# SOUND WAVES

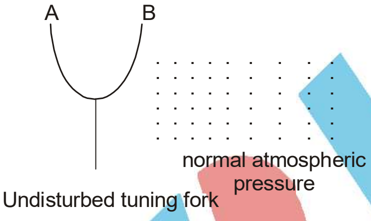
## 1. SOUND WAVES

Sound is type of longitudnal wave. In general majority of longitudinal waves are termed as sound waves. Sound is produced by a vibrating source, like when a gong of a bell is struck with a hammer, sound is produced. The vibrations produced by gong are propagated through air, Through air these vibrations reach to the ear and ear drum is set into vibrations and these vibrations are communicated to human brain. By touching the gong of bell by hand, we can feel the vibrations.

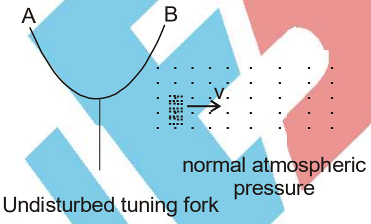
## 2. PROPAGATION OF SOUND WAVES

Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker. Being a mechanical wavs, sound needs a medium having properties of inetia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of presure, which is produced by the vibrating source.

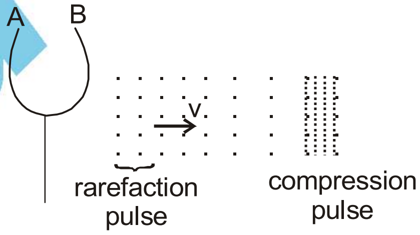
Consider a tuning fork producing sound waves.



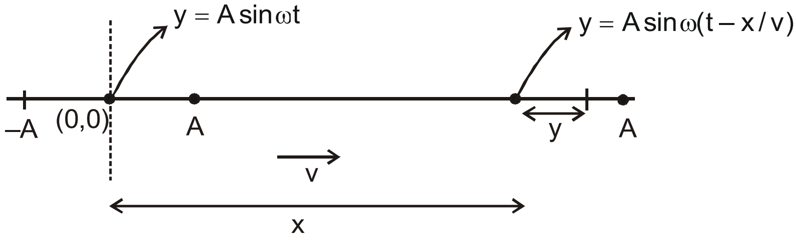

When Prong B moves outward towards right it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a compression pulse and it travels away from the prong with the speed of sound




After producing the compression pulse, the prong B reverses its motion and moves inward. This drages away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called rarefaction pulse. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.




A longitudinal wave in a fluid is described either in tems of the longitudal displacements suffered by the particles of the medium.



Consider a wave going in the x-direction in a fluid. Suppose that at a time t, the particle at the undistrubed position x suffers a displacement y in the x-direction.

$$y = A \sin \omega \left(t - \frac{x}{v}\right) \qquad \text{...(i)}$$

Position of any particle from origin at any time = x + y

x = Distance of the mean position of the particle from the origin.

y = Displacement of the particle from its mean position.

General Equation :

(0,0) $\Rightarrow$ $y = A \sin (\omega t + \phi)$

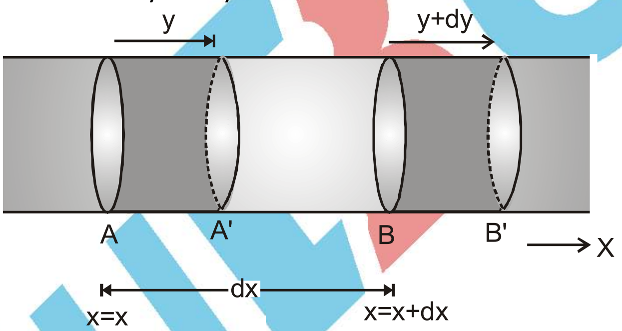
(0,x) $\Rightarrow$ $y = A \sin [(t - x/v) + \phi]$

Displacement wave $y = A \sin (\omega t - kx + \phi)$

- If we fix $x = x_0$ then we are dealing with the particle whose mean position at distance $x_0$ from origin & this particle is performin SHM of amp. A with time period T phase difference = $- kx + \phi$

## 3. COMPRESSION WAVES

When a longitudinal wave propagated in a gaseous medium, it produces compression and rarefaction in the medium periodically. The region where compression occurs, the pressure is more than the normal pressure of the medium. Thus we can also describe longitudinal waves in a gaseous medium as pressure waves and these are also termed as compression waves in which the pressure at different point of medium also varies periodically with their displacements. Let us discuss the propagation of excess pressure in a medium in longitudinal wave analytically.




Consider a longitudinal wave propagating in positive x-direction as shown in figure. Figure shows a segmentAB of the medium of width dx. In this medium let a longitudinal wave is propagating whose equation is given as

$$y = A \sin (\omega t - kx) \qquad \text{...(1)}$$

Where y is the displacement of medium particle situated at a distance x from the origin, along the direction of propagation of wave. In figure shown AB is the medium segment whose a medium particle is at position x = x and B is at x = x +dx at an instant. If after some time t medium particle at A reaches to a point A' which is displaced by y and the medium particle at b reaches to point B′ which is at a displacmenet y + dy from B. Here dy is given by equation (3.116) as

$$dy = - Ak \cos (\omega t - kx)\, dx$$

Here due to displacement of section AB to A′B′ the change in volume of it's section is given as

$dV = - S\, dy$ [S $\rightarrow$ Area of cross-section]

$= SA\, k \cos (\omega t - kx)\, dx$

The volume of section AB is $V = S\, dx$

Thus volume strain in section AB is

$$\frac{dV}{V} = \frac{-SAk\cos(\omega t - kx)dx}{S\,dx} \quad \text{or} \quad \frac{dV}{V} = -Ax\cos(\omega t - kx)$$

If B is the bulk modulus of the medium, then the excess pressure in the section AB can be given as

$$\Delta P = -B\left(\frac{dV}{V}\right) \qquad \text{...(2)}$$

$$\Delta P = BAk \cos (\omega t - kx)$$

or $\Delta P = \Delta P_0 \cos(\omega t - kx)$ ...(3)

Here $\Delta P_0$ is the pressure amplitude at a medium particle at position x from origin and $\Delta P$ is the excess pressure at that point. Equation shown that excess varies periodically at every point of the medium with pressure amplitude $\Delta P_0$, which is given as

$$\Delta P_0 = BAk = \frac{2\pi}{\lambda} AB \quad ...(4)$$

Equation shown is also termed as the equation of pressure wave in gaseous medium. We can also see that the pressure wave differs in phase is $\pi/2$ from the displacement wave and pressure maxima occurs where the displacement is zero and displacement maxima occur where the pressure is at its normal level. Remembers that pressure maxima implies that the presure at a point is pressure amplitude times more or less then the normal pressure level of the medium.

### 3.1 Velocity and Acceleration of particle :

General equation of wave is given by

$$y = A \sin(\omega t - kx)$$

$$v_p = \frac{\partial y}{\partial t} = A\,\omega \cos(\omega t - kx) \quad ...(1)$$

$$a_p = \frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx) \quad ...(2)$$

$$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) \quad ...(3)$$

Here $\frac{\partial y}{\partial x}$ = slope of (y, x) curve Now again differentiate eq. – 3

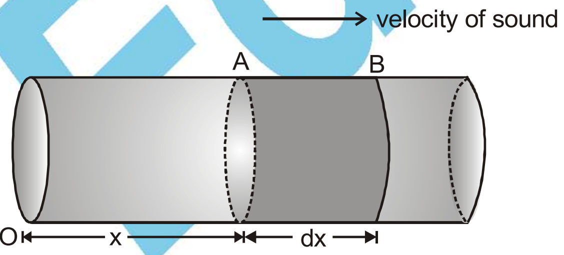
$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \quad ...(4)$$

from eq. (2) & (4)

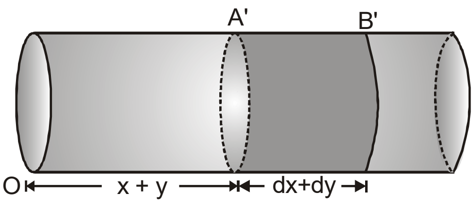
$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

## 4. VELOCITY OF SOUND/LONGITUDINAL WAVES IN SOLIDS

Consider a section AB of medium as shown in figure(a) of cross-sectional area S. Let A and B be two cross section as shown. Let in this medium sound propagation is from left to right. If wave source is at origin O and when it oscillates, the oscillations at that point propagate along the rod.



(a)



(b)

Here we say an elastic wave has propagated along the rod with a velocity determined by the physical properties of the medium. Due to oscillations say a force F is developed at every point of medium which produces a stress in rod and is the cause of strain or propagation of disturbance along the rod. This stress at any cross-sectional area can be given as

$$\text{Stress } S_1 = \frac{F}{S} \quad ...(1)$$

If we consider the section AB of medium at a general instant of time t. The end A is at a distance x from O and B is at a distance x + dx from O. Let in time dt due to oscillations, medium particles at a are displaced along the length of medium by y and those at B by y + dy. The resulting position of section and A′ and B′ shown in figure (b), Here we can say that the section AB is deformed (elongated) by a length dy. Thus strain produced in it is

Strain in section AB $\quad E = \frac{dy}{dx}$ ...(2)

If Young's modulus of the material of medium is Y, we have

$$\text{Young's Modulus } Y = \frac{\text{Stess}}{\text{Strain}} = \frac{S_1}{E}$$

From equation (1) and (2), we have $Y = \frac{F/S}{dy/dx}$

or $\quad F = YS\frac{dy}{dx}$ ...(3)

If net force acting of secting AB is dF then it is given as

$$dF = dma \quad ...(4)$$

Where *dm* is the mass of section AB and a be its acceleration, which can be given as for a medium of density $\rho$.

$$dm = \rho S dx \quad \text{and} \quad a = \frac{d^2y}{dt^2}$$

From equation (4), we have $dF = (\rho S dx)\frac{d^2y}{dx^2}$

or $\quad \frac{dF}{dx} = \rho S\frac{d^2y}{dt^2}$ ...(5)

