Wave Optics

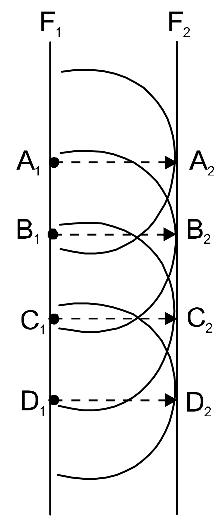
10

HUYGENS' CONSTRUCTION

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. We can guess that he must have seen water waves many times in the canals of his native place Holland. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wavefront as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets.

Huygens' principle is illustrated in (Figure) in the simple case of a plane wave.

(i) At time t = 0, we have a wavefront F_1, F_1 separates those parts of the medium which are undisturbed from those where the wave has already reached.



(ii) Each point on F_1 acts like a new source and sends out a spherical wave. After a time 't' each of these will have radius vt. These spheres are the secondary wavelets.

(iii) After a time t, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Notice that F_2 is a surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.

(iv) The secondary wavelet from the point A_1 on F_1 touches F_2 at A_2. Draw the line connecting any point A_1 on F_1 to the corresponding point A_2 on F_2. According to Huygens, $A_1 A_2$ is a ray. It is perpendicular to the wavefronts F_1 and F_2 and has length vt. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts Is the same along any ray. In our example, the speed 'v' of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.

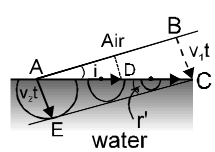
(v) This geometrical construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on. This is known as Huygens' construction.

Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbours into oscillation, and therefore acts as a secondary source. But what if there is no medium, as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction works in this case as well.

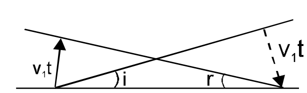
REFLECTION AND REFRACTION

We can use a modified form of Huygens' construction to understand reflection and refraction of light. Figure (a) shows an incident wavefront which makes an angle 'i' with the surface separating two media, for example, air and water. The phase speeds in the two media are v_1 and v_2. We can see that when the point A on the incident wavefront strikes the surface, the point B still has to travel a distance BC = AC sin *i*, and this takes a time $t = BC/v_1 = AC (\sin i) / v_1$. After a time t, a secondary wavefront of radius v_2t with A as centre would have travelled into medium 2. The secondary wavefront with C as centre would have just started, i.e.. would have zero radius. We also show a secondary wavelet originating from a point D in between A and C. Its radius is less than v_2t. The wavefront in medium 2 is thus a line passing through C and tangent to the circle centred on A. We can see that the angle r' made by this refracted wavefront with the surface is given by $AE = v_2t = AC \sin r'$. Hence, $t = AC (\sin r'\} / v_2$. Equating the two expressions for 't' gives us the law of refraction in the form $\sin i/\sin r' = v_1/v_2$. A similar picture is drawn in Fig. (b) for the reflected wave which travels back into medium 1. In this case, we denote the angle made by the reflected wavefront with the surface by r, and we find that i = r. Notice that for both reflection and refraction, we use secondary wavelets starting at different times. Compare this with the earlier application (Fig.) where we start them at the same time.

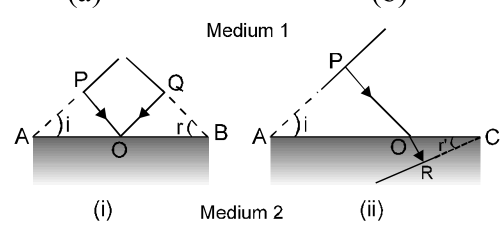
The preceding argument gives a good physical picture of how the refracted and reflected waves are built up from secondary wavelets. We can also understand the laws of reflection and refraction using the concept that the time taken by light to travel along different rays from one wavefront to another must be the same. (Fig.) Shows the incident and reflected wavefronts when a parallel beam of light falls on a plane surface. One ray POQ is shown normal to both the reflected and incident wavefronts. The angle of incidence i and the angle of reflection r are defined as the angles made by the incident and reflected rays with the normal. As shown in Fig. (c), these are also the angles between the wavefront and the surface.



(a)



(b)



(c)

(Fig.) (a) Huygens' construction for the (a) refracted wave. (b) Reflected wave. (c) Calculation of propagation time between wavefronts in (i) reflection and (ii) refraction.

We now calculate the total time to go from one wavefront to another along the rays. From Fig. (c), we have, we have Total time for light to reach from P to Q

$$= \frac{PO}{v_1} + \frac{OQ}{v_1} = \frac{AO \sin i}{v_1} + \frac{OB \sin r}{v_1}$$

$$= \frac{OA \sin i + (AB - OA)\sin r}{v_1}$$

$$= \frac{AB \sin r + OA(\sin i - \sin r)}{v_1}$$

Different rays normal to the incident wavefront strike the surface at different points O and hence have different values of OA. Since the time should be the same for all the rays, the right side of equation must actually be Independent of OA. The condition, for this to happen is that the coefficient of OA in Eq. (should be zero, i.e., sin i = sin r. We, thus. have the law of reflection, i = r. Figure also shows refraction at a plane surface separating medium 1 (speed of light v_1) from medium 2 (speed of light v_2). The incident and refracted wavefronts are shown, making angles i and r' with the boundary. Angle r' is called the angle of refraction. Rays perpendicular to these are also drawn. As before, let us calculate the time taken to travel between the two wavefronts along any ray.

Time taken from P to R $= \frac{PO}{v_1} + \frac{OR}{v_2}$

$$= \frac{OA \sin i}{v_1} + \frac{(AC - OA)\sin r'}{v_2}$$

$$= \frac{AC \sin r'}{v_2} + OA\left(\frac{\sin i}{v_1} - \frac{\sin r'}{v_2}\right)$$

This time should again be independent of which ray we consider. The coefficient of OA in Equation is, therefore, zero,. That is,

$$\frac{\sin i}{\sin r'} = \frac{v_1}{v_2} = n_{21}$$

where n_{21} is the refractive index of medium 2 with respect to medium 1. This is the Snell's law of, refraction that we have already dealt with from Eq. n_{21} is the ratio of speed of light in the first medium (v_1) to that in the second medium (v_2). Equation is, known as the Snell's law of refraction. If the first medium is vacuum, we have

$$\frac{\sin i}{\sin r'} = \frac{c}{v_2} = n_2$$

where n_2 is the refractive index of medium 2 with respect to vacuum, also called the absolute refractive index of the medium. A similar equation defines absolute refractive index n_1 of the first medium. From Eq. we then get

$$n_{21} = \frac{v_1}{v_2} = \left(\frac{c}{n_1}\right) / \left(\frac{c}{n_2}\right) = \frac{n_2}{n_1}$$

The absolute refractive index of air is about 1.0003, quite close to 1. Hence, for all practical purposes, absolute refractive index of a medium may be taken with respect to air. For water, $n_1 = 1.33$, which means $v_1 = \frac{c}{1.33}$, i.e. about 0.75 times the speed of light in vacuum. The measurement of the speed of light in water by Foucault (1850) confirmed this prediction of the wave theory.

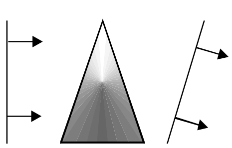
Once we have the laws of reflection and refraction, the behaviour of prisms. lenses, and mirrors can be understood. These topics are discussed in detail in the previous Chapter. Here we just describe the behaviour of the wavefronts in these three cases (Fig.)

(i) Consider a plane wave passing through a thin prism. Clearly, the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most. Since light travels more slowly in glass. This explains the tilt in the emerging wavefront.

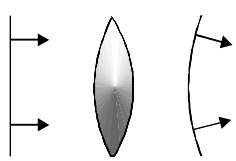
(ii) Similarly, the central part of an incident plane wave traverses the thickest portion of a convex lens and is delayed the most. The emerging wavefront has a depression at the centre. It is spherical and converges to a focus,

(iii) A concave mirror produces a similar effect. The centre of the wavefront has to travel a greater distance before and after getting reflected, when compared to the edge. This again produces a converging spherical wavefront.

(iv) Concave lenses and convex mirrors can be understood from time delay arguments in a similar manner. One interesting property which is obvious from the pictures of wavefronts is that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray (Fig.). For example, when a convex lens focuses light to form a real image, it may seem that rays going through the centre are shorter. But because of the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.



(a)



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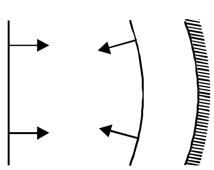


(c)

WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

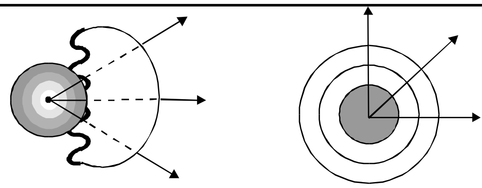
Figure (a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e.. radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

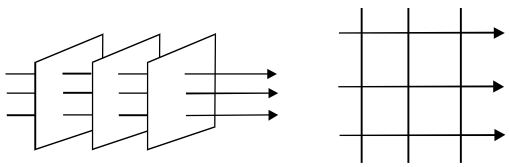
(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (b)

A linear source such as a slit Illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wave fronts may be regarded as planar.



(a)



(b)

Figure : Wavefronts and the corresponding rays in. two cases: (a) diverging spherical wave. (b) plane wave. The figure on the left shows a wave (e.g.. light) in three dimensions. The figure on the right shows a wave in two dimensions (a water surface).

COHERENT AND INCOHERENT ADDITION OF WAVES

Two sources which vibrate with a fixed phase difference between them are said to be coherent. The phase differences between light coming form such sources does not depend on time.

