NUCLEAR PHYSICS

It exists at the centre of an atom, containing entire positive charge and almost whole of mass. The electron resolve around the neuleus to form an atom. The nucleus consists of protons (+ve charge) and neutrons. A proton has positive charge equal in magnitude to that of an electron ($+ 1.6 \times 10^{-19}$ C) and a mass equal to 1840 C) and a mass equal to 1840 times that of an electron. A neutron has no charge and mass is approximately equal to that of proton.

PROPERTIES OF A NUCLEUS

(1) Nuclear Mass :

As we know that every nucleus contains protons and neutrons and so every nucleus has a define mass. However, since the mass of electron is negligible so atomic mass is roughly equal to nuclear mass.

Atomic masses are measured in atomic mass unit (a.m.u.) defiined as

$\Rightarrow$ $1u = 931.478 \text{ MeV/c}^2$

and its energy equivalent is 931.48 MeV

Thye number of protons is a nucleus of an atom is called as the atomic number (Z) of that atom. The number of protons plus neutrons (called as Nucleus) in a nucleus of an atom is called as mass number (A) of that atom.

A partiicular set of nucleons forming an atom is called as nuclide. It is represented as $_ZX^A$. The nuclides having same number of protons (Z), but different number of nucleons (A) are called as isotopes. The nuclide having same number of nucleons (A), but different number of protons (Z) are called as isobars. The nuclide having same number of neutrons (A-Z) are called as isotones.

(2) Nuclear Charge :

Since nucleus contain +vely charged protons (charge = 1.67×10^{-19} C) and neutrons (neutral) so every nucleus has a net +ve charge.

(3) Nuclear Radius :

A rough estiimate of nuclear size suggests us that the radius of the nucleus of an atom having mass number 'A' is given by

$$R = R_0A^{1/3}$$

Where R_0 is a constant found to be equal to

$$R_0 = 1.4 \times 10^{-15} \text{ m} = 1.4 \text{ fm.}$$

(4) Nuclear Density :

In spite of the fact that nuclear radius depends on mass number of the atom but nuclear density is independent of mass number because if neutrons are supposed to be of almost the same mass as that of protons then the total mass of a nucleus is proportional to A. If each nucleon are supposed to have a mass m then nuclear density is given by

$$\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3} \qquad \text{(Which is independent of A)}$$

(5) Nuclear spin and magnetic moment

Like orbial electrons in an atom, nucleons inside nucleus have well defined quantum states. Correspondingly they have angular momentum and hence a magnetic moment. Like electrons nucleons also have intrinsic angular momentum and 'magnetic' moment corresponding to their spin.

NUCLEAR FORCES

If only the electrostatic and gravitattional forces existed in the nucleus, then it would be impossible to have stable nuclei composed of protons and neutrons. The gravitational forces are much too small to hold the nucleons together compared to the electrostatic forces repelling the protons. Since stable atoms of neutrons and protons do exiist, there must be another attractive force acting within the nucleus. This force is called the nuclear force.

Properties of nuclear force

(1) They are charge independent. The nuclear force between two proton is same as that between two neutrons or between a neutron and proton. This is known as charge independent character of nuclear forces.

(2) They may be repulsive may be attractive (Repulsive at exceedingly small separation between two nucleons appreciably smaller than 10^{-13} cm. i.e. 10^{-15} m.

(3) It is a short range force. Its radius of action is of the order of 10^{-13} cm.

(4) The nuclear force is of saturation character. Each nucleon in nucleus interacts with a limited number of nucleons.

(5) Nuclear force are much stronger than electromagnetic force or gravitational attrqactive forces. It is the strongest of all the forces. This is why it is called strong interaction.

(6) Nuclear force is spin dependent. If two interacting nucleons are having parallel spins then nuclear force operative betwee them is comparatively stronger and if their spins are antiparallel, nuclear interaction is comparatively weaker.

(7) Nuclear force is a non-central force. They can not be represented as directe along the st. line connecting the centers of the interacting nucleons. Its non central nature is due to the fact that it depends also on the orientation of the nucleon spins.

MASS DEFECT

It is observed that the mass of a nucleus is slightly less than the sum of the masses of constituent nucleons. Suppose a nucleus consists of 'Z' protons and 'N' neutrons. Mass of a proton, a neutron and the resulting nucleus are respectively m_p, m_n and M then mass defect of the nucleus is given by of the nucleus is given by

$$\Delta m = Zm_p + Nm_n - M$$

If A is the mass number of the nucleus

$$\Delta m = [Zm_p(A - Z)m_n - M]$$

In terms of atomic masses we may also write mass defect as

$$\Delta m = [Zm({}_1^1H)Nm_n - m({}_z^AX)]$$

Where $m({}_1^1H)$ = mass of one hydrogen atom.

$m({}_z^AX)$ = mass of atom having atomic no. Z and mass no. A

e.g.,

mass of ${}_1^1H$ = 1.00784 u

mass of neutron = 1.00874 u

Expected mass of deuterium = 2.01654 u

but measured mass = 2.0141 u

mass defect,

Δm = 0.00244 u

NUCLEAR STABILIITY

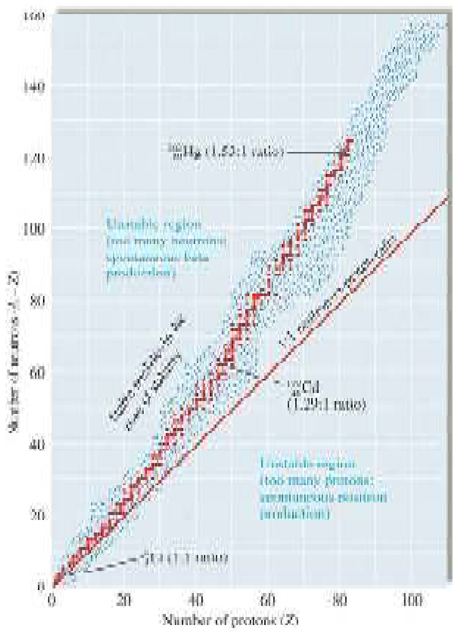


Figure shows pot of N vs Z for known nuclides. The stable nuclides are indicated by the black dots. Non-stable nuclides decay by emission of particles, or electromagnetic radiation, in a process called radioactivity.