From equation (3) on differentiating w.r. to x, we can write

$$\frac{dF}{dx} = YS\frac{d^2y}{dt^2} \quad ...(6)$$

From equation (5) and (6) we get

$$\frac{d^2y}{dx^2} = \left(\frac{Y}{\rho}\right)\frac{d^2y}{dx^2} \quad ....(7)$$

Equation (7) is the differential form of wave equation, comparing it with previous equation we get the wave velocity in the medium can be given as

$$v = \sqrt{\frac{Y}{\rho}}$$

Similar to the case of a solid in fluid, instead of Young's Modulus we use Bulk modulus of the medium hence the velocity of longitudinal waves in a fluid medium is given as

$$v = \sqrt{\frac{B}{\rho}}$$

Where B is the Bulk modulus of medium.
For a gaseous medium bulk modulus is defined as

$$B = \frac{dp}{(-dV/V)} \quad \text{or} \quad B = -V\frac{dP}{dV}$$

**4.1 Newton's Formula for velocity of Sound in Gases**

Newton assumed that during sound propagation temperature of medium remains constant hence the stated that propagation of sound in a gasesous medium is an isothermal phenomenon, thus Boyal's law can be applied in the process. So for a section of medium we use

$$PV = \text{constant}$$

Differentiating we get

$$PdV + V\,dP = 0$$

or $\quad -V\frac{dP}{dV} = P$

or bulk modulus of medium can be given as

B = P (Pressure of medium)

Newton found that during isothermal propagation of sound in a gaseous medium, bulk modulus of medium is equal to the pressure of the medium, hence sound velocity in a gaseous medium can be given as

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{P}{\rho}} \quad ...(1)$$

From gas law we have $\frac{P}{\rho} = \frac{RT}{M}$ ...(2)

From (1) & (2) we have $v = \sqrt{\frac{RT}{M}}$ ...(3)

From the expression in equation (1) if we find the sound veloicty in air at normal temperature and atompsheric pressure we have

Normal atmospheric pressure is $P = 1.01 \times 10^5$ Pa

Density of air at NTP is $\rho = 1.293$ kg/m$^3$

Now from equation (1) $v = \sqrt{\frac{P}{\rho}} \Rightarrow v = \sqrt{\frac{1.01 \times 10^5}{1.293}} = 279.45$ m/s

But the experimental value of veloicty of sound determined from various experiments gives the velocity of sound at NTP, 332 m/s. Therefore there is a difference of about 52 m/s between the theoretical and experimental values. This large difference can not be attributed to the experimental errors. Newton was unable to explain error in his formula. This correction was explained by a French Scientist Laplace.

### 4.2 Laplace Correction

Laplace explained that when sound waves propagated in a gaseous medium. There is compression and rarefaction in the particles of medium. Where there is compression, particles come near to each other and are heated up, where there is rarefaction, medium expends and there is fall of temperature. Therefore, the temperature of medium at every point does not remain constant so the process of sound propagation is not isothermal. The total quantity of heat of the system as a whole remains constant. medium does not gain or loose any heat to the surrounding. Thus in a gaseous medium sound propagation is an adiabatic process. For adiabatic process the relation in pressure and volume of a section of medium can be given as

$$PV^{\gamma} = \text{constant} \quad ...(1)$$

Here $\gamma = \frac{C_p}{C_v}$, ratio of specific heats of the medium.

Differentiating equation (1) we get,

$$dPV^{\gamma} + \gamma V^{\gamma-1}\, dVP = 0$$

or $$dP + \gamma \frac{PdV}{dV} = 0$$

or $$-V\frac{dP}{dV} = \gamma P$$

Bulk modulus of medium $B = \gamma P$

Thus Laplace found that during adiabatic propagation of sound, the Bulk modulus of gaseous medium is equal to the product of ratio of specific heats and the pressure of medium. Thus velocity of sound propagation can be given as

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

From gas law $v = \sqrt{\frac{\gamma RT}{M}}$

From above equation we find sound velocity in air at NTP, we have

Normal atmospheric pressure $P = 1.01 \times 10^5$ Pa

Density of air at NTP $P = 1.293$ kg/m$^3$

Ratio of specific heat of air $\gamma = \frac{C_p}{C_v} = 1.42$

$$\Rightarrow v = \sqrt{\frac{\gamma\rho}{\rho}} = \sqrt{\frac{1.42 \times 1.01 \times 10^5}{1.293}} = 333.04 \text{ m/s}$$

This value is in agreement with exerimental value.

Now at any temperature t°C velocity of sound $v_t = \sqrt{\frac{\gamma R(273+t°)}{M}} = \sqrt{\frac{\gamma R273}{M}}\left(1+\frac{t}{273}\right)^{1/2}$

$$v_t = v_0\left(1+\frac{t}{546}\right)$$

**4.3 Effect of Temperature on Velocity of Sound**

We have velocity of sound propagation in a gasous medium as

$$v = \sqrt{\frac{\gamma RT}{M}}$$

For a given gaseous medium $\gamma$, R and M remains constant, thus velocity of sound is directly proportional to square root of absolute temperature of the medium. Thus

$$v \propto \sqrt{T}$$

If at two different temperatures $T_1$ and $T_2$, sound velocities in medium are $v_1$ and $v_2$ then from above equation we have

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

**4.4 Effect of Pressure on Velocity of Sound**

We know form gas law

$$\frac{P}{\rho} = \frac{RT}{M}$$

If temperature of a medium remains constant then on changing pressure, density of medium proportionally changes so that the ratio $\frac{P}{\rho}$ remains constant.

Hence if in a medium, $T$ = constant

Then, $\frac{P}{\rho}$ = constant

Thus velocity of sound, $v = \sqrt{\frac{\gamma P}{\rho}}$ = constant

Therefore, the velocity of sound in air or in a gas is independent of change in pressure.

**4.5 Effect of Humidity on Velocity of Sound**

The density of water vapour at NTP is 0.8 kg/m³ whereas the density of dry air at NTP is 1.293 kg/³. Therefore water vapour has a density less than the density of dry air. As atmospheric pressure remains approximately same, the velocity of sound is more in moist air then the velocity of sound in dry air.

$v_{\text{moist air}} > v_{\text{dry air}}$ (from the previous equation)

**4.6 Effect of Wind on Velocity of Sound**

If wind is blowing in the direction of propagation of sound, it will increase the velocity of sound, On the other hand if wave propagation is opposite to the direction of propagation of wind, wave velocity is decreased. If wind blows at speed $v_\omega$ then sound velocity in the medium can be given as

$$\vec{v} = \vec{v}_s + \vec{v}_w$$

Where $\vec{v}_s$ is the velocity of sound in still air.

## 5. APPEARANCE OF SOUND TO HUMAN EAR

The appearance of sound to a human ear is characterised by three parameters (a) pitch (b) loudness and (c) quality.

### (a) Pitch and Frequency

Pitch of a sound is that sensation by which we differentiate a buffalo voice, a male voice and a female voice. We say that a buffalo voice is of low pitch, a male voice has higher pitch and a female voice has still higher pitch. This sensation primarly depends on the dominant frequency present in the sound. Higher the frequency, higher will be the pitch and vice versa.

### (b) Loudness and Intensity

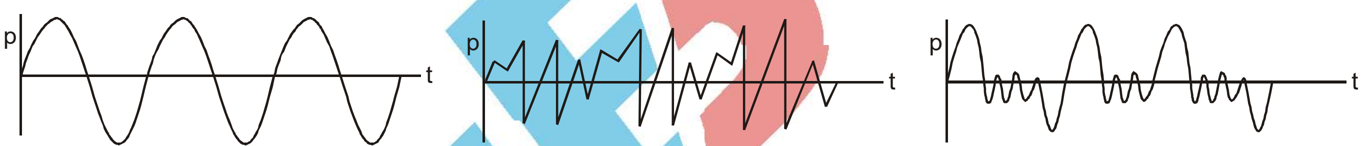
The loudness that we sense is related to the intensity of sound though it is not directly proportional to it. Our perception of loudness is beter correlated with the sound level measured in decibels (abbreviated as dB) and defined as follows.

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where I is the intensity of the sound and $I_0$ is a constant reference intensity $10^{-12}$ W/m² The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I = I_0$ , the sound level $\beta = 0$.

### (c) Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.




We differenatiate between the sound from a tabla and that from a mridang by saying that they have different quality.

### (d) Energy in sound Waves

$P_{avg} = 2\pi^2 f^2 A^2 \mu v$

Intensity $= 2\pi^2 A^2 f^2 \rho v$

$$\Delta P = \Delta P_0 \cos(\omega t - kx)$$

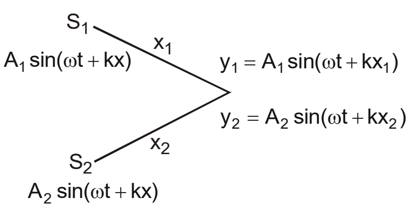
$$\Delta P_0 = \beta^3 Ak \;\Rightarrow\; A = \frac{\Delta P_0}{\beta K}$$

$$\text{Intensity} = \frac{\omega^2 A^2 \rho v}{2} = \frac{\omega^2 \Delta P_0^{\,2} \rho v}{2\beta^2 k^2} = \frac{\omega^2 \Delta P_0^{\,2} \rho v^3}{2\beta^2 \omega^2} = \frac{\Delta P_0^{\,2} \rho v^2 . v}{2\beta^2}$$

$$= \frac{\Delta P_0^{\,2} \rho \beta . v}{2\beta^2 \rho} \qquad \left(v = \sqrt{\frac{\beta}{\rho}}\right)$$

$$I = \frac{\Delta P_0^2 V}{2.\beta}$$

## 6. ANALYTICAL TREATMENT OF INTERFERENCE OF WAVES



Interference implies super position of waves. Whenever two or more than two waves superimpose each other at some position then the resutant displacement of the particle is given by the vector sum of the individual displacements.

Let the two waves coming from sources $S_1$ & $S_2$ be

$y_1 = A_1 \sin(\omega t + kx_1)$

$y_2 = A_2 \sin(\omega t + kx_2)$ respectively.

