Mechanical Properties of Fluids

DEFINITION OF FLUID

The term fluid refers to a substance that can flow and does not have a shape of its own. For example liquid and gases.

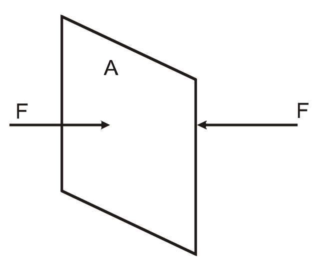
Fluid includes property $\rightarrow$ (A) Density (B) Viscosity (C) Bulk modulus of elasticity (D) pressure (E) specific gravity

PRESSURE IN A FLUID

The pressure p is defined as the magnitude of the normal force acting on a unit surface area.

$$P = \frac{\Delta F}{\Delta A}$$

ΔF = normal force on a surface area ΔA.



The pressure is a scalar quantity. This is because hydrostatic pressure is transmitted equally in all directions when force is applied, which shows that a definite direction is not associated with pressure.

Thrust. The total force exerted by a liquid on any surface in contact with it is called thrust of the liquid.

CONSEQUENCES OF PRESSURE

(i) Railway tracks are laid on large sized wooden or iron sleepers. This is because the weight (force) of the train is spread over a large area of the sleeper. This reduces the pressure acting on the ground and hence prevents the yielding of ground under the weight of the train.

(ii) A sharp knife is more effective in cutting the objects than a blunt knife.

The pressure exerted = Force/area. The sharp knife transmits force over a small area as compared to the blunt knife. Hence the pressure exerted in case of sharp knife is more than in case of blunt knife.

(iii) A camel walks easily on sand but a man cannot inspite of the fact that a camel is much heavier than man.

This is because the area of camel's feet is large as compared to man's feet. So the pressure exerted by camel on the sand is very small as compared to the pressure exerted by man. Due to large pressure, sand under the feet of man yields and hence he cannot walk easily on sand.

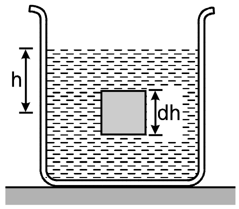
VARIATION OF PRESSURE WITH HEIGHT

Assumptions : (i) unaccelerated liquid (ii) uniform density of liquid (iii) uniform gravity

Weight of the small element dh is balanced by the excess pressure. It means $\frac{dp}{dh} = \rho g$.

$$\int_{P_a}^{P} dp = \rho g \int_{0}^{h} dh$$

$\Rightarrow$ $P = P_a + \rho gh$



PASCAL'S LAW

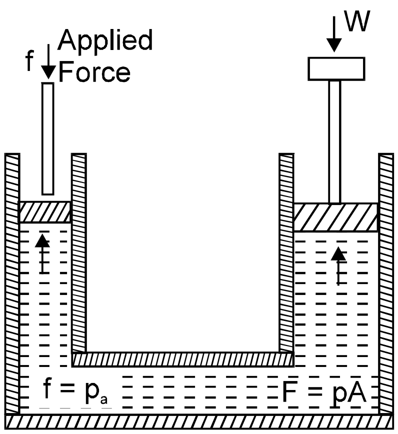
if the pressure in a liquid is changed at a particular, point the change is transmitted to the entire liquid without being diminished in magnitude. In the above case if P_a is increased by some amount than P must increase to maintained the difference $(P - P_a) = h\rho g$. This is Pascal's Law which states that Hydraulic lift is common application of Pascal's Law.

1. Hydraulic press.

$$p = \frac{f}{a} = \frac{W}{A} \text{ or } f = \frac{W}{A} \times a$$

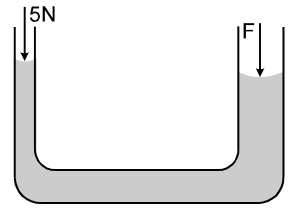
as A >> a then f << W..

This can be used to lift a heavy load placed on the platform of larger piston or to press the things placed between the piston and the heavy platform. The work done by applied force is equal to change in potential energy of the weight in hydraulic press.



Solved Examples

Ex.1. The area of cross-section of the two arms of a hydraulic press are 1 cm^2 and 10 cm^2 respectively (figure). A force of 5 N is applied on the water in the thinner arm. What force should be applied on the water in the thicker arms so that the water may remain in equilibrium?



Sol. : In equilibrium, the pressures at the two surfaces should be equal as they lie in the same horizontal level. If the atmospheric pressure is P and a force F is applied to maintain the equilibrium, the pressures are

$P_0 + \frac{5\text{ N}}{1\text{ cm}^2}$ and $P_0 + \frac{F}{10\text{ cm}^2}$ respectively.

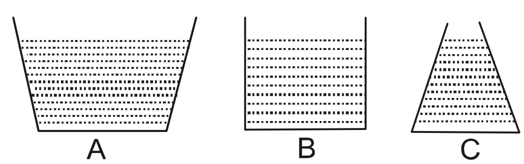
This givens F = 50 N.

2. Hydraulic Brake.

Hydraulic brake system is used in auto-mobiles to retard the motion.

HYDROSTATIC PARADOX

Pressure is directly proportional to depth and by applying pascal's law it can be seen that pressure is independent of the size and shape of the containing vessel. (In all the three cases the heights are same).



$P_A = P_B = P_C$

ATMOSPHERIC PRESSURE

Definition.

The atmospheric pressure at any point is numerically equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere.

At 0ºC, density of mercury = 13.595 g cm^{-3}, and at sea level, g = 980.66 cm s^{-2}

Now P = hρg.

Atmospheric pressure = 76 × 13.595 × 980.66 dyne cm^{-2} = 1.013 × 10^{-5} N-m^{2} (p_a)

Height of Atmosphere

The standard atmospheric pressure is 1.013 × 10^5 Pa (N m^{-2}). If the atmosphere of earth has a uniform density ρ = 1.30 kg m^{-3}, then the height h of the air column which exerts the standard atmospheric pressure is given by

$\Rightarrow$ hρg = 1.013 × 10^5

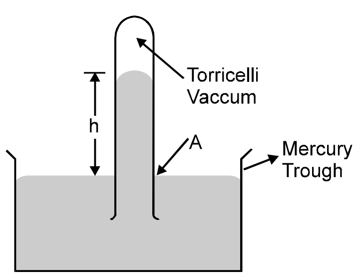
$h = \frac{1.013 \times 10^5}{\rho g} = \frac{1.013 \times 10^5}{1.13 \times 9.8}$ m = 7.95 × 10^3 m ~ 8 km.

In fact, density of air is not constant but decreases with height. The density becomes half at about 6 km high, $\frac{1}{4}$th at about 12 km and so on. Therefore, we can not draw a clear cut line above which there is no atmosphere. Anyhow the atmosphere extends upto 1200 km. This limit is considered for all practical purposes.

MEASUREMENT OF ATMOSPHERIC PRESSURE

1. Mercury Barometer.

To measure the atmospheric pressure experimentally, torricelli invented a mercury barometer in 1643.



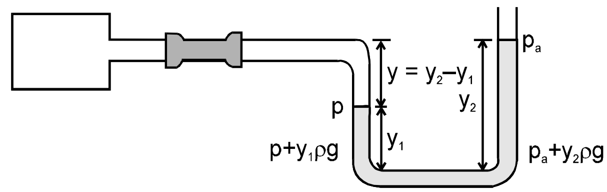
p_a = hρg

The pressure exerted by a mercury column of 1mm high is called 1 Torr.

1 Torr = 1 mm of mercury column

2. Open tube Manometer

Open-tube manometer is used to measure the pressure gauge. When equilibrium is reached, the pressure at the bottom of left limb is equal to the pressure at the bottom of right limb.



i.e. p + y_1 ρg = p_a + y_2 ρg

p – p_a = ρg (y_2 – y_1) = ρgy

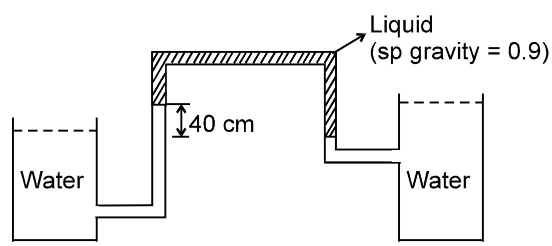
p – p_a = ρg (y_2 – y_1) = ρgy

p = absolute pressure, p – p_a = gauge pressure.

Thus, knowing y and ρ (density of liquid), we can measure the gauge pressure.

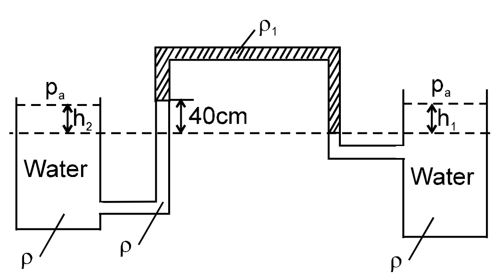
Solved Examples

Ex.2. The manometer shown below is used to measure the difference in water level between the two tanks. Calculate this difference for the conditions indicated.



Sol. : $p_a + h_1 \rho g - 40\rho_1 g + 40\rho g = p_a + h_2 \rho g$

$h_2 \rho g - h_1 \rho g = 40 \rho g - 40 \rho_1 g$



as ρ_1 = 0.9ρ

(h_2 – h_1) ρg = 40ρg – 36ρg

h_2 – h_1 = 4 cm

3. Water Barometer.

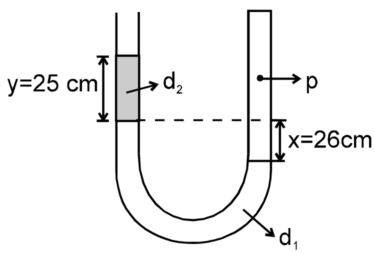
Let us suppose water is used in the barometer instead of mercury.

$h\rho g = 1.013 \times 10^5$ or $\quad h = \dfrac{1.013 \times 10^5}{\rho g}$

The height of the water column in the tube will be 10.3 m. Such a long tube cannot be managed easily, thus water barometer is not feasible.

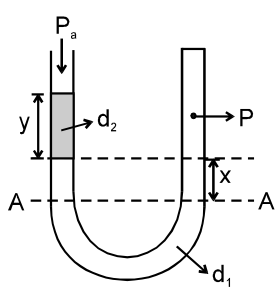
Solved Examples

Ex. 3. In a given U-tube (open at one-end) find out relation between p and p_a. Given $d_2 = 2 \times 13.6$ gm/cm^3 $d_1 = 13.6$ gm/cm^3



Sol. : Pressure in a liquid at same level is same i.e. at A – A–,

$p_a + d_2 y g + x d_1 g = p$



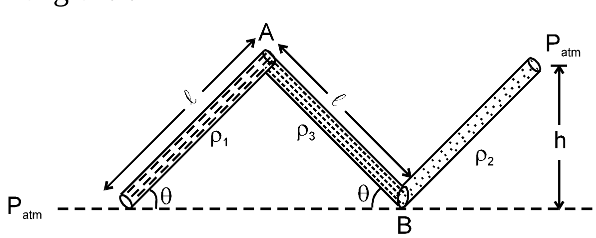
In C.G.S.

$p_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = p$

$p_a + 13.6 \times g\,[50 + 26] = p$

$2p_a = p \quad [p_a = 13.6 \times g \times 76]$

Ex. 4. Find out pressure at points A and B. Also find angle 'θ'.



Sol. : Pressure at A – $\quad P_A = P_{atm} - \rho_1 gl \sin\theta$

Pressure at B $\quad P_B = P_{atm} + \rho_2 gh\,\theta$

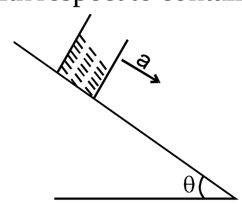
But P_B is also equal to $\quad P_B = P_A + \rho_3 gl \sin\theta$

Hence - $\quad P_{atm} + \rho_2 gh = P_A + \rho_3 gl \sin\theta$

$P_{atm} + \rho_2 gh = P_{atm} - \rho_1 gl \sin\theta + \rho_3 gl \sin\theta$

$\sin\theta = \dfrac{\rho_2 h}{(\rho_3 - \rho_1)\ell}$.