In a conventional light source, however, light comes from a large number of individual atoms, each atom emitting a pulse lasting for about 1 ns. Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently light coming from two such sources have a fixed phase relationship for about 1ns, hence interference pattern will keep changing every billionth of a second. The eye can notice intensity changes which lasts at least one tenth of a second. Hence we will observe uniform intensity on the screen which is the sum of the two individual intensities. Such sources are said to be incoherent. Light beam coming from two such independent sources do not have any fixed phase relationship and they do not produce any stationary interference pattern. For such sources, resultant intensity at any point is given by

$$I = I_1 + I_2 \qquad ...... (1)$$

PRINCIPLE OF SUPERPOSITION

When two or more waves simultaneously pass through a point, the disturbance of the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement, in case of sound wave it means pressure change, in case of E.M.W. it is electric field or magnetic field. Superposition of two light travelling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called *interference*.

1. Superposition of two sinusoidal waves :

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let, $x_1(t) = a_1 \sin \omega t$

and, $x_2(t) = a_2 \sin (\omega t + \phi)$

represent the displacement produced by each of the disturbances. Here we are assuming the displacements to be in the same direction. Now according to superposition principle, the resultant displacement will be given by,

$$x(t) = x_1(t) + x_2(t)$$

$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

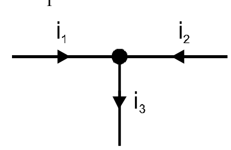
$$= A \sin (\omega t + \phi_0)$$

where $A^2 = a_1^{\,2} + a_2^{\,2} + 2a_1 \, . \, a_2 \cos \phi$ ... (1)

and $\tan \phi_0 = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$ ...(2)

Solved Examples

Ex. 1 If $i_1 = 3\sin \omega t$ and $i_2 = 4 \cos \omega t$, find i_3.



Sol. from kirchoff's current law,

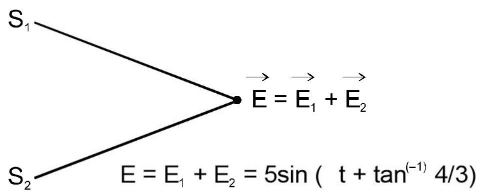
$$i_3 = i_1 + i_2.$$

$$= 3 \sin \omega t + 4 \sin (\omega t + \frac{\pi}{2})$$

$$= 5 \sin (\omega t + \tan^{-1} \;)$$

Ex. 2 S_1 and S_2 are two source of light which produce individually disturbance at point P given by $E_1 = 3\sin \omega t$, $E_2 = 4 \cos \omega t$. Assuming $\vec{E}_1$ & $\vec{E}_2$ to be along the same line, find the result of their superposition.

Sol.



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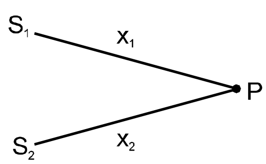
2. Superposition of progressive waves; path difference :

Let S_1 and S_2 be two sources producing progressive waves (disturbance travelling in space given by y_1 and y_2)

At point P,



$y_1 = a_1 \sin(\omega t - kx_1 + \theta_1)$

$y_2 = a_2 \sin(\omega t - kx_2 + \theta_2)$

$y = y_1 + y_2 = A \sin(\omega t + \Delta\phi)$

Here, the phase difference,

$\Delta\phi = (\omega t - kx_1 + \theta_1) - (\omega t - kx_2 + \theta_2)$

$= k(x_2 - x_1) + (\theta_1 - \theta_2) = k\Delta p + \Delta\theta$

Here $\Delta p = \Delta x$ is the path difference

Clearly, phase difference due to path difference = k (path difference)

where $k = \dfrac{2\pi}{\lambda}$

$\Rightarrow \quad \Delta\phi = k\Delta p = \dfrac{2\pi}{\lambda}\Delta x \quad ..... (3)$

For Constructive Interference :

$\Delta\phi = 2n\pi, \quad n = 0, 1, 2 ....$

or, $\Delta x = n\lambda$

$A_{max} = A_1 + A_2$

Intensity, $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2}$

$\Rightarrow \quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad ... (4)$

For Destructive interference :

$\Delta\phi = (2n + 1)\pi, \ n = 0, 1, 2 ....$

or, $\Delta x = (2n + 1)\lambda/2$

$A_{min} = |A_1 - A_2|$

Intensity, $\sqrt{I_{min}} = \sqrt{I_1} - \sqrt{I_2}$

$\Rightarrow \quad I_{max} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \quad ... (5)$

Solved Examples

Ex. 3 Light from two sources, each of same frequency and travelling in same direction, but with intensity in the ratio 4 : 1 interfere. Find ratio of maximum to minimum intensity.

Sol. $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\dfrac{\sqrt{\dfrac{I_1}{I_2}} + 1}{\sqrt{\dfrac{I_1}{I_2}} - 1}\right)^2$

$= \left(\dfrac{2+1}{2-1}\right)^2 = 9 : 1.$

YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E.)

In 1802 Thomas Young devised a method to produce a stationary interference pattern. This was based upon division of a single wavefront into two; these two wavefronts acted as if they emanated from two sources having a fixed phase relationship. Hence when they were allowed to interfere, stationary interference pattern was observed.

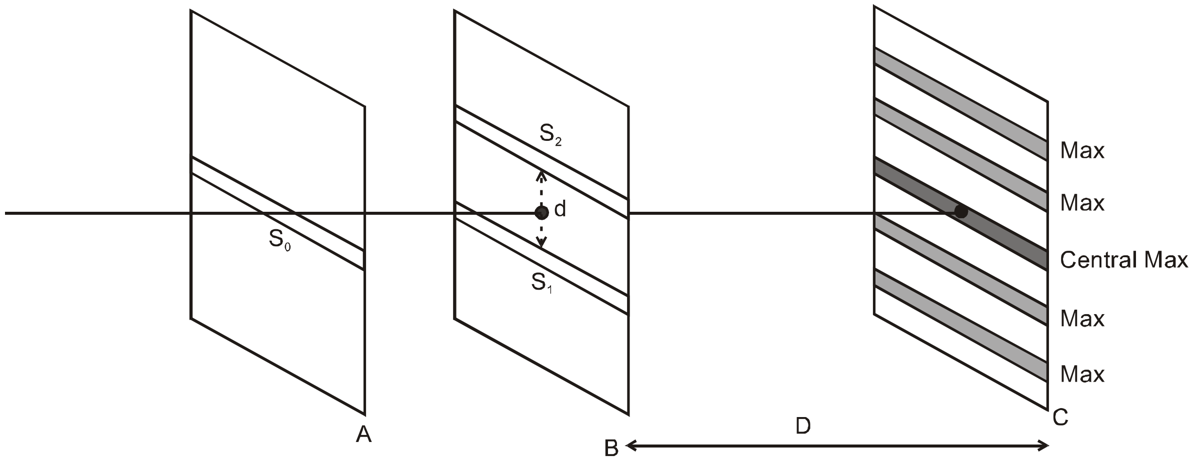


Figure : 1 : *Young's Arrangement to produce stationary interference pattern by division of wave front S_0 into S_1 and S_2*

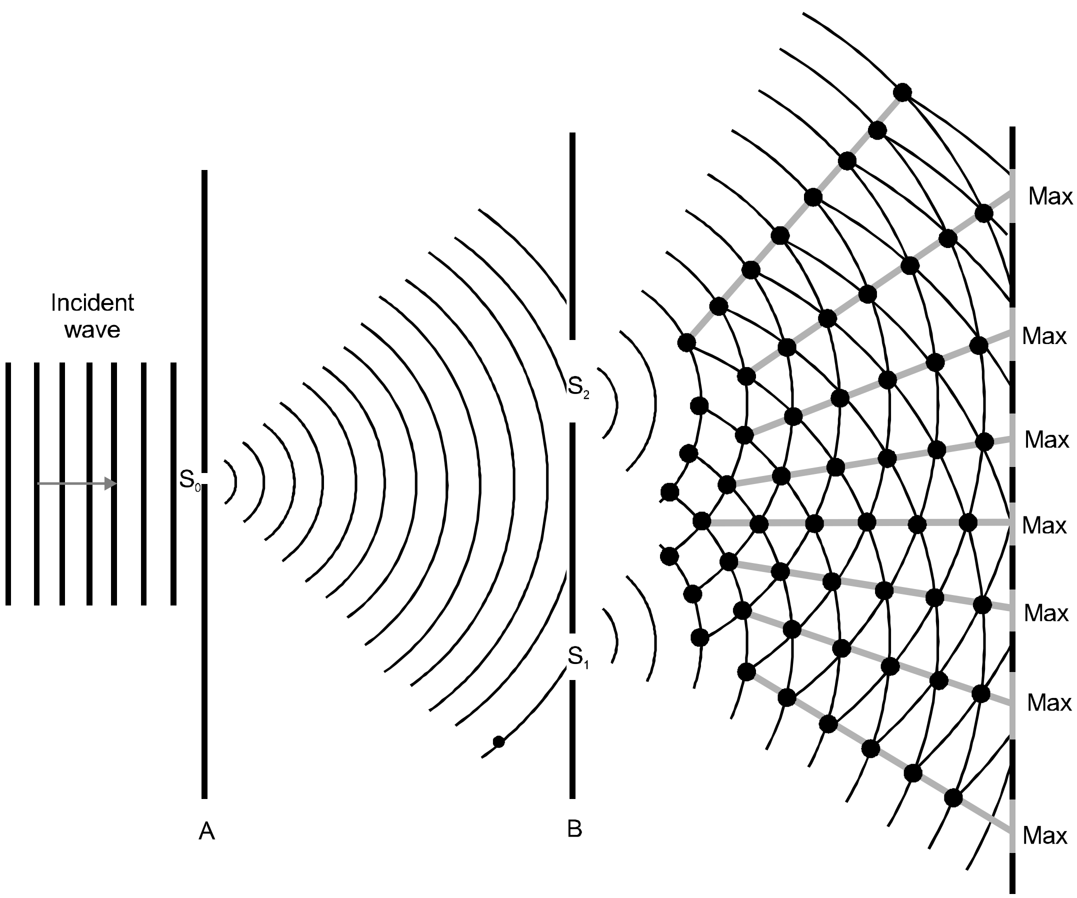


Figure 2 : In Young's interference experiment, light diffracted from pinhole S_0 excounters pinholes S1 and S2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producing an interference pattern on screen C.