Due to superposition

$y_{net} = y_1 + y_2$

$y_{net} = A_1 \sin(\omega t + kx_1) + A_2 \sin(\omega t + kx_2)$

Phase difference between $y_1$ & $y_2 = k(x_2 - x_1)$

i.e., $\Delta\phi = k(x_2 - x_1)$

As $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$ (where $\Delta x$ = path difference & $\Delta\phi$ = phase difference)

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\Rightarrow A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

$\therefore I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$ (as $I \propto A^2$)

When the two displacements are in phase, then the resultant amplitude will be sum of the two amplitude & $I_{net}$ will be maximum, this is known of constructive interference.

For $I_{net}$ to be maximum

$\cos\phi = 1 \Rightarrow \phi = 2n\pi$ where n = {0,1,2,3,4,5...........}

$\frac{2\pi}{\lambda}\Delta x = 2n\pi \Rightarrow \Delta x = n\lambda$

For constructive interference

$I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$

When $I_1 = I_2 = I$

$I_{net} = 4I$

$A_{net} = A_1 + A_2$

When superposing waves are in opposite phase, the resultant amplitude is the difference of two amplitudes & $I_{net}$ is minimum; this is known as destructive interference.

For $I_{net}$ to be minimum,

$\cos\Delta\phi = -1$

$\Delta\phi = (2n + 1)\pi$ where n = {0,1,2,3,4,5...........}

$\frac{2\pi}{\lambda}\Delta x = (2n + 1)\pi \Rightarrow \Delta x = (2n+1)\frac{\lambda}{2}$

For destructive interference

$I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$

If $I_1 = I_2$

$I_{net} = 0$

$A_{net} = A_1 - A_2$

Generally,

$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

If $I_1 = I_2 = I$

$I_{net} = 2I + 2I\cos\phi$

$I_{net} = 2I(1 + \cos\phi) = 4I\cos^2\frac{\Delta\phi}{2}$

Ratio of $I_{max}$ & $I_{min}$ = $\frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

## 7. LONGITUDINAL STANDING WAVES

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering waves are given by

$p_1 = p_0 \sin(\omega t - kx)$ and $p_2 = p_0 \sin(\omega t + kx + \phi)$

then the equation of the resultant standing wave would be given by

$$p = p_1 + p_2 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

$$\Rightarrow p = p_0' \sin\left(\omega t + \frac{\phi}{2}\right) \qquad ...(1)$$

This is equation of SHM* in which the amplitude $p_0'$ depends on position as

$$p_0' = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \qquad ...(2)$$

Points where pressure remains permanently at its average value, i.e., pressure amplitude is zero is called a pressure node, and the condition for a pressure node would be given by

$p_0' = 0$

i.e. $\cos\left(kx + \frac{\phi}{2}\right) = 0$

i.e. $kx + \frac{\phi}{2} = 2n\pi \pm \frac{\pi}{2}$, $n = 0,,1,2......$

Similarly points where pressure amplitude is maximum is called a pressure antinode and condition for a pressure antinode would be given by

$p_0' = \pm 2p_0$

i.e. $\cos\left(kx + \frac{\phi}{2}\right) = \pm 1$

or $\left(kx + \frac{\phi}{2}\right) = n\pi$, $n = 0, 1, 2,......$

**Note :**

- Note that a pressure node in a standing wave would correspond to a displacement antinode; and a pressure anti-node would correspond to a displacement node.
- (when we label eqn. (1) as SHM, what we mean that excess pressure at any point varies simple harmonically. if the sound waves were represented in terms of displacement waves, then the equation of standing wave corresponding to (1) would be

$$s = s_0' \cos\left(\omega t + \frac{\phi}{2}\right) \text{ where } s_0' = 2s_0 \sin\left(kx + \frac{\phi}{2}\right)$$

This can be easily observed to be an equation of SHM. It represents the medium particles moving simple harmonically about their mean position at x.

## 8. REFLECTION OF SOUND WAVES

Reflection of sound waves from a rigid boundary (e.g. closed end of an organ pipe) as analogous to reflection of a string wave from rigid boundary; reflection accompanied by an inversion i.e. an abrupt phase change of p. This is consistent with the requirement of displacement amplitude of remains zero at the rigid end, since a medium particle at the rigid end can not vibrate. As the excess pressure and displacement corresponding to the same sound wave vary by $\pi/2$ in term of phase, a displacement minima at the rigid end will be a point of pressure maxima. This implies that the reflected pressure waves from the rigid boundary will have same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse

On the other hand, reflection of sound wave from a low pressure region (like open end of an organ pipe) is analogies to reflection of string wave from a free end. This point corresponds to a displacment maxima, so that the incident & reflected displacement wave at this point must be in phase. This would imply that this point would be a minima for pressure wave (i.e. pressure at this point remains at its average value), and hence thereflected presure wave would be out of phase by $\pi$ with respect to the incident wave. i.e. a compression pulse is reflected as a rarefaction pulse and vice-versa.

## 9. WAVES IN A VIBRATING AIR COLUMN

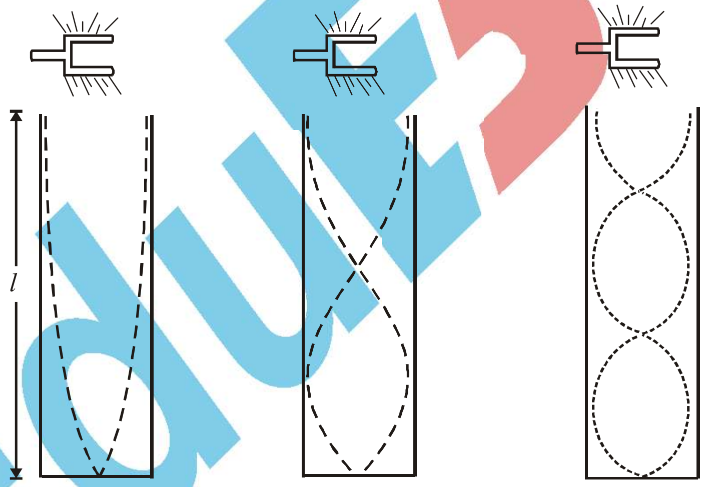
Hollow pipes have long used for making musical sounds. A hollow pipe we call organ pipe. To understand how these work, first we examine the behaviour of air in a hollow pipe that is open at both ends. If we blow air across one end, the disturbance due to the moving air at that end propagates along the pipe to the far end. When it reaches far end, a part of the wave is reflected, similar in the case when a wave is reflected along a string whose end point is free to move. Since the air particles are free to move at the open end, the end point is an antinode. if one end of the pipe is closed off, the the air is not free to move any further in that direction and closed end becomes a node. now the resonant behaviour of pipe is completely changed. Similar in the case of string, here also all harmonic frequencies are possible and resonance may take place it the frequency of external source matches with any of the one harmonic freuquency of pipe. Let us discuss in detail.

### 9.1 Vibration of Air in a Closed Organ Pipe

When a tuning fork is placed near the open end of a pipe. The air in the pipe oscillates with the same frequency as that of tuning fork. Here the open end should be an antinode and closed and should be a node for perfect reflection of waves from either end or for formation of stationary waves. Since one end is a node and other is an antinode, the lowest frequency (largest wavelength) vibration has no other nodes or antinodes between ends as shown in figure(a). This is the fundamental (minimum) frequency at which stationary waves can be formed in a closed organ pipe. Thus if the wavelength is $\lambda$ then we can see from figure (a), which shows the displacement wave of longitudinal waves in the closed organ pipe.

$$l = \frac{\lambda}{4} \quad \text{...(1)}$$

or $$\ell = 4\,l$$




(a) (b) (c)

Thus fundamental frequency of oscillations of closed organ pipe of length $l$ be given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{4l} \quad \text{...(2)}$$

Similarly first overtone of closed pipe vibrations is shown in figure (b) here wavelength $\lambda'$ and pipe length $l$ are related as

$$l = \frac{3\lambda'}{4} \quad \text{...(3)}$$

or $$\lambda' = \frac{4l}{3}$$

Thus frequency of first overtone oscillatinos of a closed organ pipe of length $l$ can be given as

$$n_2 = \frac{v}{\lambda'} = \frac{3v}{4l} \quad \text{...(4)}$$

$$= 3n_1$$

This is three times the fundamental frequency thus after fundamental only third harmonic frequency exist for a closed organ pipe at which resonance can take place or stationary waves can be formed in it. Similarly next overtone, second overtone is shown in figure(c). Here the wavelength $\lambda''$ and pipe length $l$ are related as

$$l = \frac{5\lambda''}{4}$$

or $$\lambda'' = \frac{4l}{5}$$

Thus the frequency of second overtone oscillation of a closed organ pipe of length $l$ can be given as

$$n_3 = \frac{v}{\lambda''} = \frac{5v}{4l} = 5n_1$$

This is fifth harmonic frequency of fundamental oscillations.

In general $f = \dfrac{(2n-1)v}{4\ell}$

Here frequency of oscillation is called $(2n - 1)^{th}$ harmonic and $(n - 1)^{th}$ overtone

From above analysis it is clear that the resonant frequecies of the closed organ pipe are only odd harnomics of the fundamental frequency.