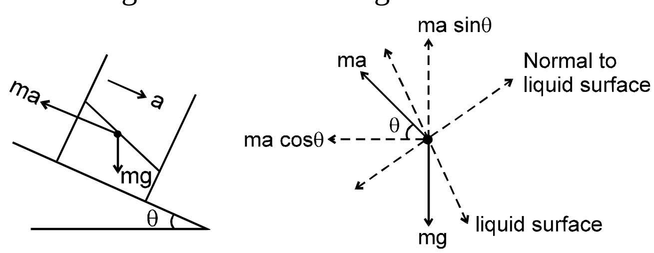
Ex. 5. In the given figure, the container slides down with acceleration 'a' on an incline of angle 'θ'. Liquid is stationary with respect to container. Find out -



(i) Angle made by surface of liquid with horizontal plane.

(ii) Angle if a = g sin θ.

Sol. : Consider a fluid particle on surface. The forces acting on it are shown in figure.



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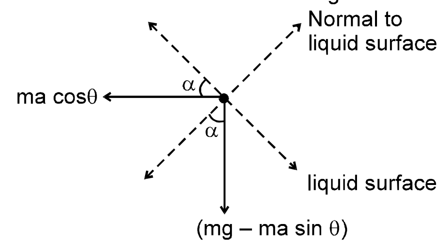


Resultant force acting on liquid surface, will always normal to it

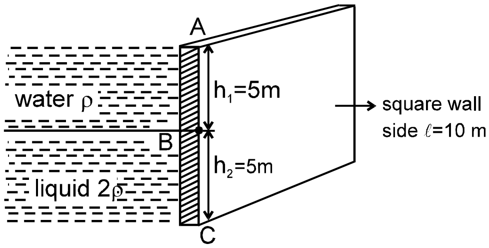
$$\tan\alpha = \frac{ma\cos\theta}{mg - ma\sin\theta} = \frac{a\cos\theta}{(g - a\sin\theta)}$$

Thus angle of liquid surface with the horizontal is equal to $\quad \alpha = \tan^{-1}\dfrac{a\cos\theta}{(g - a\sin\theta)}$

(ii) If a = g sin θ, then $\alpha = \tan^{-1}\left(\dfrac{a\cos\theta}{g - g\sin^2\theta}\right)$

$= \tan^{-1}\dfrac{g\sin\theta\cos\theta}{g\cos^2\theta} = \tan^{-1}(\tan\theta) \quad \alpha = \theta$

Ex. 6. Water and liquid is filled up behind a square wall of side ℓ. Find out



(a) Pressures at A, B and C

(b) Forces in part AB and BC

(c) Total force and point of application of force. (neglect atmosphere pressure in every calculation)

Sol. :

(a) As there is no liquid above 'A',

So pressure at A, $p_A = 0$

Pressure at B, $p_B = \rho g h_1$

Pressure at C, $p_C = \rho g h_1 + 2\rho g h_2$

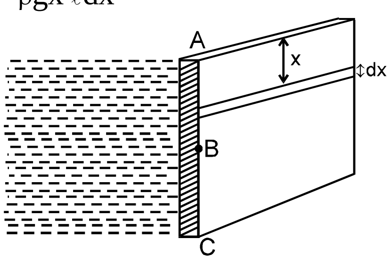
(b) Force at A = 0

Take a strip of width 'dx' at a depth 'x' in part AB.

Pressure is equal to ρgx.

Force on strip = pressure × area

$dF = \rho g x\, \ell dx$



Total force upto B

$$F = \int_0^{h_1} \rho g x \ell dx = \frac{\rho g x \ell h_1^2}{2} = \frac{1000 \times 10 \times 10 \times 5 \times 5}{2}$$

$= 1.25 \times 10^6$ N

In part BC for force take a elementary strip of width dx in portion BC. Pressure is equal to

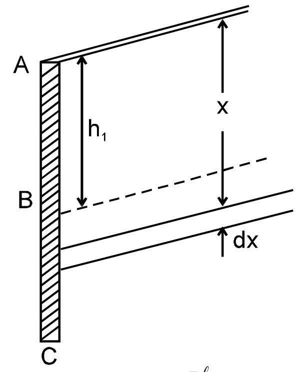
$= \rho g h_1 + 2\rho g(x - h_1)$

Force on elementary strip = pressure × area

$dF = [\rho g h_1 + 2\rho g(x - h_1)]\, \ell\, dx$

Total force on part BC

$$F = \int_{h_1}^{\ell} [\rho g h_1 + 2\rho g (x - h_1)] \ell dx$$



$$= \left[\rho g h_1 x + 2\rho g \left[\frac{x^2}{2} - h_1 x \right] \right]_{h_1}^{\ell} \ell$$

$$= \rho g h_1 h_2 \ell + 2\rho g \ell \left[\frac{\ell^2 - h_1^2}{2} - h_1 \ell + h_1^2 \right]$$

$$= \rho g h_1 h_2 \ell + \frac{2\rho g \ell}{2} [\ell^2 + h_1^2 - 2h_1 \ell]$$

$$= \rho g h_1 h_2 \ell + \rho g \ell (\ell - h_1)^2$$

$$= \rho g h_2 \ell [h_1 + h_2] = \rho g h_2 \ell^2$$

$$= 1000 \times 10 \times 5 \times 10 \times 10 = 5 \times 10^6 \text{ N}$$

(c) Total force $= 5 \times 10^6 + 1.25 \times 10^6 = 6.25 \times 10^6$ N

Taking torque about A

Total torque of force in AB $= \int dF \cdot x = \int_0^{h_1} \rho g \ell dx . x$

$$= \left[\frac{\rho g \ell x^3}{3} \right]_0^{h_1} = \frac{\rho g \ell h_1^3}{3} = \frac{1000 \times 10 \times 10 \times 125}{3}$$

$$= \frac{1.25 \times 10^7}{3} \text{ N - m}$$

Total torque of force in BC $= \int dF \cdot x$

On solving we get $= \rho g h_1 h_2 \ell [h_1 + \frac{h_2}{2}] + \rho g h_2^2 \ell [h_1 + \frac{2h_2}{3}]$

$= 1000 \times 10 \times 5 \times 5 \times 10 [5 + 2.5] + 1000 \times 10 \times 25 \times 10 [5 + \frac{10}{3}]$

$$= 2.5 \times 7.5 \times 10^6 + \frac{62.5}{3} \times 10^6 = \frac{118.75}{3} \times 10^6$$

Total torque $= \frac{11.875 \times 10^7}{3} + \frac{1.25 \times 10^7}{3}$

$$= \frac{13.125 \times 10^7}{3}$$

Total torque = total force × distance of point of application of force from top = F . x_p

$6.25 \times 10^6\, x_p = \dfrac{13.125\times10^7}{3}$ $\qquad x_p = 7\text{m}$

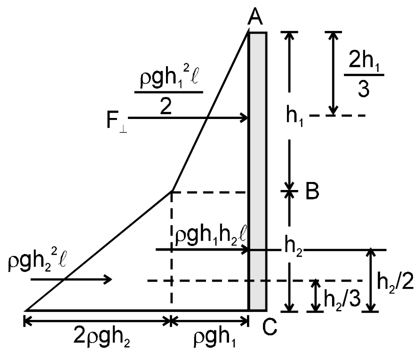
Alternatively

We can solve this problem by pressure diagram also.

Force on 'AB' part is area of triangle 'ABC'

$F_{AB} = \rho g h_1 \times \dfrac{h_1}{2} \times \ell \qquad = \dfrac{\rho g h_1^2 \ell^2}{2}$

Torque of force of AB part about A -



$\tau_{AB} = \dfrac{\rho g h_1^2 \ell}{2} \times \dfrac{2h_1}{3}$

$= \dfrac{\rho g h_1^3 \ell}{3} = \dfrac{\rho g \ell^4}{24}$

Force on 'BC' part is area of trapezium -

$F_{BC} = \rho g h_1 h_2 \ell + 2\rho g h_2 \times \dfrac{h_2}{2} \ell \; = \rho g h_1 h_2 \ell + \rho g h_2^2 \ell$

Torque of force of 'BC' part about 'A' -

$\tau_{BC} = \rho g h_1 h_2 \ell \; (h_1 + \dfrac{h_2}{2}) + \rho g h_2^2 \ell (h_1 + \dfrac{2h_2}{3})$

$= \dfrac{\rho g \ell^3}{4}\left[\dfrac{\ell}{2} + \dfrac{\ell}{4}\right] + \rho g \dfrac{\ell^3}{4}\left[\dfrac{\ell}{2} + \dfrac{\ell}{3}\right]$

$= \dfrac{\rho g \ell^3}{4}\left[\ell + \dfrac{\ell}{3} + \dfrac{\ell}{4}\right] = 19\dfrac{\rho g \ell^4}{48}$

Total force $= \dfrac{\rho g h_1^2 \ell}{2} + \rho g h_1 h_2 \ell + \rho g h_2^2 \ell$

$= \dfrac{\rho g \ell^3}{8} + \rho g \dfrac{\ell^3}{4} + \dfrac{\rho g \ell^3}{4}\left[1 + 1 + \dfrac{1}{2}\right] = \dfrac{5\rho g \ell^3}{8}$

Total torque $= \dfrac{19\rho g \ell^4}{48} + \dfrac{\rho g \ell^4}{24} = \dfrac{21\rho g \ell^4}{48}$

But $F\, x_p = \dfrac{21\rho g \ell^4}{48} \Rightarrow \dfrac{5\rho g \ell^3}{8} \times p = \dfrac{21\rho g \ell^4}{48}$

$x_p = \dfrac{21\ell}{30} = \dfrac{21\times10}{30} = 7\text{ m}$

Thus total force is acting at 7m below A point.

ARCHIMEDES' PRINCIPLE

According to this principle, when a body is immersed wholly or partially in a fluid, it loses its weight which is equal to the weight of the fluid displaced by the body.

Up thrust = buoyancy = $V\rho_\ell g$

V = volume submerged $\qquad \rho_\ell$ = density of liquid.

Relation between density of solid and liquid

weight of the floating solid = weight of the liquid displaced

$V_1\, \rho_1\, g = V_2\, \rho_2\, g \quad \Rightarrow \quad \dfrac{\rho_1}{\rho_2} = \dfrac{V_2}{V_1}$

or $\dfrac{\text{Density of solid}}{\text{Density of liquid}}$

$= \dfrac{\text{Volume of the immeresed portion of the solid}}{\text{Total Volume of the solid}}$

This relationship is valid in accelerating fluid also.

Thus, the force acting on the body are :

(i) its weight Mg which acts downward and

(ii) net upward thrust on the body or the buoyant force (mg)

Hence the apparent weight of the body = Mg – mg = weight of the body – weight of the displaced liquid.

Or Actual Weight of body – Apparent weight of body = weight of the liquid displaced.

The point through which the upward thrust or the buoyant force acts when the body is immersed in the liquid is called its centre of buoyancy. This will coincide with the centre of gravity if the solid body is homogeneous. On the other hand if the body is not homogeneous, then the centre of gravity may not lie on the line of the upward thrust and hence there may be a torque that causes rotation in the body.

If the centre of gravity of the body and the centre of buoyancy lie on the same straight line, the body is in equilibrium.

If the centre of gravity of the body does not coincide with the centre of buoyancy (i.e., the line of upthrust), then torque acts on the body. This torque causes the rotational motion of the body.

Solved Examples

Ex. 7. A copper piece of mass 10 g is suspended by a vertical spring. The spring elongates 1 cm over its natural length to keep the piece in equilibrium. A beaker containing water is now placed below the piece so as to immerse the piece completely in water. Find the elongation of the spring. Density of copper = 9000 kg/m^3. Take g = 10 m/s^2.

