LIGHT : REFRACTION

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REFRACTION OF LIGHT

◈ **DEFINITION :** When light rays travelling in a medium are incident on a transparent surface of another medium they are bent as they travel in second medium.

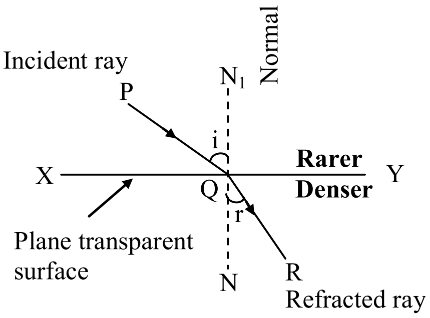


Fig. Refraction of light from a plane transparent denser surface.

◈ **SOME ASSOCIATED TERMS**

◆ **Transparent surface :** The plane surface which refracts light, is called transparent surface. In diagram, XY is the section of a plane transparent surface.

◆ **Point of incidence :** The point on transparent surface, where the ray of light meets it, is called point of incidence. In diagram, Q is the point of incidence.

◆ **Normal :** Perpendicular drawn on the transparent surface at the point of incidence, is called normal. In diagram, N_1QN_2 is the normal on surface XY.

◆ **Incident ray :** The ray of light which strikes the transparent surface at the point of incidence, is called incident ray in diagram PQ is the incident ray.

◆ **Refracted ray :** The ray of light which travels from the point of incidence into the other medium, is called refracted ray. In diagram, QR is the refracted ray.

◆ **Angle of incidence :** The angle between the incident ray and the normal on the transparent surface at the point of incidence, is called the angle of incidence. It is represented by the symbol i. In diagram, angle PQN_1 is the angle of incidence.

◆ **Angle of refraction :** The angle between the refracted ray and the normal on the transparent surface at the point of incidence, is called angle of refraction. It is represented by symbol r. In diagram angle RQN_2 is the angle of refraction.

◆ **Plane of incidence :** The plane containing the normal and the incident ray, is called plane of incidence. For the diagram, plane of book page is the plane of incidence.

- **Plane of refraction :** The plane containing the normal and the refracted ray, is called plane of refraction. For the diagram, plane of book page is the plane of refraction.

➤ LAW OF REFRACTION OF LIGHT

- **First Law :** The incident ray, the normal to the transparent surface at the point of incidence and the refracted ray, all lie in one and the same plane.
- **Second Law :** The ratio of sine of angle of incidence to the sine of the angle of refraction is constant and is called refractive index of the second medium with respect to the first medium.

$$\frac{\sin i}{\sin r} = \mu$$

➤ REFRACTIVE INDEX

(a) Refractive Index in terms of Speed of Light:

The refractive index of a medium may be defined in terms of the speed of light as follows :

$$\text{Refractive index} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

or $\mu = \frac{c}{v}$

(b) Refractive Index in terms of Wavelength :

Since the frequency (ν) remains unchanged when light passes from one medium to another, therefore,

$$\mu = \frac{c}{v} = \frac{\lambda_{vac} \times \nu}{\lambda_{med} \times \nu} = \frac{\lambda_{vac}}{\lambda_{med}}$$

(c) Relative Refractive Index :

The relative refractive index of medium 2 with respect to medium 1 is defined as the ratio of speed of light (v_1) in the medium 1 to the speed of light (v_2) in medium 2 and is denoted by ${}_1\mu_2$.

Thus, $${}_1\mu_2 = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1}$$

As refractive index is the ratio of two similar physical quantities, so it has no **unit and dimension**.

◈ FACTORS ON WHICH THE REFRACTIVE INDEX OF A MEDIUM DEPENDS ARE :

(i) Nature of the medium.
(ii) Wavelength of the light used.
(iii) Temperature
(iv) Nature of the surrounding medium.

It may be noted that refractive index is a characteristic of the pair of the media and also depends on the wavelength of light, but is independent of the angle of incidence.

