SQUARE OF A NUMBER: The Square of a number is that number raised to the power 2.

Examples: Square of 9 = $9^2 = 9 \times 9 = 81$
Square of 0.2 = $(0.2)^2 = (0.2) \times (0.2) = 0.04$

PERFECT SQUARE: A natural number is called a perfect square, if it is the square of some natural number.

Example: We have $1^2 = 1$, $2^2 = 4$, $3^2 = 9$

Some Properties of Squares of Numbers

1. The square of an even number is always an even number.

   Example: 2 is even and $2^2 = 4$, which is even.

2. The square of an odd number is always an odd number.

   Example: 3 is odd and $3^2 = 9$, which is odd.

3. The square of a proper fraction is a proper fraction less than the given fraction.

   Example: $Square\ of\ \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{1}{4}$ and we see that $\frac{1}{4} < \frac{1}{2}$

4. The square of a decimal fraction less than 1 is smaller than the given decimal.

   Example: $0.1 < 1$ and $(0.1)^2 = 0.1 \times 0.1 = 0.01 < 0.1$.

5. A number ending in 2, 3, 7 or 8 is never a perfect square.

   Example: The numbers 72, 243, 567 and 1098 end in 2, 3, 7 and 8 respectively. So, none of them is a perfect square.

6. A number ending in an odd number of zeros is never a perfect square.

   Examples: The numbers 690, 87000 and 4900000 end in one zero, three zeros and five zeros respectively. So, none of them is a perfect square.

SQUARE ROOT: The square root of a number x is that number which when multiplied by itself gives x as the product. We denote the square root of a number x by $\sqrt{x}$.

Example: Since $7 \times 7 = 49$, so $\sqrt{49} = 7$, i.e., the square root of 49 is 7.

METHODS OF FINDING THE SQUARE ROOTS OF NUMBERS

To Find the Square Root of a Given Perfect Square Number Using Prime Factorization Method:

1. Resolve the given number into prime factors
2. Make pairs of similar factors
3. The product of prime factors, chosen one out of every pair, gives the square root of the given number.

Examples: Find Square root of i) 625 and ii) 1296

5	625
5	125
5	25
5	5
	1

$625 = 5 \times 5 \times 5 \times 5$ Hence $\sqrt{625} = 5 \times 5 = 25$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
Hence $\sqrt{1296} = 2 \times 2 \times 3 \times 3 = 36$

Test for a number to be a Perfect Square: A given number is a perfect square, if it can be expressed as the product of pairs of equal factors.

Example: $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
Hence $\sqrt{1296} = 2 \times 2 \times 3 \times 3 = 36$

To Find the Square Root of a given number By Division Method

1. Mark off the digits in pairs starting with the unit digit. Each pair and remaining one digit (if any) is called a period.
2. Think of the largest number whose square is equal to or just less than the first period. Take this number as the divisor as well as quotient.
3. Subtract the product of divisor and quotient from first period and bring down the next period to the right of the remainder. This becomes the new dividend.
4. Now, new divisor is obtained by taking twice the quotient and annexing with it a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of new divisor and this digit is equal to or just less than the new dividend.

Repeat steps 2, 3 and 4 till all the periods have been taken up. Now, the quotient so obtained is the required square root of the given number.

Example: Find the square root of 467856

6	46	78	56	684
	36			
128	10	78		
	10	24		
1364		54	56	
		45	56	

SQUARE ROOT OF NUMBERS IN DECIMAL FORM

Method: Make the number of decimal places even, by affixing a zero, if necessary. Now, mark periods (starting from the right most digit) and find out the square root by the long-division method. Put the decimal point in the square root as soon as the integral part is exhausted.

Example: *Find the square root* of 204.089796

1	2	04	.08	97	96	14.286
	1					
24	1	04				
		96				
282		8	08			
		5	64			
2848		2	44	97		
		2	27	84		
28566			17	13	96	
			17	13	96	
				x		

$$\sqrt{J204.089796} = 14.286$$

Square root of numbers which are not perfect squares

Example: Find the value of $\sqrt{0.56423}$ up to 3 places of decimal.