Sol. : Let the spring constant be k. When the piece is hanging in air, the equilibrium condition gives

k (1 cm) = (0.01 kg) (10 m/s)

or k (1 cm) = 0.1 N. ....(i)

The volume of the copper piece

$= \frac{0.01\text{kg}}{9000\,\text{kg/m}^3} = \frac{1}{9} \times 10^{-5}$ m^3.

This is also the volume of water displaced when the piece is immersed in water. The force of buoyancy

= weight of the liquid displaced

$= \frac{1}{9} \times 10^{-5}$ m^3 × (1000 kg/m^3) × (10 m/s^2) = 0.011 N.

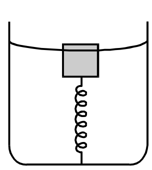
If the elongation of the spring is x when the piece is immersed in water, the equilibrium condition of the piece gives,

kx = 0.1 N – 0.011 N = 0.089 N. ....(ii)

By (i) and (ii),

$x = \frac{0.089}{0.1}$ cm = 0.89 cm.

Ex. 8. A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of wood = 800 kg/m^3 and spring constant of the spring = 50 N/m. Take g = 10 m/s^2.



Sol. : The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside ater = 3 cm – 2.4 = 0.6 cm. Suppose the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water

= volume of the block = 27 × 10^{-6} m^3.

Hence, the force of buoyancy

= (27 × 10^{-6} m^3) × 1(1000 kg/m3) × (10 m/s^2)

= 0.27 N.

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring

= 50 N/m × 0.6 cm = 0.3 N.

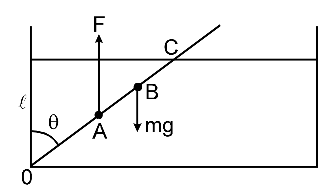
The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is

W′ = (27 × 10^{-6} m) × (800 kg/m^3) × (10 m/s^2)

= 0.22 N.

Thus, W = 0.27 N + 0.3 N – 0.22 N = 0.35 N.

Ex. 9. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0)



Sol. : The forces acting on the plank are shown in the figure. The height of water level is ℓ = 0.5m. The length of the plank is 1.0 m = 2ℓ. The weight of the plank acts through the centre B of the plank. We have OB = ℓ. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have $OA = \frac{OC}{2} = \frac{\ell}{2\cos\theta}$.

Let the mass per unit length of the plank be ρ.

Its weight mg = 2$\ell\rho$g.

The mass of the part OC of the plank = $\left(\frac{\ell}{\cos\theta}\right)\rho$.

The mass of water displaced = $\frac{1}{0.5}\frac{\ell}{\cos\theta}\rho = \frac{2\ell\rho}{\cos\theta}$.

The buoyant force F is, therefore, $F = \frac{2\ell\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So $\quad mg\,(OB)\sin\theta = F(OA)\sin\theta$

or, $(2\ell\rho)\ell = \left(\frac{2\ell\rho}{\cos\theta}\right)\left(\frac{\ell}{2\cos\theta}\right)$ or, $\cos^2\theta = \frac{1}{2}$

or, $\cos\theta = \frac{1}{\sqrt{2}}$, or, $\theta = 45°$.

Ex. 10. A cylindrical block of wood of mass M is floating in water with its axis vertical. It is depressed a little and then released. Show that the motion of the block is simple harmonic and find its frequency.

Sol. : Suppose a height h of the block is dipped in the water in equilibrium position. If r be the radius of the cylindrical block, the volume of the water diplaced $= \pi r^2 h$. For floating in equilibrium,

$\pi r^2 h\rho g = W$ ....(i)

where ρ is the density of water and W the weight of the block.

Now suppose during the vertical motion, the block is further dipped through a distance x at some instant. The volume of the displaced water is $\pi r^2 (h + x)$. The forces acting on the block are, the weight W vertically downward and the buoyancy $\pi r^2(h + x)\rho g$ vertically upward.

Net force on the block at displacement x from the equilibrium position is

$F = W - \pi r^2 (h + x)\rho g = W - \pi r^2 h\rho g - \pi r^2\rho x g$

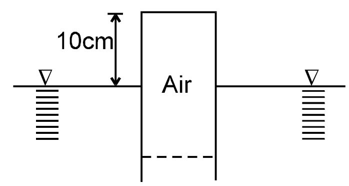
Using (i) $F = -\pi r^2 \rho g x = -kx$,

where $k = \pi r^2 \rho g$.

Thus, the block executes SHM with frequency.

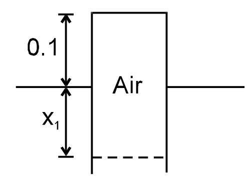
$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{\pi r^2\rho g}{M}}.$$

Ex. 11. A cylindrical bucket with one end open is observed to be floating on a water (ρ = 1000 kg/m^3) with open and down. It is of 10 N weight and is supported by air that is trapped inside it as shown below. The bucket floats with a height 10 cm above the liquid surface. If the air trapped is assumed to follow isothermal law, then determine the force F necessary just to submerge the bucket. The internal area of cross-section of bucket is 21 cm^2. The thickness of the wall is assumed to negligible and the atmospheric pressure must be neglected. (g = 10 m/sec^2)



Sol. : Weight of bucket

$W = Ax_1\rho g$ .....(1)



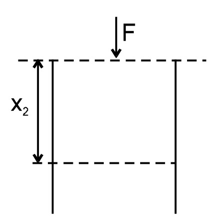
pressure at liquid - air interface = pressure of air $= \rho g x_1$

From (1) $p_1 = \rho g x_1 = \rho g \frac{W}{A\rho g} = \frac{W}{A}$

$v_1 = A[h + x_1] = A\left[h + \frac{W}{A\rho g}\right]$

Let force F is applied

downward force = F + W = Buoyant = $Ax_2\rho g$....(2)



$p_2 = x_2\rho g,\ v_2 = Ax_2$

$p_1v_1 = p_2v_2$

$\frac{W}{A} \times A\left[h + \frac{W}{A\rho g}\right] = x_2\,\rho\, g\, A\, x_2$

$\Rightarrow x_2 = \sqrt{\frac{W}{A\rho g}\left[h + \frac{W}{A\rho g}\right]}$ from (2)

$$F + W = A\rho g\sqrt{\frac{W}{A\rho g}\left[h+\frac{W}{A\rho g}\right]}$$

$$\Rightarrow F + W = \sqrt{WA\rho gh + W^2}$$

$F = \sqrt{WA\rho gh + W^2} - W$ substituting values -

$W = 10$ N, $\rho = 1000$ kg/m^3, $A = 2.1 \times 10^{-3}$ m^2

F

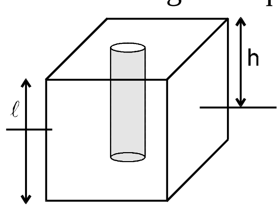
$= \sqrt{10 \times 2.1 \times 10^{-3} \times 1000 \times 10 \times 10^{-1} + 100} - 10$

$= 11 - 10 = 1$N

Ex. 12. A large block of ice cuboid of height 'ℓ' and density $\rho_{ice} = 0.9\,\rho_w$, has a large vertical hole along its axis. This block is floating in a lake. Find out the length of the rope required to raise a bucket of water through the hole.

Sol. : Let area of ice-cuboid excluding hole = A

weight of ice block = weight of liquid displaced



$A\,\rho_{ice}\,\ell\,g = A\,\rho_w\,(\ell - h)\,g$

$\frac{9\ell}{10} = \ell - h \Rightarrow \quad h = \ell - \frac{9\ell}{10} = \left(\frac{\ell}{10}\right)$

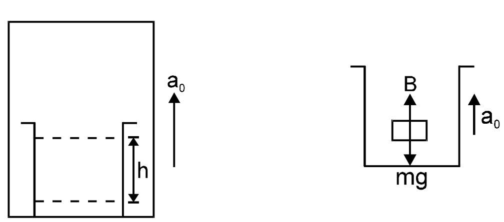
PRESSURE IN CASE OF ACCELERATING FLUID

(i) Liquid Placed in elevator :

When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth 'h' may be given by,

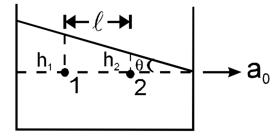
$p = h\rho\,[g + a_0]$

and force of buoyancy, $B = m\,(g + a_0)$



(ii) Free surface of liquid in horizontal acceleration:

$\tan\theta = \frac{a_0}{g}$

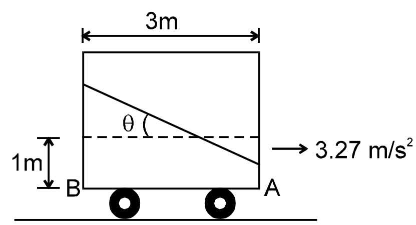


$p_1 - p_2 = \ell\rho\,a_0$ where p_1 and p_2 are pressures at point 1 & 2. Then $h_1 - h_2 = \frac{\ell a_0}{g}$

Solved Examples

Ex. 13. An open rectangular tank 1.5 m wide 2m deep and 2m long is half filled with water. It is accelerated horizontally at 3.27 m/sec^2 in the direction of its length. Determine the depth of water at each end of tank. [g = 9.81 m/sec^2]

Sol. : $\tan\theta = \frac{a}{g} = \frac{1}{3}$



Depth at corner 'A'

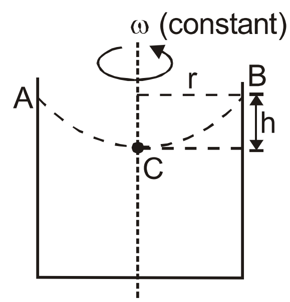
$= 1 - 1.5\tan\theta$

$= 0.5$ m **Ans.**

Depth at corner 'B' $= 1 + 1.5\tan\theta = 1.5$ m **Ans.**

(iii) Free surface of liquid in case of rotating cylinder.

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$



STREAMLINE FLOW

The path taken by a particle in flowing fluid is called its line of flow. In the case of steady flow all the particles passing through a given point follow the same path and hence we have a unique line of flow passing through a given point which is also called streamline.

CHARACTERISTICS OF STREAMLINE

1. A tangent at any point on the stream line gives the direction of the velocity of the fluid particle at that point.

2. Two steamlines never intersect each other.

Laminar flow : If the liquid flows over a horizontal surface in the form of layers of different velocities, then the flow of liquid is called Laminar flow. The particle of one layer do not go to another layer. In general, Laminar flow is a streamline flow.

Turbulent Flow : The flow of fluid in which velocity of all particles crossing a given point is not same and the motion of the fluid becomes disorderly or irregular is called turbulent flow.

REYNOLD'S NUMBER

According to Reynold, the critical velocity (v_c) of a liquid flowing through a long narrow tube is

(i) directly proportional to the coefficient of viscosity (η) of the liquid.

(ii) inversely proportional to the density ρ of the liquid and

(iii) inversely proportional to the diameter (D) of the tube.

That is $v_c \propto \dfrac{\eta}{\rho D}$ or $v_c = \dfrac{R\eta}{\rho D}$

or $= \dfrac{v_c \rho D}{\eta}$ ....(1)

where R is the Reynold number.

If R < 2000, the flow of liquid is streamline or laminar. If R > 3000, the flow is turbulent. If R lies between 2000 and 3000, the flow is unstable and may change from streamline flow to turbulent flow.

EQUATION OF CONTINUITY

The equation of continuity expresses the law of conservation of mass in fluid dynamics.

This is called equation of continuity and states that as the area of cross section of the tube of flow becomes larger, the liquid's (fluid) speed becomes smaller and vice-versa.

Illustrations -

(i) Velocity of liquid is greater in the narrow tube as compared to the velocity of the liquid in a broader tube.

(ii) Deep waters run slow can be explained from the equation of continuity i.e., av = constant. Where water is deep the area of cross section increases hence velocity decreases.

ENERGY OF A LIQUID

A liquid can posses three types of energies :

(i) Kinetic energy :

The energy possessed by a liquid due to its motion is called kinetic energy. The kinetic energy of a liquid of mass m moving with speed v is $\frac{1}{2}mv^2$.

