CIRCLE

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DEFINITIONS

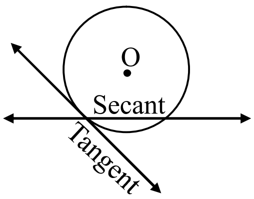
◈ **Secant :**

A line which intersects a circle in two distinct points is called a secant.

◈ **Tangent :**

A line meeting a circle only in one point is called a tangent to the circle at that point.

The point at which the tangent line meets the circle is called the point of contact.



NUMBER OF TANGENTS TO A CIRCLE

(i) There is no tangent passing through a point lying inside the circle.

(ii) There is one and only one tangent passing through a point lying on a circle.

(iii) There are exactly two tangents through a point lying outside a circle.

LENGTH OF TANGENT

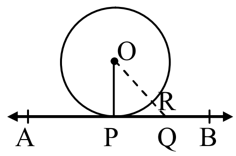
The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.

RESULTS ON TANGENTS

Theorem 1 :

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given : A circle with centre O and a tangent AB at a point P of the circle.



To prove : $OP \perp AB$.

Construction : Take a point Q, other than P, on AB. Join OQ.

Proof : Q is a point on the tangent AB, other than the point of contact P.

$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R.

Then, $OR < OQ$ [a part is less than the whole]...(i)

But, $OP = OR$ [radii of the same circle]. ....(ii)

$\therefore$ $OP < OQ$ [from (i) and (ii)].

Thus, OP is shorter than, any other line segment joining O to any point of AB, other than P.

In other words, OP is the shortest distance between the point O and the line AB.

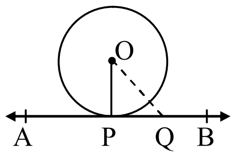
But, the shortest distance between a point and a line is the perpendicular distance.

$\therefore$ $OP \perp AB$.

Theorem 2 : (Converse of Theorem 1)

A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.

Given : A circle with centre O in which OP is a radius and AB is a line through P such that OP $\perp$ AB.



To prove : AB is a tangent to the circle at the point P.

Construction : Take a point Q, different from P, on AB. Join OQ.

Proof : We know that the perpendicular distance from a point to a line is the shortest distance between them.

$\therefore$ OP $\perp$ AB $\Rightarrow$ OP is the shortest distance from O to AB.

$\therefore$ OP < OQ.

$\therefore$ Q lies outside the circle

[$\because$ OP is the radius and OP < OQ].

Thus, every point on AB, other than P, lies outside the circle.

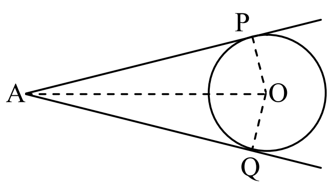
$\therefore$ AB meets the circle at the point P only.

Hence, AB is the tangent to the circle at the point P.

Theorem 3 :

The lengths of tangents drawn from an external point to a circle are equal.

Given : Two tangents AP and AQ are drawn from a point A to a circle with centre O.



To prove : AP = AQ

Construction : Join OP, OQ and OA.

Proof : AP is a tangent at P and OP is the radius through P.

$\therefore$ OP $\perp$ AP.

Similarly, OQ $\perp$ AQ.

In the right triangle OPA and OQA, we have

OP = OQ [radii of the same circle]

OA = OA [common]

$\therefore$ ΔOPA $\cong$ ΔOQA [by RHS–congruence]

Hence, AP = AQ.

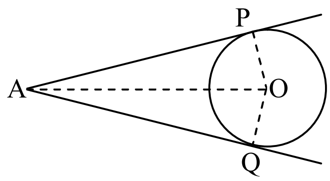
Theorem 4 :

If two tangents are drawn from an external point then

(i) They subtend equal angles at the centre, and

(ii) They are equally inclined to the line segment joining the centre to that point.

Given : A circle with centre O and a point A outside it. Also, AP and AQ are the two tangents to the circle.



To prove : $\angle$AOP =$\angle$AOQ and $\angle$OAP = $\angle$OAQ.

