CONIC SECTIONS

CIRCLE

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point is always constant. The fixed point is called the centre and constant distance is called the radius of the circle.

1. STANDARD FORMS OF EQUATION OF A CIRCLE

2.1 General Equation of a Circle –

The general equation of a circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$, Where g, f, c are constants.

(i) Centre of a general equation of a circle is $(-g, -f)$ i.e. $(-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of y)

(ii) Radius of a general equation of a circle is

$$\sqrt{g^2 + f^2 - c}$$

NOTE :

(i) The general equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle if $a = b \neq 0$ and $h = 0$.

(ii) Locus of a point P represent a circle if its distance from two points A and B is not equal i.e. $PA = kPB$ represent a circle if $k \neq 1$

(ii) General equation of a circle represents –

(a) A real circle if $g^2 + f^2 - c > 0$

(b) A point circle if $g^2 + f^2 - c = 0$

(c) An imaginary circle if $g^2 + f^2 - c < 0$

(iv) In General equation of a circle –

(a) If $c = 0 \Rightarrow$ The circle passes through origin

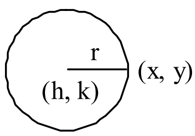
(b) If $f = 0 \Rightarrow$ The centre is on x – axis

(c) If $g = 0 \Rightarrow$ The centre is on y– axis

Central Form of Equation of a circle -

The equation of a circle having centre (h, k) and radius r is

$(x - h)^2 + (y - k)^2 = r^2$



NOTE :

(i) If the centre is origin, then the equation of the circle is $x^2 + y^2 = r^2$

(ii) If r = 0 than circle is called point circle and its equation is

$(x - h)^2 + (y - k)^2 = 0$

Diametral Form – If (x_1, y_1) and (x_2, y_2) be the extremities of a diameter, then the equation of circle is $(x - x_1)$ $(x - x_2) + (y - y_1)(y - y_2) = 0$

2. EQUATION OF A CIRCLE IN SOME SPECIAL CASES

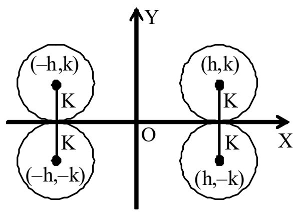
(i) If centre of circle is (h, k) and passes through origin then its equation is

$(x - h)^2 + (y - k)^2 = h^2 + k^2$

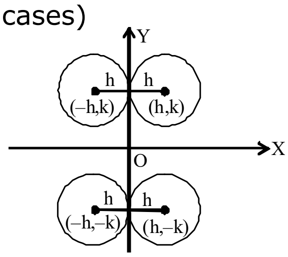
$\Rightarrow x^2 + y^2 - 2hx - 2ky = 0$

(iii) If the circle touches x axis then its equation is (Four cases)

$(x \pm h)^2 + (y \pm k)^2 = k^2$

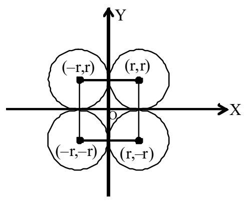


(iii) If the circle touches y axis then its equation is (Four cases)



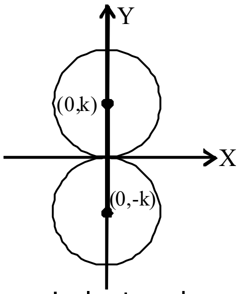
$(x \pm h)^2 + (y \pm k)^2 = h^2$

(iv) If the circle touches both the axis then its equation is (Four cases) $(x \pm r)^2 + (y \pm r)^2 = r^2$



(v) If the circle touches x axis at origin (Two cases) $x^2 + (y \pm k)^2 = k^2$

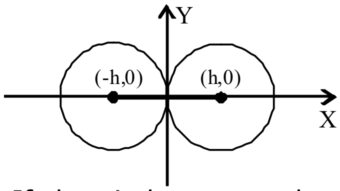
$\Rightarrow \quad x^2 + y^2 \pm 2ky = 0$



(vi) If the circle touches y axis at origin (Two cases)

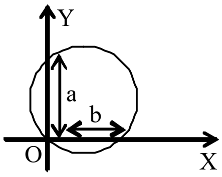
$(x \pm h)^2 + y^2 = h^2$

$\Rightarrow x^2 + y^2 \pm 2xh = 0$



(vii) If the circle passes through origin and cut intercept of a and b on axes, the equation of circle is (Four cases)

$x^2 + y^2 - ax - by = 0$ and centre is (a/2, b/2)



3. POSITION OF A POINT WITH RESPECT TO A CIRCLE

A point (x_1, y_1) lies outside, on or inside a circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ according as $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ is positive, zero or negative i.e.

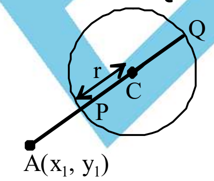
$S_1 > 0 \Rightarrow$ Point is outside the circle.

$S_1 = 0 \Rightarrow$ Point is on the circle.

$S_1 < 0 \Rightarrow$ Point is inside the circle.

4. The least and greatest distance of a point from a circle

Let S = 0 be a circle and A (x_1, y_1) be a point. If the diameter of the circle which is passing through the circle at P and Q. then



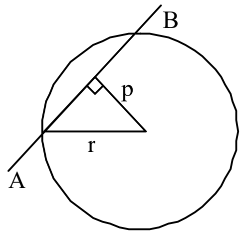
AP = AC – r = least distance

AQ = AC + r = greatest distance,

where 'r' is the radius and C is the centre of circle

5. LINE AND CIRCLE

Let L = 0 be a line and S = 0 be a circle, if 'r' be the radius of a circle and p be the length of perpendicular from the centre of circle on the line, then if



$p > r \Rightarrow$ Line is outside the circle

$p = r \Rightarrow$ Line touches the circle

$p < r \Rightarrow$ Line is the chord of circle

$p = 0 \Rightarrow$ Line is diameter of circle

NOTE :

(i) Length of the intercept made by the circle on the line is = $2\sqrt{r^2 - p^2}$

(ii) The length of the intercept made by line y = mx + c with the circle $x^2 + y^2 = a^2$ is

$$2\sqrt{\frac{a^2(1+m^2) - c^2}{1+m^2}}$$

PARABOLA

1. DEFINITION

A parabola is the locus of a point which moves in such a way that its distance from a fixed point is equal to its perpendicular distance from a fixed straight line.