### 9.2 Vibration of Air in Open Organ Pipe

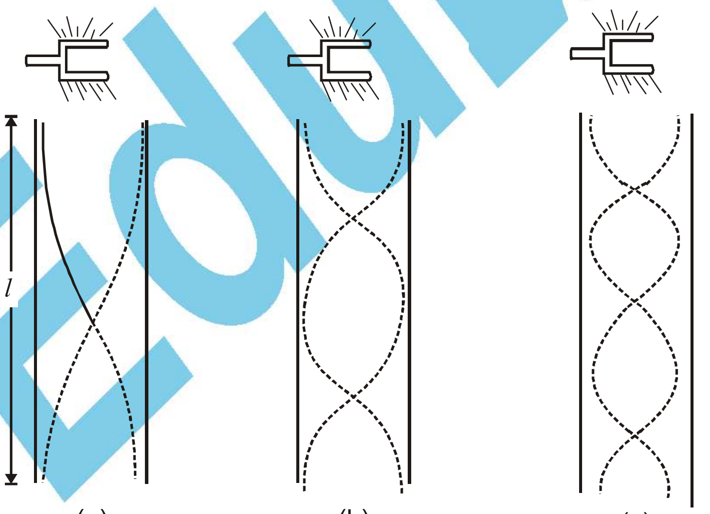
Figure shown the resonant oscillations of an open organ pipe. The least frequency at which an open organ pipe resonates is the one with longest wavelength when at both the open ends of pipe antinodes are formed and there is one node is between as shown in figure (a). In this situation the wavelengths of sound in air $\lambda$ is related to length of organ pipe as

$$l = \frac{\lambda}{2}$$

or $$\lambda = 2l \qquad ...(1)$$

Thus the fundamental frequency of organ pipe can be given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{2l}$$




(a) (b) (c)

Similarly next higher frequency at which the open organ pipe resonate is shown in figure (b) which we call first overtone. Here the wavelength $\lambda'$ is related to the length of pipe as

$$l = \lambda' \qquad ....(2)$$

Thus here resonant frequency for first overtone is given as

$$n_2 = \frac{v}{\lambda'} = \frac{v}{l} \qquad ...(3)$$

Which is second harmonic of fundamental frequency. Similarly as shown in figure (c), in second overtone oscillations, the wavelength $\lambda''$ of sound is releated to the length of pipe as

$$l = \frac{3\lambda''}{2} \quad \text{...(4)}$$

or $$\lambda'' = \frac{2l}{3} \quad \text{...(5)}$$

Thus be frequency of second overtone oscillations of an open organ pipe can given as

$$n_3 = \frac{v}{\lambda''} = \frac{3v}{2l} \quad \text{...(6)}$$

$$= 3n_1 \quad \text{...(7)}$$

Which is third harmonic of fundamental frequency.

In General $f = \frac{nv}{2\ell}$

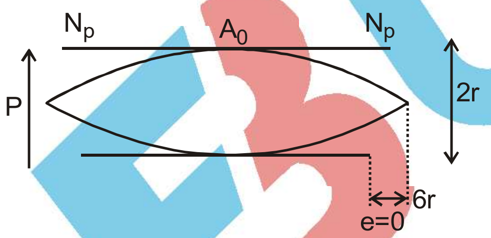
we can say frequency of oscillation is called $n^{th}$ harmonic and $(n - 1)^{th}$ overtone

The above analysis shown that resonant frequencies for formation of stationary waves includes all the possible harmonic frequencies for an open organ pipe.

**9.3 End correction**

As mentioned earlier the displacment antinode at an open end of an organ pipe lies slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by

$e = 0.6\ r$



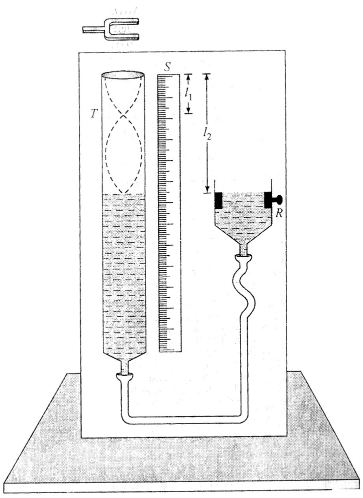

where r = radius of the organ pipe.

with end correction, the fundamental frequency of a closed pipe ($f_e$) and an open argon pipe ($f_0$) will be given by

$$f_e = \frac{v}{4(\ell + 0.6r)} \quad \text{and} \quad f_0 = \frac{v}{2(\ell + 1.2r)}$$

**9.4 Resonance Tube**

This an apparatus used to determine velocity of sound in air experimentally and also to compare frequncies of two tuning forks.



shown figure the setup of a resonance the experiment. There is a long tube T in which initially water is filled upto the top and the eaer level can be change by moving a reservoir R up and down.

A tunning fork of known frequency $n_0$ is struck gently on a rubber pad and brought near the open and tube T due to which oscillations are transferred to the air coloumn in the tube above water level. Now we gradually decrease the water level in the tube. This air column behaves like a closed organ pipe and the water level as closed end of pipe. As soon as water level reaches a position where there is a node of corresponding stationary wave, in air column, resonance takes place and maximum sound intensity is detected. Let at this position length of air column be $l_1$. If water level is further decreased, again maximum sound intensity is observed when water leel is at another node i.e. at a length $l_2$ as shown in figure. Here if we find two successive resonance lengths $l_1$ and $l_2$, we can get the wavelength of the wave as

$$l_2 - l_1 = \frac{\lambda}{2}$$

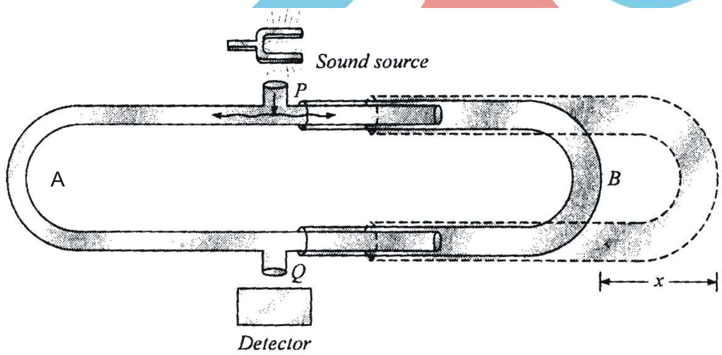
or $$\lambda = 2(l_2 - l_1)$$

Thus sound velocity in air can be given as

$$v = n_0\lambda = 2n_0(l_2 - l_1)$$

**9.5 Quink's Tube**

This is an apparatus used to demonstrate the phenomenon of interference and also used to measure velocity of sound in air. This is made up of two U-tubes A and B as shown in figure. Here the tube B can slide in and out from the tube A. There are two openings P and Q in the tube A. At opening P, a tuning fork or a sound source of known frequency $n_0$ is placed and at the other opening a detector is placed to detect the resultant sound of interference occurred due to superposition of two sound waves coming from the tubes A and B.




Initially tube B is adjusted so that detector detects a maximum. At this instant if length of paths covered by the two waves from P to Q from the side of A and side of B are $l_1$ are $l_2$ respectively then for constructive interference we must have

$$l_2 - l_2 = N\lambda \qquad ...(1)$$

If now tube B is further pulled out by a distance x so that next maximum is obtained and the length of path from the side of B is $l_2'$ then we have

$$l_2' = l_2 + 2x \qquad ...(2)$$

Where x is the displacement of the tube. For next constructive interference of sound at point Q, we have

$$l_2' - l_1 = (N+1)\lambda \qquad ..(3)$$

From equation (1), (2) and (3), we get

or $$x = \frac{\lambda}{2} \qquad ...(4)$$

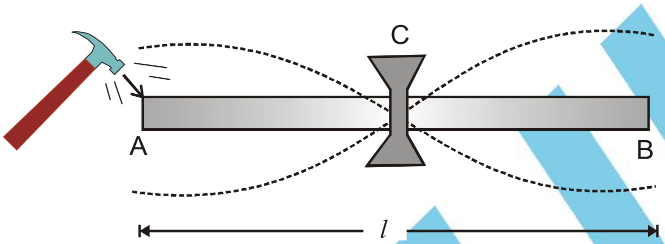
Thus by experiment we get the wavelength of sound as for two sucessive points of constructive interference, the path difference must be $\lambda$. As the tube B is pulled out by x, this introduces a path difference 2x in the path of sound wave through tube B. If the frequency of the source is known, $n_0$, the velocity of sound in the air filled in tube can be given as

$$v = n_0\lambda = 2n_0x \qquad ...(5)$$

### 9.6 Vibrations of Clamped Rod

We have discussed the resonant vibrations of a string clamped at two ends. Now we discuss the ocillations of a rod clamped at a point on its length as shown in figure. Figure shows a rod AB clamped at its middle point. If we gently hit the rod at its one end, it begin to oscillate and in the natural oscillations the rod vibrates at its lowest frequency and maximum wavelength, which we call fundamental mode of oscillations. With maximum wavelength when transverse stationary waves setup in the rod, the free ends vibrates as antinodes and the clamped end a node as shown in figure. Here if $\lambda$ be the wavelength of the wave, we have

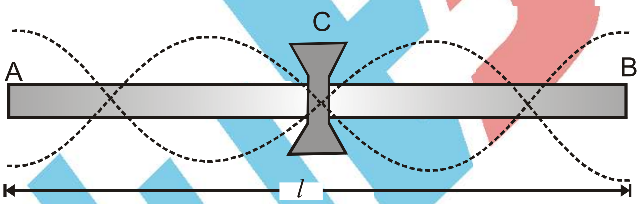
$$l = \frac{\lambda}{2} \quad \text{or} \quad \lambda = 2l$$




Thus the frequency of fundamental oscillations of a rod damped at mid point can be given as

$$n_0 = \frac{v}{\lambda} = \frac{l}{2l}\sqrt{\frac{Y}{\rho}} \qquad ...(1)$$

Where Y is the Young's modulus of the material of rod and $\rho$ is the density of the material of rod.
Next higher frequency at which rod vibrates will be then one when wave length is decreased to a value so that one node is inserted between mid point and an end of rod as shown in figure




In this case if $\lambda$ be the wavelength of the waves in rod, we have

$$l = \frac{3\lambda}{2}$$

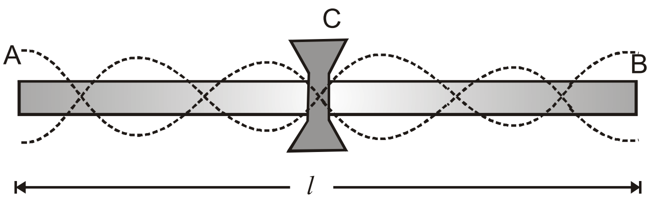
or $$\lambda = \frac{2l}{3} \qquad ...(2)$$

Thus in this case the oscillation frequency of rod can be given as

$$n_1 = \frac{v}{\lambda} = \frac{3}{2l}\sqrt{\frac{Y}{\rho}} = 3n_0 \qquad ...(3)$$

This is called first overtone frequency of the damped rod or thid harmonic frequency. Similarly, the next higher freqnecy of oscillation i.e. second overtone of the oscillating rod can be shown in figure shown. Here is $\lambda$ be the wavelength of the wave then it can be given as

$$l = \frac{5\lambda}{2} \quad \text{or} \quad \lambda = \frac{2l}{5} \qquad ...(4)$$




Thus the frequency of oscillation of rod can be given as

$$n_2 = \frac{v}{\lambda} = \frac{5}{2l}\sqrt{\frac{Y}{\rho}} = 5n_0 \qquad ..(5)$$

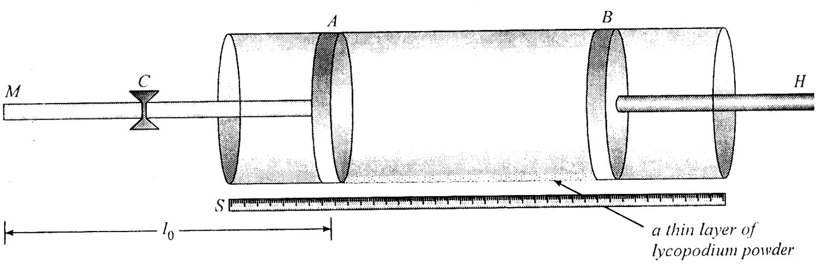
Thus the second overtone frequency is the fifth harmonic of the fundamental oscillation frequency of rod. We can also see from the above analysis that the resonant freqencies at which stationary waves are setup in a damped rod are only odd harmonics of fundamental frequency.
Thus when an external source of frequency matching with any of the harmonic of the damped rod then stationary waves are setup in the rod.