1. Analysis of Interference Pattern

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former. We have already discussed the phase-difference arising due to path difference. If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference. If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.

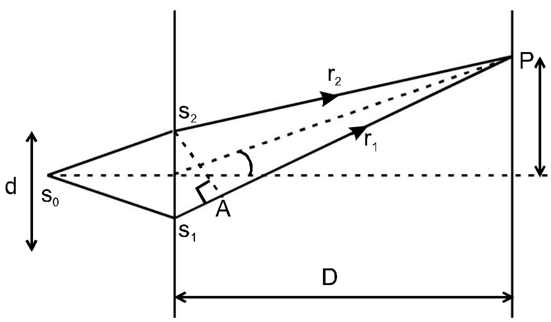


Figure : 3

Path difference $\Delta p = S_1P - S_2P$

$$= \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2} \qquad ...(1)$$

Approximation I :

For D >> d, we can approximate rays $\vec{r}_1$ and $\vec{r}_2$ as being approximately parallel, at angle θ to the principle axis.

Now, $S_1P - S_2P = S_1A = = S_1S_2 \sin\theta$

$\Rightarrow$ path difference $= d \sin\theta$ ...(2)

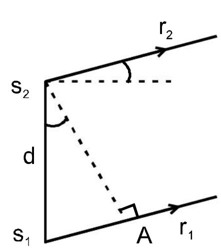


Figure : 4

Approximation II :

further if θ is small, i.e.$y << D$, $\sin\theta = \tan\theta = \frac{y}{D}$

and hence, path difference $= \frac{dy}{D}$ ...(3)

for maxima (constructive interference),

$$\Delta p = \frac{d.y}{D} = n\lambda$$

$$\Rightarrow \quad y = \frac{n\lambda D}{d}, \; n = 0, \pm 1, \pm 2, \pm 3 \quad ...(4)$$

Here n = 0 corresponds to the central maxima

n = ±1 correspond to the 1st maxima

n = ±2 correspond to the 2nd maxima and so on.

for minima (destructive interference).

$$\Delta p = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2} \pm \frac{5\lambda}{2}$$

$$\Rightarrow \Delta p = \begin{cases} (2n-1)\frac{\lambda}{2} & n = 1, 2, 3............ \\ (2n+1)\frac{\lambda}{2} & n = -1, -2, -3........ \end{cases}$$

consequently,

$$y = \begin{cases} (2n-1)\frac{\lambda D}{2d} & n = 1, 2, 3......... .... \\ (2n+1)\frac{\lambda D}{2d} & n = -1, -2, -3....... \end{cases} \quad ... (5)$$

Here n = ± 1 corresponds to first minima,

n = ± 2 corresponds to second minima and so on.

2 Fringe width :

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

fringe width $\quad \beta = \frac{\lambda D}{d}$ ... (6)

- Notice that it is directly proportional to wavelength and inversely proportional to the distance between the two slits.

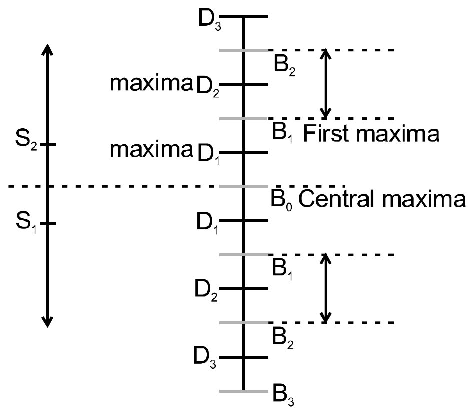


Figure : 5 fringe pattern in YDSE

3 Maximum order of Interference Fringes :

In section 1 we obtained,

$y = \frac{n\lambda D}{d}$, n = 0, ± 1, ± 2 ..... for interference maxima, but n cannot take infinitely large values, as that would violate the approximation (II)

i.e., θ is small or $y << D$

$$\Rightarrow \quad \frac{y}{D} = \frac{n\lambda}{d} << 1$$

Hence the above formula (4 & 5) for interference maxima/minima are applicable when

$$n << \frac{d}{\lambda}$$

when n becomes comparable to $\frac{d}{\lambda}$ path difference can no longer be given by equation (3) but by (2)

Hence for maxima

$$\Delta p = n\lambda$$

$$\Rightarrow \quad d\sin\theta = n\lambda$$

$$\Rightarrow \quad n = \frac{d\sin\theta}{\lambda}$$

Hence highest order of interference maxima,

$$n_{max} = \left[\frac{d}{\lambda}\right] \qquad .... (7)$$

where [ ] represents the greatest integer function.

Similarly highest order of interference minima,

$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] \qquad .... (8)$$

Aliter

$\Delta p = S_1P - S_2P$

$\Delta p \le d \Rightarrow \quad \Delta p_{max} = d$

(3rd side of a triangle is always greater than the difference in length of the other two sides)

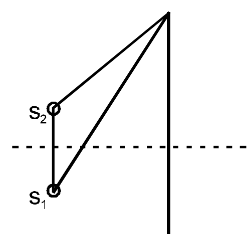


Figure : 6

4 Intensity :

Suppose the electric field components of the light waves arriving at point P(in the Figure : 3) from the two slits S_1 and S_2 vary with time as

$$E_1 = E_0 \sin \omega t$$

and $$E_2 = E_0 \sin (\omega t + \phi)$$

Here $\phi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

and we have assumed that intensity of the two slits S_1 and S_2 are same (say I_0); hence waves have same amplitude E_0.

then the resultant electric field at point P is given by,

$E = E_1 + E_2 = E_0 \sin \omega t + E_0 \sin (\omega t + \phi)$

$= E_0' \sin (\omega t + \phi')$

where $E_0'^2 = E_0^2 + E_0^2 + 2E_0 . E_0 \cos \phi$

$= 4 E_0^2 \cos^2 \phi/2$

Hence the resultant intensity at point P,

$$I = 4I_0 \cos^2 \frac{\phi}{2} \qquad ....(9)$$

$I_{max} = 4I_0$ when $\frac{\phi}{2} = n\pi$,

n = 0, ±1, ±2,.....,

$I_{min} = 0$ when $\frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi$

$n = 0, \pm 1, \pm 2$ ....

Here $\phi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

If $D >> d$, $\phi = \frac{2\pi}{\lambda} d \sin\theta$

If $D >> d$ & $y << D$, $\phi = \frac{2\pi}{\lambda} d \frac{y}{D}$

However if the two slits were of different intensities I_1 and I_2,

say $E_1 = E_{01}, \sin \omega t$

and $E_2 = E_{02}, \sin(\omega t + \phi)$

then resultant field at point P,

$$E = E_1 + E_2 = E_0 \sin(\omega t + \phi)$$

where $E_0^2 = E_{01}^2 + E_{O2}^2 + 2E_{01} E_{02} \cos\phi$

Hence resultant intensity at point P,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi \quad ...(10)$$

Solved Examples

Ex. 4 In a YDSE, D = 1m, d = 1mm and $\lambda = 1/2$ mm

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the no of maxima and minima obtained on the screen.

Sol. $D >> d$

Hence $\Delta P = d \sin\theta$

$$\frac{d}{\lambda} = 2,$$

clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\Delta p = \frac{dy}{D}$ cannot be used

for Ist maxima,

$$\Delta p = d \sin\theta = \lambda$$

$\Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{1}{2} \Rightarrow \theta = 30^o$

Hence, $y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

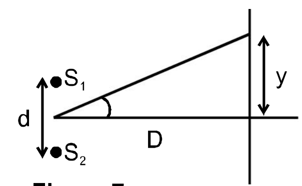


Figure 7

(ii) Maximum path difference

$\Delta P_{max} = d = 1$ mm

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$

and highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total no. of maxima = $2n_{max} + 1^* = 5$

*(central maxima).

Total no. of minima = $2n_{min} = 4$

Ex. 5 Monochromatic light of wavelength 5000 A° is used in Y.D.S.E., with slit-width, d = 1mm, distance between screen and slits, D = 1m. If intensity at the two slits are, $I_1 = 4I_0$, $I_2 = I_0$, find

(i) fringe width β

(ii) distance of 5th minima from the central maxima on the screen

(iii) Intensity at $y = \frac{1}{3}$ mm

(iv) Distance of the 1000th maxima

(v) Distance of the 5000th maxima

Sol. (i) $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5$ mm

(ii) $y = (2n-1)\frac{\lambda D}{d}$, $n = 5 \Rightarrow y = 2.25$ mm

(iii) At $y = \frac{1}{3}$ mm, $\quad y << D$

Hence $\Delta p = \frac{d.y}{D}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$$

Now resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

$$= 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi$$

$$= 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5\times10^{-6}} = 2000$

n = 1000 is not << 2000

Hence now $\Delta p = d\sin\theta$ must be used

Hence, $d\sin\theta = n\lambda = 1000\,\lambda$

$\Rightarrow \quad \sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2}$

$\Rightarrow \quad \theta = 30^{o}$

$y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

(v) Highest order maxima

$$n_{max} = \left[\frac{d}{\lambda}\right] = 2000$$

Hence, n = 5000 is not possible.

SHAPE OF INTERFERENCE FRINGES IN YDSE

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$$S_2P - S_1P = \Delta = \text{constant}. \quad ...(1)$$

If $\Delta = \pm\frac{\lambda}{2}$, the fringe represents 1st minima.

If $\Delta = \pm\frac{3\lambda}{2}$ it represents 2nd minima

If $\Delta = 0$ it represents central maxima,

If $\Delta = \pm\lambda$, it represents 1st maxima etc.