BINDING ENERGY

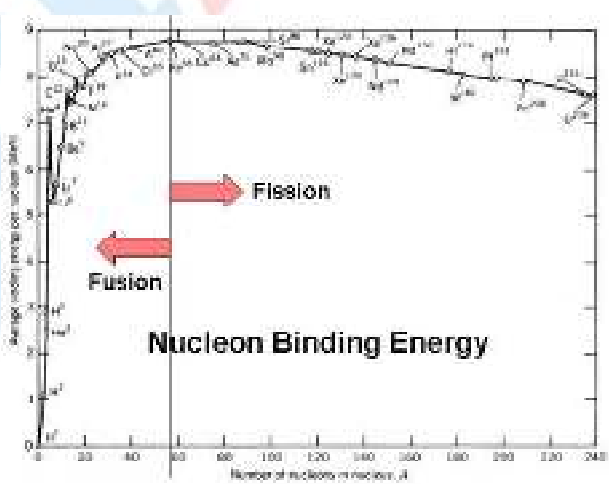
To break a nucleus into its constituent nuclei some energy is required to be supplied. This energy is called Binding Energy of the given nucleus or the energy equivalent of the missing mass of a nucleus is called the binding energy of the nucleus.

$$BE = (\Delta m)c^2 = [Zm_p + (A-Z)m_n - M]c^2$$

BE = Δm(on amu) × 931 MeV Where Δm = mass defect

Binding energy per nucleon is a measure of the stability of the nucleus. If there be n nucleons which is equal to A.

$$\frac{\text{Binding Energy}}{\text{Nucleon}} = \frac{B.E.}{A}$$



From the plot of B.E./ nucleons Vs mass number (A), we observe that :

(1) Binding energy per nucleon has low value for both heavy and liight nuclei i.e. heavy as well as light nuclei, both are unstable. B.E./nucleons increases on an average and reaches a maximum of about 8.7 MeV for A = 50–80. For more heavy nuclei, B.E./nucleons decreases slowly as A increases. For the heaviest natural element U^{238} it drops to about 7.5 MeV. From above observation, it follows that nuclei in the region of atomic 50 – 80 are most stable.

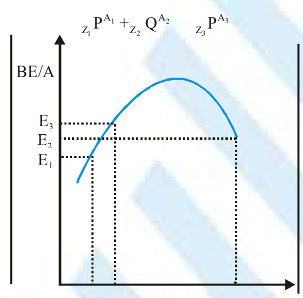
(2) The intermediate nuclei have large value of binding energy per nucleon so they are more stable.

(3) Binding energy per nucleon increases rapidly upto mass number 20 but there are peaks corresponding to ${}_{2}^{4}He, {}_{6}^{12}C, {}_{8}^{16}O$ which indicates that these nuclei are more stable than neighbours. The reason is that they may be considered to passes magic numbers i.e. their mass number is divisible by 4 and these nuclei may have ${}_{2}^{4}He$ as their constituents.

(4) The minimum value of the BE/Nucleon is in the case of deutron that is 1.11 MeV.

(5) The maximum value of the BE/Nucleon is 8.79 Mev for the nuclei ${}_{26}^{56}Fe$ which is therefore the most stable nucleus.

Ex. Using the following plot of BE/nucleon vs mass number, mention the condition for which the energy is absorbed or released for the reaction



Sol. Binding energy for reaction is $(xE_a + yE_b)$ and that for product is zE_c

Case - I :

$$\text{if } (A_1E_1 + A_2E_2) > A_3E_3$$

Energy is absorbed

Case - II :

$$\text{if } (A_1E_1 + A_2E_2) < A_3E_3$$

Energy is released

Note

(i) If we split a heavy neclues into two medium sized nucleii and total binding energy of new nuclei is greater then parent nuclei, then energy is released (Nuclear fission)

(ii) If two nuclei of small mass number combine to form a single medium size nucleus for which biinding energy is greater than the constituent nuclei, then energy is released (Nuclear fusion)

NUCLEAR COLLISIONS

We can represent a nuclear collision or reaction by the following notation, which means X (a,b) Y

$$\underset{\text{(bombarding particle)}}{a} + \underset{\text{(at rest)}}{X} \rightarrow Y + b$$

We can apply :

(i) Conservation of momentum (ii) Conservation of charge (iii) Conservation of mass–energy

For any nuclear reaction

$$\begin{array}{ccccccc} a & + & X & \rightarrow & Y & + & b \\ K_1 & & K_2 & & K_3 & & K_4 \end{array}$$

By mass energy conservation

(i) $K_1 + K_2 + (m_\alpha + m_x)c^2 = K_3 + K_4 + (m_Y + m_b)c^2$

(ii) Energy released in any nuclear reaction or collision is called Q value of the reaction

(iii) $Q = (K_3 + K_4) - (K_1 + K_2) = \Sigma K_P - \Sigma K_R = (\Sigma m_R - \Sigma m_P)c^2$

(iv) If Q is positive, energy is released and products are more stable in comparison to reactants.