1.1 Focus : The fixed point is called the focus of the Parabola.

1.2 Directrix : The fixed line is called the directrix of the Parabola.

2. TERMS RELATED TO PARABOLA

2.1 Eccentricity : If P be a point on the parabola and PM and PS are the distances from the directrix and focus S respectively then the ratio PS/PM is called the eccentricity of the Parabola which is denoted by e.

Note: By the definition for the parabola e = 1.

If $e > 1 \Rightarrow$ Hyperbola,

$e = 0 \Rightarrow$ circle,

$e < 1 \Rightarrow$ ellipse

2.2 Axis : A straight line passes through the focus and perpendicular to the directrix is called the axis of parabola.

2.3 Vertex : The point of intersection of a parabola and its axis is called the vertex of the Parabola.

NOTE: The vertex is the middle point of the focus and the point of intersection of axis and directrix

2.4 Focal Length (Focal distance) : The distance of any point P (x, y) on the parabola from the focus is called the focal length. i.e. The focal distance of P = the perpendicular distance of the point P from the directrix.

2.5 Double ordinate : The chord which is perpendicular to the axis of Parabola or parallel to Directrix is called double ordinate of the Parabola.

2.6 Focal chord : Any chord of the parabola passing through the focus is called Focal chord.

2.7 Latus Rectum : If a double ordinate passes through the focus of parabola then it is called as latus rectum.

2.7.1 Length of latus rectum :

The length of the latus rectum = 2 x perpendicular distance of focus from the directrix.

3. STANDARD FORM OF EQUATION OF PARABOLA

If we take vertex as the origin, axis as x- axis and distance between vertex and focus as 'a' then equation of the parabola in the simplest form will be-

$$y^2 = 4ax$$

3.1 Parameters of the Parabola $y^2 = 4ax$

(i) Vertex A $\Rightarrow$ (0, 0)

(ii) Focus S $\Rightarrow$ (a, 0)

(iii) Directrix $\Rightarrow$ $x + a = 0$

(iv) Axis $\Rightarrow$ $y = 0$ or x– axis

(v) Equation of Latus Rectum $\Rightarrow$ $x = a$

(vi) Length of L.R. $\Rightarrow$ 4a

(vii) Ends of L.R. $\Rightarrow$ (a, 2a), (a, – 2a)

(viii) The focal distance $\Rightarrow$ sum of abscissa of the point and distance between vertex and L.R.

(ix) If length of any double ordinate of parabola $y^2 = 4ax$ is 2ℓ then coordinates of end points of this Double ordinate are

$$\left(\frac{\ell^2}{4a}, \ell\right) \text{ and } \left(\frac{\ell^2}{4a}, -\ell\right).$$

4. REDUCTION TO STANDARD EQUATION

If the equation of a parabola is not in standard form and if it contains second degree term either in y or in x (but not in both) then it can be reduced into standard form. For this we change the given equation into the following forms-

$(y-k)^2 = 4a(x-h)$ or $(x-p)^2 = 4b(y-q)$

And then we compare from the following table for the results related to parabola.

5. GENERAL EQUATION OF A PARABOLA

If (h,k) be the locus of a parabola and the equation of directrix is $ax + by + c = 0$, then its equation is given by

$$\sqrt{(x-h)^2+(y-k)^2} = \frac{ax+by+c}{\sqrt{a^2+b^2}}$$ which gives

$(bx-ay)^2 + 2gx + 2fy + d = 0$ where g, f, d are the constants.

Note:

(i) It is a second degree equation in x and y and the terms of second degree forms a perfect square and it contains at least one linear term.

(ii) The general equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a parabola, if

(a) $h^2 = ab$

(b) $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$

6. EQUATION OF PARABOLA WHEN ITS VERTEX AND FOCUS ARE GIVEN

6.1 If both lie on either of the coordinate axis :

In this case first find distance 'a' between these points and taking vertex as the origin suppose the equation as $y^2 = 4ax$ or $x^2 = 4ay$. Then shift the origin to the vertex.

6.2 When both do not lie on any coordinate axes

In this case first find the coordinates of Z and equation of the directrix, then write the equation of the parabola by the definition.

ELLIPSE

1 DEFINITION

An ellipse is the locus of a point which moves in such a way that its distance form a fixed point is in constant ratio to its distance from a fixed line. The fixed point is called the **focus** and fixed line is called the **directrix** and the constant ratio is called the **eccentricity of a ellipse** denoted by (e).

In other word, we can say an ellipse is the locus of a point which moves in a plane so that the sum of it distances from fixed points is constant.

2. EQUATION OF AN ELLIPSE

2.1 Standard Form of the equation of ellipse

Let the distance between two fixed points S and S' be 2ae and let C be the mid point of SS′. Taking CS as x- axis, C as origin.

Let P(h,k) be the moving point Let SP+ SP′ = 2a (fixed distance) then

SP+S'P

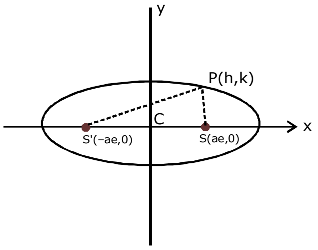
$= \sqrt{\{(h-ae)^2+k^2\}} + \sqrt{\{(h+ae)^2+k^2\}} = 2a$

$h^2(1- e^2) + k^2 = a^2(1- e^2)$

Hence Locus of P(h, k) is given by.

$x^2(1- e^2) + y^2 = a^2(1- e^2)$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$



Let us assume that $a^2(1- e^2) = b^2$

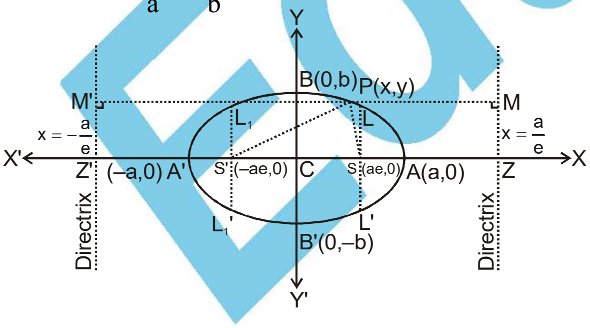
∴ The standard equation will be given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

2.1.1 Various parameter related with standard ellipse :

Let the equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b$$



(i) Vertices of an ellipse : The point of which ellipse cut the axis x-axis at A(a,0) & A′(– a, 0) and y- axis at B(0, b) &

B′(0, – b) is called the vertices of an ellipse.