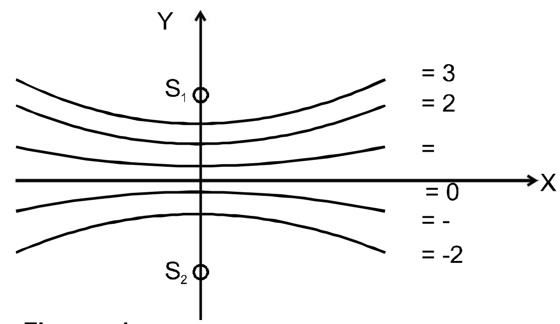


Figure : 1

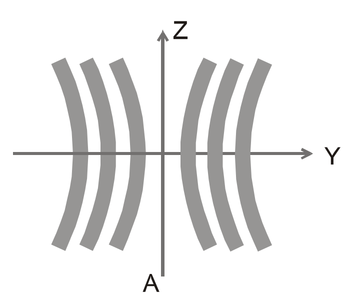
Equation (1) represents a hyperbola with its two foci at S_1 and S_2

The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

A. If the screen is $\perp$ er to the X axis, i.e. in the YZ plane, as is generally the case, firinges are hyperbolic with a straight central section.

B. If the screen is in the XY plane, again fringes are hyperbolic.

C. If screen is $\perp$ er to Y axis (along S_1S_2), ie in the XZ plane, fringes are concentric circles with center on the axis S_1S_2; the central fringe is bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n-1)\frac{\lambda}{2}$.



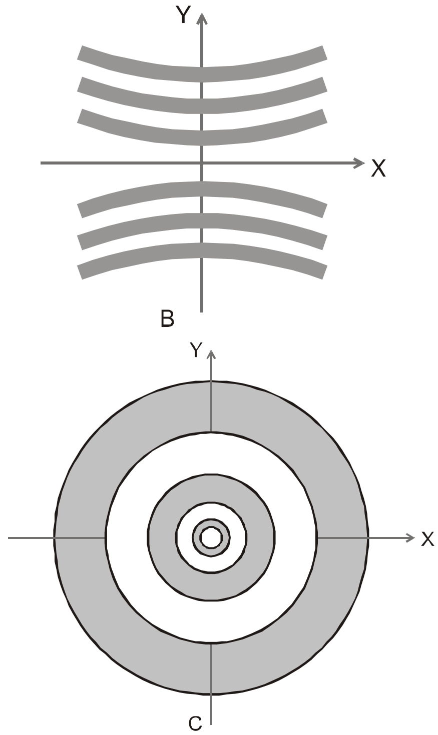


Figure : 2

YDSE WITH WHITE LIGHT

The central maxima will be white because all wavelengths will constructively interference here. However slightly below (or above) the position of central maxima fringes will be coloured. for example if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190 \text{ nm},$$

completely destructive interference will occur for violet light. Hence we will have a line devoid of violet colour that will appear reddish. And if

$$S_2P–S_1P = \frac{\lambda_{red}}{2} \simeq 350 \text{ nm},$$

completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe; for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination. for example if

$$S_2P - S_1 P = 3000 \text{ nm},$$

then constructive interference will occur for wavelengths $\lambda = \frac{3000}{n}$ nm. In the visible region these wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish–yellow), 430 nm (violet). Clearly such a light will appear white to the unaided eye.

Thus with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

Solved Examples

Ex. 6 A beam of light consisting of wavelengths 6000Å and 4500Å is used in a YDSE with D = 1m and d = 1 mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

Sol. $\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6 \text{ mm}$

$\beta_2 = \frac{\lambda_2 D}{d} = 0.45 \text{ mm}$

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

then, $y = n_1P_1 = n_2P_2$ = LCM of β_1 and β_2

$\Rightarrow$ y = LCM of 0.6 cm and 0.45 mm

y = 1.8 mm **Ans.**

At this point 3rd maxima for 6000 Å & 4th maxima for 4500 Å coincide

Ex. 7 White light is used in a YDSE with D = 1m and d = 0.9 mm. Light reaching the screen at position y = 1 mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

Sol. $\Delta p = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3}$ m

= 900 nm

for minima $\Delta p = (2n - 1)\lambda/2$

$\Rightarrow \quad \lambda = \frac{2\Delta P}{(2n-1)} = \frac{1800}{(2n-1)}$

$= \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7}$ ....

of these 600 nm and 360 nm lie in the visible range. Hence these will be missing lines in the visible spectrum.

GEOMETRICAL PATH & OPTICAL PATH

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation

$E = E_0 \sin(\omega t - kx + \phi)$

If the light travels by Δx, its phase changes by

$k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$.

Consequently, change in phase,

$$\Delta\phi = k\Delta x = \frac{\omega}{c}(\mu\Delta x)$$

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. i.e. a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

The quantity $\mu\Delta x$ is called the optical path length of light, Δx_{opt}. And in terms of optical path length, phase difference would be given by,

$$\Delta\phi = \frac{\omega}{c}\Delta x_{opt} = \frac{2\pi}{\lambda_0}\Delta x_{opt} \quad .... (1)$$

where λ_0 = wavelength of light in vacuum.

However in terms of the geometrical path length Δx,

$$\Delta\phi = \frac{\omega}{c}(\mu\Delta x) = \frac{2\pi}{\lambda}\Delta x \quad .....(2)$$

where λ = wavelength of light in the medium ($\lambda = \frac{\lambda_0}{\mu}$).

1 **Displacement of fringe** on introduction of a glass slab in the path of the light coming out of the slits–

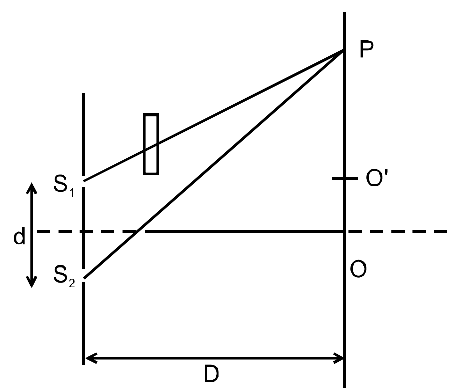


Figure : 1

On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by $t(\mu - 1)$. Now the path difference between waves coming form S_1 and S_2 at any point P is

$\Delta p = S_2P - (S_1P + t(\mu - 1))$

$= (S_2P - S_1P) - t(\mu - 1)$

$\Rightarrow \quad \Delta p = d \sin\theta - t(\mu - 1)$ if $d << D$

and $\quad \Delta p = \frac{yd}{D} - t(\mu - 1)$ If $y << D$ as well.

for central bright fringe,

$$\Delta p = 0$$

$$\Rightarrow \quad \frac{yd}{D} = t(\mu - 1).$$

$$\Rightarrow \quad y = OO' = (\mu - 1)t\,\frac{D}{d} = (\mu - 1)\,t\,.\,\frac{\beta}{\lambda}$$

The whole fringe pattern gets shifted by the same distance

$$\Delta = (\mu - 1)\,.\,\frac{D}{d} = (\mu - 1)t\,\frac{B}{\lambda}.$$

* Notice that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

Solved Examples

Ex. 8 In a YDSE with d = 1mm and D = 1m, slabs of ($t = 1\mu m$, $\mu = 3$) and ($t = 0.5\ \mu m$, $\mu = 2$) are introduced in front of upper and lower slit respectively. Find the shift in the fringe pattern.

Sol. Optical path for light coming from upper slit S_1 is

$$S_1P + 1\mu m\,(2 - 1) = S_2P + 0.5\ \mu m$$

Similarly optical path for light coming from S_2 is

$$S_2P + 0.5\ \mu m\,(2 - 1) = S_2P + 0.5\ \mu m$$

Path difference :

$$\Delta p = (S_2P + 0.5\ \mu m) - (S_1P + 2\mu m)$$

$$= (S_2P - S_1P) - 1.5\ \mu m.$$

$$= \frac{yd}{D} - 1.5\ \mu m$$

for central bright fringe $\Delta p = 0$

$$\Rightarrow \quad y = \frac{1.5\mu m}{1mm} \times 1m = 1.5\ mm.$$

The whole pattern is shifted by 1.5 mm upwards.

YDSE WITH OBLIQUE INCIDENCE

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up

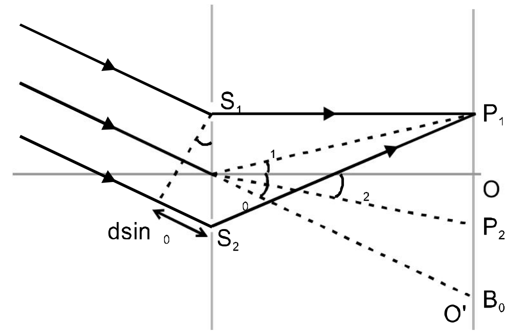


Figure : 1

for points above the central point on the screen, (say for P_1)

$$\Delta p = d\sin\theta_0 + (S_2P_1 - S_1P_1)$$

$$\Rightarrow \quad \Delta p = d\sin\theta_0 + d\sin\theta_1$$

(If d << D)

and for points below O on the screen, (say for P_2)

$$\Delta p = |(d\sin\theta_0 + S_2P_2) - S_1P_2|$$

$$= |d\sin\theta_0 - (S_1P_2 - S_2P_2)|$$

$$\Rightarrow \quad \Delta p = |d\sin\theta_0 - d\sin\theta_2|$$

(if d << D)

We obtain central maxima at a point where,

$$\Delta p = 0.$$

$$(d\sin\theta_0 - d\sin\theta_2) = 0$$

$$\text{or} \quad \theta_2 = \theta_0.$$

This corresponds to the point O' in the diagram.

Hence we have finally for path difference.