(v) If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

$$Q = \Sigma(B.E.)_{product} - \Sigma(B.E.)_{reactants}$$

Ex. Let us find the Q value of fusion reaction

$$^4He + {}^4He \rightarrow {}^8Be,\ \text{if}\ \frac{B.E.}{A}\ \text{of He} = X\ \text{and}\ \frac{B.E.}{A}\ \text{of Be} = Y \Rightarrow Q = 8Y - 8X$$

Q value for α decay

$${}_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 \quad \Rightarrow \quad Q = K_\alpha + K_Y \quad ....(i)$$

Momentum conservation, $p_Y = p_\alpha$ ...(ii)

$$K_\alpha = \frac{p^2}{2 \times m \times 4} \quad \Rightarrow \quad K_Y = \frac{p^2}{2m(A-4)} = \frac{4K_\alpha}{A-4}$$

$$Q = K_\alpha + \frac{4K_\alpha}{A-4} = \frac{A}{A-4}K_\alpha \quad \Rightarrow \quad K_\alpha = \frac{A-4}{A}Q$$

For α decay $A > 210$ which means maximum part of released energy is associated with K.E. of α. If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction is called Threshold energy E_{th}, given by

$$E_{th} = -Q\left(\frac{m_1}{m_2} + 1\right) \text{ where } m_1 = \text{mass of the projectile.}$$

E_{th} = minimum kinetic energy of the projectile to initiate the nuclear reaction

m_2 = mass of the target

Ex. How much energy must a bombarding proton possess to cause the reaction ${}_3Li^7 + {}_1H^1 \rightarrow {}_4Be^7 + {}_0n^1$
(Mass of ${}_3Li^7$ atom is 7.01600, mass of ${}_1H^1$ atom is 1.0783, mass of ${}_4Be^7$ atom is 7.01693)

Sol. Since the mass of an atom includes the masses of the atomic electrons, the appropriate number of electron masses must be subtracted from the given values.

Reactants : Total mass $= (7.01600 - 3\,m_e) + (1.0783 - 1\,m_e) = 8.0943 - 4m_e$

Products : Total mass $= (7.01693 - 4m_e) + 1.0087 = 8.02563 - 4m_e$

The energy is supplied as kinetic energy of the bombarding proton. The incident proton must have more than this energy because the system must possess some kinetic energy even after the reaction, so that momentum is conserved with momentum conservation taken into account, the minimum kinetic energy that the incident particle must possess can be found with the formula.

where, $Q = -[(8.02563 - 4m_e) - (8.0943 - 4\,m_e)]\ 931.5\ \text{MeV} = -63.96\ \text{MeV}$

$$E_{th} = -\left(1 + \frac{m}{M}\right) Q = -\left(1 + \frac{1}{7}\right)(-63.96) = 73.1\ \text{MeV}$$

NUCLEAR FISSION

In 1938 by Hahn and Strassmann. By attack of a particle splitting of a heavy nucleus (A > 230) into two or more lighter nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

$$A + p \longrightarrow \text{excited nucleus} \longrightarrow B + C + Q$$

Hahn and Strassmann done the first fision of nucleus of U^{235}).

When U^{235} is bombarded by a neutron it splits into two fragments and 2 or 3 secondary neutrons and releases about 190 MeV ($\approx$ 200 MeV) energy per fission (or from single nucleus)

Fragments are uncertain but each time energy released is almost same.

Possible reactions are

$$U^{235} + {}_0n^1 \longrightarrow Ba + Kr + 3\,{}_0n^1 + 200\ \text{MeV} \qquad \text{or} \qquad U^{235} + {}_0n^1 \longrightarrow Xe + Sr + 2\,{}_0n^1 + 200\ \text{MeV}$$

and many other reactions are possible.

(i) The average number of secondary neutrons is 2.5.

(ii) Nuclear fission can be explained by using "liquid drop model" also.

(iii) The mass defect Δm is about 0.1% of mass of fissioned nucleus

(iv) About 93% of released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ – rays.

NUCLEAR CHAIN REACTION :

The equation of fission of U^{235} is $\quad U^{235} + {}_0n^1 \longrightarrow Ba + Kr + 3\,{}_0n^1 + Q.$

These three secondary neutrons produced in the reaction may cause of fission of three more U^{235} and give 9 neutrons, which in turn, may cause of nine more fission of U^{235} and so on.

Thus a continuous 'Nuclear Chain reaction' would start.

If there is no control on chain reaction then in a short time ($\approx 10^{-6}$ sec.) a huge amount of energy will be released. (This is the principle of 'Atom bomb'). If chain is controlled then produced energy can be used for peaceful purposes. For example nuclear reactor (Based on fission) are generating electricity.

NATURAL URANIUM :

It is mixture of U^{235} (0.7%) and U^{238} (99.3%).

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV. But U^{238} is fissionable with fast neutrons.

Note : Chain reaction in natural uranium can't occur. To improve the quality, percentage of U^{235} is increased to 3%. The proposed uranium is called 'Enriched Uranium' (97% U^{238} and 3% U^{235})

LOSSES OF SECONDARY NEUTRONS :

Leakage of neutrons from the system : Due to their maximum K.E. some neutrons escape from the system.

Absorption of neutrons by U^{238} : Which is not fissionable by these secondary neutrons.

CRITICAL SIZE (OR MASS) :

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called critical size.

REPRODUCTION FACTOR :

$$(K) = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

(i) If size of Uranium used is 'Critical' then K = 1 and the chain reaction will be steady or sustained (As in nuclear reaction)

(ii) If size of Uranium used is 'Super critical' then K > 1 and chain reaction will accelerate resulting in a explosion (As in atom bomb)

(iii) If size of Uranium used is 'Sub Critical' then K < 1 and chain reaction will retard and will stop.

NUCLEAR REACTOR (K = 1) : Credit $\rightarrow$ To Enricho Fermi

Construction :

(i) Nuclear Fuel : Commonly used are U^{235} , Pu^{239}. Pu^{239} is the best. Its critical size is less than critical size of U^{235}. But Pu^{239} is not naturally available and U^{235} is used in most of the reactors.

(ii) Moderator : Its function is to slow down the fast secondary neutrons. Because only slow neutrons can bring the fission of U^{235}. The moderator should be light and it should not absorb the neutrons. Commonly, Heavy water (D_2O, molecular weight 20 gm.) Graphite etc. are used. These are rich of protons. Neutrons collide with the protons and interchange their energy. Thus neutrons get slow down.

(iiii) Control rods :They have the ability to capture the slow neutrons and can control the chain reaction at any stage. Boron and Cadmium are best absorber of neutrons.