➤ REFRACTION THROUGH GLASS SLAB

(a) Refraction through a rectangular glass slab and principle of reversibility of light :

Consider a rectangular glass slab, as shown in figure. A ray AE is incident on the face PQ at an angle of incidence i. On entering the glass slab, it bends towards normal and travels along EF at an angle of refraction r. The refracted ray EF is incident on face SR at an angle of incidence r′. The emerged ray FD bends away from the normal at an angle of refraction **e**.

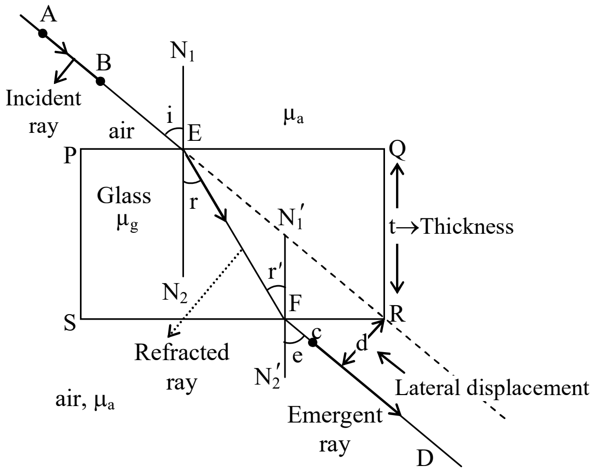
Thus the emergent ray FD is parallel to the incident rays AE, but it has been laterally displaced with respect to the incident ray. There is shift in the path of light on emerging from a refracting medium with parallel faces.

Lateral shift :

Lateral shift is the perpendicular distance between the incident and emergent rays when light is incident obliquely on a refracting slab with parallel faces.

Factors on which lateral shift depends are :

(i) Lateral shift is directly proportional to the thickness of glass slab.
(ii) Lateral shift is directly proportional to the incident angle.
(iii) Lateral shift is directly proportional to the refractive index of glass slab.
(iv) Lateral shift is inversely proportional to the wavelength of incident light.



Proof for i = e

Case-I :
For light going from air to glass at point E.

$\mu_a \sin i = \mu_g \sin r$ ..... (1)

Case-II :
For light going from glass to air at point F.

$\mu_g \sin r = \mu_a \sin e$ .....(2)

From (1) & (2) we can say that i = e

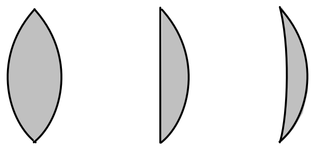
⇒ incident & emergent rays are parallel to each other.

➢ SPHERICAL LENS

◆ **Definition :** A piece of a transparent medium bounded by at least one spherical surface, is called a spherical lens.

◆ **Types :** There are two types of spherical lenses:

(i) **Convex or Converging Lenses :** These are thick in the middle and thin at the edges.



(a) (b) (c)

Fig. Three types of convex lenses

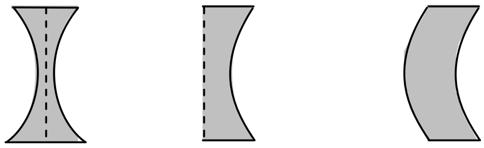
(a) **Double Convex Lens :** It has both the surfaces convex.

(b) **Plano–Convex Lens :** It has one surface plane and the other surface convex.

(c) **Concavo–Convex Lens :** It has one surface concave and the other surface convex.

(ii) **Concave or Diverging Lenses :** These are thin in the middle and thick at the edges.

There are three types of concave lenses :



(a) (b) (c)

Fig. Three types of concave lenses

(a) **Double Concave Lens :** It has both the surfaces concave. (Fig.)

(b) **Plano–Concave Lens :** It has one surface plane and the other surface concave. (fig.)

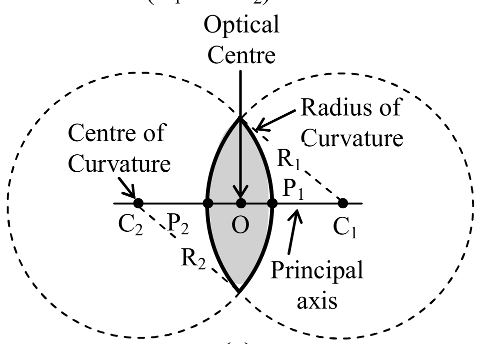
(c) **Convexo–Concave Lens :** It has one surface convex and the other surface concave. (fig.)