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) - \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) - \text{for points between O \& O'} \\ d(\sin\theta - \sin\theta_0) - \text{for points below O'} \end{cases} \quad ..(1)$$

Solved Examples

Ex. 9 In YDSE with D = 1m, d = 1mm, light of wavelength 500 nm is incident at an angle of 0.57º w.r.t. the axis of symmetry of the experimental set up. If centre of symmetry of screen is O as shown.

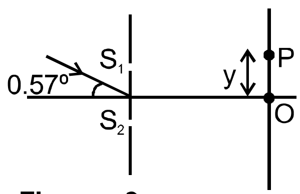


Figure : 2

(i) find the position of central maxima

(ii) Intensity at point O in terms of intensity of central maxima I_0.

(iii) Number of maxima lying between O and the central maxima.

Sol. (i) $\theta = \theta_0 = 0.57º$

$\Rightarrow \quad y = -D \tan\theta \simeq -D\theta$

$= -1 \text{ meter} \times \left(\frac{0.57}{57}\text{rad}\right)$

$\Rightarrow \quad y = -1\text{cm}.$

(ii) for point 0, $\theta = 0$

Hence, $\Delta p = d \sin\theta_0 \simeq d\theta_0$

$= 1 \text{ mm} \times (10^{-2} \text{ rad})$

$= 10{,}000 \text{ nm} = 20 \times (500 \text{ nm})$

$\Rightarrow \quad \Delta p = 20\,\lambda$

Hence point O corresponds to 20th maxima

$\Rightarrow$ intensity at O = I_0

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

THIN-FILM INTERFERENCE

In YDSE we obtained two coherent source from a single (incoherent) source by division of wave-front. Here we do the same by division of Amplitude (into reflected and refracted wave).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d then waves reflected from the upper surface interfere with waves reflected from the lower surface.

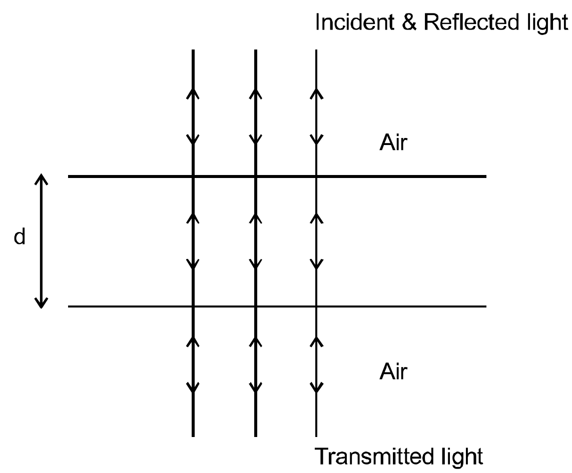


Figure : 1

Clearly the wave reflected from the lower surface travel an extra optical path of 2μd, where μ is refractive index of the film.

Further if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

Consequently condition for constructive and destructive interference in the reflected light is given by,

$2\mu d = n\lambda$ for destructive interference

and $2\mu d = (n + \frac{1}{2})\lambda$ for constructive interference ....(1)

where $n = 0, 1, 2$ ....,

and λ = wavelength in free space.

Interference will also occur in the transmitted light and here condition of constructive and destructive interference will be the reverse of (1)

i.e. $2\mu d =$

$$\begin{cases} n\lambda & \text{for constructive interference} \\ (n+\frac{1}{2}\lambda) & \text{for destructive interference} \end{cases} \quad ..(2)$$

This can easily be explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light) However the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

In deriving equation (1) we assumed that the medium surrounding the thin film on both sides is rarer compared to the medium of thin film.

If medium on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of π in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by equation 1.

However if medium on one side of the film in denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of π in both reflection from upper and lower surface. Now the condition for constructive and destructive interference in the reflected light would be given by equation 2 and not equation 1.

Solved Examples

Ex. 10 White light, with a uniform intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on a water film, of index of refraction $\mu = 1.33$ and thickness $d = 320$ nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

Sol. This situation is like that of Figure (1), for which euqation (1) gives the interference maxima. Solving for λ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{m+1/2} = \frac{(2)(1.33)(320\,nm)}{m+1/2} = \frac{851nm}{m+1/2}$$

for m = 0, this give us λ = 1700 nm, which is in the infrared region. For m = 1, we find l = 567 nm, which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 nm, which is in the ultraviolet region. So the wavelength at which the light seen by the observer is brightest is

λ = 567 nm.

Ex. 11 A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (figure). The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible specturm (λ = 550 nm)? Assume the light is approximately perpendicular to the lens surface.

Sol. The situation here differs from figure (9.1) in that $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same (see figure 2). Unwanted reflections from glass can be, suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference 2L within the film must be equal to an odd number of half wavelengths:

$$2L = (m + 1/2)\lambda_{n2},$$

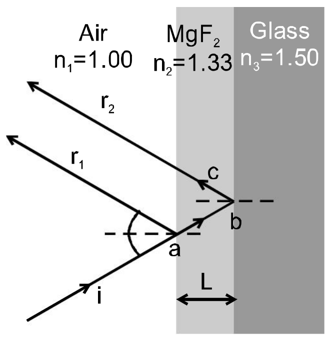


Figure : 2

or, with $\lambda n_2 = \lambda/n_2$,

$$2n_2 L = (m + 1/2)\lambda.$$

We want the least thickness for the coating, that is, the smallest L. Thus we choose m = 0, the smallest value of m. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550\,\text{nm}}{(4)(1.38)} = 96.6 \text{ nm}$$

DIFFRACTION

MEANING OF DIFFRACTION

It is the spreading of waves round the corners of an obstacle, of the order of wave length.

DEFINITION OF DIFFRACTION

The phenomenon of bending of light waves around the sharp edges of opaque obstacles or aperture and their encroachment in the geometrical shadow of obstacle or aperture is defined as diffraction of light.

NECESSARY CONDITIONS OF DIFFRACTION OF WAVES

The size of the obstacle (a) must be of the order of the wavelength of the waves (λ).

$$\frac{a}{\lambda} \leq 1$$

Note : Greater the wave length of wave higher will be its degree of diffraction. This is the reason that diffraction of sound & radio waves is easily observed but for diffraction of light, additional arrangement have to be arranged.

$$\lambda_{sound} > \lambda_{light}$$

Wave length of sound is nearly equal to size of obstacle. If size of obstactle is a & wavelength is λ then,

S.No.	a V/S λ	Diffraction
(1)	$a << \lambda$	Not possible
(2)	$a >> \lambda$	Not possible
(3)	$a \approx \lambda$	Possible

INTERPRETATION OF DIFFRACTION

As a result of diffraction maxima & minima of light intensities are which has unequal intensities. Diffraction is the result of superposing of waves from infinite number of coherent sources on the same wavefront has been distorted by the obstacle.

EXAMPLE OF DIFFRACTION

(1) When an intense source of light is viewed with the partially opened eye, colours are observed in the light.

(2) Sound produced in one room can be heard in the nearby room.

(3) Apperance of a shining circle around the section of sun just before sun rise.

(4) Coloured spectrum is observed if a light source at far distance is seen through a thin cloth.

TWO TYPE OF DIFFRACTION

Fresnel Diffraction : Fresenel diffraction which involves non-plane (spehrical) wavefronts, so that the sources and the point p (where diffraction effect is to be observed) are to be at a finite distance from the diffracting obstacle.

Fraunhofer Diffraction : Fraunhofer diffraction deals with wavefronts that are plane on arrival and an effective viewing distance of infinity. If follows that fraunhofer diffraction is an important special case of fresnel diffraction. In youngs double slit experiment, we assume the screen to be relatively distant, that we have fraunhofer condition.

(1) Fresnel Diffraction :

According to fresnel principal a wavefront can be divided into a number of small parts which are known as fresnel's half period zones.

Each point on the wavefront is a source of secondary wavelets, so that the wave from two consecutive zones reach the point of observation in opposite phase corresponding to a path difference of $\lambda/2$.

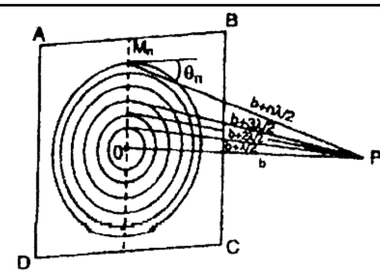
Mathematical Interpretation of Half Period Zones (HPZ) :

ABCD is a plane wavefront emitted by a monochromatic source of light. We want to determine its effect at a point of observation P distant b from centre. Taking P as centre and distance.

b, $b + \lambda/2$, $b + 2\lambda/2$, ... etc as radii we draw a large number of cocentric spheres.

The area of first circle is known as the first half period zone. The area between first & second circle is known as second HPZ.

Thus, the peripheral area enclosed between the nth circle & $(n-1)^{th}$ circle is defined as the n^{th} HPZ.