$\therefore$ K.E. per unit mass $= \dfrac{\frac{1}{2}mv^2}{m} = \dfrac{1}{2}v^2$.

(ii) Potential energy :

The potential energy of a liquid of mass m at a height h is m g h.

$\therefore$ P.E. per unit mass $= \dfrac{mgh}{m} = gh$

(iii) Pressure energy :

The energy possessed by a liquid by virtue of its pressure is called pressure energy.

Consider a vessel fitted with piston at one side (figure). Let this vessel is filled with a liquid. Let 'A' be the area of cross section of the piston and P be the pressure experienced by the liquid.

The force acting on the piston = PA

If dx be the distance moved by the piston, then work done by the force = PA dx = PdV

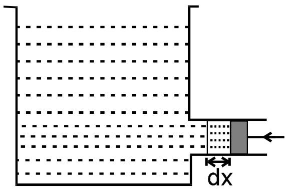
where dV = Adx, volume of the liquid swept.

This work done is equal to the pressure energy of the liquid.

$\therefore$ Pressure energy of liquid in volume dV = PdV.

The mass of the liquid having volume dV = ρdV,

ρ is the density of the liquid.



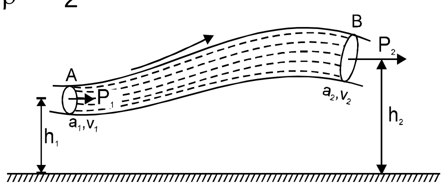
$\therefore$ Pressure energy per unit mass of the liquid

$= \dfrac{PdV}{\rho dV} = \dfrac{P}{\rho}$.

BERNOULLI'S THEOREM

It states that the sum of pressure energy, kinetic energy and potential energy per unit mass or per unit volume or per unit weight is always constant for an ideal (i.e. incompressible and non-viscous) fluid having stream-line flow.

i.e. $\dfrac{P}{\rho} + \dfrac{1}{2}v^2 + gh$ = constant.

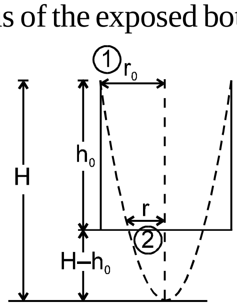


Solved Examples

Ex. 14. A circular cylinder of height h_0 = 10 cm and radius r_0 = 2cm is opened at the top and filled with liquid. It is rotated about its vertical axis. Determine the speed of rotation so that half the area of the bottom gets exposed. (g = 10 m/sec^2).

Sol. : Area of bottom = πr_0^2

If r is radius of the exposed bottom, then



$\pi r^2 = \dfrac{1}{2}\pi r_0^2 \qquad r = \dfrac{r_0}{\sqrt{2}}$

Applying Bernoulli's equation between points (1) and (2) -

$$P_{atm} + \frac{1}{2}\rho v_1^2 - \rho gH = P_{atm} + \frac{1}{2}\rho v_2^2 - \rho g(H - h_0)$$

$$-\rho gh_0 = \frac{1}{2}\rho(v_2^2 - v_1^2) \Rightarrow \quad 2gh_0 = [v_1^2 - v_2^2]$$

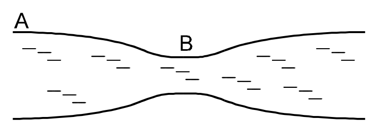
$$= [w^2 r_0^2 - w^2 r^2]$$

$$r_0 = 2 \times 10^{-2}\ m \quad \Rightarrow \quad 2gh_0 = w^2 [r_0^2 - r^2]$$

$$w = \frac{2}{r_0}\sqrt{gh} = \frac{2}{2 \times 10^{-2}}\sqrt{10 \times 0.1}$$

= 100 radian / sec.

Ex. 15. Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between A and B where the areas of cross-section are 30cm^2 and 15cm^2 respectively. Find the rate of flow of water through the tube.



Sol : Let the velocity at A = v_A and that at B = v_B.

By the equation of continuity, $\dfrac{v_B}{v_A} = \dfrac{30\,cm^2}{15\,cm^2} = 2$.

By Bernoulli's equation,

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

or, $P_A - P_B = \dfrac{1}{2}\rho(2v_A)^2 - \dfrac{1}{2}\rho v_A^2 = \dfrac{3}{2}\rho v_A^2$

or, $600\ \dfrac{N}{m^2} = \dfrac{3}{2}\left(1000\dfrac{kg}{m^3}\right) v_A^2$

or, $v_A = \sqrt{0.4\ m^2/s^2} = 0.63$ m/s.

The rate of flow = (30 cm^2) (0.63 m/s) = 1800 cm^3/s.

APPLICATION OF BERNOULLI'S THEOREM

(i) Bunsen burner

(ii) Lift of an airfoil.

(iii) Spinning of a ball (Magnus effect)

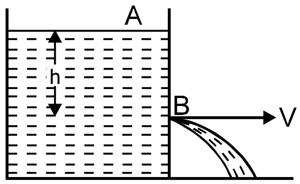
(iv) The sprayer.

(v) A ping-pong ball in an air jet

(vi) Torricelli's theorem (speed of efflux)

At point A, $P_1 = P$, $v_1 = 0$ and $h_1 = h$

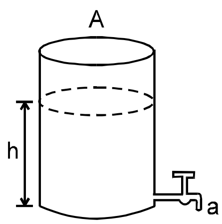
At point B, $P_2 = P$, $v_2 = v$ (speed of efflux) and $h = 0$



Using Bernoulli's theorem $\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 = \frac{1}{2}v_2^2$, we have $\frac{P}{\rho} + gh + 0 = \frac{P}{\rho} + 0 + \frac{1}{2}v^2 \Rightarrow \frac{1}{2}v^2 = gh$ or $v = \sqrt{2gh}$

Solved Examples

Ex. 16. A cylindrical container of cross-section area, A is filled up with water upto height 'h'. Water may exit through a tap of cross section area 'a' in the bottom of container. Find out



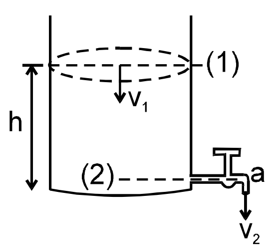
(a) Velocity of water just after opening of tap.

(b) The area of cross-section of water stream coming out of tape at depth h_0 below tap in terms of 'a' just after opening of tap.

(c) Time in which container becomes empty.

(Given : $\left(\frac{a}{A}\right)^{1/2} = 0.02$, h = 20 cm, h_0 = 20 cm)

Sol. : Applying Bernoulli's equation between (1) and (2) -



$$P_a + \rho gh + \frac{1}{2}\rho v_1^2 = P_a + \frac{1}{2}\rho v_2^2$$

Through continuity equation :

$$Av_1 = av_2,\ v_1 = \frac{av_2}{a} \qquad \rho gh + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$$

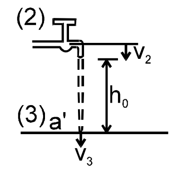
on solving - $v_2 = \sqrt{\dfrac{2gh}{1-\dfrac{a^2}{A^2}}} = 2\text{m/sec.}$ ....(1)

(b) Applying Bernoulli's equation between (2) and (3)

$$\frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho v_3^2$$

Through continuity equation -

$$av_2 = a'v_3 \Rightarrow v_3 = \frac{av_2}{a'}$$



$$\Rightarrow \frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho\left(\frac{av_2}{a'}\right)^2$$

$$\frac{1}{2} \times 2 \times 2 + gh_0 = \frac{1}{2}\left(\frac{a}{a'}\right)^2 \times 2 \times 2$$

$$\left(\frac{a}{a'}\right)^2 = 1 + \frac{9.8 \times .20}{2} \Rightarrow \left(\frac{a}{a'}\right)^2 = 1.98$$

$$\Rightarrow a' = \frac{a}{\sqrt{1.98}}$$

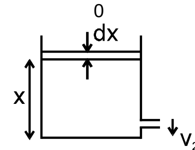
(c) From (1) at any height 'h' of liquid level in container, the velocity through tap,

$$v = \sqrt{\frac{2gh}{0.98}} = \sqrt{20h}$$

we know, volume of liquid coming out of tap = decrease in volume of liquid in container.

For any small time interval 'dt' $av_2dt = -A \cdot dx$

$$a\sqrt{20x}\ dt = -A\,dx \Rightarrow \int_0^t dt = -\frac{A}{a}\int_h^0 \frac{dx}{\sqrt{20x}}$$



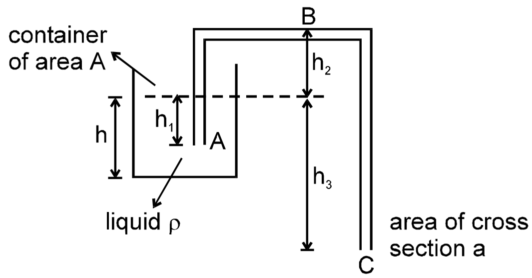
$$t = \frac{A}{a\sqrt{20}}\left[2\sqrt{x}\right]_h^0 \Rightarrow t = \frac{A}{a\sqrt{20}}\ 2\sqrt{h}$$

$$= \frac{A}{a} \times 2 \times \sqrt{\frac{h}{20}} = \frac{2A}{a}\sqrt{\frac{0.20}{20}} = \frac{2A}{a} \times 0.1$$

Given $\left(\frac{a}{A}\right)^{1/2} = 0.02$ or $\frac{A}{a} = \frac{1}{0.0004} = 2500$

Thus t = 2 × 2500 × 0.1 = **500 second**.

Ex. 17.



In a given arrangement

(a) Find out velocity of water coming out of 'C'

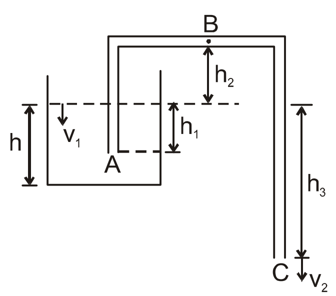
(b) Find out pressure at A, B and C.

Sol. :

(a) Applying Bernoulli's equation between liquid surface and point 'C'.

$$p_a + \frac{1}{2}\rho v_1^2 = p_a - \rho g h_3 + \frac{1}{2}\rho v_2^2$$

through continuity equation



$$Av_1 = av_2,\ v_1 = \frac{av_2}{A} \Rightarrow \frac{1}{2}\rho\frac{a^2}{A^2}v_2^2 = -\rho g h_3 + \frac{1}{2}\rho v_2^2$$

$$v_2^2 = \frac{2gh_3}{1-\frac{a^2}{A^2}}, \qquad v_2 = \sqrt{\frac{2gh_3}{1-\frac{a^2}{A^2}}}$$

(b) Pressure at A just outside the tube , $p_A = p_{atm} + \rho g h_1$

For pressure at B,

$$P_A + 0 + 0 = p_B + \rho g h_2 + \frac{1}{2}\rho v_B^2$$

$$P_B = P_A - \rho g h_2 - \frac{1}{2}\rho\left(\frac{2gh_3}{1-\frac{a^2}{A^2}}\right)$$

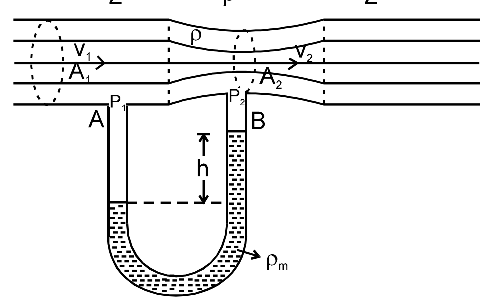
Pressure at C, $p_C = p_{atm}$

(VII) Venturimeter.

It is a gauge put on a flow pipe to measure the flow of speed of a liquid (Fig). Let the liquid of density ρ be flowing through a pipe of area of cross section A_1. Let A_2 be the area of cross section at the throat and a manometer is attached as shown in the figure. Let v_1 and P_1 be the velocity of the flow and pressure at point A, v_2 and P_2 be the corresponding quantities at point B.