◈ **SOME ASSOCIATED TERMS :**

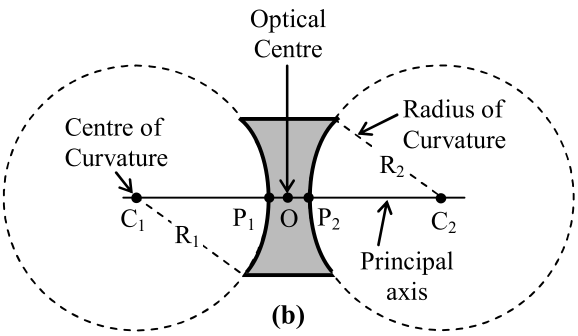
(i) Centre of curvature (C) :
The centre of curvature of the surface of a lens is the centre of the sphere of which it forms a part, because a lens has two surfaces, so it has two centres of curvature. In figure (a) and (b) points, C_1 and C_2 are the centres of curvature.

(ii) Radius of curvature (R) :
The radius of curvature of the surface of a lens is the radius of the sphere of which the surface forms a part. R_1 and R_2 in the figure (a) and (b) represents radius of curvature.

(iii) Principal axis (C_1C_2) :
It is the line passing through the two centres of curvature (C_1 and C_2) of the lens.



(a)



(b)

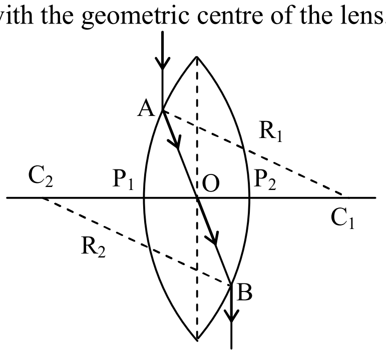
Figure : Characteristics of convex and concave lenses

(iv) Optical centre :

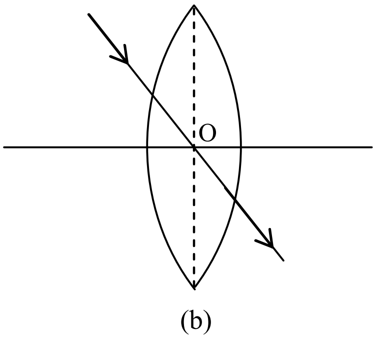
If a ray of light is incident on a lens such that after refraction through the lens the emergent ray is parallel to the incident ray, then the point at which the refracted ray intersects, the principal axis is called the optical centre of the lens.

$$\frac{OP_1}{OP_2} = \frac{P_1C_1}{P_2C_2} = \frac{R_1}{R_2}$$

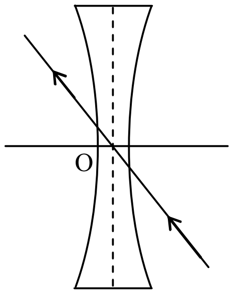
If the radii of curvature of the two surfaces are equal, then the optical centre coincides with the geometric centre of the lens.



(a)



(b)

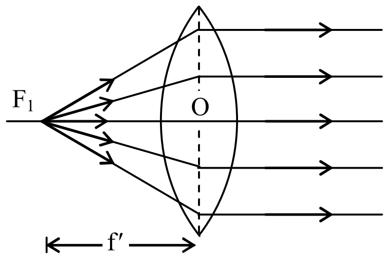


(c)

(v) Principal foci and focal length :

(A) First principal focus and first focal length :

It is a fixed point on the principal axis such that rays starting from this point (in convex lens) or appearing to go towards this point (concave lens), after refraction through the lens, become parallel to the principal axis. It is represented by F_1.