(iv) Coolant : A substance which absorb the produced heat and transfers it to water for further use. Generally coolant is water at high pressure

FAST BREADER REACTORS

The atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy. The amount of produced fuel (Pu^{239}) is more than consumed fuel (U^{235})

(i) Fuel : Natural Uranium.

(ii) Process: During fission of U^{235}, energy and secondary neutrons are produced. These secondary neutrons are absorbed by U^{238} and U^{239} is formed. This U^{239} converts into Pu^{239} after two beta decay. This Pu^{239} can be separated, its half life is 2400 years.

$${}_{92}U^{238} + {}_{0}n^{1} \longrightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239} \text{ (best fuel of fission)}$$

This Pu^{239} can be used in nuclear weapons because of its small critical size than U^{235}.

(iii) Moderator : Are not used in these reactors.

(iv) Coolant : Liquid sodium

NUCLEAR FUSION :

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$$A + B \rightarrow C + Q \text{ (Energy)}$$

The product (C) is more stable then reactants (A and B) & $m_C < (m_a + m_b)$

and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = \Delta m\, 931$ MeV

The total binding energy and binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_c - (E_a + E_b)$$

Fusion of four hydrogen nuclei into helium nucleus :

$$4({}_1H^1) \rightarrow {}_2He^4 + 2\,{}_{+}\beta^0 + 2\,\nu + 26\text{ MeV}$$

❖ Energy released per fission >> Energy released per fusion.

❖ Energy per nucleon in fission $\left[= \frac{200}{235} ;\ 0.85\text{MeV}\right]$ << energy per nucleon in fusion $\left[= \frac{24}{4} ;\ 6\text{MeV}\right]$

REQUIRED CONDITION FOR NUCLEAR FUSION

(i) High temperature :

Which provide kinetic energy to nuclei to overcome the repulsive electrostatic force between them.

(ii) High Pressure (or density) :

Which ensure frequent collision and increases the probability of fusion. The required temperature and pressure at earth (lab) are not possible. These condition exist in the sun and in many other stars. The source of energy in the sun is nuclear fusion, where hydrogen is in plasma state and there protons fuse to form helium nuclei.

HYDROGEN BOMB

It is based on nuclear fusion and produces more energy than an atom bomb.

Pair production	Pair Annihilation
A γ–photon of energy more than 1.02 MeV, when interact with a nucleus produces pair of electron (e^-) and positron (e^+). The energy equivalent to rest mass of e^- (or e^+)=0.51 MeV. The energy equivalent to rest mass of pair ($e^- + e^+$) = 1.02 MeV. For pair production Energy of photon $\geq$ 1.02 MeV. If energy of photon is more than 1.02 MeV, the extra energy (E–1.02) MeV divides approximately in equal amount to each particle as the kinetic energy or $(KE)_{e^- \text{ or } e^+} = \left[\frac{E_{Ph} - 1.02}{2}\right]$MeV If E < 1.02 MeV, pair will not produce.	When electron and positron combines they annihilates to each other and only energy is released in the form of two gama photons. If the energy of electron and positron are negligible then energy of each γ–photon is 0.5/MeV

Ex. In a nuclear reactor, fission is produced in 1 g for U^{235} (235.0439) in 24 hours by slow neutrons (1.0087 u). Assume that ${}_{35}Kr^{92}$ (91.8973 u) and ${}_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost.

(i) Write the complete reaction **(ii)** Calculate the total energy produced in kilowatt hour. Given 1u = 931 MeV.

Sol. The nuclear fission reaction is ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1$

Mass defect $\Delta m = [(m_u + m_n) - (m_{Ba} + m_{Kr} + 3m_n)] = 256.0526 - 235.8373 = 0.2153$ u

Energy released Q = 0.2153 × 931 = 200 MeV. Number of atoms in 1 g = $\frac{6.02 \times 10^{23}}{235} = 2.56 \times 10^{21}$

Energy released in fission of 1 g of U^{235} is E $= 200 \times 2.56 \times 10^{21} = 5.12 \times 10^{23}$ MeV

$$= 5.12 \times 10^{23} \times 1.6 \times 10^{-13} = 8.2 \times 10^{10}\text{ J}$$

$$= \frac{8.2 \times 10^{10}}{3.6 \times 10^{6}}\text{ kWh} = 2.28 \times 10^{4}\text{ kWh}$$

Ex. It is proposed to use the nuclear fusion reaction : ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4$ in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with at 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (Mass of ${}_1H^2$ is 2.0141 u and mass of ${}_2He^4$ is 4.0026 u)

Sol. Energy released in the nuclear fusion is $Q = \Delta mc^2 = \Delta m(931)$ MeV (where Δm is in amu)

$Q = (2 \times 2.0141 - 4.0026) \times 931$ MeV $= 23.834$ MeV $= 23.834 \times 10^6$ eV

Since efficiency of reactor is 25%

So effective energy used $= \frac{25}{100} \times 23.834 \times 10^6 \times 1.6 \times 10^{-19}$ J $= 9.534 \times 10^{-13}$ J

Since the two deuterium nuclei are involved in a fusion reaction,

therefore, energy released per deuterium is $\frac{9.534 \times 10^{-13}}{2}$.

For 200 MW power per day, number of deuterium nuclei required $= \frac{200 \times 10^6 \times 86400}{\frac{9.534 \times 10^{-23}}{2}} = 3.624 \times 10^{25}$

Since 2g of deuterium constitute 6×10^{23} nuclei, therefore amount of deuterium required is

$$= \frac{2 \times 3.624 \times 10^{25}}{6 \times 10^{23}} = 120.83 \text{ g/day.}$$

RADIOACTIVITY

INTRODUCTION

Among about 2500 known nuclides, fewer than 300 are stable. The others are unstable structures that decay to form other nuclides by spontaneously emitting particles and electromagnetic radiation, a process called radioactivity. The time scale of these decay processes ranges from a small fraction of a microsecond to billions of years. The substances which emit these radiations are called as radioactive substances. It was discovered by Henry Becquerel for atoms of Uranium. Later iit was discovered that many naturally occurring compounds of heavy elements like radium, thorium etc. also emit radiations.

