GRAVITATION

CONTENTS

➤ FORCE OF GRAVITATION

Any two particles in the universe attract each other. This force is called the force of gravitation.

This concept was given by **Newton**.

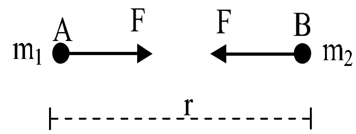
➤ NEWTON'S LAW OF GRAVITATION

According to newton, "Any two bodies in universe attract each other with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them."

◆ **Mathematical expression**

Lat A and B be two particle of mass m_1 and m_2 respectively. Let the distance AB = r.

By the law of gravitational, the particle A attracts the particle B with a force F such that.



$F \propto m_1 m_2$ (for a given pair of particles)

$F \propto \frac{1}{r^2}$ (for a given separation between the particles)

So $F \propto \frac{m_1 m_2}{r^2}$

or $F = G\frac{m_1 m_2}{r^2}$

Here G is a constant known as the universal constant of gravitation. $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

◆ G is independent of the masses of the bodies and the distance between them.

◆ Newton's law of gravitation is applicable for everybody in the universe.

Note : The force between any two bodies in the universe is called the force of gravitation while the force with which earth attracts a body is called the force of gravity.

◈ **Some Scientific Phenomenon Based on Gravitational Force :**

◆ The gravitational force between the sun and the earth keeps the earth moving around the sun.

◆ The gravitational force b/w the earth and the moon keeps the moon moving around the earth.

◆ Existence of our solar system is due to gravitational force.

- Gravitation force of the sun and the moon on the earth's water surface is responsible for the tides in sea.
- Atmosphere above the earth is held due to gravitational force of the earth.
- Gravitational force between the sun and planet keeps the planet moving around the sun.
- Gravitational force is responsible for providing the centripetal force required by the planets.
- The attractive force of the earth is responsible for providing the centripetal force required by moon.

Newton's Third Law of Motion and Law of Gravitation :

- Newton's third law of motion is applicable to gravitation also.

  Ex. if the earth exerts a force of attraction on a body, the body also exerts an equal and opposite force of attraction on the earth.

- As a = F/m

  mass of the body is larger, acceleration produced will be smaller and vice versa.

Ex. 1 Calculate the force between two masses of 100 kg and 1000 kg separated by a distance of 10 m ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2).

Sol. According to Newton's law of gravitation, force of attraction between two bodies is

$$F = \frac{Gm_1m_2}{r^2}$$

Here, $m_1 = 100$ kg; $m_2 = 1000$ kg ;

$r = 10$ m ; $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

$$\therefore \quad F = \frac{6.67\times10^{-11}\times100\times1000}{(10)^2}$$

$$= 6.67 \times 10^{-8}\ \text{N}$$

Ex. 2 Given mass of earth = 6×10^{24} kg, radius of earth = 6.4×10^6 m. Calculate the force of attraction experienced by a man of mass 50 kg.

Sol. Force of gravitation is given by the expression, $F = \frac{Gm_1m_2}{r^2}$

Here, mass of earth, $m_1 = 6 \times 10^{24}$ kg;

mass of man, $m_2 = 50$ kg

Distance between them is to be taken equal to the radius of earth.

$\therefore \quad r = 6.4 \times 10^6$ m

Substituting these values, we get

$$F = \frac{6.67\times10^{-11}\times6\times10^{24}\times50}{(6.4\times10^6)^2} = 488.5\ \text{N}$$

Ex. 3 Compare the gravitational forces exerted by the sun and the moon on earth. Which exerts a greater force on earth ?