Using Bernoulli's theorem :

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2, \text{ we get}$$



$$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2$$

(Since $h_1 = h_2 = h$)

$$\text{or } (P_1 - P_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad ....(1)$$

According to continuity equation, $A_1 v_1 = A_2 v_2$

$$\text{or } v_2 = \left(\frac{A_1}{A_2}\right)v_1$$

Substituting the value of v_2 in equation (1) we have

$$(P_1 - P_2) = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right] \quad \frac{1}{2}\rho\ v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

Since $A_1 > A_2$, therefore, $P_1 > P_2$

$$\text{or } v_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]} = \frac{2A_2^2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}$$

where $(P_1 - P_2) = \rho_m gh$ and h is the difference in heights of the liquid levels in the two tubes.

$$v_1 = \sqrt{\frac{2\rho_m gh}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

The flow rate (R) i.e., the volume of the liquid flowing per second is given by $R = v_1 A_1$.

(viii) During wind storm,

The velocity of air just above the roof is large so according to Bernoulli's theorem, the pressure just above the roof is less than pressure below the roof. Due to this pressure difference an upward force acts on the roof which is blown of without damaging other parts of the house.

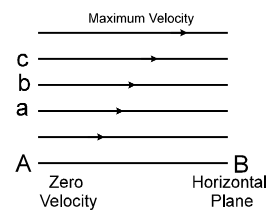
(ix) When a fast moving train cross a person standing near a railway track, the person has a tendency to fall towards the train. This is because a fast moving train produces large velocity in air between person and the train and hence pressure decreases according to Bernoulli's theorem. Thus the excess pressure on the other side pushes the person towards the train.

VISCOSITY

When a solid body slides over another solid body, a frictional-force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig.). The layer of the liquid which is in contact with the surface is at rest, while the velocity of other layers increases with distance from the fixed surface. In the Fig., the lengths of the arrows represent the increasing velocity of the layers. Thus there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers a, b and c. Their velocities are in the increasing order. The layer a tends to retard the layer b, while b tends to retard c. Thus each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. **The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.**

The property of viscosity is seen in the following examples :



(i) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerine, etc. have a larger viscosity than thinner ones like water. If we pour coaltar and water on a table, the coaltar will stop soon while the water will flow upto quite a large distance.

(ii) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.

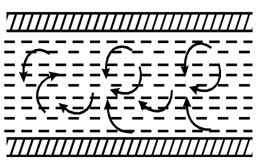
(iii) We can walk fast in air, but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.

(iv) The cloud particles fall down very slowly because of the viscosity of air and hence appear floating in the sky.

Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

FLOW OF LIQUID IN A TUBE : CRITICAL VELOCITY

When a liquid flows 'in a tube, the viscous forces oppose the flow of the liquid, Hence a pressure difference is applied between the ends of the tube which maintains the flow of the liquid. If all particles of the liquid passing through a particular point in the tube move along the same path, the flow" of the liquid is called 'stream-lined flow'. This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'. When the velocity of flow exceeds the critical velocity, the flow is no longer stream-lined but becomes turbulent. In this type of flow, the motion of the liquid becomes zig-zag and eddy-currents are developed in it.



Reynold proved that the critical velocity for a liquid flowing in a tube is $v_c = k\eta/\rho a$. where ρ is density and η is viscosity of the liquid, a is radius of the tube and k is 'Reynold's number' (whose value for a narrow tube and for water is about 1000). When the velocity of flow of the liquid is less than the critical velocity, then the flow of the liquid is controlled by the viscosity, the density having no effect on it. But when the velocity of flow is larger than the critical velocity, then the flow is mainly governed by the density, the effect of viscosity becoming less important. It is because of this reason that when a volcano erupts, then the lava coming out of it flows speedly inspite of being very thick (of large viscosity).

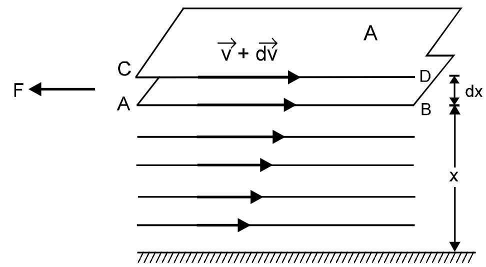
VELOCITY GRADIENT AND COEFFICIENT OF VISCOSITY

The property of a liquid by virtue of which an opposing force (internal friction) comes into play when ever there is a relative motion between the different layers of the liquid is called viscosity. Consider a flow of a liquid over the horizontal solid surface as shown in fig. Let us consider two layers AB and CD moving with velocities $\vec{v}$ and $\vec{v} + d\vec{v}$ at a distance x and (x + dx) respectively from the fixed solid surface.

According to Newton, the viscous drag or back ward force (F) between these layers depends.

(i) directly proportional to the area (A) of the layer and (ii) directly proportional to the velocity gradient $\left(\frac{dv}{dx}\right)$ between the layers.

i.e. $F \propto A\frac{dv}{dx}$ or $F = -\eta A\frac{dv}{dx}$ ...(1)



η is called Coefficient of viscosity. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.

SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Similarities

Viscosity and solid friction are similar as

1. Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.
2. Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.
3. Both are due to molecular attractions.

Differences between them →

Viscosity		Solid Friction	
(i)	Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	(i)	Friction between two solids is independent of the area of solid surfaces in contact.
(ii)	Viscous drag is proportional to the relative velocity between two layers of liquid.	(ii)	Friction is independent of the relative velocity between two surfaces.
(iii)	Viscous drag is independent of normal reaction between two layers of liquid.	(iii)	Friction is directly proportional to the normal reaction between two surfaces in contact.

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as under →

1. As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season.
2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.
3. The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.
4. The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.
5. It finds an important use in the circulation of blood through arteries and veins of human body.

UNITS OF COEFFICIENT OF VISCOSITY

From the above formula, we have

$$\eta = \frac{F}{A(\Delta v_x / \Delta z)}$$

$\therefore$ dimensions of η

$$= \frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \frac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$$

Its unit is kg/(meter-second)*

In C.G.S. system, the unit of coefficient of viscosity is dyne s cm^{-2} and is called poise. In SI the unit of coefficient of viscosity is N sm^{-2} and is called decapoise.

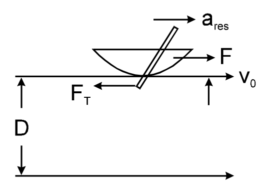
1 decapoise = 1 N sm^{-2} = (10^5 dyne) × s × (10^2 cm)$^{-2}$ = 10 dyne s cm^{-2} = 10 poise

Solved Examples

Ex. 18. A man is rowing a boat with a constant velocity 'v_0' in a river the contact area of boat is 'A' and coefficient of viscosity is η. The depth of river is 'D'. Find the force required to row the boat.

Sol. $F - F_T = m\,a_{res}$

As boat moves with constant velocity $a_{res} = 0$



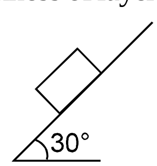
$F = F_T$

But $F_T = \eta A \frac{dv}{dz}$, but $\frac{dv}{dz} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$

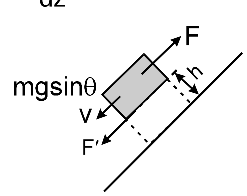
then $F = F_T = \frac{\eta A v_0}{D}$

Ex. 19. A cubical block (of side 2m) of mass20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/sec. ($g = 10$ m/sec^2)

find out the thickness of layer of liquid.



Sol. $F = F' = \eta A \frac{dv}{dz} = mg \sin\theta \qquad \frac{dv}{dz} = \frac{v}{h}$

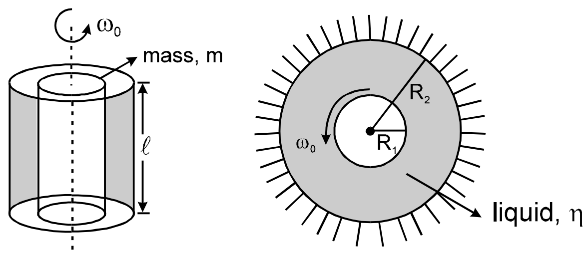


$20 \times 10 \times \sin 30° = \eta \times 4 \times \frac{10}{h}$

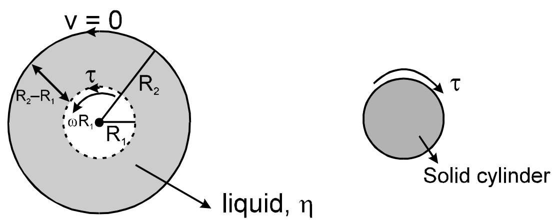
$h = \frac{40 \times 10^{-2}}{100}$ – [$\eta = 10^{-1}$ poise = 10^{-2} N-sec-m^{-2}]

$= 4 \times 10^{-3}$ m = 4 mm

Ex. 20. As per the shown figure the central solid cylinder starts with initial angular velocity ω_0. Find out the time after which the angular velocity becomes half.



Sol. $F = \eta A \frac{dv}{dz}$, where $\frac{dv}{dz} = \frac{\omega R_1 - 0}{R_2 - R_1}$



$F = \eta \frac{2\pi R_1 \ell \omega R_1}{R_2 - R_1}$ and $\tau = FR_1 = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$

$I\alpha = \frac{2\pi\eta R_1^3 \omega\ell}{R_2 - R_1} \Rightarrow \frac{MR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\eta R_1^3 \omega\ell}{R_2 - R_1}$

or $-\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \frac{4\pi\eta R_1 \ell}{m(R_2 - R_1)} \int_0^t dt$

$\Rightarrow t = \frac{m(R_2 - R_1)\ell n2}{4\pi\eta\ell R_1}$

EFFECT OF TEMPERATURE ON THE VISCOSITY

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature. That is, $\eta \propto \frac{1}{\sqrt{T}}$. On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$.

STOKE'S LAW

Stokes proved that the viscous drag (F) on a spherical body of radius r moving with velocity v in a fluid of viscosity η is given by $F = 6\pi\eta r v$. This is called Stokes' law.

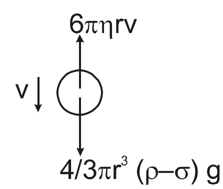
TERMINAL VELOCITY

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

Calculation of Terminal Velocity

Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas), whose density is σ and coefficient of viscosity η. When it attains a terminal velocity v. It is subjected to two forces :

(i) effective force acting downward



$= V(\rho - \sigma)g = \frac{4}{3}\pi r^3(\rho - \sigma)g$,

(ii) viscous force acting upward = $6\pi\eta rv$.

Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

$6\pi\eta rv = \frac{4}{3} p\, r^3(\rho - \sigma)g$

or $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

Thus, terminal velocity of the ball is directly proportional to the square of its radius

Important point

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is Negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

Solved Examples

Ex. 21. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Sol. Rate of heat loss = power = F × v = 6 π η r v × v

$= 6\,\pi\,\eta\,r\,v^2 = 6p\,\eta\,r\left[\dfrac{2}{9}\dfrac{gr^2(\rho_0-\rho_\ell)}{\eta}\right]^2$

Rate of heat loss $\alpha\, r^5$

Ex. 22. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg/(m-s), what will be the terminal velocity of the drop? (density of water = 1.0×10^3 kg/m^2 and g = 9.8 N/kg.) Density of air can be neglected.

Sol. By Stoke's law, the terminal velocity of a water drop of radius r is given by

$$\nu = \frac{2}{9}\ \frac{r^2(\rho-\sigma)g}{\eta}$$

where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and r = 0.0015 mm = 1.5×10^{-3} mm = 1.5×10^{-6} m. Substituting the values :

$$\nu = \frac{2}{9} \times \frac{(1.5\times10^{-6})^2\times(1.0\times10^3)\times9.8}{1.8\times10^{-5}}$$

= **2.72×10^{-4} m/s**

Ex. 23. A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg/m^3 enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given: coefficient of viscosity of water = 1.0×10^{-3} N-s/m^2, g = 10 m/s^2 and density of water = 1.0×10^3 kg/m^3.