At present, it is known that all the naturally occurring elements having atomic number greater than 82 are radioactive. For example some of them are; radium, polonium, thorium, actinium, uranium, radon etc. Later on Rutherford found that emission of radiation always accompained by transformation of one element (transmutation) into another. In actual radioactivity is the result of disintegration of an unable nucleus. Rutherford studied the nature of thesed radiations and found that these mainly consist of α,β,γ particles (rays).

α-Particles : $({}_2He^4)$

These carry a charge of +2e and mass equal to $4m_p$. These are nuclei of helium atoms. The energies of α-particles very form 5 Me V to 9 Me V ; theiir velocities vary from 0.01 – 0..1 times of c (velocity of light). They can be deflected by electric and magnetic field and have lower penetrating power but high ionising power.

β-Particles : $({}_1e^0)$

These are fast moving electrons having charge equal to e and mass $m_e = 9.1 \times 10^{-31}$ kg. Their velocities very from 1% to 99% of the velocity of light (c). They can also be deflected by electric and magnetic fields. They have low ionising power but high penetrating power.

γ-particles : $({}_0\gamma^0)$

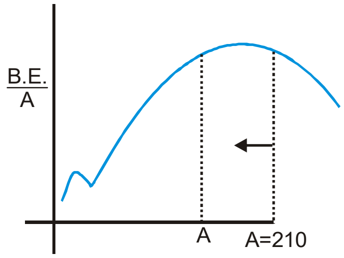
These are electro-magnetic waves of nuclear origin and of very short wavelength. They have no masss. They have maximum penetrating power and minimum ionising power. The energy released in a nuclear reaction is mainly emitted in from these γ-radiations.

RADIOACTIVE DECAYS

Generally, there are three types of radioactive decays **(i)** α decay **(ii)** β^- and β^+ decay **(iii)** γ decay

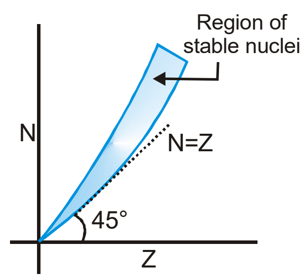
(i) α decay

In α decay, the unstable nucleus emits an α particle. By emitting α particle, the nucleus decreases it's mass energy number and move towards stability. Nucleus having A>210 shows α decay.By releasing α particle, it can attain higher stability and Q value is positive.

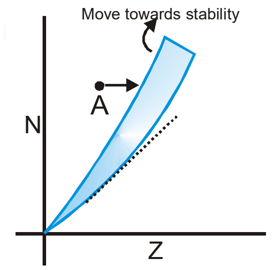


(ii) β decay

In beta decay (N/Z) ratio of nucleus is changed. This decay is shown by unstable nuclei. In beta decay, either a neutron is converted into proton or proton is converted into neutron. For better understanding we discuss N/Z graph. There are two type of unstable nuclides



❖ A type



For A type nuclides $(N/Z)_A > (N/Z)$ stable

To achieve stability, it increases Z by conversion of neutron into proton

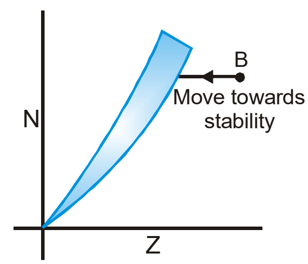
$${}_0n^1 \rightarrow {}_1p^1 + e^{-1} + \bar{\nu},\ {}_ZX^A \rightarrow {}_{Z+1}Y^A + \underset{(\beta\ \text{particle})}{e^{-1}} + \bar{\nu}$$

This decay is called β^{-1} decay.

Kinetic energy available for e^{-1} and $\bar{\nu}$ is, $Q = K_\beta + K_{\bar{\nu}}$

K.E. of β satisfies the condition $0 < K_\beta < Q$

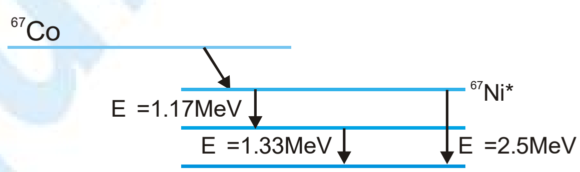
❖ B type



For B type nuclides $(N/Z)_B < (N/Z)$ stable

To achieve stability it decreases Z by the conversion of a proton into neutron. That is, $p \rightarrow n + \underset{(\text{positron})}{e^+} + \underset{(\text{neutrino})}{\nu},\ {}_ZX^A \rightarrow {}_{Z-1}Y^A + \underset{(\beta^+\text{particle})}{e^+} + \nu$

(iii) γ decay : when an α or β decay takes place, the daughter nucleus is usually in higher energy state, such a nucleus comes to ground state by emitting a photon or photons.



Order of energy of γ photon is 100 KeV e.g. ${}^{67}_{27}Co \rightarrow \underset{(\text{higher energy state})}{{}^{67}_{28}Ni^*} + \beta^- + \bar{\nu},\ {}^{67}_{28}Ni^* \rightarrow {}^{67}_{28}Ni + \gamma$ photon

Properties of α, β and γ rays

Features	α–particles	β–particles	γ–rays
Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electrons ($_{-1}\beta^0$ or β^-)	Electromagnetic wave (photons)
Charge	Twice of proton (+2e)	Electronic (– e)	Neutral
Mass	$\approx 4m_p$ (rest mass of β) m_p–mass of proton	rest mass = 0 = (rest mass of electron)	
Speed	1.4×10^7 m/s. to 2.2×10^7 m/s. (Only certain value between this range). Their speed depends on nature of the nucleus. So that it is a characteristic speed.	1% of c to 99% of c (All possible values between this range) β–particles come out with different speeds from the same type of nucleus. So that it can not be a characteristic speed.	Only $c = 3 \times 10^8$ m/s γ–photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.
K.E.	$\approx$ MeV	$\approx$ MeV	$\approx$ MeV
Energy spectrum	Line and discrete (or linear)	Continuous (or linear)	Line and discrete
Ionization power ($\alpha>\beta>\gamma$)	10,000 times of γ–rays	100 times of γ–rays (or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
Penetration power ($\gamma>\beta>\alpha$)	$\frac{1}{10000}$ times of γ–rays	$\frac{1}{100}$ times of γ–rays (100 times of α)	1(100 times of β)
Effect of electric or magnetic field	Deflection	Deflection (More than α)	No deflection
Explanation of emission	By Tunnel effect (or quantum mechanics)	By weak nuclear interactions	With the help of energy levels in nucleus

Laws of Radioactive Decay

1. The radioactive decay is a spontaneous process with the emission of α, β and γ rays. It is not influenced by external conditions such as temperature, pressure, electric and magnetic fields.