7	0.56	42	30	0.751
	49			
145	7	42		
	7	25		
1501		17	30	
		15	01	

$$\sqrt{0.56423} = \overline{\overset{2\qquad 29}{\sqrt{0.564230}}} = 0.751$$

SQUARE ROOTS OF FRACTIONS: For any positive real numbers *a* and *b*, we have:

i. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Example: Find the square root of $\frac{\sqrt{441}}{\sqrt{1849}} = \frac{21}{43}$

ii. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example: $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

CUBE OF A NUMBER: The cube of a number is that number raised to the power 3.

Example: Cube of 2 = $2^3 = 2\times2\times2 = 8$

PERFECT CUBE: A natural number is said to be a perfect cube, if it is the cube of some natural number.

Example: $1^3 = 1$, $2^3 = 8$, $3^3 = 27$ and so on…

CUBE ROOT: The cube root of a number *x* is that number which when multiplied by itself three times gives *x* as the product. We denote the cube root of a number *x* by $\sqrt[3]{x}$

Example: Since 5 x 5 x 5 = 125, therefore $\sqrt[3]{125} = 5$

METHOD OF FINDING THE CUBE ROOT OF NUMBERS: Cube Root of a Given Number by Prime Factorization Method

1. Resolve the given number into prime factors.
2. Make groups in triplets of similar factors.
3. The product of prime factors, chosen one out of ever triplet, gives the cube root of the given number.

Example: Find Cube of 17576.
17576 = 2 x 2 x 2 x 13 x 13 x 13
Therefore $\sqrt[3]{17576} = 2 \times 13 = 26$

Test for a Number to be a Perfect Cube

A given natural number is a perfect cube if it can be expressed as the product of triplets of equal factors.

Cube Roots of Fractions and Decimals

Example: Find cube root of:

i. $\sqrt[3]{\frac{2197}{343}} = \frac{\sqrt[3]{2197}}{\sqrt[3]{343}} = \frac{\sqrt[3]{13\times13\times13}}{\sqrt[3]{7\times7\times7}} = \frac{13}{7}$

ii. $\sqrt[3]{19.683} = \frac{\sqrt[3]{19686}}{\sqrt[3]{1000}} = \frac{27}{10}$

SQUARE ROOTS BY USING TABLES

A table showing the square roots of all natural numbers from 1 to 100 has been given, each approximating to 3 places of decimal using this table, we can find the square roots of numbers, larger than 100, as illustrated in the following examples.

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	21	4.583	41	6.403	61	7.810	81	9.000
2	1.414	22	4.690	42	6.481	62	7.874	82	9.055
3	1.732	23	4.796	43	6.557	63	7.937	83	9.110
4	2.000	24	4.899	44	6.633	64	8.000	84	9.165
5	2.236	25	5.000	45	6.708	65	8.062	85	9.220
6	2.449	26	5.099	46	6.782	66	8.124	86	9.274
7	2.646	27	5.196	47	6.856	67	8.185	87	9.327
8	2.828	28	5.292	48	6.928	68	8.246	88	9.381
9	3.000	29	5.385	49	7.000	69	8.307	89	9.434
10	3.162	30	5.477	50	7.071	70	8.367	90	9.487
11	3.317	31	5.568	51	7.141	71	8.426	91	9.539
12	3.464	32	5.657	52	7.211	72	8.485	92	9.592
13	3.606	33	5.745	53	7.280	73	8.544	93	9.644
14	3.742	34	5.831	54	7.348	74	8.602	94	9.695
15	3.873	35	5.916	55	7.416	75	8.660	95	9.747
16	4.000	36	6.000	56	7.483	76	8.718	96	9.798
17	4.123	37	6.083	57	7.550	77	8.775	97	9.849
18	4.243	38	6.164	58	7.616	78	8.832	98	9.899
19	4.359	39	6.245	59	7.681	79	8.888	99	9.950
20	4.472	40	6.325	60	7.746	80	8.944	100	10.000