Sol. The velocity attained by the sphere in falling freely from a height h is

$\nu = \sqrt{2gh}$ ....(i)

This is the terminal velocity of the sphere in water. Hence by Stoke's law, we have

$$\nu = \frac{2}{9}\ \frac{r^2(\rho-\sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere

σ (= 1.0×10^3 kg/m^3) is the density of water and η is coefficient of viscosity of water.

$$\therefore\ \nu = \frac{2\times(1.0\times10^{-3})^2(1.0\times10^4-1.0\times10^3)\times10}{9\times1.0\times10^{-3}}$$

= 20 m/s

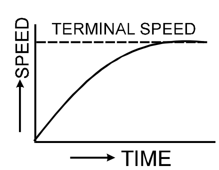
from equation (i), we have $h = \dfrac{\nu^2}{2g} = \dfrac{20\times20}{2\times10} = 20\,\text{m}$

Applications of Stokes' Formula

(i) In determining the Electronic Charge by Millikan's Experiment : Stokes' formula is used in Millikan's method for determining the electronic charge. In this method the formula is applied for finding out the radii of small oil-drops by measuring their terminal velocity in air.

(ii) Velocity of Rain Drops : Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning the raindrops are very small in size and so they fall with such a small velocity that they appear floating in the sky as cloud. As they grow in size by further condensation, then they reach the earth with appreciable velocity,

(iii) Parachute : When a soldier with a parachute jumps from a flying aeroplane, he descends very slowly in air.



In the beginning the soldier falls with gravity acceleration g, but soon the acceleration goes on decreasing rapidly until in parachute is fully opened. Therefore, in the beginning the speed of the falling soldier increases somewhat rapidly but then very slowly. Due to the viscosity of air the acceleration of the soldier becomes ultimately zero and the soldier then falls with a constant terminal speed. In Fig graph is shown between the speed of the falling soldier and time.

EXPLANATION OF SOME OBSERVED PHENOMENA

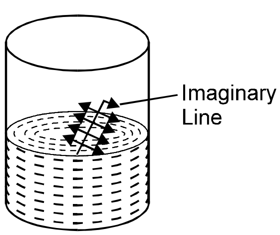
1. Lead balls are spherical in shape.
2. Rain drops and a globule of mercury placed on glass plate are spherical.
3. Hair of a shaving brush/painting brush, when dipped in water spread out, but as soon as it is taken out. Its hair stick together.
4. A greased needle placed gently on the free surface of water in a beaker does not sink.
5. Similarly, insects can walk on the free surface of water without drowning.
6. Bits of Camphor gum move irregularly when placed on water surface.

SURFACE TENSION

Surface Tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface Tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in the figure.

i.e. Surface tension



$$(T) = \frac{\text{Total force on either of the imginary line (F)}}{\text{Length of the line } (\ell)}$$

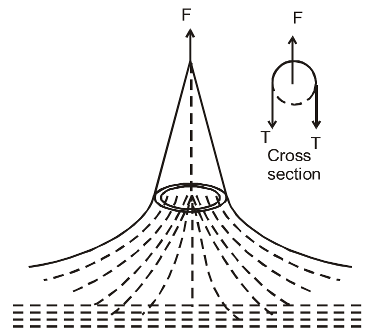
Units of Surface Tension.

In C.G.S. system the unit of surface tension is dyne/cm (dyne cm^{-1}) and Sl system its units is Nm^{-1}

Solved Examples

Ex. 24. A ring is cut form a platinum tube of 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water? (g = 980 cm/s^2).

Sol.



The ring is in contact with water along its inner and outer circumference; so when pulled out the total force on it due to surface tension will be

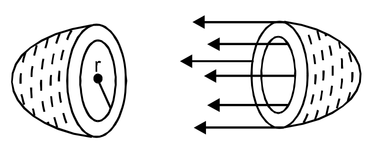
$F = T\,(2\pi\, r_1 + 2\pi\, r_2)$

So, $T = \dfrac{mg}{2\pi(r_1 + r_2)}$ [$\because$ F = mg]

i.e., $T = \dfrac{3.97 \times 980}{3.14 \times (8.5 + 8.7)} = 72.13$ dyne/cm

EXCESS PRESSURE INSIDE A LIQUID DROP AND A BUBBLE

1. **Inside a bubble :** Consider a soap bubble of radius r.



Let p be the pressure inside the bubble and p_a outside. The excess pressure = $p - p_a$. Imagine the bubble broken into two halves, and consider one half of it as shown in Fig. Since there are two surfaces, inner and outer, so the force due to surface tension is

F = surface tension x length = T x 2 (circumference of the bubble) = T x 2 (2 Tπ r) ... (1)

The excess pressure $(p - p_a)$ acts on a cross-sectional area πr^2, so the force due to excess pressure is

$\Rightarrow F = (p - p_a)\pi r^2$ .... (2)

The surface tension force given by equation (1) must balance the force due to excess pressure given by equation (2) to maintain the equilibrium. i.e. $(p - p_a)\pi r^2 = T \times 2(2\pi r)$ or $(p - p_a) = \frac{4T}{r} = p_{excess}$

above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.

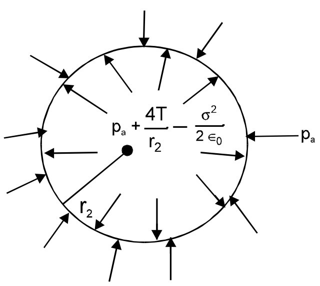
2. **Inside the drop :** In a drop, there is only one surface and hence excess pressure can be written as

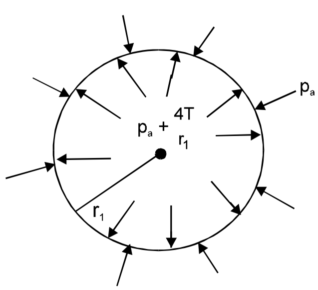
$$(p - p_a) = \frac{2T}{r} = p_{excess}$$

3. **Inside air bubble in a liquid :** $(p - p_a) = \frac{2T}{r} = p_{excess}$

4. **A charged bubble :** If bubble is charged, it's radius increases.

Bubble has pressure excess due to charge too. Initially pressure inside the bubble $= p_a + \frac{4T}{r_1}$





For charge bubble, pressure inside $= p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}$, where σ surface is surface charge density.

Taking temperature remains constant, then from Boyle's law

$$\left(p_a + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 = \left[p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}\right]\frac{4}{3}\pi r_2^3$$

From above expression the radius of charged drop may be calculated. It can conclude that radius of charged bubble increases, i.e. $r_2 > r_1$

Solved Examples

Ex. 25. A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

Sol. The total pressure inside the bubble at depth h_1 is (P is atmospheric pressure)

$$= (P + h_1\rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is $= (P + h_2\rho g) + \frac{2T}{r_2} = P_2$

Now, according to Boyle's Law

$P_1V_1 = P_2V_2$ where $V_1 = \frac{4}{3}\pi r_1^3$,

and $V_2 = \frac{4}{3}\pi r_2^3$

Hence we get $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3$

$$= \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^3$$

or, $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]r_2^3$

Given that : $h_1 = 100$ cm, $r_1 = 0.1$ mm $= 0.01$ cm, $r_2 = 0.126$ mm $= 0.0126$ cm, $T = 567$ dyne/cm, $P = 76$ cm of mercury. Substituting all the values, we get **h_2 = 9.48 cm**.

THE FORCE OF COHESION

The force of attraction between the molecules of the same substance is called cohesion.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Examples.

(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

FORCE OF ADHESION

The force of attraction between molecules of different substances is called adhesion.

Examples.

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Adhesive force helps us to write on the paper with ink.

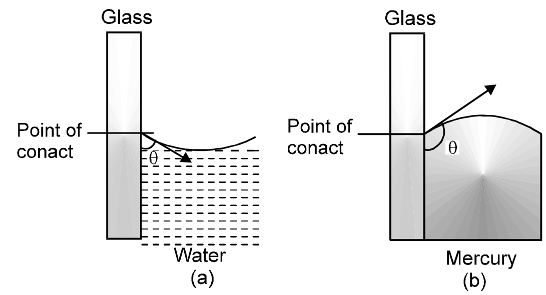
(iii) Large force of adhesion between cement and bricks helps us in construction work.

(iv) Due to force of adhesive, water wets the glass plate.

(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

ANGLE OF CONTACT

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact. Those liquids which wet the walls of the container (say in case of water and glass) have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids which don't wet the walls of the container (say in case of mercury and glass) have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass about 140°, whereas the angle of contact of water with glass is about 8°. But, for pure water, the angle of contact θ with glass is taken as 0°.



SHAPE OF LIQUID MENISCUS

When a capillary tube or a tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. This curved surface is due to the two forces i.e.

(i) due to the force of cohesion and

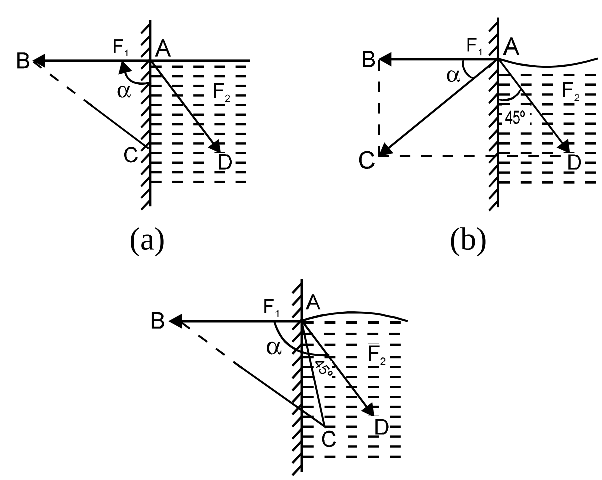
(ii) due to the force of adhesion. The curved surface of the liquid is called **meniscus of the liquid.** Various forces acting on molecule A are:

(iii) Force F_1 due to force of adhesion which acts outwards at right angle to the wall of the tube. This force is represented by AB.

(iv) Force F_2 due to force of cohesion which acts at an angle of 45° to the vertical. This force is represented by AD.

(v) The weight of the molecule A which acts vertically downward along the wall of the tube.

Since the weight of the molecule is negligible as compared to F_1 and F_2 and hence can be neglected. Thus, there are only two forces (F_1 and F_2) acting on the liquid molecules. These forces are inclined at an angle of 135°.



(a) (b) (c)

The resultant force represented by AC will depend upon the values of F_1 and F_2. Let the resultant force makes an angle α with F_1.

According to parallelogram law of vectors tan α

$$= \frac{F_2 \sin 135^o}{F_1 + F_2 \cos 135^o} = \frac{F_2/\sqrt{2}}{F_1 - F_2/\sqrt{2}} = \frac{F_2}{\sqrt{2}F_1 - F_2}$$

Special cases :

(i) If $F_2 = \sqrt{2}\, F_1$, then $\tan\alpha = \infty \therefore \alpha = 90°$

Then the resultant force will act vertically downward and hence the meniscus will be plane or horizontal shown in figure (a). Example; pure water contained in silver capillary tube.

(ii) If $F_2 < \sqrt{2}\, F_1$, then tan α is positive $\therefore$ α is acute angle

Thus, the resultant will be directed outside the liquid and hence the meniscus will be concave upward shown in figure (b). This is possible in case of liquids which wet the walls of the capillary tube. Example ; water in glass capillary tube.

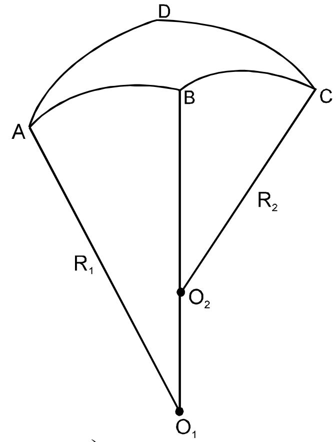
(iii) If $F_2 > \sqrt{2}\, F_1$, then tan α is negative $\therefore$ α is obtuse angle.