(Given : mass of sun, $M_S = 4 \times 10^{31}$ kg; mass of moon, $M_m = 6.3 \times 10^{22}$ kg; distance between sun and earth, $r_{se} = 1.3 \times 10^{12}$ m and the distance between moon and earth, $r_{me} = 4.5 \times 10^8$m)

Sol. If mass of sun is M_S and mass of earth is M_e and distance between the sun and earth is r_{se}, then force exerted by the sun on earth is

$$F_S = \frac{GM_sM_e}{(r_{se})^2} \quad ...(1)$$

Similarly, if mass of moon is M_m, mass of earth is M_e, the distance between moon and earth is r_{me}, then force exerted by moon on the earth is

$$F_m = \frac{GM_mM_e}{(r_{me})^2} \quad ...(2)$$

Dividing equation (1) by equation (2), we get

$$\frac{F_s}{F_m} = \frac{GM_sM_e}{(r_{se})^2} \times \frac{r_{me}^2}{GM_mM_e}$$

$$= \frac{M_s}{M_m} \times \frac{(r_{me})^2}{(r_{se})^2}$$

$$= \frac{4\times10^{31}}{6.3\times10^{22}} \times \left(\frac{4.5\times10^8}{1.3\times10^{12}}\right)^2 = 76.07$$

$\therefore$ The force exerted by the sun on earth is about 76 times the force exerted by the moon on earth.

Ex. 4 If mass and radius of earth is 6.0×10^{24} kg and 6.4×10^{6} m respectively, calculate the force exerted by earth on a body of mass 1 kg. Also, calculate :

(i) acceleration produced in the body of mass 1 kg, and

(ii) acceleration produced in the earth

Sol. From Newton's law of gravitation, we know that the force of attraction between two bodies is given by

$$F = \frac{Gm_1m_2}{r^2}$$

Here, m_1 = mass of earth = 6.0×10^{24} kg;

m_2 = mass of body = 1 kg

r = distance between the two bodies

= radius of earth = 6.4×10^{6} m

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2

$$\therefore \quad F = \frac{6.67\times10^{-11}\times6.0\times10^{24}\times1}{(6.4\times10^{6})^2} = 9.8 \text{ N}$$

This shows that earth exerts a force of 9.8 N on a body of mass of 1 kg. The body will exert an equal force of attraction of 9.8 N on earth.

(i) Acceleration produced in the body of mass 1kg

Force = mass × acceleration

$$\therefore \quad \text{Acceleration, } a = \frac{F}{m} = \frac{9.8}{1} = 9.8 \text{ m/s}^2$$

Thus, the acceleration produced in a body of mass 1 kg due to attraction of earth is 9.8 m/s^2, which is quite large. Thus, when a body is released, it falls towards the earth with an acceleration of 9.8 m/s^2, which can be easily observed.

(ii) Acceleration produced in the earth

Similarly, acceleration of earth is given by

$$= \frac{\text{Force}}{\text{Mass of earth}} = \frac{9.8}{6.0\times10^{24}}$$

$= 1.63 \times 10^{-24}$ m/s^2

This shows that the acceleration produced in the earth by a body of mass 1 kg is 1.63×10^{-24} m/s^2 which is very small and cannot be observed.

EARTH'S GRAVITATIONAL FORCE

- The force which earth exerts on a body is called 'force of gravity'. i.e. $F = \dfrac{GMm}{R^2}$

  Where M = mass of the earth, R = radius of the earth.
- Due to this force, a body released from some height on the earth's surface falls towards the earth with its velocity increasing at a constant rate.

Acceleration due to Gravity :

- The acceleration produced in a body due to attraction of earth is called the acceleration due to gravity and is denoted by 'g'.

  $$g = \frac{GM}{R^2} = 9.8 \text{ m/s}^2$$

  near the earth surface

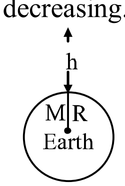
  $$g \text{ on moon } \approx \frac{g_e}{6} = \frac{9.8}{6} \text{ m/s}^2$$
- A body moving upwards with some initial velocity, experiences a retardation of 9.8 m/s^2 & its velocity decreases continuously unless it becomes zero.
- After this, it again starts falling towards the earth with the same acceleration of 9.8 m/s^2.
- The value of g is minimum at equator and maximum at poles.
- The value of g does not depend upon the mass of the body falling towards the earth.