(A) Radius of n^{th} Half Period Zone (HPZ)

Radius of n^{th} HPZ = Radius of nth circle.

$$r_n = \sqrt{(b + n\lambda/2)^2 - b^2}$$

$$r_n = \sqrt{nb\lambda + \frac{n^2\lambda^2}{4}}$$

If λ^2 is very small we can neglect it.

$$r_n = \sqrt{nb\lambda}$$

$\therefore$ r_n depends on :-

(i) $r_n \propto \sqrt{n}$ (ii) $r_n \propto \sqrt{b}$ (iii) $r_n \propto \sqrt{\lambda}$

If b & λ remains constant for wavefront

$$r_1 : r_2 : r_3 = \sqrt{1} : \sqrt{2} : \sqrt{3}$$

Radius of HPZ is proportional to square root of natural numbers

(B) Average Distance of Point P from n^{th} HPZ

$$d_n = \frac{1}{2}\begin{bmatrix} \text{DIstance of } n^{th} \text{ circle from point P} + \\ \text{Distance of } (n-1)^{th} \text{ circle from point P} \end{bmatrix}$$

$$d_n = \frac{1}{2}\left[(b + n\lambda/2) + (b + \left(\frac{n-1}{2}\right)\lambda)\right]$$

$$d_n = b + \frac{(2n-1)}{4}\lambda$$

(C) Area of n^{th} Half Period Zone (HPZ) :

A_n = Area of n^{th} HPZ = Area of n^{th} circle = Area of $(n-1)^{th}$ circle

$$A_n = \pi r_n^2 - \pi r_{n-1}^2$$

$$A_n = \pi \left(b\lambda + \frac{(2n-1)\lambda^2}{4}\right)$$

If $\frac{\lambda^2}{4} <<< b\lambda$

$A_n = \pi b\lambda$

Note : Area of HPZ dowes not depend on value of n

$\therefore\ A_1 : A_2 : A_3 ....A_n : : 1 : 1 : ......1$

or $\mathbf{A_1 = A_2 = A_3 = ... A_n = \pi b\lambda}$

if $\frac{\lambda^2}{4} <<< b\lambda$, area of HPZ increases with increasing value of n

(D) Amplitude at point P due to nth HPZ :

(i) inversely proportional to distance $\Rightarrow R_n \alpha \frac{1}{d_n}$

(ii) Proportional to area $\Rightarrow R_n\ \alpha\ A_n$

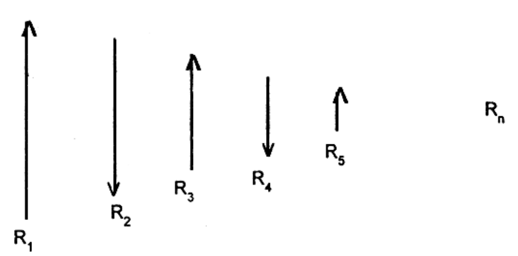
(iii) proportional to area $\Rightarrow R_n\ \alpha\ (1 + \cos\theta_n)$

$$R_n\ \alpha\ \frac{A_n}{d_n}(1+\cos\theta_n)$$

The phase difference between the wavelets originating from two consecutive HPZ and reaching the point P is π(or path difference λ/2 & time difference T/2).

The wavelets originating from two consecutive HPZ meet at point P in the opposite phase i.e., the amplitude of any two consecutive wavelets are of opposite signs.

Amplitude of Light of Consecutive HPZ may be shown by Vector Diagram :



Resultant Amplitude :

$R = R_1 - R_2 + R_3 - R_4 + R_5 - R_6 .... (-1)^{n-1} R_n$

It is clear from figure :

$R_2 = \frac{R_1 + R_2}{2},\ R_3 = \frac{R_2 + R_4}{2}$

$\frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = \frac{R_5}{R_4}$ ....= constant (< 1)

Resultant amplitude R at Point P due to Whole wavefront :

$R = R_1 - R_2 + R_3 - R_4$ ....$= R_1/2$

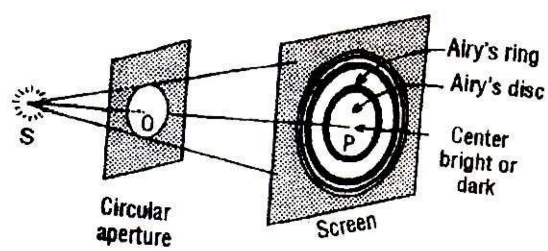
$I\ \alpha\ R^2 \Rightarrow \left(\frac{R_1}{2}\right)^2 = \frac{R_1^2}{4} = \frac{I_1}{4} \Rightarrow I = \frac{I_1}{4}$

Resultant intensity due to whole wavefront is equal to 25% of intensity due to first HPZ.

Note 1 : Alternative HPZ has phase difference 2π & path difference λ. They remains in phase at point P.

Note 2 : Secondary waves emitted from even number of HPZ reaches in phase at point P. Also secondary waves emitted from odd number of HPZ reaches in opposite phase at point P.

DIFFRACTION DUE TO CIRCULAR APERTURE



Let r = radius of aperture

b = distance from screen

(1) Then circular aperture allows only few HPZ to pass through which depends on r and b.

If λ is the wavelength of light used

$n\pi\ b\lambda = \pi r^2$

$\Rightarrow\ n = \frac{\pi r^2}{\pi b\lambda}$

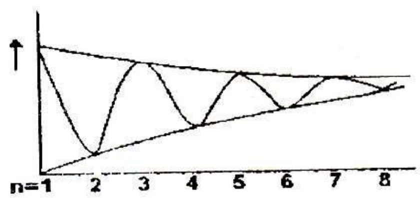
(2) If odd number of HPZ are allowed to pass through, the central fringe will be bright.

(3) The central fringe is brightness only one HPZ is allowed to pass. Then if n increases brightness of brightness central goes down.

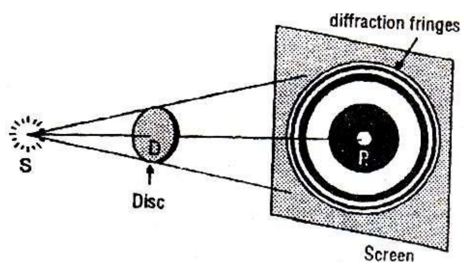
(4) Even number of HPZ are allowed to pass through the aperture, the central point will be dark.

(5) As n increases the darkness of dark central fringe goes dows. When n = 2 central fringe is darkest.

(6) If one screen is brought near to the aperture then b decreases n increases and bright 8 dark will be observed in alternate order.



DIFFRACTION DUE TO CIRCULAR DISC



(1) Central fringe of the shadow is always bright irrespective of number of HPZ covered by the disc.

(2) Around the central bright fringe bright & dark rings are observed in alternate order which is an effect interference.

(3) No. of HPZ = Area of disc

$$n\pi b\lambda = \pi r^2$$

$$n = \frac{r^2}{b\lambda}$$

The intensity at the centre P of the shadow region will be due to those HPZ which are not obstructed by the disc. Let n = 1, then the resultant amplitude at P will be

$$R = -R_2 + R_3 - R_4 \ldots\ldots$$

so the intensity $= -\frac{R_2}{2}$

so the intensity $I = \frac{KR_2^2}{2}$

If in the absence of the disc, the intensity of the light at P is

$$I_0 = \frac{KR_2^2}{4} \qquad \text{then} \qquad I = I_0\left(\frac{R_2}{R_1}\right)^2$$

DIFFERENCE BETWEEN INTERFERENCE & DIFFRACTIONM OF LIGHT

S.No.	Interference	Diffraction
1.	Two coherent sources are necessary	One coherent source is necessary
2.	All fringes have same width	Fringes have unequal width
3.	Width of central bright fringe is equal to other fringes	Width of central bright fringe is just double to other fringes
4.	Intensity of dark fringe is zero	Intensity of dark fringe is not zero
5.	All bright fringes have equal intensity	As order of bright fringes increases, intensity goes down.
6.	**For bright fringe :** (a) Path difference $\Delta = n\lambda$ (b) Phase difference $\delta = 2n\pi$	**For bright fringe :** $D = (2n - 1)\lambda/2$ $\delta = (2n - 1)\pi$
7.	**For dark fringe :** (a) Path difference $D = (2n - 1)\lambda/2$ (b) Phase difference $d = (2N - 1)\pi$	**For dark fringe :** $D = 2n\lambda/2$ $d = 2n\pi$

ZONE PLATE

Zone plate is a transperent film in which alternate HPZs have been blocked out so that the light coming from transparent (unblocked out) HPZs arrive in the same phase Thus, the zoneplate is a special

diffracting obstacle, the result of which is that the amplitudfe at P will be

$$\mathbf{R = R_1 + R_3 + R_5 + \ldots\ldots\ldots R_{n-1}}$$

$$\mathbf{R = R_2 + R_4 + R_6 + R_{2n}}$$

(1) If 2, 4, 6....... HPZ are black then zone plate is positive.

(2) If 1, 3, 5 ..... HPZ are black then zone plate is negative

(3) This is fundamentally used for monochromatic light

(4) This proves diffraction phenomenon

Focus on zone plate :

The first principal focus of zone plate lies at a distance b for which

$$r = \sqrt{b\lambda}$$

Thus the first focal length or principal focal length is ($f_1 = b$)

$$f_1 = \frac{r^2}{\lambda}$$

The zone plate has other focii also and thus n^{th} focal length

$$f = \frac{r^2}{(2n+1)\lambda} \qquad n = 0, 1, 2, 3$$

Further, the following relation connecting objects distacne u, image distance v and focal length f

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Is also valid for a zone plate.

Comments : DIfference between a zone plate and a convex lens.

(1) A convex lens has only one focus, while a zone plate has many foci.

(2) For a convex lens $f_{blue} < f_{red}$ but for a zone plate $f_{blue} > f_{red}$

(3) The surface of a convex lens are curved (spherical) but the durface of a zone plate are plane.

RESOLVING POWER (R.P.)

A large number of images are formed as a consequence of light diffraction from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring points.

Linear R.P. = $d/\lambda D$

Angular R.P. = d/λ

D = Observed distance

d = Distance between two points.

Telescope :

Limit of resolution $= \theta = \sin^{-1}\frac{1.22\lambda}{a}$; For small angles $\theta = \frac{1.22\lambda}{a}$

Resolving power = $\frac{1}{\text{limit of reslution}}$

Microscope :

Limit of resolution (the smallest distacne between the two object) $= x_{min} = \frac{1.22\lambda}{2\mu \sin\theta}$

Prism :

R.P. = $t\,(d\mu/d\lambda) = \lambda/d\lambda$

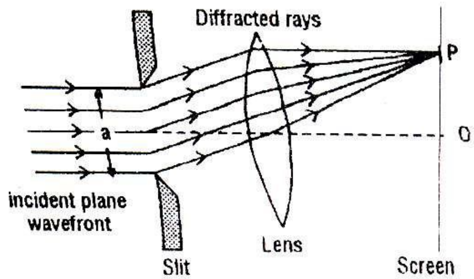
Diffraction Grating :

R.P. = $\lambda/d\lambda = N \times n$ (N is total number of lines & n is the order of spectrum)

Eye :

The limit of resolution of human eye is ‘1’ of arc (One minte of arc)

FRAUNHOFER DIFFRACTION FOR SINGLE SLIT



In this diffraction pattern central maxima is bright on the both side of it, maxima & minima occurs symmetrically.