Proof : In ΔAOP and ΔAOQ, we have

AP = AQ [tangents from an external point are equal]

OP = OQ [radii of the same circle]

OA = OA [common]

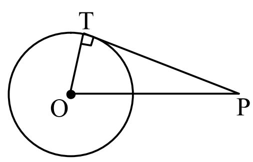
$\therefore$ ΔAOP ΔAOQ [by SSS–congruence].

Hence, $\angle$AOP = $\angle$AOQ and $\angle$OAP = $\angle$OAQ.

❖ EXAMPLES ❖

Ex.1 From a point P, 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle.

Sol. Let O be the centre of the given circle and let P be a point such that



$OP = 10$ cm.

Let PT be the tangent such that PT = 8 cm.

Join OT.

Now, PT is a tangent at T and OT is the radius through T.

$\therefore$ $OT \perp PT$.

In the right ΔOTP, we have

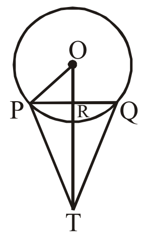
$OP^2 = OT^2 + PT^2$ [by Pythagoras' theorem]

$\Rightarrow$ $OT = \sqrt{OP^2 - PT^2} = \sqrt{(10)^2 - (8)^2}$ cm

$= \sqrt{36}$ cm = 6 cm.

Hence, the radius of the circle is 6 cm.

Ex.2 In the given figure, PQ is a chord of length 8cm of a circle of radius 5cm. The tangents at P and Q intersect at a point T. Find the length TP.



Sol. Join OP and OT Let OT intersect PQ at a point R.

Then, TP = TQ and $\angle PTR = \angle QTR$.

$\therefore$ $TR \perp PQ$ and TR bisects PQ.

$\therefore$ PR = RQ = 4 cm.

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$ cm

$= \sqrt{25 - 16}$ cm $= \sqrt{9}$ cm = 3cm.

Let TP = x cm and TR = y cm.

From right ΔTRP, we get

$TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + 16 \;\Rightarrow x^2 - y^2 = 16$ .... (i)

From right ΔOPT, we get

$TP^2 + OP^2 = OT^2$

$\Rightarrow$ $x^2 + 5^2 = (y + 3)^2$ [$\because$ $OT^2 = (OR + RT)^2$]

$\Rightarrow$ $x^2 - y^2 = 6y - 16$ ....(ii)

From (i) and (ii), we get

$$6y - 16 = 16 \;\Rightarrow 6y = 32 \Rightarrow y = \frac{16}{3}.$$

Putting $y = \frac{16}{3}$ in (i), we get

$$x^2 = 16 + \left(\frac{16}{3}\right)^2 = \left(\frac{256}{9} + 16\right) = \frac{400}{9}$$

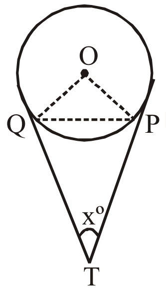
$$\Rightarrow \; x = \sqrt{\frac{400}{9}} = \frac{20}{3}.$$

Hence, length TP = x cm = $\left(\frac{20}{3}\right)$cm

= 6.67 cm.

Ex.3 Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

Sol. **Given :** A circle with centre O and an external point T from which tangents TP and TQ are drawn to touch the circle at P and Q.



To prove : $\angle PTQ = 2\angle OPQ$.

Proof : Let $\angle PTQ = x^\circ$. Then,

$\angle TQP + \angle TPQ + \angle PTQ = 180^\circ$

[$\because$ sum of the $\angle$s of a triangle is 180°]

$\Rightarrow$ $\angle TQP + \angle TPQ = (180^\circ - x)$ ....(i)

We know that the lengths of tangent drawn from an external point to a circle are equal.

So, TP = TQ.