VARIATION IN THE VALUE OF GRAVITATIONAL ACCELERATION (g)

(A) Variation with altitude or height :

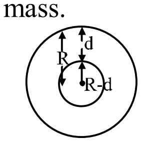
- When a body moves above the earth's surface the distance of the body from the centre of earth increases there by decreasing the force of attraction.
- $g = \dfrac{GM}{R^2}$; at the earth's surface.
- $g = \dfrac{GM}{(R+h)^2}$; at a height h above the earth's surface.

- As we go above the earth's surface the value of *g* goes on decreasing.



(B) Variation with depth d :

- As we go deeper inside the earth, the body gets attracted by the core of the earth which is smaller in mass.



- As we go inside the earth, the value of g decreases.
- Force of attraction decreases and thus decreasing the value of g and becoming zero at the centre.

(C) Variation due to rotation of the earth :

- Due to the rotation of the earth, the weight of a body is maximum at the poles and minimum at the equator.

➢ MASS & WEIGHT

	Mass	Weight
1.	Mass of a body is defined as the quantity of matter contained in it.	Weight of a body is the force with which it is attracted towards the centre of the earth. W = mg
2.	Mass of a body remains constant and does not change from place to place.	Weight of a body changes from place to place. It depends upon the value of *g*. Weight of a body on another planet will be different.
3.	Mass is measured by a pan balance.	Weight is measured by a spring balance.
4.	Unit of mass is kg.	Unit of weight is newton or kg-wt.
5.	Mass of a body cannot be zero.	Weight of a body can be zero. Ex. astronauts experience weightlessness in spaceships.
6.	Mass is a scalar quantity.	Weight is a vector quantity.

Ex. 5 Given mass of earth is 6×10^{24} kg and mean radius of earth is 6.4×10^{6} m. Calculate the value of acceleration due to gravity (g) on the surface of the earth.

Sol. The formula for the acceleration due to gravity is given by

$$g = \frac{GM}{R^2}$$

Here, $G = 6.67 \times 10^{-11}$ Nm^2/kg^2;
M = mass of earth = 6×10^{24} kg;
R = radius of earth = 6.4×10^{6} m

$$g = \frac{6.67\times10^{-11}\times6\times10^{24}}{(6.4\times10^{6})^2} = 9.8 \text{ m/s}^2$$

Ex. 6 Calculate the value of acceleration due to gravity on a planet whose mass is 4 times as that of the earth and radius is 3 times as that of the earth.

Sol. If M is the mass of the earth and R is the radius of earth, the value of acceleration due to gravity on the earth (g_e) is given by

$$g_e = \frac{GM}{R^2} \quad ...(1)$$

Let us consider a planet such that mass of the planet is equal to 4 times the mass of earth.

$$M_p = 4M$$

Radius of the planet is equal to 3 times the radius of earth.

$$R_e = 3R$$

Then, acceleration due to gravity on this planet(g_p) is

$$g_p = \frac{G\times(4M)}{(3R)^2} = \frac{4}{9}\cdot\frac{GM}{R^2} \quad ...(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{g_p}{g_e} = \frac{\frac{4}{9}\times\frac{GM}{R^2}}{\frac{GM}{R^2}} \text{ or } \frac{g_p}{g_e} = \frac{4}{9}$$

or $g_p = \frac{4}{9}(g_e)$

Since $g_e = 9.8$ m/s^2

$\therefore$ $g_p = \frac{4}{5} \times 9.8 = 4.35$ m/s^2

Thus, acceleration due to gravity on the given planet is 4.35 m/s^2.

Ex. 7 Given the mass of the moon = 7.35×10^{22} kg and the radius of the moon = 1740 km. Calculate the acceleration experienced by a particle on the surface of the moon due to the gravitational force of the moon. Find the ratio of this acceleration to that experienced by the same particle on the surface of the earth.