(ii) Major & Minor axis : The straight line AA′ is called major axis and BB′ is called minor axis. The major and minor axis taken together are called the principal axes and its length will be given by

Length of major axis → 2a

Length of minor axis → 2b

(iii) Centre : The point which bisect each chord of an ellipse is called centre (0,0) denoted by 'C'.

(iv) Directrix : ZM and Z′ M′ are two directrix and their equation are x= a/e and x = – a/e.

(v) Focus : S (ae, 0) and S′ (–ae,0) are two foci of an ellipse.

(vi) Latus Rectum : Such chord which passes through either focus and perpendicular to the major axis is called its latus rectum.

Length of Latus Rectum :

If L is (ae,ℓ) then 2ℓ is the length of Latus Rectum.

Length of Latus rectum is given by $\frac{2b^2}{a}$.

(vii) Relation between constant a, b, and e

$b^2 = a^2(1- e^2) \Rightarrow e^2 = \frac{a^2-b^2}{a^2}$

$\therefore e = \frac{\sqrt{a^2-b^2}}{a^2}$

Result :

(a) Centre C is the point of intersection of the axes of an ellipse. Also C is the mid point of AA′.

(b) Another form of standard equation of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when a < b.

In this case major axis is BB′= 2b which is along y- axis and minor axis is AA′= 2a along x- axis. Focus S(0,be) and S′(0,–be) and directrix are y = b/e and y = –b/e.

2.2 General equation of the ellipse

The general equation of an ellipse whose focus is (h,k) and the directrix is the line ax + by + c = 0 and the eccentricity will be e. Then let $P(x_1,y_1)$ be any point on the ellipse which moves such that SP = ePM

$\Rightarrow (x_1-h)^2 + (y_1-k)^2$

$$= \frac{e^2(ax_1+by_1+c)^2}{a^2+b^2}$$

Hence the locus of (x_1,y_1) will be given by

$(a^2 + b^2) [(x - h)^2 + (y-k)^2]$

$= e^2(ax + by + c)^2$

Which is the equation of second degree from which we can say that any equation of second degree represent equation of an ellipse.

Note : Condition for second degree in X & Y to represent an ellipse is that if

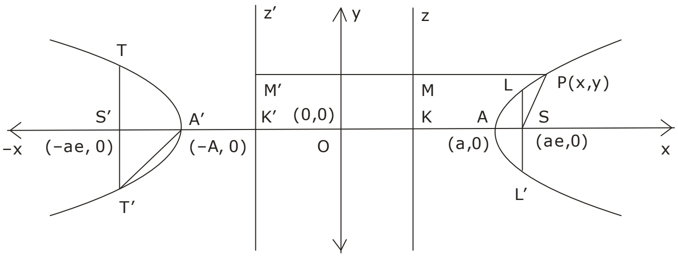
$h^2 = ab < 0$ & $\Delta = abc + 2$

$fgh - af^2 - bg^2 - ch^2 \neq 0$

HYPERBOLA

1. STANDARD EQUATION AND DEFINITIONS

Standard Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



(i) Definition hyperbola : A **Hyperbola** is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distance from a fixed line (called directrix) is always constant which is always greater than unity.

(ii) Vertices : The point A and A′ where the curve meets the line joining the foci S and S′ are called vertices of hyperbola.

(iii) Transverse and Conjugate axes : The straight line joining the vertices A and A′ is called transverse axes of the hyperbola. Straight line perpendicular to the transverse axes and passes through its centre called conjugate axes.

(iv) Latus Rectum : The chord of the hyperbola which passes through the focus and is perpendicular to its transverse axes is called latus rectum. Length of latus rectum $= \frac{2b^2}{a}$.

(v) Eccentricity : For the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,\ b^2 = a^2(e^2 - 1)$

$$e = \sqrt{1 + \left(\frac{2b}{2a}\right)^2} = \sqrt{1 + \left(\frac{\text{Conjugate axes}}{\text{Transverse axes}}\right)^2}$$

(vi) Focal distance : The distance of any point on the hyperbola from the focus is called the focal distance of the point.

Note : The difference of the focal distance of a point on the hyperbola is constant and is equal to the length of the transverse axes.

$|S'P - SP| = 2a$ (const.)

2. CONJUGATE HYPERBOLA

The hyperbola whose transverse and conjugate axes are respectively the conjugate and transverse axes of a given hyperbola is called conjugate hyperbola.

Note :

(i) If e_1 and e_2 are the eccentricities of the hyperbola and its conjugate then $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

(ii) The focus of hyperbola and its conjugate are concyclic.

Length of transverse axis :

The length of transverse axis = 2b = 8

Length of conjugate axis :

The length of conjugate axis = 2a = 6

Eccentricity : $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Foci : the co- ordinates of the foci are (0,± be), i.e., (0, ± 4)

Length of Latus rectum :

The length of latus rectum

$= \frac{2a^2}{b} = \frac{2(3)^2}{4} = \frac{9}{2}$

Equation of directrices :

The equation of directrices are

$y = \pm \frac{b}{e}$

$y = \pm \frac{4}{(5/4)} = \pm \frac{16}{5}$

SOLVED PROBLEMS

CIRCLE

Ex.1 Find the area of a circle which passes through (4, 6) and its centre is (1, 2).

Sol. $\Rightarrow$ Radius r

$= \sqrt{(4-1)^2 + (6-2)^2} = 5$

Area = $\pi r^2 = \pi . 5^2 = 25\pi$

Ex.2 Find the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$

Sol. $2g = -2 \quad \Rightarrow \quad g = -1$

$2f = 4 \quad \Rightarrow \quad f = 2$

$\Rightarrow$ Centre is (1, –2)

Ex.3 Find the radius of the circle $2(x^2 + y^2) + 4x - 3y + 1 = 0$

Sol. First making the coefficient of x^2 and y^2, 1 by dividing the equation with 2

$\Rightarrow \quad x^2 + y^2 + 2x - \frac{3}{2}y + \frac{1}{2} = 0$

$2g = 2 \quad \Rightarrow \quad g = 1$

$2f = -\frac{3}{2} \quad \Rightarrow \quad f = -\frac{3}{4}$

$c = 1/2$

$r = \sqrt{(1)^2 + \left(\frac{3}{4}\right)^2 - \frac{1}{2}} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$

Ex.4 If (6, –3) is the one extremity of diameter to the circle $x^2 + y^2 - 3x + 8y - 4 = 0$ then find its other extremity

Sol. Centre of circle is (3/2, –4)

Let the other extremity is (h, k)

$\therefore \left(\frac{6+h}{2}\right) = \frac{3}{2}, \left(\frac{3+k}{2}\right) = -4 \Rightarrow (-3, -5)$

Ex.5 If $y = 2x + K$ is a diameter to the circle $2(x^2 + y^2) + 3x + 4y - 1 = 0$, then find K

Sol. Centre of circle = (–3/4, –1)

this lies on diameter $y = 2x + K$

$\Rightarrow -1 = -3/4 \times 2 + K$

$\Rightarrow K = 1/2$

Ex.6 Find the equation of a circle whose centre is (2, –1) and radius is 3.