Thus, the resultant will be directed inside the liquid and hence the meniscus will be convex upward shown in figure (c). This is possible in case of liquids which do not wet the walls of the capillary tube. Example ; mercury in glass capillary tube.

RELATION BETWEEN SURFACE TENSION, RADII OF CURVATURE AND EXCESS PRESSURE ON A CURVED SURFACE.

Let us consider a small element ABCD (fig.) of a curved liquid surface which is convex on the upper side. R_1 and R_2 are the maximum and minimum radii of curvature respectively, They are called the 'principal radii of curvature' of the surface. Let p be the excess pressure on the concave side.

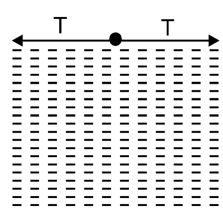
then $p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. If instead of a liquid surface, we have a liquid film, the above expression will be



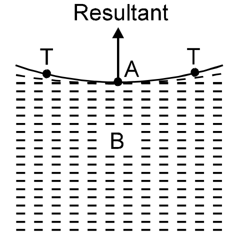
$p = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, because a film has two surface.

EXCESS OF PRESSURE INSIDE A CURVED SURFACE

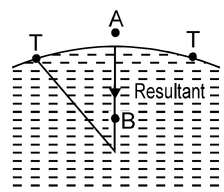
1. **Plane Surface :** If the surface of the liquid is plane [as shown in Fig.(a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore, on the liquid surface is normal.
2. **Concave Surface :** If the surface is concave upwards [as shown in Fig.(b)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side in the downward direction to balance the resultant force of surface tension $p_A - p_B = \frac{2T}{r}$.



(a) Plane Surface



(a) Concave Surface



(c) Convex Surface

3. **Convex Surface :** If the surface is convex [as shown in Fig.(c)], the resultant force due to surface tension acts in the downward direction. Since the molecule on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension, Hence there is always an excess of pressure on concave side of a curved surface over that on the convex side.

$$p_B - p_A = \frac{2T}{r}$$

Solved Examples

Ex. 26. A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in the narrow tube is 0.2 m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m and g = 9.8 m/s^2.

Sol. Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2.

Then pressure just below the mensiscus in the wide and narrow tubes respectively are

$\left(P_1 - \frac{2T}{r_1}\right)$ and $\left(P_2 - \frac{2T}{r_2}\right)$ [excess pressure = $\frac{2T}{r}$].

Difference in these pressures

$$= \left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$$

$\therefore$ True pressure difference = $P_1 - P_2$

$$= h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

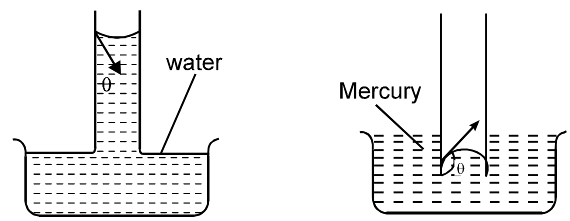
$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}$$

$$\left[\frac{1}{1.44 \times 10^{-3}} - \frac{1}{7.2 \times 10^{-4}}\right]$$

$= 1.86 \times 10^3 =$ **1860 N/m²**

CAPILLARITY

A glass tube of very fine bore throughout the length of the tube is called capillary tube. If the capillary tube is dipped in water, the water wets the inner side of the tube and rises in it [shown in figure (a)]. If the same capillary tube is dipped in the mercury, then the mercury is depressed [shown in figure (b)]. The phenomenon of rise or fall of liquids in a capillary tube is called capillarity.



(a) (b)

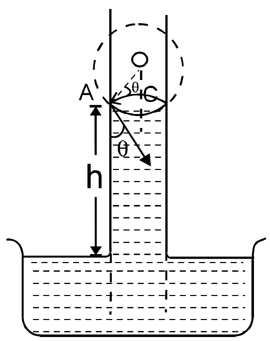
PRACTICAL APPLICATIONS OF CAPILLARITY

1. The oil in a lamp rises in the wick by capillary action.
2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tin or nib continuously.
3. Sap and water rise upto the top of the leaves of the tree by capillary action.
4. If one end of the towel dips into a bucket of water and the Other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.
5. Ink is absorbed by the blotter due to capillary action.
6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture m the soil, capillaries must be. broken up. This is done by ploughing and leveling the fields
8. Bricks are porous and behave like capillaries.

CAPILLARY RISE (HEIGHT OF A LIQUID IN A CAPILLARY TUBE) ASCENT FORMULA

Consider the liquid which wets the walls of the tube, forms a concave meniscus shown in figure. Consider a capillary tube of radius r dipped in a liquid of surface tension T and density p. Let h be the height through which the liquid rises in the tube. Let p be the pressure on the concave side of the meniscus and p_a be the pressure on the convex side of the meniscus. The excess pressure

$(p - p_a)$ is given by $(p - p_a) = \dfrac{2T}{R}$



Where R is the radius of the meniscus. Due to this excess pressure, the liquid will rise in the capillary tube till it becomes equal to the hydrostatic pressure hpg. Thus in equilibrium state.

Excess pressure = Hydrostatic pressure or $\dfrac{2T}{R} = hpg$

Let θ be the angle of contact and r be the radius of the capillary tube shown in the fig.

From ΔOAC, $\dfrac{OC}{OA} = \cos\theta$ or $R = \dfrac{r}{\cos\theta}$

$\Rightarrow h = \dfrac{2T\cos\theta}{r\rho g}$

This expression is called **Ascent formula.**

Discussion.

(i) For liquids which wet the glass tube or capillary tube, angle of contact $\theta < 90°$. Hence $\cos\theta$ = positive. $\Rightarrow$ h = positive. It means that these liquids rise in the capillary tube.

Hence, **the liquids which wet capillary tube rise in the capillary tube**. For example, water, milk, kerosene oil, patrol etc.

Solved Examples

Ex. 27. A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0^o.

Sol. The surface tension of the liquid is

$$T = \frac{rh\rho g}{2}$$

$$= \frac{(0.025\,cm)\,(3.0\,cm)\,(1.5\,gm/cm^3)\,(980\,cm/sec^2)}{2}$$

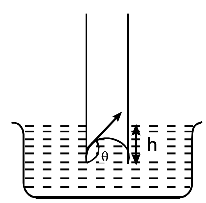
= 55 dyne/cm.

Hence excess pressure inside a spherical bubble

$$p = \frac{4T}{R} = \frac{4 \times 55\,dyne/cm}{(0.5\,cm)} = \mathbf{440\ dyne/cm^2}.$$

(ii) For liquids which do not wet the glass tube or capillary tube, angle of contact $\theta > 90°$.

Hence $\cos\theta$ = negative $\Rightarrow$ h = negative. Hence, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.

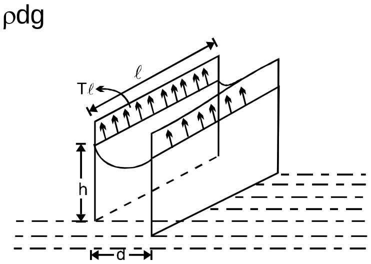


(iii) T, θ, ρ and g are constant and hence $h \propto \dfrac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. This is called **Jurin's Law.**

(iv) If two parallel plates with the spacing 'd' are placed in water reservoir, then height of rise

$\Rightarrow 2T\ell = \rho\ell hdg$

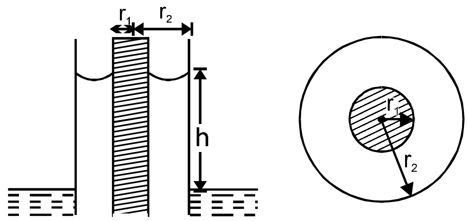
$$h = \frac{2T}{\rho dg}$$



(v) If two concentric tubes of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise

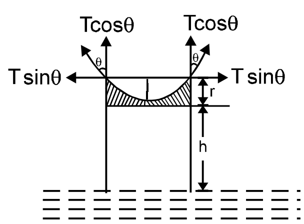
$\Rightarrow$ $T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g$

$$h = \frac{2T}{(r_2 - r_1)\rho g}$$



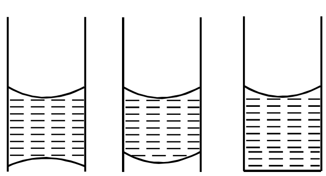
(vi) If weight of the liquid in the meniscus is to be consider:

$$T\cos\theta \times 2\pi r = [\pi r^2 h + \frac{1}{3}\pi r^2 \times r]\rho g$$



$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$

(vii) When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = 2T/R_1$ where R_1 = radius of curvature of upper meniscus.



(a) (b) (c)

The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards.

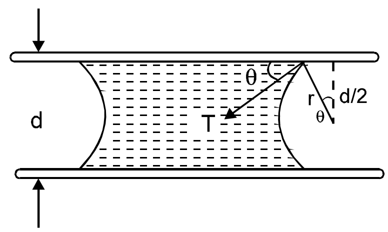
If $p_1 > p_2$ i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (fig.a). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

If $p_1 < p_2$, i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (fig. b).

The radius of lower meniscus can be given by $\frac{2T}{R_2} = p_2 - p_1$.

If $p_1 = p_2$, then is no resulting pressure. then, $p_1 - p_2 = \frac{2T}{R_2} = 0$ or, $R_2 = \infty$ i. e. lower surface will be FLAT. (fig.c).

(viii) Liquid between two Plates - When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let 'R' and 'r' be the radii of curvature of the enclosed film in two perpendicular directions.



Hence the pressure inside the film is less than the atmospheric pressure outside it by an amount p given by $p = T\left(\frac{1}{r} + \frac{1}{R=\infty}\right)$ and we have. $p = \frac{T}{r}$.

If d be the distance between the two plates and θ the angle of contact for water and glass, then, from the figure, $\cos\theta = \frac{\frac{1}{2}d}{r}$ or $\frac{1}{r} = \frac{2\cos\theta}{d}$.

Substituting for $\frac{1}{r}$ in , we get $p = \frac{2T}{d}\cos\theta$.

θ can be taken zero for water and glass, i.e. $\cos\theta = 1$. Thus the upper plate is pressed downward by the atmospheric pressure minus $\frac{2T}{d}$. Hence the resultant downward pressure acting on the upper plate is $\frac{2T}{d}$.

If A be the area of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by F = resultant pressure × area $= \frac{2TA}{d}$.

For very nearly plane surface, d will be very small and hence the pressing force F very large. Therefore it will be difficult to separate the two plates normally.

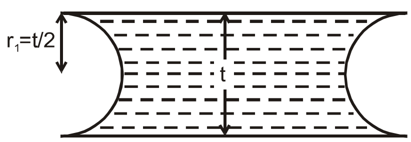
Solved Examples

Ex. 28. A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to seperate out the two glass plates in newton.

Sol. Pressure inside the film is less than outside by an amount, $P = T\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$, where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = t/2$ and $r_2 = \infty$, then the force required to separate the two glass-plates, between which a liquid film is enclosed (figure) is, $F = P \times A = \frac{2AT}{t}$, where t is the thickness of the film, A = area of film.

$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}}$$

= 45 N



Ex. 29. A glass plate of length 10 cm, breadth 1.54 cm and thickness 0.20 cm weighs 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water = 73 dyne per cm, g = 980 cm/sec^2.

Sol. Volume of the portion of the plate immersed in water is

$10 \times \frac{1}{2}(1.54) \times 0.2 = 1.54$ cm^3.

Therefore, if the density of water is taken as 1, then upthrust

= wt. of the water displaced

= 1.54 × 1 × 980 = 1509.2 dynes.

Now, the total length of the plate in contact with the water surface is 2(10 + 0.2) = 20.4 cm,

$\therefore$ downward pull upon the plate due to surface tension

= 20.4 × 73 = 1489.2 dynes

$\therefore$ resultant upthrust = 1509.2 – 1489.2

= 20.0 dynes = $\frac{20}{980}$ = = 0.0204 gm-wt.