(1) For Diffraction Maxima :

a sin θ = (2n – 1) λ/2

(2) For diffraction Minima :

a sin θ = nλ

(3) The maxima of mina is observed due to the superposition of waves emerging from infinte secondary sources between A and B points of slit.

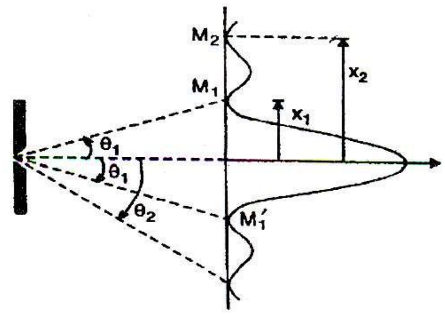
(4) Fringe width :

The distacne between two secondary minima formed on two sides of central maxima is known as the width of central maximum $W = \frac{2f\lambda}{a}$

f = focal distance of conves lense

a = width of slit

Angular width = $W_\theta = \frac{2\lambda}{a}$



DIFFRERNCE BETWEEN FRESNEL & FRAUNHOFFER DIFFRACTION

S.No.	Fresnel	Fraunhoffer
1.	The source is near and on one side of obstacle and screen also near to obstacle and on the other side.	Source & screen are both effectively at infinite distances from obstacle.
2.	Wavefront : spherical cylindrical is used.	Plane wavefront is used.
3.	No sophisticasted equipment is required.	Spectrometer is required.
4.	No lense is required	Convex lense is required.
5.	Diffraction pattern may have both bright & dark central fringe	Only bright central fringe is possible
6.	Example : Circular aperture, disc, ring etc.	Single slit, double slit, grating etc.

COMPARATIVE STUDY OF DIFFRACTION OF LIGHT & SOUND

(1) Sound travels in form of waves, that's wa\hy it is also diffracted. generally diffraction of sound waves is easily observed rather than light becuse wavelength of sound waves is the order of obstacle but wavelength of light is very small in comparisom to obsacle.

(2) **(a)** Ordinary audible sound has wavelength of the order of 1m & size of ordinary obstacle has same order that's why diffraction is easily observed.

(b) Ordinary light has wavelength of 10^{-7} m & ordinary obstacle has greater size in comparison to its wavelength that's why diffraction pattern is not observed.

(3) Generally diffraction of ultrasonic waves are not observed because its wavelength has order of 1 cm.

RECTILINEAR MOTION OF LIGHT

(1) Rectilinear motion of light can be explained by diffraction of light.

(2) If size of obstacle is the order of wavelength of light, then diffraction of light takes place & its rectilnear motion of light is not possible.

(3) If size of obstavle is must greater than wave length of light, then rectilnear motion of light is observed.

POINTS TO REMEMBER

(1) Greymaldy discovered the diffraction.

(2) Resultant intensity of light by whole wavefront is one fourth of intensity obtained by first HPZ.

(3) Intensity of diffraction pattern decreases, if size of obstacle is increased.

(4) Superposition of waves causes both diffraction & interference. Superposition of secondary waves originated from two coherent sources generates interference. Superposition of secondary wavelets generated from same wavefront, is called diffraction.

(5) Interference fringes has equal width but diffraction fringes has unequal width.

(6) In diffraction pattern, intensity of bright fringes is different but for interferecne, it is equal.

(7) intensities of dark fringes of interference is zero but for diffraction, it is not equal to zero.

(8) Effect of diffraction can be observed in only geometrical shadow of end region.

(9) In diffraction bright central fringe has double width, in comparison to others.

(10) In fresnel diffraction nor wavefront is plane neither rays are parallel.

(11) Image formed by zone plate has les intensity than formed by convex lense.

(12) Both convex lense & zone plate shows dispersion.

Solved Examples

Ex.12 What should be the size of the aperture of the objective of telescope which can just resolve the two stars of angular width of 10^{-3} degree by light of wavelength 5000Å

(A) 3.5 cm (B) 3.5 mm

(C) 3.5 m (D) 3.5 km

Sol. $d\theta = \frac{1.22\lambda}{a}$ or $a = \frac{1.22\lambda}{d\theta}$

According to question

$d\theta = 10^{-3}$ degree $= \frac{10^{-3} \times \pi}{180}$ Radian,

$\lambda - 5 \times 10^{-5}$

$$a = \frac{1.22 \times 5 \times 10^{-5} \times 180}{10^{-3} \times 3.14}$$

a = 3.5 cm

Hence the correct answer will be (A).

Ex.13 What will be the radius of 10^{th} zone of a zone plate for wavelength 5000 Å if its focal length is 20 cm

(A) 0.1 cm (B) 10 cm

(C) 0.1 m (D) 0.1 mm

Sol. $f = \frac{r_n^2}{n\lambda}$ .....(1)

According to question

n = 10,

$\lambda = 5000 \times 10^{-8}$ cm ......(2)

f = 20 cm, r_{10} = ?

From equation (1) and (2)

$r_{10} = \sqrt{20 \times 5 \times 10^{-5} \times 10}$

= 0.1 cm

Hence the correct answer will be (A).

Ex.14 A diffraction pattern is produced by a single slit of width 0.5 mm with the help of a convex lens of focal length 40cm. If the wavelength of light used is 5896Å then the distance of first dark fringe from the axis will be-

(A) 0.047 cm (B) 0.047 m

(C) 0.047 mm (D) 47 cm

Sol. $\theta = \frac{\lambda}{a}$ .....(1)

$\theta = \frac{x}{f}$ .....(2)

From eq^n. (1) and (2)

$\frac{\lambda}{a} = \frac{x}{f}$

$x = \frac{f\lambda}{a}$ .....(3)

According to question x = ?, f = 40cm

$\lambda = 5896 \times 10^{-8}$ cm

$a = 0.5 \times 10^{-1}$ cm ....(4)

From eq^n. (3) and (4)

$x = \frac{40 \times 5896 \times 10^{-8}}{5 \times 10^{-2}}$

= 0.047 cm,

Hence correct answer is (A).

Ex.15 Fraunhoffer diffraction pattern is observed at a distance of 2m on screen, when a plane-wavefront of 6000Å is incident perpendicular on 0.2 mm wide slit. Width of central maxima is

(A) 10 mm (B) 6mm

(C) 12mm (D) none of these

Sol. Width of central maxima = $\frac{2f\lambda}{a}$

$= \frac{2 \times 2 \times 6000 \times 10^{-10}}{0.2 \times 10^{-3}} = 12\text{mm}$,

Hence correct answer is (C)

Ex.16 Red light of wavelength 6500Å from a distant source falls on a slit 0.5 mm wide. What is the distance between two dark bands on each side of central bright band of diffraction pattern observed on a screen placed 1.8 m from the slit.

(A) 4.68×10^{-3} cm (B) 4.68×10^{-3} mm

(C) 4.68×10^{-3} nm (D) 4.68×10^{-3} m

Sol. Here, λ 6500Å = 6.5 × 10–7 m,

$a = 0.5$ mm $= 5 \times 10^{-4}$ m,

D = 1.8 m

Angular separation of two dark bands on each side of central bright band $2\theta = 2\lambda/a$

Actual distance between them,

$2x = 2\lambda/a \times D$

$2x = \frac{2 \times 6.5 \times 10^{-7} \times 1.8}{5 \times 10^{-4}}$

$2x = 4.68 \times 10^{-3}$ m,

Hence correct answer is (D).

Ex.17 The first diffraction minima due to a single slit diffraction is at θ = 30° for a light wavelength 5000Å. The width of the slit is

(A) 5×10^{-5} cm

(B) 1.0×10^{-4} cm

(C) 2.5×10^{-5} cm

(D) 1.25×10^{-5} cm

Sol. The distance of first diffraction minimum from the central principal maximum $x = \lambda D/d$

$\therefore \sin\theta = \frac{x}{D} = \frac{\lambda}{d}$

$\Rightarrow d = \frac{\lambda}{\sin\theta}$

$\Rightarrow d = \frac{5000 \times 10^{-8}}{\sin 30°} = 2 \times 5 \times 10^{-5}$

$\Rightarrow d = 1.0 \times 10^{-4}$ cm,

Hence correct answer is (B).

POLARISATION

UNPOLARISED LIGHT

(1) An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$. However, because of all directions are equally probable the resulting electromagnetic wave is a superposition of waves produced by individual atomic sources. This wave is called unpolarised light.

(2) All the vibrations of an unpolarised light at a given instant can be resolved in two mutually perpendicular directions and hence an unpolarised light is equivalent to superposition of two mutually perpendicular identical plane polarised light.

PLANE POLARISED LIGHT

(1) If somehow we confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion, the light is said to be plane polarised and the plane containing the direction of vibration and wave motion is called plane of polarisation.

(2) If an unpolarised light is converted into plane polarised light, its intensity reduces to half.

(3) Polarisation is a convincing proof of wave nature of light.

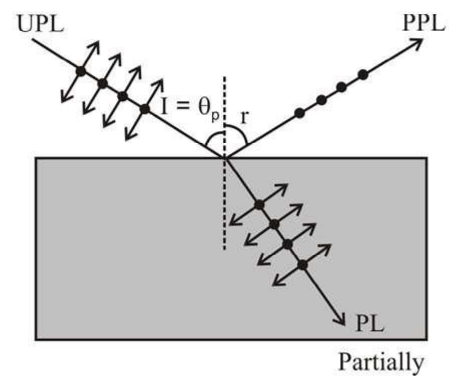
PARTIALLY POLARISED LIGHT

If in case of unpolarised light, electric vector in some plane is either more or less; then in its perpendicular plane, the light is said to be partially polarised.