Sol. $(x - 2)^2 + (y + 1)^2 = 3^2$

$\Rightarrow \quad x^2 - 4x + 4 + y^2 + 2y + 1 = 9$

$\Rightarrow \quad x^2 + y^2 - 4x + 2y - 4 = 0$

Ex.7 Find the equation of a circle with centre at the origin and which passes through (7, –2).

Sol. Radius

$r = \sqrt{7^2 + (-2)^2} = \sqrt{53}$

Equation of circle is $x^2 + y^2 = 53$

Ex.8 Find the equation of the circle touches y axis and having centre is (–2, –3)

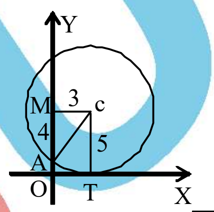
Sol. Here radius of circle | –2 | = 2

$\therefore$ Equation is $(x + 2)^2 + (y + 3)^2 = 2^2$

or $\quad x^2 + y^2 + 4x + 6y + 9 = 0$

Ex.9 A circle touches x- axis at +3 distance and cuts an intercept of 8 in +ve direction of y-axis. Then find its equation

Sol. From figure.



Radius of Circle = $\sqrt{3^2 + 4^2} = 5$ and centre is (3,5)

Hence equation is $(x-3)^2 + (y - 5)^2 = 5^2$

$\Rightarrow \quad x^2 + y^2 - 6x - 10y + 9 = 0$

Ex.10 If the line $x - y + 1 = 0$ is a chord of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ then find the length of this chord -

Sol. Centre of the circle = (1, –2)

Radius = $\sqrt{1+4+4} = 3$

Here p = $\frac{1+2+1}{\sqrt{2}} = 2\sqrt{2}$

$\therefore$ Length of chord $= 2\sqrt{a^2 - p^2}$.

$= \sqrt{9-8} = 2$

Ex.11 Find the length of intercept on y axis, by a circle whose diameter is the line joining the points (–4, 3) and (12, –1)

Sol. Here equation of the circle

$(x + 4)(x - 12) + (y - 3)(y + 1) = 0$

or $x^2 + y^2 - 8x - 2y - 51 = 0$

Hence intercept on y axis

$= \quad 2\sqrt{f^2 - c}$

$= \quad 2\sqrt{1-(-51)} = 4\sqrt{13}$

PARABOLA

Ex.12 Find the vertex of the parabola
$y^2 + 6x - 2y + 13 = 0$

Sol. We have : $y^2 + 6x - 2y + 13 = 0$
$\Rightarrow y^2 - 2y = -6x - 13$
$\Rightarrow (y - 1)^2 = -6(x + 2)$
Clearly, the vertex of this parabola is (–2,1)

Ex.13 If vertex of parabola is (2, 0) and directrix is y-axis, then find its focus

Sol. Since the axis of the parabola is the line which passes through vertex and perpendicular to the directrix, therefore x-axis is the axis of the parabola.
Obviously $Z \equiv (0, 0)$.
Let focus of the parabola is S (a, 0). Since vertex (2,0) is mid point of ZS, therefore

$\frac{a+0}{2} = 2 \Rightarrow a = 4.$

$\therefore$ Focus is (4, 0)

Ex.14 If the focus of a parabola is (1, 0) and its directrix is x+y=5, then find its vertex

Sol. Since axis is a line perpendicular to directrix, so it will be x – y = k. It also passes from focus, therefore k = 1.
So equation of axis is x – y = 1.
Solving it with x + y = 5, we get
$Z \equiv (3, 2)$.
If vertex is (a, b), then a = 2, b = 1.
Hence vertex is (2, 1).

Ex.15 Find the directrix and axis of the parabola $4y^2 - 6x - 4y = 5$

Sol. Here $4y^2 - 4y = 6x + 5$

$\Rightarrow 4\left(y-\frac{1}{2}\right)^2 = 6\,(x + 1)$

Put $y - \frac{1}{2} = Y$, $x + 1 = X$

The equation in standard form

$Y^2 = \frac{3}{2}X$

$4a = \frac{3}{2}$ $\Rightarrow a = \frac{3}{8}$

Directrix, $X + a = 0$

$\Rightarrow x + 1 + \frac{3}{8} = 0$ $\Rightarrow 8x + 11 = 0$

Axis is Y = 0 $\Rightarrow y - \frac{1}{2} = 0$

$\Rightarrow 2y - 1 = 0$

Ex.16 Find the length of latus rectum of a parabola, whose focus is (2, 3) and directrix is the line x – 4y + 3 = 0

Sol. The length of latus rectum
= 2 × perp. from focus to the directrix

$= 2 \times \frac{2-4(3)+3}{\sqrt{(1)^2+(4)^2}} = \frac{-14}{\sqrt{17}}$

The numerical length = $\frac{14}{\sqrt{17}}$

Ex.17 Find the coordinates of an endpoint of the latus rectum of the parabola
$(y - 1)^2 = 4(x + 1)$

Sol. Shifting the origin at (–1, 1) we have

$\left.\begin{aligned} x &= X-1 \\ y &= Y+1 \end{aligned}\right\}$ ...(i)

Using (i), the given parabola becomes.
$Y^2 = 4X$
The coordinates of the endpoints of latus rectum are
(X = 1, Y = 2) and (X =1, Y= –2)
Using (i), the coordinates of the end point of the latus rectum are (0,3) and (0, –1)

Ex.18 If focus of a parabola is (3,–4) and directrix is x+y–2= 0, then find its vertex