$\therefore$ apparent weight of the plate in water

= weight of the plate in air – resultant upthrust

= 8.2 – 0.0204 = 8.1796 gm **Ans.**

Ex. 30. A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Given : Outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension of water 73 dyne/cm and g = 980cm/sec^2.

Sol. Let ℓ be the length of the tube inside water. The forces acting on the tube are :

(i) Upthrust of water acting upward

$= \pi r^2 \ell \times 1 \times 980 = \frac{22}{7} \times (0.14)^2\, \ell \times 980$

= 60.368 ℓ dyne.

(ii) Weight of the system acting downward

= mg = 0.2 × 980 = 196 dyne.

(iii) Force of surface tension acting downward

$= 2\pi rT = 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24$ dyne.

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,

60.368 ℓ = 196 + 64.24 = 260.24.

$\therefore\ \ell = \frac{260.24}{60.368}$ = **4.31 cm.**

Ex. 31. A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07 nm^{-1}. Assume that the angle of contact between water and glass is 0^o.

Sol. Suppose pressures at the points A, B, C and D be P_A, P_B, P_C and P_D respectively.

The pressure on the concave side of the liquid surface is greater than that on the other side by 2T/R.

An angle of contact θ is given to be 0^o, hence

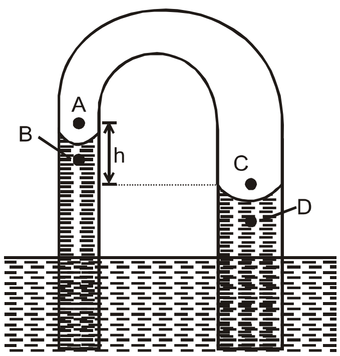
$R \cos 0^o = r$ or $R = r$

$\therefore\ P_A = P_B + 2T/r_1$ and $P_C = P_D + 2T/r_2$

where r_1 and r_2 are the radii of the two limbs

But $P_A = P_C$

$$\therefore\ P_B + \frac{2T}{r_1} = P_D + \frac{2T}{r_2}$$



$$\text{or } P_D - P_B = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where h is the difference in water levels in the two limbs

$$\text{Now,} \qquad h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Given that $T = 0.07\ Nm^{-1}$, $\rho = 1000\ kgm^{-3}$

$r_1 = \frac{3}{2}$ mm $= \frac{3}{20}$ cm $= \frac{3}{20 \times 100}$ m $= 1.5 \times 10^{-3}$ m,

$r_2 = 3 \times 10^{-3}$ m

$$\therefore\ h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right)$$

m $= 4.76 \times 10^{-3}$ m $= 4.76$ mm

Ex. 32. Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is $7.3 \times 10^{-2}\ Nm^{-1}$. Take the angle of contact to be zero, and density of water to be $1.0 \times 10^3\ kg\ m^{-3}$ ($g = 9.8\ ms^{-2}$).

Sol. Given that $r_1 = \frac{3.0}{2} = 1.5$ mm $= 1.5 \times 10^{-3}$ m,

$r_2 = \frac{6.0}{2} = 3.0$ mm $= 3.0 \times 10^{-3}$ m,

$T = 7.3 \times 10^{-2}\ Nm^{-1}$, $\theta = 0^o$ $\rho = 1.0 \times 10^3\ kg\ m^{-1}$, $g = 9.8\ ms^{-2}$

When angle of contact is zero degree, the radius of the meniscus equals radius of bore.

Excess pressure in the first bore, $P_1 = \frac{2T}{r_2}$

$$= \frac{2 \times 7.3 \times 10^{-2}}{1.5 \times 10^{-3}} = 97.3 \text{ Pascal}$$

Excess pressure in the second bore, $P_2 = \frac{2T}{r_2}$

$$= \frac{2 \times 7.3 \times 10^{-2}}{3 \times 10^{-3}} = 48.7 \text{ Pascal}$$

Hence, pressure difference in the two limbs of the tube

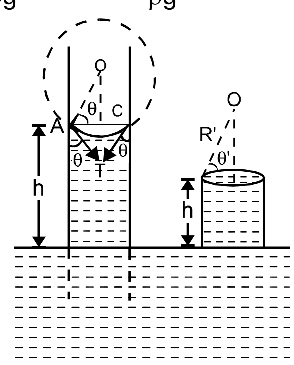
$\Delta P = P_1 - P_2 = h\rho g$

$$\text{or } h = \frac{P_1 - P_2}{\rho g} = \frac{97.3 - 48.7}{1.0 \times 10^3 \times 9.8} = 5.0 \text{ mm}$$

CAPILLARY RISE IN A TUBE OF INSUFFICIENT LENGTH

We know, the height through which a liquid rises in the capillary tube of radius r is given by

$$\therefore\ h = \frac{2T}{R\rho g} \text{ or } hR = \frac{2T}{\rho g} = \text{constant}$$



When the capillary tube is cut and its length is less then h (i.e. h'), then the liquid rises upto the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus increases and it becomes more flat so that hR = h'R' = Constant. Hence the liquid does not overflow.

If $h' < h$ then $R' > R$ or $\frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$

$\Rightarrow \cos\theta' < \cos\theta \quad \Rightarrow \quad \theta' > \theta$

Solved Examples

Ex. 33. If a 5 cm long capillary tube with 0.1 mm internal diameter open at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary. (ii) Water will rise up to the upper end of capillary (iii) Water will overflow out of the upper end of capillary/ Explain your answer.

Sol. Given that surface tension of water, T = 75 dyne/cm

Radius r = $\frac{0.1}{2}$ mm = 0.05 mm = 0.005 cm,

density ρ = 1 gm/cm^3, angle of contact, $\theta = 0°$.

Let h be the height to which water rise in the capillary tube. Then

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 75 \times \cos 0°}{0.005 \times 1 \times 981} \text{ cm} = \mathbf{30.58\ cm}.$$

But length of capillary tube, h' = 5 cm

(i) Because $h > \frac{h'}{2}$ therefore the first possibility does not exist.

(ii) Because the tube is of insufficient length therefore the water will rise upto the upper end of the tube.

(iii) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary.

The liquid meniscus will adjust its radius of curvature R' in such a way that

$$R'h' = Rh \qquad \left[\because \quad hR = \frac{2T}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length

$$\therefore\ R' = \frac{Rh}{h'} = \frac{rh}{h'} \quad \left[\because \quad R = \frac{r}{\cos\theta} = \frac{r}{\cos 0°} = r\right]$$

$$= \frac{0.005 \times 30.58}{5} = \mathbf{0.0306\ cm}$$

APPLICATIONS OF SURFACE TENSION

(i) The wetting property is made use of in detergents and waterproofing. When the detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptics spreads properly over the wound. The lubricating oils and paints also have low surface tension. So they can spread properly.

(iii) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(iv) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(v) A rough sea can be calmed by pouring oil on its surface.

EFFECT OF TEMPERATURE AND IMPURITIES ON SURFACE TENSION

The surface tension of a liquid decreases with the rise in temperature and vice versa. According to Ferguson, $T = T_0\left(1 - \frac{\theta}{\theta_c}\right)^n$ where T_0 is surface tension at 0ºC, θ is absolute temperature of the liquid, θ_c is the critical temperature and n is a constant varies slightly from liquid and has mean value 1.21. This formula shows that the surface tension becomes zero at the critical temperature, where the interface between the liquid and its vapour disappears. It is for this reason that hot soup tastes better while machinery parts get jammed in winter.

The surface tension of a liquid changes appreciably with addition of impurities. For example, surface tension of water increases with addition of highly soluble substances like NaCI, $ZnSO_4$ etc. On the other hand surface tension of water gets reduced with addition of sparingly soluble substances like phenol, soap etc.

SURFACE ENERGY

We know that the molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy. Unit of surface energy is erg cm^{-2} in C.G.S. system and Jm^{-2} in Sl system. Dimensional formula of surface energy is $[ML^{\circ}T^{-2}]$ Surface energy depends on number of surfaces e.g. a liquid drop is having one liquid air surface while bubble is having two liquid air surface.

RELATION BETWEEN SURFACE TENSION AND SURFACE ENERGY

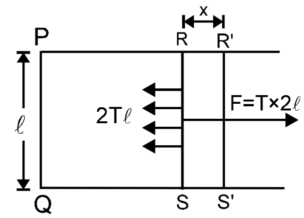
Consider a rectangular frame PQRS of wire, whose arm RS can slide on the arms PR and QS. If this frame is dipped in a soap solution, then a soap film is produced in the frame PQRS in fig. Due to surface tension (T), the film exerts a force on the frame (towards the interior of the film). Let ℓ be the length of the arm RS, then the force acting on the arm RS towards the film is. $F = T \times 2\ell$ [Since soap film has two surfaces, that is why the length is taken twice].

Let the arm RS be displaced to a new position R'S' through a distance x

$\therefore$ work done, $W = Fx = 2T\ell x$

Increase in potential energy of the soap film.

$= EA = 2E\ell x$ = work done in increasing the area (ΔW) where E = surface energy of the soap film per unit area.



According the law of conservation of energy, the work done must be equal to the increase in the potential energy

$\therefore 2T\ell x = 2E\ell x$ or $T = E = \dfrac{\Delta W}{A}$

Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area.

Solved Examples

Ex. 34. A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded if surface tension of mercury is 35×10^{-3} N/m.

Sol. If drop of radius R is sprayed into n droplets of equal radius r, then as a drop has only one surface, the initial surface area will be $4\pi R^2$ while final area is $n(4\pi r^2)$. So the increase in area

$\Delta S = n(4\pi r^2) - 4\pi R^2$

So energy expended in the process,

$W = T\Delta S = 4\pi T\,[nr^2 - R^2]$ .... (1)

Now since the total volume of n droplets is the same as that of initial drop, i.e.,

$\frac{4}{3}\pi R^3 = n[(4/3)\,\pi r^3]$ or $r = R/n^{1/3}$ ....(2)

Putting the value of r from equation (2) in (1)

$\mathbf{W = 4\pi R^2 T\,(n)^{1/3} - 1].}$

Ex. 35. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given be

$\dfrac{3T}{J}\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

where T is the surface tension of water and J is the mechanical equivalent of heat.

Sol. Let n be the number of little droplets.

Since volume will remain constant, hence volume of n little droplets = volume of single drop

$\therefore\ n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $nr^3 = R^3$

Decrease in surface area = $n \times 4\pi r^2 - 4\pi R^2$

or $\Delta A = 4\pi\,[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right] = 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$

Energy evolved $W = T \times$ decrease in surface area $= T \times 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$

Heat produced, $Q = \frac{W}{J} = \frac{4\pi TR^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$

But $Q = ms\,d\theta$

where m is the mass of big drop, s is the specific heat of water and $d\theta$ is the rise in temperature.

$\therefore\ \frac{4\pi TR^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$ = volume of big drop × density of water × sp. heat of water × $d\theta$

or, $\frac{4}{3}\pi R^3 \times 1 \times 1 \times d\theta = \frac{4\pi TR^3}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$ or,

$d\theta = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$

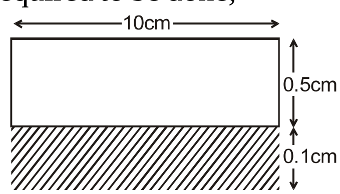
Ex. 36. A film of water is formed between two straight parallel wires each 10 cm long and at a separation 0.5 cm. Calculate the work required to increase 1 mm distance between them.

Surface tension of water = 72×10^{-3} N/m.

Sol. Here the increase in area is shown by shaded portion in the figure.

Since this is a water film, it has two surfaces, therefore increase in area, $\Delta S = 2 \times 10 \times 0.1 = 2\ cm^2$

$\therefore$ Work required to be done,



$W = \Delta S \times T = 2 \times 10^{-4} \times 72 \times 10^{-3}$

$= 144 \times 10^{-7}$ joule $= 1.44 \times 10^{-5}$ joule