### 9.7 Natural Oscillation of Organ Pipes

When we initiate some oscillations in an organ pipe, which harmonics are excited in the pipe depends on how initial disturbance is produced in it. For example, if you gently blow across the top of an organ pipe it resonates softly at its fundamental frequency. But if you blow must harder you hear the higher pitch of an overtone because the faster airsteam higher frequencies in the exciting disturbance. This sound effect can also be achieved by increasing the air pressure to an organ pipe.

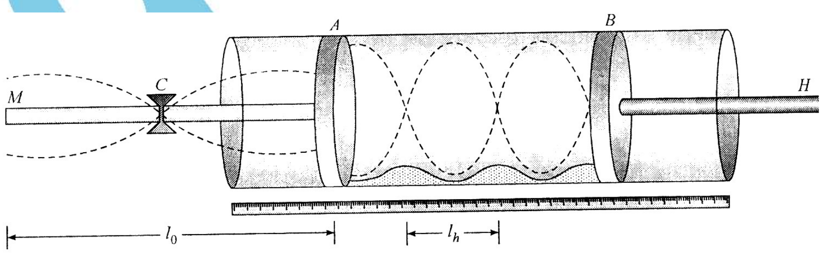
### 9.8 Kundt's Tube

This is an apparatus used to find velocity of sound in a gaseous medium or in different materials. It consists of a glass tube as shown in figure. one end of which a piston B is fitted which is attached to a wooden handle H and can be moved inside and outside the tube and fixed, the rod M of the required material is fixed at clamp C in which the velocity of sound is required, at one end of rod a disc A is fixed as shown.




In tube air is filled at room temperature and a thin layer of lycopodium powder is put along the length of the tube. It is a very fine powder particles of which can be displaced by the air particles also.
When rod M is gently rubbed with a resin cloth or hit gently, it starts oscillating in fundamental mode as shown in figure, frequency of which can be given as

$$n_{rod} = \frac{v}{\lambda} = \frac{1}{2l_0}\sqrt{\frac{Y}{\rho}} \qquad [\text{As } \lambda_0 = \frac{\lambda}{2}]$$




## 10. BEATS

When two sources of sound that have almost the same frequency are sounded together, an interesting phenomenon occurs. A sound with a frequency average of the two is heard and the loudness of sound repeatedly grows and then decays, rather than being constant. Such a repeated variation in amplitude of sound are called 'beats".
If the frequency of one of the source is changed, there is a corresponding change in the rate at which the amplitude varies. This rate is called beat fequency. As the frequencies come close together, the beat frequency becomes slower. A musician can tune a guitar to another source by listening for the beats while increasing or decreasing the tension in each string, eventually the beat frequency becomes very low so that effectively no beats are heard, and the two sources are then in tune.

We can also expalin the phenomenon of beat mathematically. Let us consider the two superposing waves have frequencies $n_1$ and $n_2$ then their respective equations of oscillation are

$$y_1 = A \sin 2\pi n_1 t \qquad ...(1)$$

and $$y_2 = A \sin 2\pi n_2 t \qquad ...(2)$$

On superposition at a point, the displacement of the medium particle is given as

$$y = y_1 + y_2$$

$$y = A \sin 2\pi n_1 t + A \sin 2\pi n_2 t$$

$$y = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t \sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t \qquad ...(3)$$

$$y = R\sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t \qquad ...(4)$$

There equation (4) gives the displacement of medium particle where susperposition takes place, it shows that the particle executes SHM with frequency $\frac{n_1 + n_2}{2}$, average of the two superposing frequencies and with amplitude R which varies with time, given as

$$R = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t \qquad ...(5)$$

Here R becomes maximum when

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = \pm 1$$

or $$2\pi\left(\frac{n_1 - n_2}{2}\right)t = N\pi \qquad [N \in I]$$

or $$t = \frac{N}{n_1 - n_2}$$

or at time $$t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, .........$$

At all the above time instants the sound of maximum loudness is heard, similarly we can find the time instans when the loudness of sound is minimum, it occurs when

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = 0$$

or $$2\pi\left(\frac{n_1 - n_2}{2}\right)t = (2N+1)\frac{\pi}{2} \qquad [N \in I]$$

or $$t = \frac{2N+1}{2(n_1 + n_2)}$$

or at time instants $$t = \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, ........$$

Here we can see that these time instants are exactly lying in the middle of the instans when loudest sound is heard. Thus on superposition of the above two frequencies at a medium particle, the sound will be increasing, decreasing, again increasing and decreasing and so on. This effect is called beats. Here the time between two successive maximum or minimum sounds is called beat period, which is given as

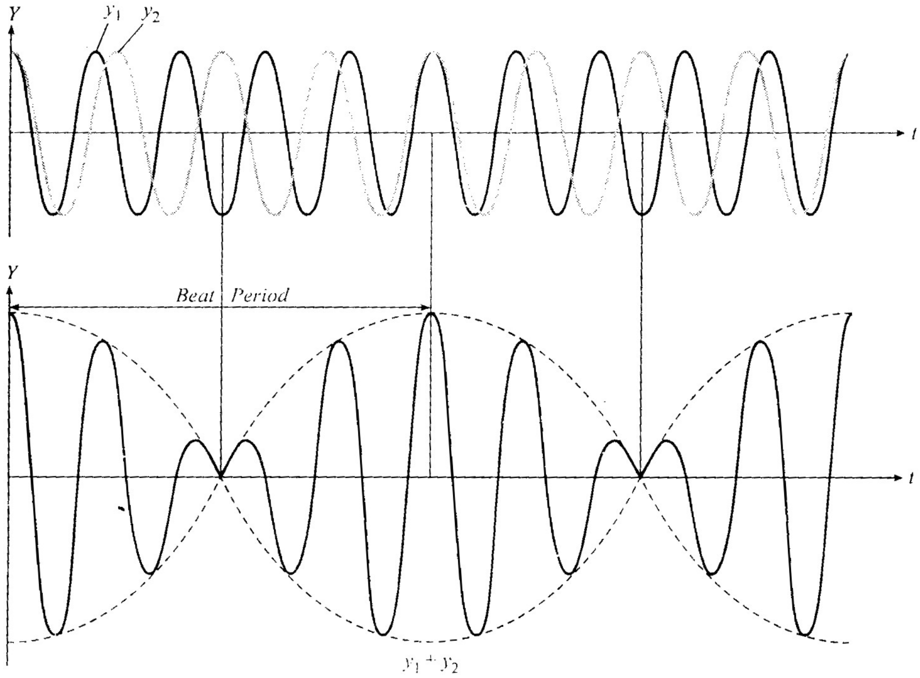
Beat Period $T_B$ = time between two successive maxima = time between two successive minima

$$= \frac{1}{n_1 - n_2}$$

Thus beat frequency or number of beats heard per second can be given as

$$f_B = \frac{1}{T_B} = n_1 - n_2$$

The superposition of two waves of slightly different frequencies is graphically shown in figure. The resulting envelope of the wave formed after superposition is also shown in figure (b). Such a wave when propagates, produces "beat" effect at the medium particles.




### 10.1 Echo

The repetition of sound produced due to reflection by a distant extended surface like a different, hill well, building etc. is called an echo. The effect of sound on human ear remains for approximately one tenth of a second. If the sound is reflected back in a time less then 1/10 of a second, no echo is heard. Hence human ears are not able to distinguish a beat frequency of 10 Hz or more than 10 Hz.

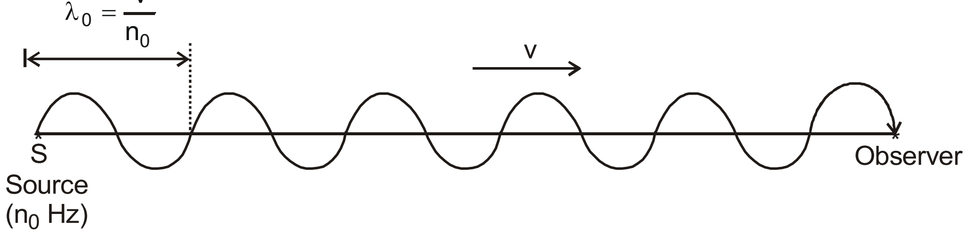
## 11. DOPPLER'S EFFECT

When a car at rest on a road sounds its high frequency horn and you are also standing on the road near by, you'll hear the sound of same frequency it is sounding but when the car approaches you with its horn sounding, the pich (frequency) of its sound seems to drop as the car passes. This phenomenon was first described by an Austrain Scientist Christien Doppler, is called the Doppler effect, He explained that when a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. Lets discuss the Doppler effect in detail for different cases.