Now, TP = TQ

$\Rightarrow \angle TQP = \angle TPQ$

$$= \frac{1}{2}(180^\circ - x) = \left(90^\circ - \frac{x}{2}\right)$$

$\therefore \quad \angle OPQ = (\angle OPT - \angle TPQ)$

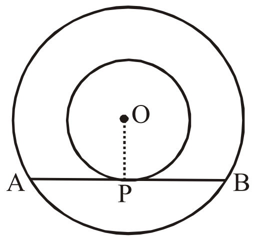
$$= 90^o - \left(90^o - \frac{x}{2}\right) = \frac{x}{2}$$

$\Rightarrow \angle OPQ = \frac{1}{2} \ \angle PTQ$

$\Rightarrow \angle PTQ = 2\angle OPQ.$

Ex.4 Prove that in two concentric circles, the chord of the larger circle which touches the smaller circle, is bisected at the point of contact.

Sol. **Given :** Two circles with the same centre O and AB is a chord of the larger circle which touches the smaller circle at P.



To prove : AP = BP.

Construction : Join OP.

Proof : AB is a tangent to the smaller circle at the point P and OP is the radius through P.

$\therefore \quad OP \perp AB.$

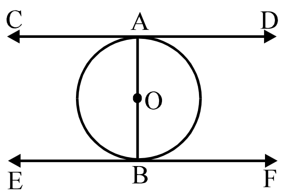
But, the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ OP bisects AB. Hence, AP = BP.

Ex.5 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol. **Given :** CD and EF are the tangents at the end points A and B of the diameter AB of a circle with centre O.

To prove : CD || EF.



Proof : CD is the tangent to the circle at the point A.

$\therefore \quad \angle BAD = 90°$

EF is the tangent to the circle at the point B.

$\therefore \quad \angle ABE = 90°$

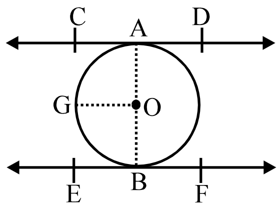
Thus, $\angle BAD = \angle ABE$ (each equal to 90°).

But these are alternate interior angles.

$\therefore$ CD || EF

Ex.6 Prove that the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

Sol. **Given :** CD and EF are two parallel tangents at the points A and B of a circle with centre O.



To prove : AOB is a diameter of the circle.

Construction : Join OA and OB.

Draw OG || CD

Proof : OG || CD and AO cuts them.

$\therefore \quad \angle CAO + \angle GOA = 180°$

$\Rightarrow \quad 90° + \angle GOA = 180° \ \ [OA \perp CD]$

$\Rightarrow \quad \angle GOA = 90°$

Similarly, $\angle GOB = 90°$

$\therefore \angle GOA + \angle GOB = (90° + 90°) = 180°$

$\Rightarrow$ AOB is a straight line

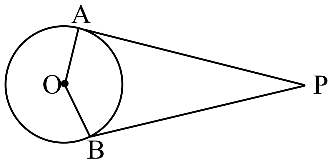
Hence, AOB is a diameter of the circle with centre O.

Ex.7 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the pointsof contact to the centre.

Sol. **Given :** PA and PB are the tangent drawn from a point P to a circle with centre O. Also, the line segments OA and OB are drawn.

To Prove : $\angle APB + \angle AOB = 180°$

Proof : We know that the tangent to a circle is perpendicular to the radius through the point of contact.



$\therefore$ PA $\perp$ OA $\Rightarrow \angle$OAP = 90°, and

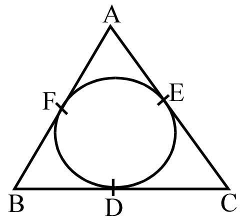
PB $\perp$ OB $\Rightarrow \angle$OBP = 90°.

$\therefore$ $\angle$OAP + $\angle$OBP = 90°.

Hence, $\angle$APB + $\angle$AOB = 180°

[$\because$ sum of the all the angles of a quadrilateral is 360°]

Ex.8 In the given figure, the incircle of ΔABC touches the sides BC, CA and AB at D, E, F respectively.



Prove that AF + BD + CE = AE + CD + BF

$= \frac{1}{2}$ (perimeter of ΔABC)

Sol. We know that the lengths of tangents from an exterior point to a circle are equal.