Examples: $\sqrt[2]{20} = 4.472$; $\sqrt[2]{54} = 7.348$; $\sqrt[2]{83} = 9.110$; $\sqrt[2]{99} = 9.950$

CUBE ROOTS OF NUMBERS, USING CUBE ROOT TABLE

The table given below shows the values of where x is a natural number. Using this table, we may find the cube root of any given natural number.

x	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$	x	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$	x	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$
1	1.000	2.154	4.642	35	3.271	7.047	15.183	69	4.102	8.837	19.038
2	1.260	2.714	5.848	36	3.302	7.114	15.326	70	4.121	8.879	19.129
3	1.442	3.107	6.694	37	3.332	7.179	15.467	71	4.141	8.921	19.220
4	1.587	3.420	7.368	38	3.362	7.243	15.605	72	4.160	8.963	19.310
5	1.710	3.684	7.937	39	3.391	7.306	15.741	73	4.179	9.004	19.399
6	1.817	3.915	8.434	40	3.420	7.368	15.874	74	4.198	9.045	19.487
7	1.913	4.121	8.879	41	3.448	7.429	16.005	75	4.217	9.086	19.574
8	2.000	4.309	9.283	42	3.476	7.489	16.134	76	4.236	9.126	19.661
9	2.080	4.481	9.655	43	3.503	7.548	16.261	77	4.254	9.166	19.747
10	2.154	4.642	10.000	44	3.530	7.606	16.386	78	4.273	9.205	19.832
11	2.224	4.791	10.323	45	3.557	7.663	16.510	79	4.291	9.244	19.916
12	2.289	4.932	10.627	46	3.583	7.719	16.631	80	4.309	9.283	20.000
13	2.351	5.066	10.914	47	3.609	7.775	16.751	81	4.327	9.322	20.083
14	2.410	5.192	11.187	48	3.634	7.830	16.869	82	4.344	9.360	20.165
15	2.466	5.313	11.447	49	3.659	7.884	16.985	83	4.362	9.398	20.247
16	2.520	5.429	11.696	50	3.684	7.937	17.100	84	4.380	9.435	20.328
17	2.571	5.540	11.935	51	3.708	7.990	17.213	85	4.397	9.473	20.408
18	2.621	5.646	12.164	52	3.733	8.041	17.325	86	4.414	9.510	20.488
19	2.668	5.749	12.386	53	3.756	8.093	17.435	87	4.431	9.546	20.567
20	2.714	5.848	12.599	54	3.780	8.143	17.544	88	4.448	9.583	20.646
21	2.759	5.944	12.806	55	3.803	8.193	17.652	89	4.465	9.619	20.724
22	2.802	6.037	13.006	56	3.826	8.243	17.758	90	4.481	9.655	20.801
23	2.844	6.127	13.200	57	3.849	8.291	17.863	91	4.498	9.691	20.878
24	2.884	6.214	13.389	58	3.871	8.340	17.967	92	4.514	9.726	20.954
25	2.924	6.300	13.572	59	3.893	8.387	18.070	93	4.531	9.761	21.029
26	2.962	6.383	13.751	60	3.915	8.434	18.171	94	4.547	9.796	21.105
27	3.000	6.463	13.925	61	3.936	8.481	18.272	95	4.563	9.830	21.179
28	3.037	6.542	14.095	62	3.958	8.527	18.371	96	4.579	9.865	21.253
29	3.072	6.619	14.260	63	3.979	8.573	18.469	97	4.595	9.899	21.327
30	3.107	6.694	14.422	64	4.000	8.618	18.566	98	4.610	9.933	21.400
31	3.141	6.768	14.581	65	4.021	8.662	18.663	99	4.626	9.967	21.472
32	3.175	6.840	14.736	66	4.041	8.707	18.758	100	4.642	10.000	21.544
33	3.208	6.910	14.888	67	4.062	8.750	18.852				
34	3.240	6.980	15.037	68	4.082	8.794	18.945				

Example: $\sqrt[3]{5} = 1.710;\ \sqrt[3]{50} = 3.684;\ \sqrt[3]{500} = 7.937$