METHODS OF POLARISATION

(a) **By Reflection :** Brewster discovered that when light is incident at a particular angle on a transparent substance, the reflected light is completely plane polarised with vibrations in a plane perpendicular to the plane of incidence. This specific angle of incidence is called polarising angle θ_P and is related to the refractive index μ of the material through the relation :

$$\tan \theta_P = \mu \quad ....(1)$$



This is known as Brewster's law.

In case of polarisation by reflection :

(1) For $I = \theta_P$, refracted light is plane polarised.

(2) For $I = \theta_P$, reflected and refracted rays are perpendicular to each other.

(3) For $<$ or $>$ θ_P, both reflected and refracted light become partially polarised.

Solved Examples

Ex.18 Image of sun formed due to reflection at air water interface is found to be very highly polarised. Refractive index of water being $\mu = 4/3$, find the angle of sun above the horizon.

Sol. Since the reflected light is very highly polarised, it implies that incident light falls at polarising angle of incidence θ_P. From Brewster's law,

$$\mu = \tan \theta_P$$

$$\therefore \quad \theta_P = \tan^{-1}(\mu) = \tan^{-1}(4/3) = 53.1^\circ$$

Since θ_P is the angle which the rays from sun make with the normal to the interface, angle with the interface will be $90^\circ - 53.1^\circ = 36.9^\circ$.

Ex.19 When light of a certain wavelength is incident on a plane surface of a material at a glancing angle 30º, the reflected light is found to be completely plane polarised. Determine

(a) refractive index of given material and

(b) angle of refraction.

Sol. Angle of incident light with the surface is 30º. Hence angle of incidence = 90º - 30º = 60º. Since reflected light is completely polarised, therefore, incidence takes place at polarising angle of incidence θ_P.

(a) $\therefore \quad \theta_P = 60^\circ$

Using Brewster's law

$$\mu = \tan \theta_P = \tan 60^\circ$$

$$\therefore \quad \mu = \sqrt{3}$$

(b) From Snell's law

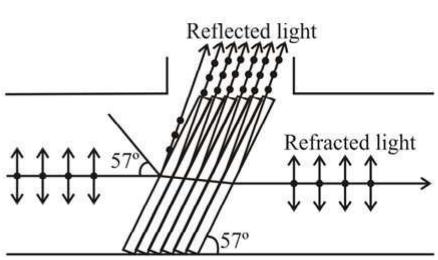
$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \quad \sqrt{3} = \frac{\sin 60^\circ}{\sin r}$$

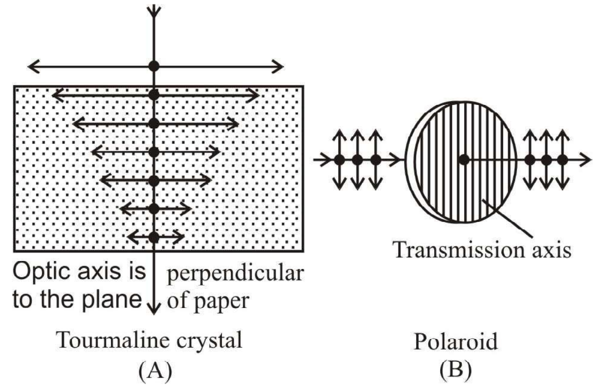
or $$\sin r = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$$

$$r = 30^\circ$$

(b) By Refraction : In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident at polarising angle. Since, in one reflection about 15 % of the light with vibration perpendicular to plane of paper is reflected, therefore after passing through a number of plates as shown in fig. emerging light will become plane polarised with vibrations in the plane of paper.

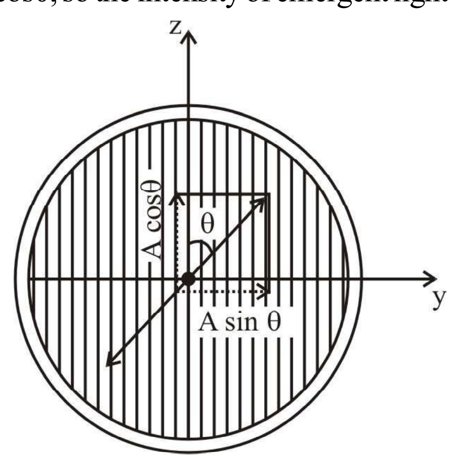


(c) By Dichroism : Some crystals such as tourmaline and sheets of iodosulphate of quinone have the property of strongly absorbing the light with vibrations perpendicular to a specific direction (called transmission axis) transmitting the light with vibrations parallel to it. This selective absorption of light is called dichroism. So if unpolarised light passes through proper thickness of these, the transmitted light will be plane polarised with vibrations parallel to transmission axis. Polariods work on this principle.



INTENSITY OF LIGHT EMERGING FROM A POLARIOD

If plane polarised light of intensity $I_0 (= KA^2)$ is incident on a polariod and its vibrations of amplitude A make an angle θ with transmission axis, as polariod will pass only those vibrations which are parallel to its transmission axis, i.e., A cosθ, so the intensity of emergent light will be



$$I = K(A \cos\theta)^2 = KA^2 \cos^2 \theta$$

or $$I = I_0 \cos^2\theta \quad [\text{as } I_0 = KA^2] \qquad ....(2)$$

This law is called Malus law. From this it is clear that :

(1) If the incident light is unpolarised, then as vibrations are equally probable in all directions (in a plane perpendicular to the direction of wave-motion), θ can have any value from 0 to 2π and

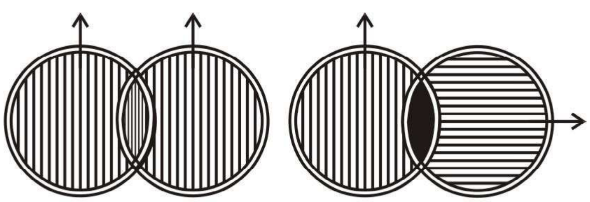
$$(\cos^2\theta)_{av} = \frac{1}{2}. \qquad I = \frac{1}{2} I_0$$

i.e., If an unpolarised light is converted into plane polarised light its intensity becomes half.

(2) If light of intensity I_1 emerging from one polaroid called polariser is incident on a second polariod (usually called analyser) the intensity of the light emerging from the second polaroid in accordance with Malus law will be given by

$I_2 = I_1 \cos^2\theta$

where θ' is the angle between the transmission axis of the two polariods.



Parallel polaroids Crossed polaroids

So if the two polaroids have their transmission axes parallel to each other,

i.e., $\theta' = 0^o$,

$I_2 = I_1 \cos^2 0^o = I_1$

and if the two polaroids are crossed, i.e., have their transmission axes perpendicular to each other, $\theta' = 90^o$,

$I_2 = I_1 \cos^2 90^o = 0$

So, if an analyser is rotated from 0º to 90º with respect to polariser, the intensity of emergent light changes from maximum value I_1 to minimum value zero.

Solved Examples

Ex.20 Two polaroids as oriented with their planes perpendicular to incident light and transmission axis making an angle of 30º with each other. What fraction of incident unpolarised light is transmitted?

Sol. If unpolarised light is passed through a polariod P_1, its intensity will become half.

So $I_1 = \frac{1}{2} I_0$ with vibrations parallel to the axis of P_1.

Now this light will pass through second polaroid P_2 whose axis is inclined at an a angle of 30º to the axis of P_1 and hence, vibrations of I_1. So in accordance with Malus law, the intensity of light emerging from P_2 will be

$$I_2 = I_1 \cos^2 30^o = \left(\frac{1}{2} I_0\right)\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

$$\frac{I_2}{I_0} = \frac{3}{8} = 37.5\ \%$$

Ex.21 Unpolarised light of intensity 32 Wm^{-2} passes through three polarisers such that the transmission axis of the last polariser is crossed with the first. If the intensity of the emerging light is 3 Wm^{-2}, what is the angle between the transmission axes of the first two polarisers? At what angle will the transmitted intensity be maximum?

Sol. If θ is the angle between the transmission axes of first polaroid P_1 and second P_2 while ϕ between the transmission axes of second polaroids P_2 and P_3, then according to given problem,

$$\theta + \phi = 90^o \text{ or } \phi = (90^o - \theta) \qquad ....(1)$$

Now, if I_0 is the intensity of unpolarised light incident on polaroid P_1, the intensity of light transmitted through it,

$$I_1 = \frac{1}{2} I_0 = I0 = \frac{1}{2}(32) = 16\ \frac{W}{m^2} \qquad ...(2)$$

Now as angle between transmission axes of polaroids P_1 and P_2 is θ, in accordance with Malus law, intensity of light transmitted through P_2 will be

$I_2 = I_1 \cos^2\theta = 16 \cos^2\theta$ [from Eq. (2)] .....(3)

And as angle between transmission axes of P_2 and P_3 is , light transmitted through P_3 will be

$I_3 = I_2 \cos^2\phi = 16 \cos^2\theta \cos^2\phi$ [from Eq.(3)]

Above equation in the light of (1) becomes,

$I_3 = 16 \cos^2\theta \cos2 (90^o - \theta)$

$= 4(\sin2\theta)^2$ ....(4)

According to given problem, $I_3 = 3 \text{ W/m}^2$

So, $4(\sin 2\theta)^2 = 3$ i.e., $\sin2\theta = (\sqrt{3}/2)$

or $2\theta = 60^o$ i.e. $\theta = 30^o$

Further in accordance with Eq. (4), I_3 will be max. when $\sin 2\theta$ = max., i.e.,

$\sin 2\theta = 1$ or $2\theta = 90^o$, i.e., $\theta = 45^o$