Sol. If M_m is the mass of the moon and R_m is its radius, then the acceleration experienced by a body on its surface is given by

$$a = \frac{GM_m}{R_m^2}$$

Here, $M_m = 7.3 \times 10^{22}$ kg;
$R_m = 1740$ km $= 1.74 \times 10^6$ m

$$\therefore \quad a = \frac{6.67\times10^{-11}\times7.3\times10^{22}}{(1.74\times10^6)^2} = 1.57 \text{ m/s}^2$$

While the acceleration due to gravity on the surface of the earth, is given by

$$g_e = \frac{GM_e}{R_e^2} = \frac{6.67\times10^{-11}\times6\times10^{24}}{(6.4\times10^6)^2} = 9.8 \text{ m/s}^2$$

Comparing acceleration due to gravity on moon to that on the earth is

$$\frac{a}{g} = \frac{1.57}{9.8} = 0.16$$

Ex. 8 At what height above the earth's surface the value of g will be half of that on the earth's surface ?

Sol. We know that the value of g at earth's surface is

$$g = \frac{GM}{R^2} \quad ...(1)$$

While the value of g at a height h above the earth's surface is given by

$$g' = \frac{GM}{(R+h)^2} \quad ...(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 \text{ or } g' = g\left(\frac{R}{R+h}\right)^2$$

Here, $g' = \frac{g}{2}$

$$\therefore \quad \frac{g}{2} = g\left(\frac{R}{R+h}\right)^2$$

or $\frac{R+h}{R} = \sqrt{2}$

or $R + h = \sqrt{2}R$

or $h = (\sqrt{2} - 1)R$

or $h = (1.41 - 1) \times 6400 = 0.41 \times 6400$
$= 2624$ km

Ex. 9 Given mass of the planet Mars is 6×10^{23} kg and radius is 4.3×10^6 m. Calculate the weight of a man whose weight on earth is 600 N. (Given g on earth = 10 m/s^2)

Sol. Weight of the man on earth, W = mg
or $600 = m \times 10$ or $m = 60$ kg

So the mass of the man is 60 kg which will remain the same everywhere.
Now acceleration due to gravity on Mars,

$$g_m = \frac{GM_m}{R_m^2}$$

Here, $M_m = 6 \times 10^{23}$ kg; $R_m = 4.3 \times 10^6$m

$$\therefore \quad g_m = \frac{6.67\times10^{-11}\times6\times10^{23}}{(4.3\times10^6)^2} = 2.17 \text{ m/s}^2$$ Now,

weight of the man on Mars will be

$$W_m = m \times g_m = 60 \times 2.17 = 130.2 \text{ N}$$

➢ EQUATIONS OF MOTION FOR FREELY FALLING OBJECT

Since the freely falling bodies fall with uniformly accelerated motion, the three equations of motion derived earlier for bodies under uniform acceleration can be applied to the motion of freely falling bodies. For freely falling bodies, the acceleration due to gravity is 'g', so we replace the acceleration 'a' of the equations by 'g' and since the vertical distance of the freely falling bodies is known as height 'h', we replace the distance 's' in our equations by the height 'h'. This gives us the following modified equations for the motion of freely falling bodies.

General equation of motion	Equations of motion for freely falling bodies
(i) $v = u + at$ changes to	$v = u + gt$
(ii) $s = ut + \frac{1}{2}at^2$ changes to	$h = ut + \frac{1}{2}gt^2$
(iii) $v^2 = u^2 + 2$ as changes to	$v^2 = u^2 + 2gh$

We shall use these modified equations to solve numerical problems. Before we do that, we should remember the following important points for the motion of freely falling bodies.

(i) When a body is dropped freely from a height, its initial velocity 'u' becomes zero

(ii) When a body is thrown vertically upwards, its final velocity 'v' becomes zero

(iii) The time taken by body to rise to the highest point is equal to the time it takes to fall from the same height.

(iv) The distance travelled by a freely falling body is directly proportional to the square of time of fall.