### 11.1 Stationary Source and Stationary Observer

Figure shown a stationary sources of frequency $n_0$ which produces sound waves in air of wavelength $\lambda_0$ given as

$$\lambda_0 = \frac{v}{n_0} \qquad \text{[v = speed of sound in air]}$$



Although sound waves are longitudinal, here we represent sound weaves by the transverse displacement curve as shown in figure to understand the concept in a better way. As source produces waves, these waves travel towards, stationary observer O in the medium (air) with speed v and wavelength $\lambda_0$. As observer is at rest here it will observe the same wavelength $\lambda_0$ is approaching it with speed v so it will listen the frequency n given as

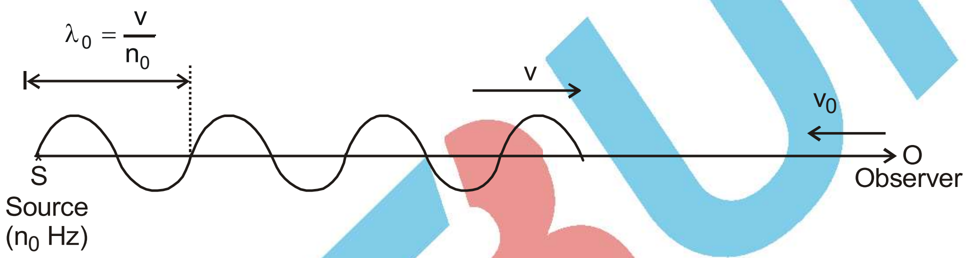
$$n = \frac{v}{\lambda_0} = n_0 \qquad \text{[same as that of source]} \qquad ...(1)$$

This is why when a stationary observer listends the sound from a stationary source of sound, it detects the same frequency sound which the source is producting. Thus no Doppler effect takes place if there is no relative motion between source and observer.

### 11.2 Stationary Source and Moving Observer

Figure shown the case when a stationary sources of frequency $n_0$ produces sound waves which have wavelength in air given as

$$\lambda_0 = \frac{v}{n_0}$$




These waves travel toward moving observer with velocity $v_0$ towards, the source. When sound waves approach observer, it will receive the waves of wavelength $\lambda_0$ with speed v + $v_0$ (relative speed). Thus the frequency of sound heard by observer can be given as

Apparent frequency $\qquad n_{ap} = \frac{v + v_0}{\lambda_0}$

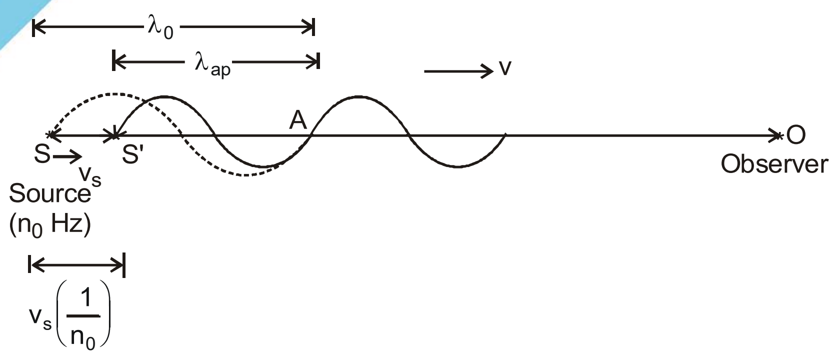
$$= \frac{v + v_0}{\left(\frac{v}{n_0}\right)} = n_0\left(\frac{v + v_0}{v}\right) \qquad ...(2)$$

Similarly we can say that if the obsever is receding away from the source the apparent frequency heard by the observer will be given as

$$n_{ap} = n_0\left(\frac{v - v_0}{v}\right) \qquad ...(3)$$

### 11.3 Moving Source and Stationary Observer

Figure shows the situation when a moving source S of frequency $n_0$ produces sound waves in medium (air) and the waves travel toward observer with velocity v.



Here if we carefully look at the initial situation when source starts moving with velocity $v_s$ as well as it starts productiong waves. The period of one oscillation is $\left(\frac{1}{n_0}\right)$ sec and in this duration source emits one wavelength $\lambda_0$ in the direction of propagation of waves with speed v, but in this duratin the source will also move forward by a distance $v_s\left(\frac{1}{n_0}\right)$. Thus the effective wavelength of emitted sound in air is slightly compressed by this distance as shown in figure. This is termed as apparent wavelength of sound in medium (air) by the moving source. This is given as

Apparent wavelength $\lambda_{ap} = \lambda_0 - v_s\left(\frac{1}{n_0}\right)$ ...(1)

$$= \frac{\lambda_0 n_0 - v_s}{n_0} = \frac{v - v_s}{n_0}$$

Now this wavelength will approach observer with speed v ( O is at rest). Thus the frequency of sound heard by observer can be given as

Apparent frequecy $n_{ap} = \frac{v}{\lambda_{ap}}$

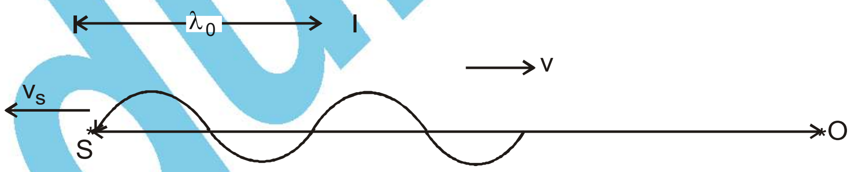
$$= \frac{v}{(v - v_s)/n_0} = n_0\left(\frac{v}{v - v_s}\right) \quad ...(2)$$

Similarly if source is receding away from observer, the apparent wavelength emitted by source in air toward observer will be slightly expanded and the apparent frequency heard by the stationary observer can be given as

$$n_{ap} = n_0\left(\frac{v}{v + v_s}\right) \quad ...(3)$$

**11.4 Moving Source and Moving Observer**

Let us consider the situation when both source and observer are moving in same direction as shown in figure at speeds $v_s$ and $v_0$ respectively.




In this case the apparent wavelength emitted by the source behind it is given as

$$\lambda_{ap} = \frac{v + v_s}{n_0}$$

Now this wavelength will approach the observer at relative speed v + $v_0$ thus the apparent frequency of sound heard by the observer is given as

$$n_{ap} = \frac{v + v_0}{\lambda_{ap}} = n_0\left(\frac{v + v_0}{v + v_s}\right) \quad ...(1)$$

By looking at the expression of apparent frequency given by equation, we can easily develop a general relation for finding the apparent frequency heard by a moving observer due to a moving source as

$$n_{ap} = n_0\left[\frac{v \pm v_0}{v \mp v_s}\right] \quad ...(2)$$

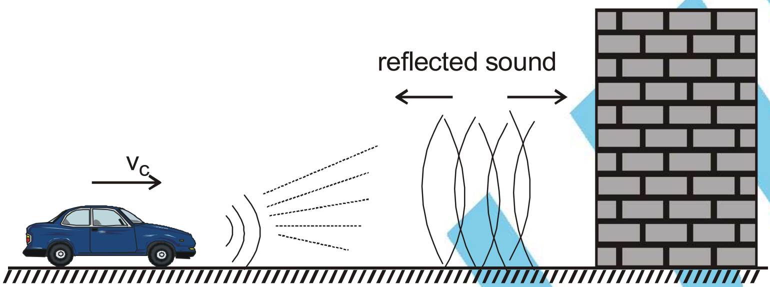
Here + and – signs are chosen according to the direction of motion of source and observer. The sign covention related to the motion direction can be stated as :

(i) For both source and observer $v_0$ and $v_s$ are taken in equation with –ve sign if they are moving in the direction of $\vec{v}$ i.e. the direction of propagation of sound from source to observer.

(ii) For both source and observer $v_0$ and $v_s$ are taken in equation (2) with +ve sign if they are moving in the direction opposite to $\vec{v}$ i.e. opposite to the direction of propagation of sound from source to observer.

### 11.5 Doppler Effect in Reflected Sound

When a car is moving toward a stationary wall as shown in figure. If the car sounds a horn, wave travels towards the wall and is reflected from the wall. When the reflected wave is heard by the driver, it appears to be of relatively high pitch. If we wish to measure the frequency of reflected sound then the problem must be handled in two steps.




First we treat the stationary wall as stationary observer and car as a moving source of sound of frequency $n_0$. In this case the frequency received by the wall is given as

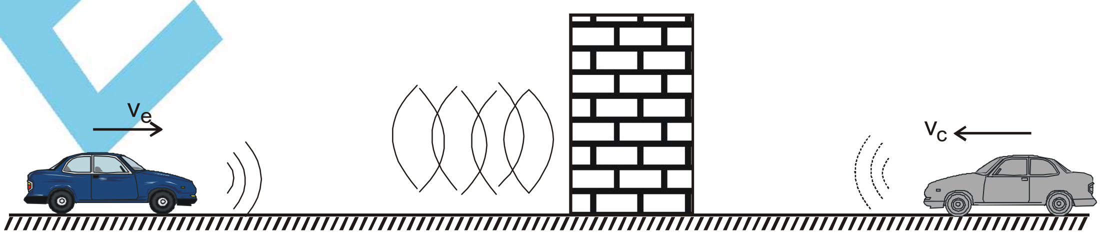
$$n_1 = n_0\left(\frac{v}{v - v_c}\right) \qquad ...(1)$$

Now wall reflects this frequency and behaves like a stationary source of sound of frequency $n_1$ and car (driver) behave like a moving observer with velocity $v_c$. Here the apparent frequency heard by the car driver can be given as

$$n_{ap} = n_1\left(\frac{v + v_c}{v}\right)$$

$$= n_0\left(\frac{v}{v - v_c}\right) \times \left(\frac{v + v_c}{v}\right) = n_0\left(\frac{v + v_c}{v - v_c}\right) \qquad ...(2)$$

Same problem can also be solved in a different manner by using method of sound images. In this procedure we assume the image of the sound source behind the reflector. In previous example we can explain this by situation shown in figure.




Here we assume that the sound which is reflected by the stationary wall is coming from the image of car which is at the back of it and coming toward it with velocity $v_c$. Now the frequency of sound heard by car driver can directly be given as

$$n_{ap} = n_0\left[\frac{v + v_c}{v - v_c}\right] \qquad ...(3)$$

This method of images for solving problems of Doppler effect is very convenient but is used only for velocities of source and observer which are very small compared to the speed of sound and it should not be used frequenctly when the reflector of sound is moving.