$\therefore$ AF = AE .... (i) [tangents from A]

BD = BF ..... (ii) [tangents from B]

CE =CD ..... (iii) [tangents from C]

Adding (i), (ii) and (iii), we get

(AF + BD + CE) = (AE + BF + CD) = k (say)

Perimeter of ΔABC = (AF + BD +CE) + (AE + BF + CD)

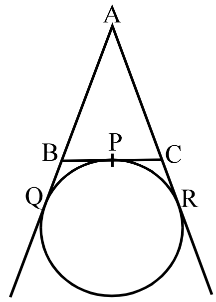
= (k + k) = 2k

$\therefore$ k = $\frac{1}{2}$ (perimeter of ΔABC).

Hence AF + BD + CE = AE + CD + BF

$= \frac{1}{2}$ (perimeter of ΔABC)

Ex.9 A circle touches the side BC of a ΔABC at P, and touches AB and AC produced at Q and R respectively, as shown in the figure.



Show that AQ = $\frac{1}{2}$ (perimeter of ΔABC)

Sol. We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ AQ = AR .... (i) [tangents from A]

BP = BQ .... (ii) [tangents from B]

CP = CR .... (iii) [tangents from C]

Perimeter of ΔABC

= AB + BC + AC

= AB + BP + CP + AC

= AB + BQ + CR + AC [using (ii) and (iii)]

= AQ + AR

= 2AQ [using (i)].

Hence, AQ = $\frac{1}{2}$ (perimeter of ΔABC)

Ex.10 Prove that there is one and only one tangent at any point on the circumference of a circle.

Sol. Let P be a point on the circumference of a circle with centre O. If possible, Let PT and PT′ be two tangents at a point P of the circle.

Now, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

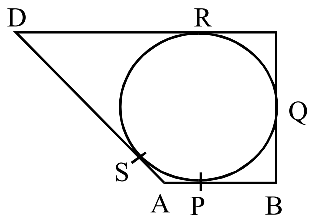
$\therefore$ OP $\perp$ PT and similarly, OP$\perp$PT′

$\Rightarrow$ $\angle$OPT = 90° and $\angle$OPT′ = 90°

$\Rightarrow$ $\angle$OPT = $\angle$OPT′

This is possible only when PT and PT′ coincide. Hence, there is one and only one tangent at any point on the circumference of a circle.

Ex.11 A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure.



Prove that AB + CD = AD + BC

Sol. We known that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ AP = AS ....(i) [tangents from A]

BP = BQ ....(ii) [tangents from B]

CR = CQ ....(iii) [tangents from C]

DR = DS ....(iv) [tangents from D]

$\therefore$ AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

[using (i), (ii), (iii), (iv)]

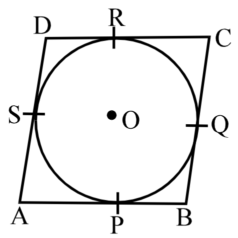
= (AS + DS) + (BQ + CQ)

= (AD + BC).

Hence, (AB + CD) = (AD + BC)

Ex.12 Prove that the paralleogram circumscribing a circle, is a rhombus.

Sol.



Given : A parallelogram ABCD

circumsribes a circle with centre O.

To prove : AB = BC = CD = AD

Proof : we know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ AP = AS .... (i) [tangents from A]

BP = BQ .... (ii) [tangents from B]

CR = CQ .... (iii) [tangents from C]

DR = DS .... (iv) [tangents from D]

$\therefore$ AB + CD = AP + BP + CR + DR

= AS + BQ + CQ + DS

[From (i), (ii), (iii), (iv)]

= (AS + DS) + (BQ + CQ)

= AD + BC

Hence, (AB + CD) = (AD + BC)

$\Rightarrow$ 2AB = 2AD

[$\because$ opposite sides of a parallelogram are equal]

$\Rightarrow$ AB = AD

$\therefore$ CD = AB = AD = BC

Hence, ABCD is a rhombus

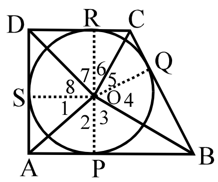
Ex.13 Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol. **Given :** A quadrilateral ABCD circumscribes a circle with centre O.