2. The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

Rate of decay = λ (number of nuclei) i.e. $\frac{dN}{dt} = -\lambda N$

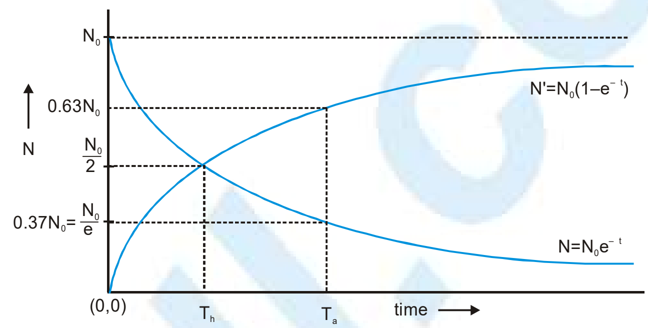
where λ is called the decay constant. This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t$$

where N_0 is the number of parent nuclei at t=0. The number that survives at time t is therefore

$N=N_0 e^{-\lambda t}$ and $t = \frac{2.303}{\lambda}\log_{10}\left(\frac{N_0}{N_t}\right)$ this function is plotted in figure.

Graph : Time versus N (or N')



(1) Half life (Th) : It is the time during which number of active nuclei reduce to half of initial value.

If at $t = 0$ no. of active nuclei N_0 then at $t = T_h$ number of active nuclei will be $\frac{N_0}{2}$

From decay equation $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_h} \Rightarrow T_h = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \approx \frac{0.7}{\lambda}$$

(2) Mean or Average Life (Ta) : It is the average of age of all active nuclei i.e.

$$T_a = \frac{\text{sum of times of existance of all nuclei in a sample}}{\text{initial number of active nuclei in that sample}} = \frac{1}{\lambda}$$

(i) At $t = 0$, number of active nuclei $= N_0$ then number of active nuclei at

$$t = T_a \text{ is } N = N_0 e^{-\lambda T_a} = N_0 e^{-1} = \frac{N_0}{e} = 0.37 N_0 = 37\% \text{ of } N_0$$

(ii) Number nuclei which have been disintegrated within duration T_a is

$N' = N_0 - N = N_0 - 0.37\,N_0 = 0.63\,N_0 = 63\%$ of N_0

(a) $T_a = \frac{1}{\lambda} = \frac{T_h}{\ln(2)} = \frac{T_h}{0.693} = 1.44\,T_h$

(b) Within duration $T_h \Rightarrow$ 50% of N_0 decayed and 50% of N_0 remains active

(c) Within duration $T_a \Rightarrow$ 63% of N_0 decayed and 37% of N_0 remains active

Activity of a sample (or decay rate)

It is the rate of decay of a radioactive sample $R = -\frac{dN}{dt} = N\lambda$ or $R = R_0 e^{-\lambda t}$

(i) Activity of a sample at any instant depends upon number of active nuclei at that instant.

$R \propto N$ (or active mass) , $R \propto m$

(ii) R also decreases exponentially w.r.t. time same as the number of active nuclei decreases.

(iii) R is not a constant with N, m and time while λ, T_h and T_a are constant

(iv) At $t = 0$, $R = R_0$ then at $t = T_h$ $\Rightarrow$ $R = \frac{R_0}{2}$ and at $t = T_a \Rightarrow R = \frac{R_0}{e}$ or $0.37\, R_0$

(v) Similarly active mass of radioactive sample decreases exponentially. $m = m_0 e^{-\lambda t}$

(vi) Activity of m gm active sample (molecular weight M_w) is $R = \lambda N = \frac{0.693}{T_h}\left[\frac{N_{AV}}{M_W}\right] m$

here N_{AV} = Avogadro number = 6.023×10^{23}

SI unit of R : 1 becquerel (1 Bq)= 1 decay/sec

Other unit is curie : 1 Ci = 3.70×10^{10} decays/sec

1 Rutherford : (1 Rd) =10^6 decays/s

Specific activity : Activity of 1 gm sample of radioactive substance. Its unit is Ci/gm

e.g. specific activity of radium (226) is 1 Ci/gm.

Ex. The half–life of cobalt–60 is 5.25 yrs. After how long does its activity reduce to about one eight of its original value?

Sol. The activity is proportional to the number of undecayed atoms: In each half–life, the remaining sample decays to half of its initial value. Since $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{1}{8}$, therefore, three half–lives or 15.75 years are required for the sample to decay to 1/8th its original strength.

Ex. A count rate meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute.

(i) Find the decay constant. **(ii)** Also, find the half–life of the sample.