### 11.6 Doppler's Effect for Accelerated Motion

For the case of a moving source and a moving observer, we known the apparent frequency observer can be given as

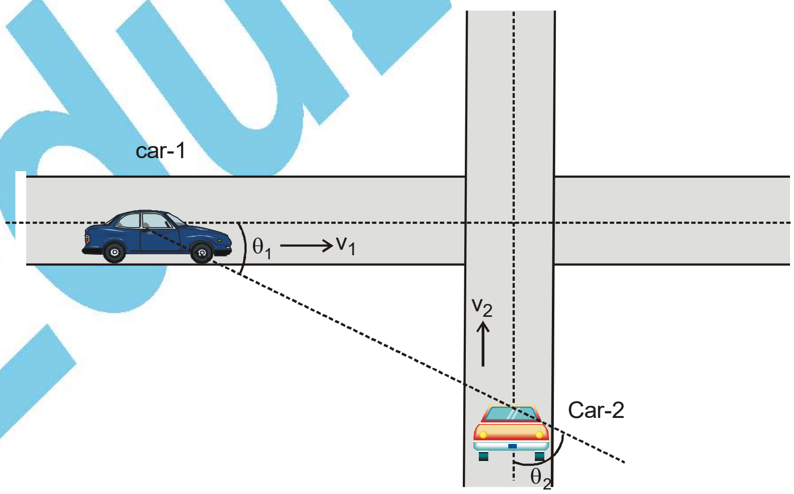
$$n_{ap} = n_0 \left[ \frac{v \pm v_0}{v \mp v_s} \right] \qquad ...(4)$$

Here v is the velocity of sound and $v_0$ and $v_s$ are the velocity of observer and source respectively.
When a source of observer has accelerated or ratarded motion then in equation (4) we use that value of $v_0$ at which observer receies the sound and for source, we use that value of $v_s$ at which it has emitted the wave.
The alternative method of solving this case is by the traditional method of compressing or expending wavelength of sound by motion of source and using relative velocity of sound with respect to observer

### 11.7 Doppler's Effect when Source and Observer are not in Same Line of Motion

Consider the situation shown in figure. Two cars 1 and 2 are moving along perpendicular roads at speed $v_1$ and $v_2$. When car - 1 sound a horn of frequency $n_0$, it emits sound in all directions and say car - 2 is at the position, shown in figure. when it receives the sound. In such cases we use velocity components of the cars along the line joining the source and observer thus the apparent frequency of sound heard by car-2 can be given as

$$n_{ap} = n_0 \left[ \frac{v + v_2 \cos\theta_2}{v - v_1 \cos\theta_1} \right] \qquad ...(6.266)$$



# SOLVED EXAMPLE

***Ex.1 On an average a human heart is found to beat 75 times in a minute. Calculate its frequency and period.***

**Ans.** The beat frequency of heart = 75/(1 min)

= 75/(60 s) = 1.25 $s^{-1}$ = 1.25 Hz

The time period T = 1/(1.25 s–1) = 0.8 s

***Ex.2 Which of the following functions of time represent (a) periodic and (b) non-periodic motion? Give the period for each case of periodic motion [ω is any positive constant].***

***(i) sin ωt + cos ωt***

***(ii) sin ωt + cos 2 ωt + sin 4 ωt***

***(iii) $e^{-\omega t}$***

***(iv) log (ωt)***

**Ans.** (i) sin ωt + cos ωt is a periodic function, it can also be written as 2 sin (ωt + π/4).

Now $\sqrt{2}$ sin (ωt + π/4)= $\sqrt{2}$ sin (ωt + π/4 + 2π)

= $\sqrt{2}$ sin [ω(t + 2π/ω) + π/4]

The periodic time of the function is 2π/ω. (ii) This is an example of a periodic motion. It can be noted that each term represents a periodic function with a different angular frequency. Since period is the least interval of time after which a function repeats its value, sin ωt has a period $T_0$ = 2π/ω; cos 2ωt has a period π/ω =$T_0$/2; and sin 4ωt has a period 2π/4ω = $T_0$/4. The period of the first term is a multiple of the periods of the last two terms. Therefore, the smallest interval of time after which the sum of the three terms repeats is $T_0$, and thus the sum is a periodic function with a period 2π/ω. (iii) The function $e^{-\omega t}$ is not periodic, it decreases monotonically with increasing time and tends to zero as tπω and thus, never repeats its value. (iv) The function log(ωt) increases monotonically with time t. It, therefore, never repeats its value and is a non-periodic function. It may be noted that as log(ωt) diverges to ?. It, therefore, cannot represent any kind of physical displacement.

***Ex.3 Which of the following functions of time represent (a) simple harmonic motion and (b) periodic but not simple harmonic? Give the period for each case.***

***(1) sin ωt – cos ωt (2) sin2 ωt***

**Ans.**

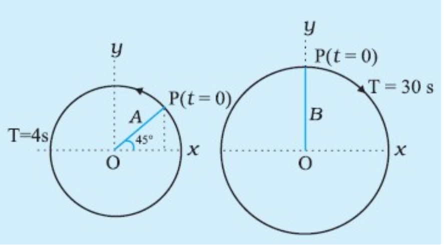
(a) sin ωt – cos ωt = sin ωt – sin (π/2 – ωt)

= $\sqrt{2}$ cos (π/4) sin (ωt – π/4) = 2 sin (ωt – π/4)

This function represents a simple harmonic motion having a period T = 2π/ω and a phase angle (–π/4) or (7π/4) (b) $\sin^2$ ωt = ½ – ½ $\cos^2$ ωt

The function is periodic having a period T = π/ω. It also represents a harmonic motion with the point of equilibrium occurring at ½ instead of zero.

***Ex.4 Figure depicts two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated on the figures. Obtain the simple harmonic motions of the x-projection of the radius vector of the rotating particle P in each case.***




**Ans.**

(a) At t = 0, OP makes an angle of 45º = π∈/4 rad with the (positive direction of ) x-axis. After time t, it covers an angle t T 2π∈ in the anticlockwise sense,

and makes an angle of $\frac{2\pi}{T}$ t + π/4 with the x-axis.

The projection of OP on the x-axis at time t

is given by, (t) = A cos ($\frac{2\pi}{T}$t + π/4) for T = 4s, x(t)

= A cos (2π/4 + π/4)

which is a SHM of amplitude A, period 4 s, and an initial phase = $\pi/4$

(b) In this case at t = 0, OP makes an angle of with the x-axis. After a time t, it covers an angle of $90^o = 2\pi/2$ in the clockwise sense and makes an angle of $\frac{2\pi}{T}$ t with the x-axis. The projection of OP on the x-axis at time t is given by x(t) = B cos 2

Writing this as x (t ) = B cos $\left(-\frac{\pi}{15}t - \frac{\pi}{2}\right)$

$x(t) = A \cos(\omega t + 4)$

comparing with. We find that this represents a SHM of amplitude B, period 30 s, and an initial phase of $-\frac{\pi}{2}$

***Ex.5 A body oscillates with SHM according to the equation (in SI units), x = 5 cos [2π∈t + π∈/4]. At t = 1.5 s, calculate the (a) displacement, (b) speed and (c) acceleration of the body.***

**Ans.** The angular frequency $\omega$ of the body

= $2\pi\in s^{-1}$ and its time period T = 1 s.

At t = 1.5 s

(a) displacement

= (5.0 m) cos [$(2\pi\in s^{-1}) \times 1.5$ s + $\pi\in/4$]

= (5.0 m) cos [$(3\pi + \pi/4)$]

= –5.0 × 0.707 m

= –3.535 m

(b) Using Eq. (14.9), the speed of the body

= – (5.0 m)($2\pi\ s^{-1}$) sin [($2\pi\ s^{-1}$) ×1.5 s + $\pi/4$]

= – (5.0 m)($2\pi\ s^{-1}$) sin [($3\pi + \pi/4$)]

= $10\pi \times 0.707\ m\ s^{-1}$

= $22\ m\ s^{-1}$

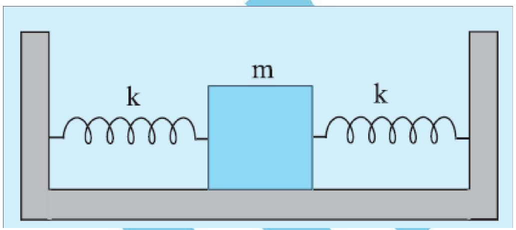
(c) Using Eq., v(t) = $\frac{d}{dt}$ x(t) the acceleration of the body

= $-(2\pi\ s^{-1})^2 \times$ displacement

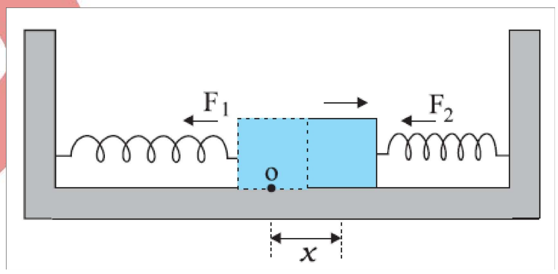
= $-(2\pi\ s^{-1})^2 \times (-3.535\ m)$

= $140\ m\ s^{-2}$

***Ex.6 Two identical springs of spring constant k are attached to a blockof mass m and to fixed supports as shown in Fig. Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations.***




**Ans.** Let the mass be displaced by a small distance x to the right side of the equilibrium position, as shown in Fig. Under thissituation the spring on the left side gets




elongated by a length equal to x and that on the right side gets compressed by the same length. The forces acting on the mass are then, $F_1$ = –k x (force exerted by the spring onthe left side, trying to pull the mass towards the mean position)

$F_2$ = –k x (force exerted by the spring on the right side, trying to push the mass towards the mean position)

