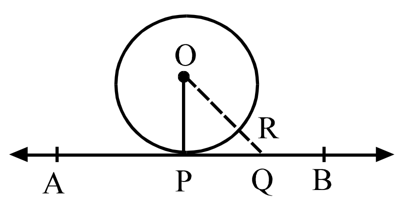
The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.

Results on Tangents

Theorem 1 :

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given : A circle with centre O and a tangent AB at a point P of the circle.



To prove : $OP \perp AB$.

Construction : Take a point Q, other than P, on AB. Join OQ.

Proof : Q is a point on the tangent AB, other than the point of contact P.

$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R.

Then, $OR < OQ$ [a part is less than the whole] .... (i)

But, $OP = OR$ [radii of the same circle]. .... (ii)

$\therefore$ $OP < OQ$ [from (i) and (ii)].

Thus, OP is shorter than, any other line segment joining O to any point of AB, other than P.

In other words, OP is the shortest distance between the point O and the line AB.

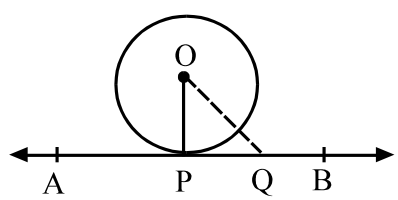
But, the shortest distance between a point and a line is the perpendicular distance.

$\therefore$ $OP \perp AB$.

Theorem 2 : (Converse of Theorem 1)

A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.

Given : A circle with centre O in which OP is a radius and AB is a line through P such that $OP \perp AB$.



To prove : AB is a tangent to the circle at the point P.

Construction : Take a point Q, different from P, on AB. Join OQ.

Proof : We know that the perpendicular distance from a point to a line is the shortest distance between them.

$\therefore$ $OP \perp AB$ Þ OP is the shortest distance from O to AB.

$\therefore$ $OP < OQ$.

$\therefore$ Q lies outside the circle

[$\because$ OP is the radius and OP < OQ].

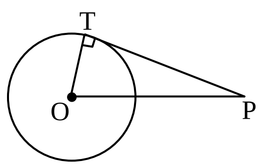
Thus, every point on AB, other than P, lies outside the circle.

$\therefore$ AB meets the circle at the point P only.

Hence, AB is the tangent to the circle at the point P.

Ex.1 From a point P, 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle.

Sol. Let O be the centre of the given circle and let P be a point such that



OP = 10 cm.

Let PT be the tangent such that PT = 8 cm.

Join OT.

Now, PT is a tangent at T and OT is the radius through T.

$\therefore$ OT $\perp$ PT.

In the right ΔOTP, we have

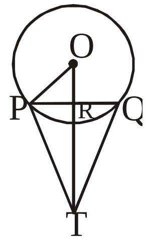
$OP^2 = OT^2 + PT^2$ [by Pythagoras' theorem]

$\Rightarrow OT = \sqrt{OP^2 - PT^2} = \sqrt{(10)^2 - (8)^2}$ cm

$= \sqrt{36}$ cm = 6 cm.

Hence, the radius of the circle is 6 cm.

Ex.2 In the given figure, PQ is a chord of length 8cm of a circle of radius 5cm. The tangents at P and Q intersect at a point T. Find the length TP.



Sol. Join OP and OT Let OT intersect PQ at a point R.

Then, TP = TQ and $\angle PTR = \angle QTR$.

$\therefore$ $TR \perp PQ$ and TR bisects PQ.

$\therefore$ PR = RQ = 4 cm.

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$ cm

$= \sqrt{25 - 16}\,\text{cm} = \sqrt{9}\,\text{cm} = 3\text{cm}.$

Let TP = x cm and TR = y cm.

From right ΔTRP, we get

$TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + 16 \quad \Rightarrow x^2 - y^2 = 16$ .... (i)

From right ΔOPT, we get

$TP^2 + OP^2 = OT^2$

$\Rightarrow\ x^2 + 5^2 = (y + 3)^2 \quad [\because OT^2 = (OR + RT)^2]$

$\Rightarrow\ x^2 - y^2 = 6y - 16$ ....(ii)

From (i) and (ii), we get

$6y - 16 = 16 \quad \Rightarrow 6y = 32 \Rightarrow y = \frac{16}{3}.$

Putting $y = \frac{16}{3}$ in (i), we get

$x^2 = 16 + \left(\frac{16}{3}\right)^2 = \left(\frac{256}{9} + 16\right) = \frac{400}{9}$

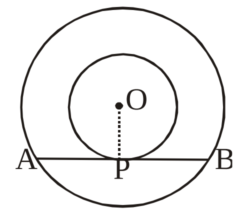
$\Rightarrow\ x = \sqrt{\frac{400}{9}} = \frac{20}{3}.$

Hence, length TP = x cm = $\left(\frac{20}{3}\right)$cm

= 6.67 cm.

Ex.3 Prove that in two concentric circles, the chord of the larger circle which touches the smaller circle, is bisected at the point of contact.

Sol. **Given :** Two circles with the same centre O and AB is a chord of the larger circle which touches the smaller circle at P.



To prove : AP = BP.

Construction : Join OP.

Proof : AB is a tangent to the smaller circle at the point P and OP is the radius through P.

$\therefore$ OP $\perp$ AB.

But, the perpendicular drawn from the centre of a circle to a chord bisects the chord.

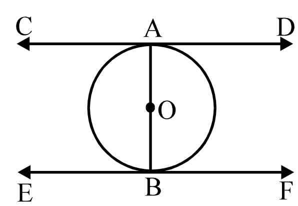
$\therefore$ OP bisects AB.

Hence, AP = BP

Ex.4 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol. **Given :** CD and EF are the tangents at the end points A and B of the diameter AB of a circle with centre O.

To prove : CD || EF.



Proof : CD is the tangent to the circle at the point A.

$\therefore$ $\angle BAD = 90°$

EF is the tangent to the circle at the point B.

$\therefore$ $\angle ABE = 90°$

Thus, $\angle BAD = \angle ABE$ (each equal to 90°).

But these are alternate interior angles.

$\therefore$ CD || EF