Sol. First we find the equation of axis of parabola, which is perpendicular to directrix. So its equation is
x – y + k = 0. It passes through focus S (3, –4)
$\Rightarrow$ 3 – (–4) + k = 0 $\Rightarrow$ k = –7
Let Z is the point of intersection of axis and directrix.
Solving equation x + y –2 = 0 and x –y–7=0 gives Z (9/2, –5/2)
Vertex A is the mid point of Z and S

$\Rightarrow A\left(\frac{3+\frac{9}{2}}{2}, \frac{-4-\frac{5}{2}}{2}\right) = A\left(\frac{15}{4}, -\frac{13}{4}\right)$

Ex.19 Find the equation of the parabola whose vertex is (–3, 0) and directrix is x + 5 = 0

Sol. A line passing through the vertex (–3, 0) and perpendicular to directrix x + 5 = 0 is x-axis which is the axis of the parabola by definition.
Let focus of the parabola is (a, 0). Since vertex, is the middle point of Z(–5, 0) and

focus S, therefore

$$-3 = \frac{(a-5)}{2} \Rightarrow a = -1$$

$\therefore$ Focus = (–1,0)

Thus the equation to the parabola is

$(x + 1)^2 + y^2 = (x + 5)^2$

$\Rightarrow$ $y^2 = 8(x + 3)$

Ex.20 Find the equation of the directrix of the parabola $y^2 = 12x$

Sol. Here a = 3, so the equation of the directrix is given by

$x = -a \Rightarrow x = -3$

$\Rightarrow x + 3 = 0$

Ex.21 Find the equation of the latus rectum of the parabola $x^2 = -12y$

Sol. Here a = 3 so the equation of the L.R. is given by $y = -a \Rightarrow y = -3$

Ex.22 Find the axis of the parabola $9y^2 - 16x - 12y - 57 = 0$

Sol. $9y^2 - 12y = 16x + 57$

$\Rightarrow (3y - 2)^2 = 16x + 57$

$$\Rightarrow \left(y - \frac{2}{3}\right)^2 = \frac{16}{9}\left(x + \frac{61}{16}\right)$$

Which shows that equation of the axis is $y - 2/3 = 0$ or $3y = 2$

Ex.23 Find the Vertex, focus, latus rectum, length of the latus rectum and equation of directrix of the parabola $y^2 = 4x + 4y$

Sol. Given parabola $y^2 = 4x + 4y$

or $(y - 2)^2 = 4(x + 1)$

or $Y^2 = 4X$

Here $X = x + 1$, $Y = y - 2$

vertex = (X = 0, Y = 0)

or $(x + 1 = 0, y - 2 = 0) \Rightarrow (-1, 2)$

Focus (X = 1, Y = 0)

or $(x + 1 = 1, y - 2 = 0) \Rightarrow (0, 2)$

Latus rectum $\Rightarrow X = 1 \Rightarrow x = 0$

Length of Latus rectum = 4

equation of the directrix is

$X = -1 \Rightarrow x + 1 = -1 \Rightarrow x = -2$

Ex.24 Find the vertex of the parabola $x^2 - 8y - x + 19 = 0$

Sol. The given equation of Parabola can be written as

$$\left(x - \frac{1}{2}\right)^2 - 8y + 19 - \frac{1}{4} = 0$$

$$\left(x - \frac{1}{2}\right)^2 = 8y - \frac{76-1}{4}$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 = 8\left(y - \frac{75}{32}\right)$$

$$\therefore \text{ vertex } = \left(\frac{1}{2}, \frac{75}{32}\right)$$

Ex.25 If (0,4) and (0,2) are vertex and focus of parabola respectively, then find its equation

Sol. Since vertex and focus are on y-axis, so y-axis is the axis of the parabola.

Distance between vertex and focus

= 'a' = 2.

So on taking vertex as origin, the equation of the parabola is $x^2 = -4ay$

(negative because vertex lies above the focus)

or $x^2 = -8y$.

Now shifting the origin to its original position, the required equation will be

$x^2 = -8(y - 4)$

$\Rightarrow x^2 + 8y = 32$

ELLIPSE

Ex.26 Find the equation of the ellipse which passes through origin and has its foci at the points (1, 0) and (3, 0)

Sol: Centre being mid point of the foci is

$$\left(\frac{1+3}{2}, 0\right) = (2, 0)$$

Distance between foci 2ae = 2

$ae = 1$ or $a^2 - b^2 = 1$ ...(i)

If the ellipse $\frac{(x-2)^2}{a^2} + \frac{y^2}{b^2} = 1$, then as it passes from (0, 0)

$$\frac{4}{a^2} = 1 \Rightarrow a^2 = 4$$

from (i) $b^2 = 3$

$$\text{Hence } \frac{(x-2)^2}{4} + \frac{y^2}{3} = 1$$

or $3x^2 + 4y^2 - 12x = 0$

Ex.27 A man running round a racecourse notes that the sum of the distance of two flag posts from him is always 10 meters and the distance between the flag posts is 8 meters. Find the area of the path he encloses

Sol.: The race course will be an ellipse with the flag posts as its foci. If a and b are the semi major and minor axes of the ellipse, then sum of focal distances 2a = 10 and 2ae = 8

a = 5, e = 4/5

$\therefore \quad b^2 = a^2(1 - e^2) \quad = 25\left(1-\frac{16}{25}\right) = 9$

Area of the ellipse = $\pi ab \quad = \pi.5.3 = 15\pi$

Ex.28 Find the equation of the ellipse whose eccentricity is 1/2, the focus is (–1, 1) and the directrix is x – y + 3 = 0.

Sol. Let P (x,y) be any point on the ellipse whose focus is S(–1,1) and eccentricity e =1/2. Let PM be perpendicular from P on the directrix. Then,

$SP = ePM \Rightarrow SP = \frac{1}{2}(PM)$

$\Rightarrow 4\,(SP)^2 = PM^2$

$\Rightarrow 4\,[(x+1)^2 + (y-1)^2] = \left(\frac{x-y+3}{\sqrt{1^2+(-1)^2}}\right)^2$

$\Rightarrow 8\,(x^2 + y^2 + 2x - 2y + 2) = (x - y + 3)^2$

$\Rightarrow 7x^2 + 7y^2 + 10x - 10y + 2xy + 7 = 0$

This is the required equation of the ellipse.

Ex.29 Find the equation of the ellipse whose axes are along the coordinate axes, vertices are (± 5,0) and foci at (± 4,0).

Sol. Let the equation of the required ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ..(1)$

The coordinates of its vertices and foci are (± a, 0) and (± ae,0) respectively.