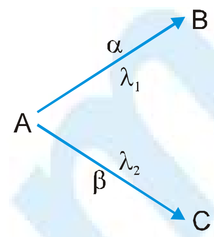
Sol. Initial activity $A_i = \left.\frac{-dN}{dt}\right|_{t=0} = \lambda N_0 = 4750$ ...**(i)** Final activity $A_f = \left.\frac{-dN}{dt}\right|_{t=5} = \lambda N = 2700$ ...**(ii)**

Dividing **(i)** by **(ii)**, we get $\frac{4750}{2700} = \frac{N_0}{N_t}$

The decay constant is given by $\lambda = \frac{2.303}{t}\log\frac{N_0}{N_t} = \frac{2.303}{5}\log\frac{4750}{2700} = 0.113\ \text{min}^{-1}$

Half–life of the sample is $T = \frac{0.693}{\lambda} = \frac{0.693}{0.113} = 6.14$ min

Parallel radioactive disintegration



Let initial number of nuclei of A is N_0 then at any time number of nuclei of

A, B & C are given by $N_0 = N_A + N_B + N_C \Rightarrow \frac{dN_A}{dt} = \frac{-d}{dt}(N_B + N_C)$

A disintegrates into B and C by emitting α, β particle.

Now, $\frac{dN_B}{dt} = -\lambda_1 N_A$ and $\frac{dN_C}{dt} = -\lambda_2 N_A \Rightarrow \frac{d}{dt}(N_B + N_C) = -(\lambda_1 + \lambda_2)N_A$

$$\Rightarrow \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2)N_A \Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2 \Rightarrow t_{eff} = \frac{t_1 t_2}{t_1 + t_2}$$

Ex. The mean lives of a radioactive substances are 1620 and 405 years for α–emission and β–emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both by α–emission and β–emission simultaneously.

Sol. When a substance decays by α and β emission simultaneously, the average rate of disintegration λ_{av} is given by $\lambda_{av} = \lambda_\alpha + \lambda_\beta$ when λ_α = disintegration constant for α–emission only λ_β = disintegration constant for β–emission only

Mean life is given by $T_m = \frac{1}{\lambda}$, $\lambda_{av} = \lambda_\alpha + \lambda_\beta \Rightarrow \frac{1}{T_m} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$

$$\lambda_{av} \times t = 2.303 \log \frac{N_0}{N_t},\ \frac{1}{324} t = 2.303 \log \frac{100}{25} \Rightarrow t = 2.303 \times 324 \log 4 = 449 \text{ years.}$$

Ex. A radioactive decay is given by $A \xrightarrow[t_{1/2}=8\ yrs]{} B$

Only A is present at t=0. Find the time at which if we are able to pick one atom out of the sample, then probability of getting B is 15 times of getting A.

Sol.

	A	$\longrightarrow$	B
at $t = 0$	N_0		0
at $t = t$	N		$N_0 - N$

Probability of getting A, $P_A = \frac{N}{N_0}$

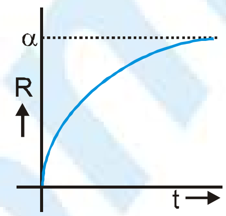
Probability of getting B, $P_B = \frac{N_0 - N}{N_0} \Rightarrow P_B = 15 P_A \Rightarrow \frac{N_0 - N}{N_0} = 15 \frac{N}{N_0} \Rightarrow N_0 = 16N \Rightarrow N = \frac{N_0}{16}$

Remaining nuclei are $\frac{1}{16}$ th of initial nuclei, hence required time $t = 4$ half lives =32 years

Radioactive Disintegration with Successive Production

$$\xrightarrow[(\alpha = rate\text{ of production})]{\alpha} A \xrightarrow{\lambda} B$$

$$\frac{dN_A}{dt} = \alpha = \lambda N_A \quad \text{....(i)}$$



when N_A in maximum $\frac{dN_A}{dt} = 0 = \alpha - \lambda N_A = 0$, $N_A \max = \frac{\alpha}{\lambda} = \frac{\text{rate of production}}{\lambda}$

By equation (i) $\int_0^t \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt$, Number of nuclei is $N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$

Ex. $\xrightarrow{10^{21}\text{ per sec}} A \xrightarrow{\lambda = \frac{1}{30}} B$

A shows radioactive disintegration and it is continuously produced at the rate of 10^{21} per sec. Find maximum number of nuclei of A.

Sol. At maximum, $r_{production} = r_{decay} \Rightarrow 10^{21} = \frac{1}{30} N \Rightarrow N = 30 \times 10^{21}$

Soddy and Fajan's Group Displacement Laws :

(i) **α–decay :** The emission of one α–particle reduces the mass number by 4 units and atomic number by 2 units. If parent and daughter nuclei are represented by symbols X and Y respectively then,

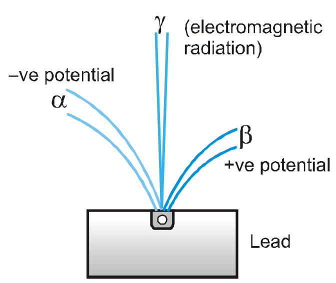
$$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}(\alpha)$$

(ii) **β–decay :** Beta particles are said to be fast moving electrons coming from the nucleus of a radioactive substance. Does it mean that a nucleus contains electrons? No, it is an established fact that nucleus does not contain any electrons. When a nucleus emits a beta particle, one of its neutrons breaks into a proton, an electron (i.e., β–particle) and an antineutrino $n \rightarrow p + e + \bar{\nu}$

where n= neutron p = proton e = β–particle

Thus emission of a beta particle is caused by the decay of a neutron into a proton. The daughter nucleus thus has an atomic number greater than one (due to one new proton in the nucleus) but same mass number as that of parent nucleus. Therefore, representing the parent and daughter nucleus by symbols X and Y respectively, we have $_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + \beta + \bar{\nu}$

(iii) **γ–decay :** When parent atoms emit gamma rays, no charge is involved as these are neutral rays. Thus there is no effect on the atomic number and mass number of the parent nucleus. However the emission of γ–rays represents energy. Hence the emission of these rays changes the nucleus from an excited (high energy) state to a less excited (lower energy) state.