To Prove : $\angle$AOB+ $\angle$COD = 180°

and $\angle$BOC + $\angle$AOD = 180°

Construction : Join OP, OQ, OR and OF



Proof : We know that the tangents drawn from an external point of a circle subtend equal angles at the centre.

$\therefore$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$

and $\angle 7 = \angle 8$

And, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6$

$+ \angle 7 + \angle 8 = 360°$ [$\angle$s at a point]

$\Rightarrow 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360°$ and

$2(\angle 1 + \angle 8) + 2(\angle 4 + \angle 5) = 360°$

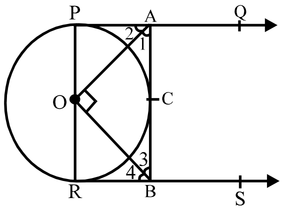
$\Rightarrow\ \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$ and

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

$\Rightarrow\ \angle AOB + \angle COD = 180°$ and

$\angle AOD + \angle BOC = 180°$

Ex.14 In the given figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects PQ at A and RS at B.



Prove that $\angle AOB = 90°$

Sol. **Given :** PQ and RS are two parallel tangents to a circle with centre O and AB is a tangent to the circle at a point C, intersecting PQ and RS at A and B respectively.

To prove : $\angle AOB = 90°$

Proof : Since PA and RB are tangents to the circle at P and R respectively and POR is a diameter of the circle, we have

$\angle OPA = 90°$ and $\angle ORB = 90°$

$\Rightarrow\ \angle OPA + \angle ORB = 180°$

$\Rightarrow$ PA || RB

We know that the tangents to a circle from an external point are equally inclined to the line segment joining this point to the centre.

$\therefore\quad \angle 2 = \angle 1$ and $\angle 4 = \angle 3$

Now, PA|| RB and AB is a transversal.

$\therefore\quad \angle PAB + \angle RBA = 180°$

$\Rightarrow\ (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180°$

$\Rightarrow\ 2\angle 1 + 2\angle 3 = 180°$

$[\ \therefore\ \angle 2 = \angle 1$ and $\angle 4$ and $\angle 3\ ]$

$\Rightarrow\ 2(\angle 1 + \angle 3) = 180°$

$\Rightarrow\ \angle 1 + \angle 3 = 90°$

From ΔAOB, we have

$\angle AOB + \angle 1 + \angle 3 = 180°$

[$\because$ sum of the $\angle$s of a triangle is 180°]

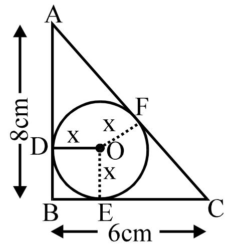
$\Rightarrow\ \angle AOB + 90° = 180°$

$\Rightarrow\ \angle AOB = 90°$

Hence, $\angle AOB = 90°$

Ex.15 ABC is a right triangle, right angled at B. A circle is inscribed in it. The lengths of the two sides containing the right angle are 6 cm and 8 cm. Find the radius of the incircle.

Sol.



Let the radius of the in circle be x cm.

Let the in circle touch the side AB, BC and CA at D, E, F respectively. Let O be the centre of the circle.

Then, OD = OE = OF = x cm.

Also, AB = 8 cm and BC = 6 cm.

Since the tangents to a circle from an external point are equal, we have

AF = AD = (8 – x) cm, and

CF = CE = (6 – x) cm.

$\therefore\quad AC = AF + CF = (8 - x)$ cm $+ (6 - x)$ cm $= (14 - 2x)$ cm.

Now , $AC^2 = AB^2 + BC^2$

$\Rightarrow\ (14 - 2x)^2 = 8^2 + 6^2 = 100 = (10)^2$

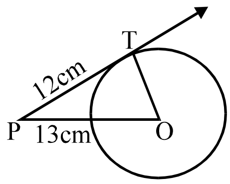
$\Rightarrow\ 14 - 2x = \pm 10 \qquad \Rightarrow x = 2$ or $x = 12$

$\Rightarrow\ x = 2$ [neglecting x = 12].

Hence, the radius of the in circle is 2cm.

Ex.16 A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12cm. Find the radius of the circle.

Sol. Since tangent to a circle is perpendicular to the radius through the point of contact.