$\therefore$ a = 5 and ae = 4 $\Rightarrow$ e = 4/5.

Now, $b^2 = a^2\,(1 - e^2)$

$\Rightarrow \quad b^2 = 25\left(1-\frac{16}{25}\right) = 9.$

Substituting the values of a^2 and b^2 in (1) , we get $\frac{x^2}{25} + \frac{y^2}{9} = 1,$

which is the equation of the required ellipse.

Ex.30 Find the centre, the length of the axes and the eccentricity of the ellipse $2x^2+3y^2-4x-12y+13 = 0.$

Sol. The given equation can be rewritten as

$2[x^2 - 2x] + 3\,[y^2 - 4y] + 13 = 0$

or $2\,(x - 1)^2 + 3\,(y - 2)^2 = 1$

or $\frac{(x-1)^2}{(1/\sqrt{2})^2} + \frac{(y-2)^2}{(1/\sqrt{3})^2} = 1,$

or $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$

$\therefore$ Centre X = 0, Y = 0 i.e. (1,2).

Length of major axis = 2a = $\sqrt{2}$

Length of minor axis = 2b = $2/\sqrt{3}$ and

$e = \sqrt{(a^2-b^2)/a} = 1/\sqrt{3}$

Ex.31 Find the radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having its centre (0,3)

Sol. $e = 1 - \frac{b^2}{a^2} = 1 - \frac{9}{16} \therefore e = \frac{\sqrt{7}}{4}$

$\therefore$ Foci are (± ae, 0) or ($\pm\sqrt{7}$,0).

Centre is (0,3)

$\therefore$ Radius = $\sqrt{7+9} = 4$

Ex.32 Find the eccentricity of the ellipse represented by the equation

$25x^2 + 16y^2 - 150\,x - 175 = 0$

Sol. $25(x^2 - 6x + 9) + 16y^2 = 175 + 225$

or $25(x - 3)^2 + 16y^2 = 400$

or $\frac{X^2}{16} + \frac{Y^2}{25} = 1$. Form $\frac{X^2}{b^2} + \frac{Y^2}{a^2} = 1$

$\therefore$ Major axis lies along y- axis. ;

$\therefore \quad e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{16}{25}\,; \therefore e = \frac{3}{5}$

Ex.33 Find the equation of the ellipse having its centre at the point (2,–3), one focus at (3,– 3) and one vertex at (4,–3).

Sol. C ≡ (2, – 3), S ≡ (3, – 3) and A ≡ (4, – 3)

Now CA = $\sqrt{(4-2)^2+(-3+3)^2} = 2$

$\therefore$ a = 2

Again CS = $\sqrt{(3-2)^2+(-3+3)^2} = 1$

$\because$ ae = 1 $\quad \therefore e = \frac{1}{a} = \frac{1}{2}$

Let the directrix cut the major- axis at Q.

Then $\frac{AS}{AQ} = e = \frac{1}{2}$

If Q ≡ (α, β), then

SA : AQ = e : 1 = 1 : 2

$\therefore$ A $\equiv \left(\frac{\alpha+6}{3}, \frac{\beta-6}{3}\right)$ = (4, – 3)

$\Rightarrow \frac{\alpha+6}{3} = 4, \frac{\beta-6}{3} = -3$

$\therefore \alpha = 6, \beta = -3$

Slope of CA = 0, therefore directrix will be parallel to y- axis.

Since directrix is parallel to y- axis and it passes through Q(6, – 3)

equation of the directrix is x = 6.

Let P(x,y) be any point on the ellipse, then

$$e = \frac{1}{2} = \frac{PS}{PM} = \frac{\sqrt{(x-3)^2 + (y+3)^2}}{\left|\frac{x-6}{\sqrt{1^2}}\right|}$$

$$\therefore \frac{1}{4} = \frac{(x-3)^2 + (y+3)^2}{(x-6)^2} ;$$

or $(x - 6)^2 = 4[(x-3)^2 + (y+3)^2]$

or $x^2 - 12x + 36 =$

$4[x^2 - 6x + 9 + y^2 + 6y + 9]$

or $3x^2 + 4y^2 - 12x + 24y + 36 = 0$

Ex.34 Find the centre, the length of the axes, eccentricity and the foci of the ellipse.

$12x^2 + 4y^2 + 24x - 16y + 25 = 0$

Sol. The given equation can be written in the form $12(x+1)^2 + 4(y-2)^2 = 3$

$$\Rightarrow \frac{(x+1)^2}{1/4} + \frac{(y-2)^2}{3/4} = 1 \quad .(1)$$

Co- ordinates of centre of the ellipse are given by x + 1 = 0 and y – 2 = 0

Hence centre of the ellipse is (– 1, 2)

If a and b be the lengths of the semi major and semi-minor axes, then

$a^2 = 3/4, b^2 = 1/4$

$\therefore$ Length of major axis = 2a = $\sqrt{3}$,

Length of minor axis = 2b = 1

$\therefore a = \frac{\sqrt{3}}{2}, b = \frac{1}{2}$

Since $b^2 = a^2(1 - e^2)$ $\therefore 1/4 = 3/4(1 - e^2)$

$\Rightarrow e = \sqrt{2/3}$ $\therefore ae = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{\sqrt{2}}$

Co-ordinates of foci are given by

$x + 1 = 0, y - 2 = \pm ae$

Thus foci are $\left(-1, 2 \pm \frac{1}{\sqrt{2}}\right)$

Ex.35 Find the equation of an ellipse whose focus is (– 1,1), eccentricity is 1/2 and the directrix is x – y + 3 = 0.

Sol. Let P (x,y) be any point on the ellipse whose focus is S (–1,1) and the directrix is x – y + 3 = 0.

PM perpendicular from P (x,y) on the directrix x –y + 3 = 0.

Then by definition

SP = ePM $\Rightarrow (SP)^2 = e^2 (PM)^2$

$$\Rightarrow (x+1)^2 + (y-1)^2 = \frac{1}{4}\left\{\frac{x-y+3}{\sqrt{2}}\right\}^2$$

$\Rightarrow 8(x^2 + y^2 + 2x - 2y + 2)$

$= x^2 + y^2 + 9 - 2xy + 6x - 6y$

$\Rightarrow 7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

which is the required equation of the ellipse.

Ex.36 The foci of an ellipse are (± 2, 0) and its eccentricity is 1/2, find its equation.

Sol. Let the equation of the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, Then coordinates of foci are (± ae, 0).