1. NUCLEAR COLLISIONS

We can represent a nuclear collision or reaction by the following notation, which means X (a, b) Y

$$\underset{\text{(bombarding particle)}}{a} + \underset{\text{(at rest)}}{X} \rightarrow Y + b$$

We can apply :

(a) Conservation of momentum **(b)** Conservation of charge **(c)** Conservation of mass- energy

For any nuclear reaction $\underset{K_1}{a} + \underset{K_2}{X} \rightarrow \underset{K_3\ K_4}{Y + b}$

By mass energy conservation

(a) $K_1 + K_2 + (m_\alpha + m_x)c^2 = K_3 + K_4 + (m_Y + m_b)c^2$

(b) Energy released in any nuclear reaction or collision is called Q value of the reaction

(c) $Q = (K_3 + K_4) - (K_1 + K_2) = \sum K_p - \sum K_R = \left(\sum m_R - \sum m_p\right)c^2$

(d) If Q is positive, energy is released and products are more stable in comparison to reactants.

(e) If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

$$Q = \sum (B.E)_{\text{products}} - \sum (B.E)_{\text{reactants}}$$

2. Nuclear Fission

In 1938 by Hahn and Strassmann. By attack of a particle splitting of a heavy nucleus (A > 230) into two or more lighter nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

$$A + p \rightarrow \text{excited nucleus} \rightarrow B + C + Q$$

3. Nuclear Fusion

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$$A + B \rightarrow C + Q \text{ (Energy)}$$

The product (C) is more stable then reactants (A and B) and $m_c < (m_a + m_b)$ and mass defect

$$\Delta m = \left[(m_a + m_b) - m_c\right] \text{ amu}$$

Energy released is $E = \Delta m\ 931$ MeV

The total binding energy and binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_c - (E_a + E_b)$$

4. Radioactivity

Radioactive Decays : Generally, there are three types of radioactive decays

(a) α decay **(b)** β^- and β^+ dacay **(c)** γ decay

(a) **α decay :** By emitting α particle, the nucleus decreases it's mass number and move towards stability. Nucleus having A > 210 shows a decay.

(b) **β decay :** In beta decay, either a neutron is converted into proton or proton is converted into neutron.

(c) **γ decay :** When an α or β decay takes place, the daughter nucleus is usually in higher energy state, such a nucleus comes to ground state by emitting a photon or photons.

Order of energy of γ photon is 100 keV

5. **Laws of Radioactive Decay :** The rate of disintegration isdirectly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

Rate of decay = λ (number of nuclei) i.e., $\frac{dN}{dt} = -\lambda N$

where λ is called the decay constant.

This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t$$

where N_0 is the number of parent nuclei at t = 0. The number that survives at time t is therefore $N = N_0 e^{-\lambda t}$ and

$$t = \frac{2.303}{\lambda} \log_{10}\left(\frac{N_0}{N_t}\right)$$

$N = N_0 e^{-\lambda t}$ where λ = decay constant

(a) Half life $t_{1/2} = \frac{\ln 2}{\lambda}$

(b) Average life $t_{av} = \frac{1}{\lambda}$

Within duration $t_{1/2} \Rightarrow$ 50 % of N_0 decayed and 50 % of N_0 remains active

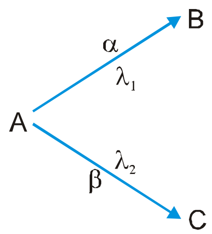
Within duration $t_{av} \Rightarrow$ 63 % of N_0 decayed and 37 % of N_0 remains active

(c) Activity $R = \lambda N = R_0 e^{-\lambda t}$
(d) 1 Bq = 1 decay/s.
(e) 1 curie $= 3.7 \times 10^{10}$ Bq.
(f) 1 rutherford = 10^6 Bq

(g) After n half lives Number of nuclei left $= \frac{N_0}{2^n}$

Probability of a nucleous for survival of time $t = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

6. **Parallel radioactive disintergration**



Let initial number of nuclei of A is N_0 then at any time number of nuclei of A, B & C are given by $N_0 = N_A + N_B + N_C$

$$\Rightarrow \frac{dN_A}{dt} = \frac{-d}{dt}(N_B + N_C)$$

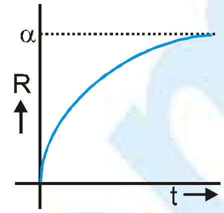
A disintegrates into B and C by emitting α, β particle.

Now, $\frac{dN_B}{dt} = -\lambda_1 N_A$ and $\frac{dN_C}{dt} = -\lambda_2 N_A \Rightarrow \frac{d}{dt}(N_B + N_C) = -(\lambda_1 + \lambda_2) N_A$

$$\Rightarrow \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2) N_A \Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2 \Rightarrow t_{eff} = \frac{t_1 t_2}{t_1 + t_2}$$

7. Radioactive Disintegration with Successive

Production $\xrightarrow[(\alpha=\text{rate of production})]{\alpha} A \xrightarrow{\lambda} B$



$$\frac{dN_A}{dt} = \alpha - \lambda N_A \quad ......\text{(i)}$$

When N_A is maximum $\frac{dN_A}{dt} = 0 \Rightarrow \alpha - \lambda N_A = 0$,

$$N_{A\,max} = \frac{\alpha}{\lambda} = \frac{\text{rate of production}}{\lambda}$$

By equation (i) $\int_0^t \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt$, Number of nuclei is $N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$

8. Equivalence of mass and energy

$E = mc^2$

Note :- $1u = 1.66 \times 10^{-27}$ kg $= 931$ MeV or $c^2 = 931.5$ MeV/u

9. Binding energy of ${}_ZX^A$

$BE = \Delta mc^2 = [Zm_p + (A - Z)m_n - m_x]c^2 = [Zm_H + (A - Z)m_n - m_x)c^2$

10. Q-value of a nuclear reaction

For $a + X \rightarrow Y + b$ or X (a, b) Y ; $Q = (M_a + M_x - M_Y - M_b)c^2$

11. Radius of the nucleus

$R = R_0 A^{1/3}$ where $R_0 = 1.3\ f_m = 1.3 \times 10^{-15}$ m

From Bohr Model

$n_1 = 1, \quad n_2 = 2, 3, 4......$K series
$n_1 = 2, \quad n_2 = 3, 4, 5......$L series
$n_1 = 3, \quad n_2 = 4, 5, 6......$M series