$\therefore \quad \angle OTP = 90°$

In right triangle OTP, we have

$OP^2 = OT^2 + PT^2$

$\Rightarrow \; 13^2 = OT^2 + 12^2$

$\Rightarrow \; OT^2 = 13^2 - 12^2$

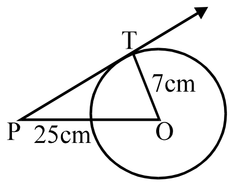
$= (13 - 12)(13 + 12) = 25$

$\Rightarrow \; OT = 5$

Hence, radius of the circle is 5 cm.

Ex.17 Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that the radius of the circle is 7 cm.

Sol. Let P be the given point, O be the centre of the circle and PT be the length of tangent from P. Then, OP = 25 cm and OT = 7 cm.



Since tangent to a circle is always perpendicular to the radius through the point of contact.

$\therefore \quad \angle OTP = 90°$

In right triangle OTP, we have

$OP^2 = OT^2 + PT^2$

$\Rightarrow \; 25^2 = 7^2 + PT^2$

$\Rightarrow \; PT^2 = 25^2 - 7^2$

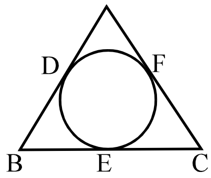
$= (25 - 7)(25 + 7)$

$= 576$

$\Rightarrow \; PT = 24$ cm

Hence, length of tangent from P = 24 cm

Ex.18 In Fig., if AB = AC, prove that BE = EC



Sol. Since tangents from an exterior point to a circle are equal in length

$\therefore$ AD = AF [Tangents from A]

BD = BE [Tangents from B]

CE = CF [Tangents from C]

Now,

AB = AC

$\Rightarrow$ AB – AD = AC – AD

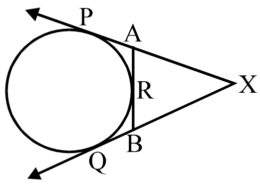
[Subtracting AD from both sides]

$\Rightarrow$ AB – AD = AC – AF [Using (i)]

$\Rightarrow$ BD = CF $\Rightarrow$ BE = CF [Using (ii)]

$\Rightarrow$ BE = CE [Using (iii)]

Ex.19 In fig. XP and XQ are tangents from X to the circle with centre O. R is a point on the circle.



Prove that, XA + AR = XB + BR.

Sol. Since lengths of tangents from an exterior point to a circle are equal.

$\therefore$ XP = XQ .... (i) [From X]

AP = AR .... (ii) [From A]

BQ = BR .... (iii) [From B]

Now, XP = XQ

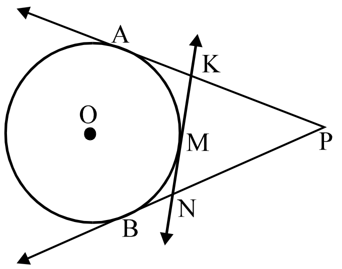
$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR

[Using equations (i) and (ii)]

Ex.20 PA and PB are tangents from P to the circle with centre O. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that KN = AK + BN.

Sol. We know that the tangents drawn from an external point to a circle are equal in length.



$\therefore$ PA = PB .... (i) [From P]

KA = KM .... (ii) [From K]

and, NB = NM .... (iii) [From N]

Adding equations (ii) and (iii), we get

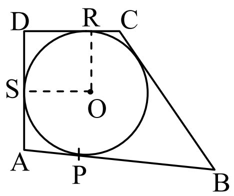
KA + NB = KM + NM

$\Rightarrow$ AK + BN = KM + MN $\Rightarrow$ AK + BN = KN

Ex.21 ABCD is a quadrilateral such that $\angle D = 90°$. A circle (O, r) touches the sides AB, BC, CD and DA at P,Q,R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find r.

Sol. Since tangent to a circle is perpendicular to the radius through the point.

$\therefore$ $\angle ORD = \angle OSD = 90°$



It is given that $\angle D = 90°$ Also, OR = OS. Therefore,

ORDS is a square.

Since tangents from an exterior point to a circle are equal in length.