$\therefore ae = 2 \Rightarrow a \times \frac{1}{2} = 2 \quad \left[\because e = \frac{1}{2}\right]$

$\Rightarrow a = 4$

We have $b^2 = a^2(1 - e^2)$

$\therefore b^2 = 16\left(1 - \frac{1}{4}\right) = 12$

Thus, the equation of the ellipse is

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

HYPERBOLA

Ex.37 If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then find the value of b^2

Sol. For hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{81}{144} = \frac{225}{144}$

$e = \frac{15}{12} = \frac{5}{4}$ i.e., e > 1

Hence the foci are

$(\pm ae, 0)\left(\pm\frac{12}{5}.\frac{5}{4}, 0\right) = (\pm 3, 0)$

Now the foci coincide therefore for ellipse

$ae = 3$ or $a^2e^2 = 9$

or $a^2\left(1-\frac{b^2}{a^2}\right) = 9$

$a^2 - b^2 = 9$

or $16 - b^2 = 9$

$\Rightarrow$ $b^2 = 7$

Ex.38 Find the eccentricity of the conic represented by $x^2-y^2-4x+4y+16 = 0$

Sol. We have $x^2 - y^2 - 4x + 4y + 16 = 0$

or $(x^2 - 4x) - (y^2 - 4y) = -16$

or $(x^2 - 4x + 4) - (y^2 - 4y + 4) = -16$

or $(x - 2)^2 - (y - 2)^2 = -16$

or $\frac{(x-2)^2}{4^2} - \frac{(y-2)^2}{4^2} = -1$

Shifting the origin at (2, 2), we obtain

$\frac{x^2}{4^2} - \frac{y^2}{4^2} = -1,$

where $x = X + 2$, $y = Y + 2$

This is rectangular hyperbola, whose eccentricity is always $\sqrt{2}$.

i.e. $e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{4^2}{4^2}$;

$e = \sqrt{2}$

Ex.39 The equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represent a hyperbola then find

(A) The length of the transverse axes

(B) Length of latus rectum

(C) Equation of directrix

Sol. We have

$9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$9(x^2 - 2x) - 16(y^2 - 2y) = 151$

$9(x^2-2x+1) - 16(y^2 - 2y + 1) = 144$

$9(x - 1)^2 - 16(y - 1)^2 = 144$

$\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

Shifting the origin at (1, 1) without rotating the axes

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

where $x = X + 1$ and $y = Y + 1$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

where $a^2 = 16$ and $b^2 = 9$ so

The length of the transverse axes $= 2a = 8$

The length of the letus rectum

$= \frac{2b^2}{a} = \frac{a}{2}$

The equaiton of the directrix

$x = \pm\frac{a}{e}$

$x - 1 = \pm\frac{16}{5} \Rightarrow x = \pm\frac{16}{5} + 1$

$x = \frac{21}{5}$; $x = -\frac{11}{5}$

EXERCISE

CIRCLE

Q.1 Find the equation of a circle with centre (2, 4) and radius 5

Q.2 Find the centre and radius of $(x-3)^2+(y-1)^2=9$

Q.3 Find the equation of the circle whose centre is (2,–5) and which passes through the point (3, 2).

Q.4 Find the equation of the circle of radius 5 cm, whose centre lies on the y-axis and which passes through the point (3, 2).

Q.5 Find the equation of the circle whose centre is (2, –3) and which passes through the intersection of the lines $3x + 2y = 11$ and $2x + 3y = 4$.

Q.6 Find the equation of the circle passing through the point (–1, 3)and having its centre at the point of intersection of the lines $x - 2y = 4$ and $2x + 5y + 1 = 0$.

Q.7 If two diameters of a circle lie along the lines $x-y=9$ and $x-2y=7$, and the area of the circle is 38.5 sq cm, find the equation of the circle.

Q.8 Find the equation of the circle, the coordinates of the end points of one of whose diameter are A(3, 2) and B(2, 5)

Q.9 The sides of a rectangle are given by the equations $x = -2$, $x = 4$, $y = -2$ and $y = 5$. Find the equation of the circle drawn on the diagonal of this rectangle as its diameter.

Q.10 Show that the equation $x^2+y^2-4x+6y-5= 0$ represents a circle. Find its centre and radius.

Q.11 Show that the equation $x^2+y^2-3x+3y+10=0$ does not represent a circle.

Q.12 Find the equation of the circle which is circumscribed about the triangle whose vertices are A(–2, 3), B(5, 2) and C(6, –1). Find the centre and radius of this circle.

Q.13 Find the equation of the circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point P(5, 4).

Q.14 Show that the points A(1, 0), B(2, –7), C(8, 1) and D(9, –6) all lie on the same circle. Find the equation of this circle, its centre and radius.

Q.15 Find the equation of the circle which passes through the points (1, 3) and (2, –1), and has its centre on the line $2x + y - 4 = 0$.

Q.16 Find the equation of the circle concentric with the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ and with touches the y-axis.

Q.17 Find the equation of the circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and of double its area.

Q.18 Prove that the centres of the three circles $x^2 + y^2 - 4x - 6y - 12 = 0$, $x^2 + y^2 + 2x + 4y - 5 = 0$ and $x^2 + y^2 - 10x - 16y + 7 = 0$ are collinear.

Q.19 Find the equation of a circle passing through the origin and intercepting lengths a and b on the axes.

Q.20 Find the equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

Q.21 One end of a diameter of the circle $x^2 + y^2 - 6x + 5y - 7 = 0$ is A(–1, 3). Find the coordinates of the other end of this diameter.

PARABOLA

Q.22 Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

(i) $y^2 = 12x$ **(ii)** $y^2 = -8x$

(iii) $x^2 = 16y$ **(iv)** $x^2 = -8y$

Q.23 Find the equation of the parabola with vertex at the origin and focus at F(–2, 0).

Q.24 Find the equation of the parabola with focus F(4, 0) and directrix $x = -4$.

Q.25 Find the equation of the parabola with vertex at the origin and focus F(0, 5).

Q.26 Find the equation of the parabola with vertex at the origin, passing through the point P(5, 2) and symmetric with respect to the y-axis.

Q.27 Find the equation of the parabola which is symmetric about the y-axis and passes through the point P(2, –3).

ELLIPSE

Q.28 Find the
(i) lengths of the major and minor axes,
(ii) coordinates of the vertices
(iii) coordinates of the foci
(iv) eccentricity
(v) length of the latus rectum of $\frac{x^2}{25}+\frac{y^2}{9}=1$

Q.29 Find the equation of the ellipse whose vertices are at (±6, 0) and foci at (±4, 0).