The net force, F, acting on the mass is then given by,

F = –2kx

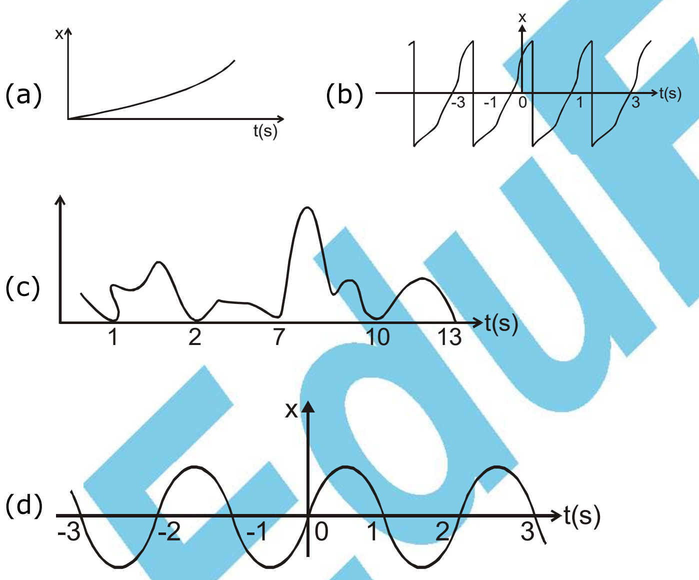
Hence the force acting on the mass is proportional to the displacement and is directed towards the mean position; therefore, the motion executed by the mass is simple harmonic. The time period of oscillations is,

$T = \sqrt{\frac{m}{2k}}$

## Exercise - I UNSOLVED PROBLEMS

**Q.1.** Which of the following examples represent periodic motion ?
(a) a swimmer completion one (return) trip from one bank of a river to the other and back.
(b) a freely suspended bar magnet displaced from its N-S direction and released.
(c) a hydrogen molecule rotating about its center of mass.
(d) an arrow released from a bow.

**Q.2.** Which of the following examples represent (nearly) simply harmonic motion and which represent periodic but not simple harmonic motion?
(a) the rotation of earth about its axis.
(b) motion of an oscillating mercury column in a U-Tube.
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.
(d) general vibrations of a poly atomic molecule about its equilibrium position




**Q.3.** Figure 14,27 depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion )?

**Q.4.** Which of the following function of time represent (a) harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion ? Give period for each case of periodic mouton :($\omega$ is any positive constant).
(a) $\sin\omega t - \cos\omega t$ (b)$\sin^3 \omega t$
(c)$3\cos(\pi/4 - 2\omega t)$
(d) $\cos\omega t + \cos 3\omega t + \cos 5\omega t$
(e)$\exp(-\omega^2 t^2)$ (f) $1 + \omega t + \omega^2 t^2$

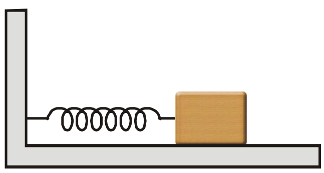
**Q.5.** A particle is inlinear simple harmonic motin between tow points, A and B, 10 cm apart. Take the directin from A to B as the positive direction and give the signs of velocity, accelerating and force on the particle when it is
(a) at the end A (b) at the ent B,
(c) at the mid-point of AB going towards A.
(d) at 2 cm away from b going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from A going towards A.

**Q.6.** Which of the following relationship between the acceleration a and the displacement x of a particle involve harmonic motion.
(a) $\alpha = 0.7x$ (b) $\alpha = 200x$
(C) $\alpha = 10x$ (d) $\alpha = 100\ x^2$

**Q.7.** The motion of a particle execution simple harmonic motion is described by the displacement function.
$x(t) = A\cos(\omega t + \theta)$
If the initial ( t =0) position of the particle is 1 cm and its initial velocity is $\omega$ cm/s, what are its amplitude and initial phase angle ? The angular frequency of the particle is $\pi\ s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B\sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

**Q.8.** A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released oscillates with a period of 0.6 s. What is the weight of the body?

**Q.9.** A spring having with a spring constant 1200 N $m^{-1}$ is mounted on a horizontal table as shown in fig. 14.28. A mass of 3 kg is attached to the free end of the spring. The masses then pulled sideways to a distance of 2.0 cm and released.
Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass and (iii) the maximum speed of the mass.



**Q.10.** In Exercise 14.9, let us take the position of mass when the spring is un stretched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the

oscillating mass if at the moment we start the stopwatch (t = 0 ), the mass is
(a) at the mean position
(b) at the maximum stretched position, and
(c) at the maximum compressed position.

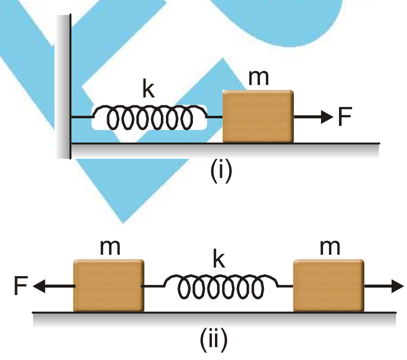
In what way do these function for SHM differ from each other, in frequency, in amplitude or the initial phase?

**Q.11.** Figures 14.29 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution (i.e. clockwise or anti- clockwise) are indicated on each figure.

Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

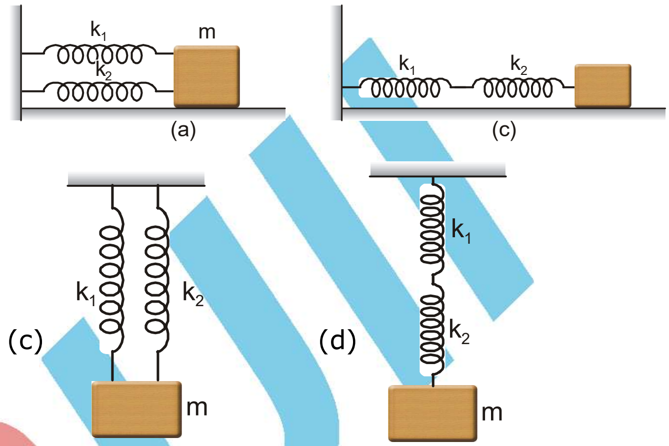
**Q.12.** Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ( t = 0 ) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).
(a) $x = -2 \sin (3t + \pi / 3)$
(b) $x = \cos (\pi / 6 - t)$
(c) $x = 3 \sin (2\pi t + \pi / 3)$
(d) $x = 2 \cos \pi t$

**Q.13** Figure (i) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force **F** applied at the free end stretches the spring. Figure (ii) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in figure (ii) is stretched by the same force **F**.

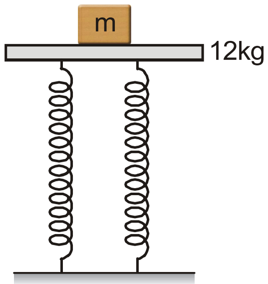



(a) What is the maximum extension of the spring in the two cases?
(b) If the mass in figure (i) and the two masses in Figure (ii) are released free, what is the period of oscillation in each case?

**Q.14.** Figure 14.31 shows four different spring arrangement. If the mass in each arrangement is displaced form its equilitbrium position and released , what is the resulting frequency of vibration in each case? Neglect the mass of the spring. [Figs. (a) and (b) represent an arrangement of springs in parallel, and (c) and (d) represent 'springs in series' ]




**Q.15.** A tray or mass 12 kg. is supported by two indentical springs as shown in Fig. 14.32. When the tray is pressed down pressed down slightly and then released, it executes SHM with a time period of 1.5 s. What is the spring constant of each spring ? When a block of mass m is placed on the tray, the period of SHM changes to 3.0 s. What is the mass of the block




**Q.16.** The piston in the cylinder head of a locomotive has a stroke ( twice the amplitude of 1.0 m. If the piston moves with simple harmonic motion with angular frequency of 200 rev/ min, what its maximum speed?

**Q.17.** The acceleration due to gravity on the surface of moon is 1.7 m $s^{-2}$. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s ? (g on the surface of earth is 9.8 m $s^{-2}$ )

**Q.18.** Answer the following questions:
(a)Time period of a particle in SHM depends on the force constant k and mass m of the pariticle:

$T = 2\pi\sqrt{\frac{m}{k}}$. A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motin of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that $T$ is greater than $2\pi\sqrt{\frac{l}{g}}$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

(d) What is the frequency of osculation of a simple pendulum mounted in a cabin that is freely falling under gravity?

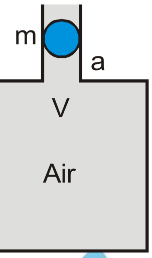
**Q.19.** A simple pendulum of lenght $l$ and having a bob of mass $m$ is suspended in a car. The car is moving on a circular track of radius $R$ with a uniform speed $\upsilon$. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period ?

**Q.20.** A cylindrical piece of cork of base area $A$ and height $h$ floats in a liquid of density $\rho$ the cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$ where $\rho$ is the density of cork. ( Ignore damping due to viscosity of the liquid).

**Q.21.** A trolley of mass 3.0 kg. as shown in Fig. 14.33, is connected to two springs, each of spring constant 600 n $m^{-1}$. If the trolley is displaced from its euilibrium position by 5.0 cm and released, what is (a) the period of ensuing oscillations, and (b) the maximum sped of the trolley ? How much energy is dissipated as heat by the time the trolley comes to rest due to damping forces ?

**Q.22.** One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two column. Show that, when the suctinpump is removed, the column of mercury in the U-tube executes simple harmonic motion .

**Q.23** An air chamber of volume V has a neck area of cross section a into which a ball of mass m can move up and down without any friction (figure). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal (see figure).




**Q.24.** You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The amplitude of oscillation 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation . Estimate the values of (a) the spring constant $k$ and (b) the damping constant $b$ for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

**Q.25.** A 1500 kg car carrying four 75 kg people move over a 'washboard' dirt road with corrugating 4.0 m apart, which cause the car to bounce on its spring suspension. The car bounces with maximum amplitude when its speed is 20 km $h^{-1}$ . The car now stops and the four people get out. By how much the car body rise on its suspension owing to this decrease in mass

**Q.26.** Show that for a particle inlinear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

**Q.27.** A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. the period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constat $\alpha$ is defined by the relation $j = -\alpha\,\theta$, where $J$ is the restoring couple and $\theta$ the angle of twist).