$\therefore$ BP = BQ

CQ = CR and DR = DS

Now, BP = BQ

$\Rightarrow$ BQ = 27 [$\because$ BP = 27 cm (Given)]

$\Rightarrow$ BC – CQ = 27

$\Rightarrow$ 38 – CQ = 27 [$\because$ BC = 38 cm]

$\Rightarrow$ CQ = 11cm

$\Rightarrow$ CR = 11cm [$\because$ CR = CQ]

$\Rightarrow$ CD – DR = 11

$\Rightarrow$ 25 – DR = 11 [$\because$ CD = 25cm]

$\Rightarrow$ DR = 14 cm

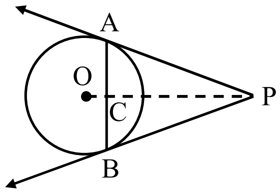
But, ORDS is a square.

Therefore, OR = DR = 14 cm

Hence, r = 14 cm

Ex.22 Prove that the tangents at the extremities of any chord make equal angles with the chord.

Sol. Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose the tangents meet at P. Join OP. Suppose OP meets AB at C. We have to prove that $\angle$ PAC = $\angle$ PBC In triangles PCA and PCB, we have



PA = PB

$\left[\because \text{Tangents from an external point are equal}\right]$

$\angle$ APC = $\angle$ BPC

[$\because$ PA and PB are equally inclined to OP]

and, PC = PC [Common]

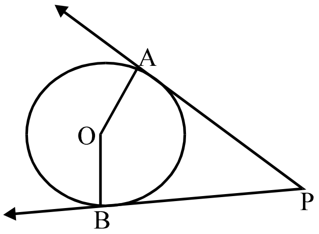
So, by SAS - criterion of congruence, we have

Δ PAC $\cong$ Δ PBC

$\Rightarrow$ $\angle$ PAC = $\angle$ PBC

Ex.23 In fig., O is the centre of the circle, PA and PB are tangent segments. Show that the quadrilateral AOBP is cyclic.

Sol.



Since tangent at a point to a circle is perpendicular to the radius through the point.

$\therefore$ OA$\perp$AP and OB$\perp$BP

$\Rightarrow$ $\angle$OAP = 90° and $\angle$OBP = 90°

$\Rightarrow$ $\angle$OAP + $\angle$OBP = 90° + 90° = 180° ....(i)

In quadrilateral OAPB, we have

$\angle$OAP + $\angle$APB + $\angle$AOB + $\angle$OBP = 360°

$\Rightarrow$ $(\angle APB + \angle AOB) + (\angle OAP + \angle OBP) = 360°$

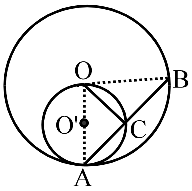
$\Rightarrow$ $\angle$APB + $\angle$AOB + 180° = 360°

$\angle$APB + $\angle$AOB = 180° ....(ii)

From equations (i) and (ii),we can say that the quadrilateral AOBP is cyclic.

Ex.24 In fig., circles C(O,r) and C(O′, r/2) touch internally at a point A and AB is a chord of the circle C (O,r) intersecting C(O′, r/2) at C, Prove that AC = CB.

Sol. Join OA, OC and OB. Clearly, $\angle$OCA is the angle in a semi-circle.



$\therefore$ $\angle OCA = 90°$

In right triangles OCA and OCB, we have

OA = OB = r

$\angle OCA = \angle OCB = 90°$

and OC = OC

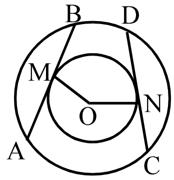
So, by RHS criterion of congruence, we get

Δ OCA $\cong$ Δ OCB

$\Rightarrow$ AC = CB

Ex.25 In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length.

Sol. Let AB and CD be two chords of the circle which touch the inner circle at M and N respectively.



Then, we have to prove that

AB = CD

Since AB and CD are tangents to the smaller circle.

$\therefore$ OM = ON = Radius of the smaller circle

Thus, AB and CD are two chords of the larger circle such that they are equidistant from the centre. Hence, AB = CD.