Q.30 Find the equation of the ellipse the ends of whose major and minor axes are (±4, 0) and (0, ±3) respectively.

Q.31 Find the equation of the ellipse whose foci are (±2, 0) and the eccentricity is $\frac{1}{2}$.

Q.32 Find the equation of the ellipse whose foci are at (0, ±4) and e = $\frac{4}{5}$.

Q.33 Find the equation of the ellipse with centre at the origin, major axis on the x-axis and passing through the points (4, 3) and (–1, 4).

Q.34 Find the equation of the ellipse which passes through the point (4, 1) and having its foci at (±3, 0).

Q.35 Find the equation of an ellipse whose eccentricity is $\frac{2}{3}$, the latus rectum is 5 and the centre is at the origin.

HYPERBOLA

Q.36 Find the
(i) length of the axes
(ii) coordinates of the vertices
(iii) coordinates of the foci
(iv) eccentricity
(v) length of the latus Rectum of $\frac{x^2}{9}-\frac{y^2}{16}=1$

Q.37 Find the equation of the hyperbola with vertices at (± 6, 0) and foci at (± 8, 0).

Q.38 Find the equation of the hyperbola with vertices at (0, ±5) and foci at (0, ±8).

Q.39 Find the equation of the hyperbola whose foci are ($\pm\sqrt{29}$, 0) and the transverse axis is of the length 10.

Q.40 Find the equation of the hyperbola whose foci are (±5, 0) and the conjugate axis is of the length 8. Also, find its eccentricity.

Q.41 Find the equation of the hyperbola whose foci are ($\pm 3\sqrt{5}$, 0) and the length of the latus rectum is 8 units.

Q.42 Find the equation of the hyperbola whose vertices are (± 2, 0) and the eccentricity is 2.

Q.43 Find the equation of the hyperbola whose foci are ($\pm\sqrt{5}$, 0) and the eccentricity is $\sqrt{\frac{5}{3}}$.

Q.44 Find the equation of the hyperbola, the length of whose latus rectum is 4 and the eccentricity is 3.

Q.45 Find the equation of the hyperbola with eccentricity $\sqrt{2}$ and the distance between whose foci is 16.

Q.46 Find the equation of the hyperbola whose foci are (0, ±13) and the length whose conjugate axis is 24.

Q.47 Find the equation of the hyperbola whose foci are (0, ± 10) and the length of whose latus rectum is 9 units.

Q.48 Find the equation of the hyperbola having its foci at (0, $\pm\sqrt{14}$) and passing through the point P(3, 4).

ANSWERS

1. $x^2+y^2-4x-8y-5 = 0$ **2.** Centre (3, 1), radius = 3

3. $x^2 + y^2 - 4x + 10y - 21 = 0$

4. $(x^2 + y^2 - 12y + 11 = 0)$ or $(x^2 + y^2 + 4y - 21 = 0)$

5. $x^2+y^2-4x+6y+3 = 0$ **6.** $x^2+y^2-4x+2y-20 = 0$

7. $4x^2+4y^2-88x-16y+451=0$

8. $x^2 + y^2 - 5x - 7y + 16 = 0$

9. $x^2+y^2-2x-3y-18 = 0$

10. Centre (2, –3), radius $3\sqrt{2}$

12. $x^2+y^2-2x+2y-23 = 0$, centre (1,–1) and radius=5

13. $x^2 + y^2 + 4x + 6y - 85 = 0$

14. $x^2+y^2-10x+6y+9=0$, centre C(5,–3) and radius=5

15. $x^2 + y^2 - 3x - 2y - 1 = 0$

16. $x^2 + y^2 - 4x - 6y + 9 = 0$

17. $x^2 + y^2 - 6x + 12y - 15 = 0$

19. $x^2 + y^2 - ax - by = 0$

20. $x^2 + y^2 - 17x - 19y + 50 = 0$

21. B(7, - 8)

22. **(i)** F(3, 0), O(0, 0), x + 3 = 0, y = 0, 12 units
(ii) F(–2, 0), O(0, 0), x = 2, y = 0, 8 units
(iii) F(0, 4), O(0, 0), y + 4 = 0, x = 0, 16 units
(iv) F(0, –2), O(0, 0), y = 2, x = 0, 8 units

23. $y^2 = -8x$ **24.** $y^2=16x$ **25.** $x^2= 20y$

26. $2x^2=25y$ **27.** $3x^2=-4y$

28. **(i)** 10 units, 6 units **(ii)** A(–5, 0) and B(5, 0)

(iii) $F_1(-4, 0)$ and $F_2(4, 0)$ **(iv)** e = $\frac{4}{5}$ (v) 3.6 units

29. $\frac{x^2}{36}+\frac{y^2}{20}=1$ **30.** $\frac{x^2}{16}+\frac{y^2}{9}=1$

31. $\frac{x^2}{16}+\frac{y^2}{12}=1$ **32.** $\frac{x^2}{9}+\frac{y^2}{25}=1$

33. $7x^2 + 15y^2 = 247$ **34.** $\frac{x^2}{18}+\frac{y^2}{9}=1$

35. $\frac{4x^2}{81}+\frac{4y^2}{45}=1$ **36.** **(i)** 6 units, 8 units

(ii) A(–3, 0), B(3, 0) **(iii)** $F_1(-5, 0)$, $F_2(5, 0)$

(iv) e = $\frac{5}{3}$ **(v)** $10\frac{2}{3}$ units

37. $\frac{x^2}{36}-\frac{y^2}{28}=1$ **38.** $\frac{y^2}{25}-\frac{x^2}{39}=1$

39. $\frac{x^2}{25}-\frac{y^2}{4}=1$ **40.** $\frac{x^2}{9}-\frac{y^2}{16}=1, e=\frac{5}{3}$

41. $\frac{x^2}{25}-\frac{y^2}{20}=1$ **42.** $\frac{x^2}{4}-\frac{y^2}{12}=1$

43. $\frac{x^2}{3}-\frac{y^2}{2}=1$ **44.** $16x^2 - 2y^2 = 1$

45. $x^2-y^2=32$ **46.** $\frac{y^2}{25}-\frac{x^2}{144}=1$

47. $\frac{y^2}{64}-\frac{x^2}{36}=1$ **48.** $y^2- x^2 = 7$