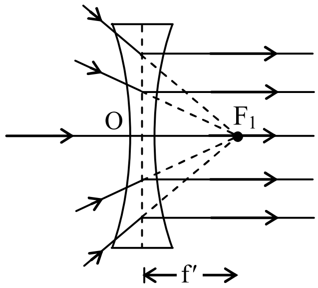
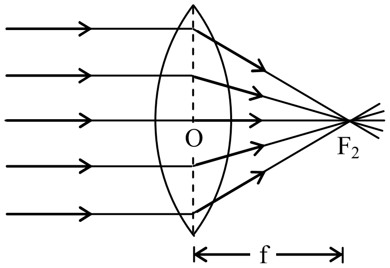


Figure : Ray diagram showing first principal focus

(B) Second principal focus and second focal length :

It is a fixed point on the principal axis such that the light rays incident parallel to the principal axis, after refraction through the lens, either converge to this point (in convex lens) or appear to diverge from this point (in concave lens). It is denoted by F_2.



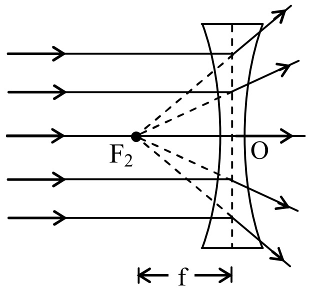


Figure : Ray diagram showing second principal focus

If the medium on both sides of a lens is same, then the numerical values of the first and second focal lengths are equal. Thus

$$f = f'$$

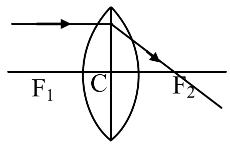
(vi) Aperture :

It is the diameter of the circular boundary of the lens.

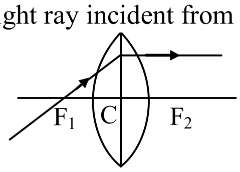
➢ RULE FOR IMAGE FORMATION BY RAY DIAGRAM METHOD

◈ THREE SPECIAL RAYS FOR CONVEX LENS

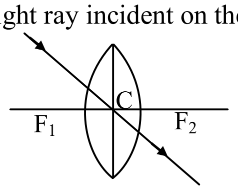
- ◆ When light ray incident parallel to principal axis.



- ◆ When light ray incident from focus.

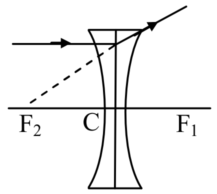


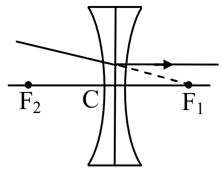
- ◆ When light ray incident on the pole.

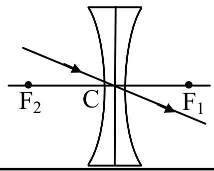


◈ THREE SPECIAL RAYS FOR CONCAVE LENS

- ◆ When light ray incident parallel to principal axis.







➢ IMAGE FORMATION BY LENS

- ◆ **Introduction :** From lens formula, we find that for a lens of a fixed focal length as object distance u changes, image distance ν also changes. Moreover, as u decreases or increases, this changes the position, the nature and the size of the image.

  Different cases, are as given below with their ray diagrams.

◈ CONVEX LENS IN DIFFERENT CASES

Case 1 : Object at Infinity

- ◆ **A point object lying on the principal axis**

  Rays come parallel to the principal axis and after refraction from the lens, actually meet at the second principal focus F_2.

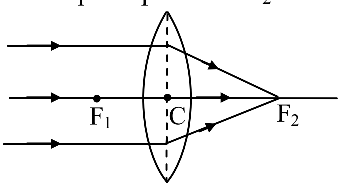


Fig. Convex lens point object at infinity, image at focus.

The image is formed at focus F_2. It is real and point sized.

- ◆ **A big size object with its foot on the principal axis**

Parallel rays come inclined to the principal axis. Image of foot is formed at the focus.

Image is formed at the second principal focus F_2. It is real inverted and diminished (smaller in size than the object). (Fig.)

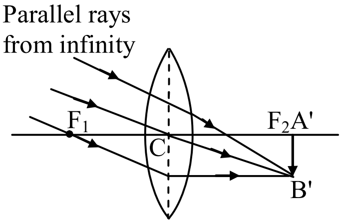


Fig. Convex lens : big size object at infinity, image at focus

Case 2 : Object at distance more than twice the Focal Length

Real object AB has its image A′B′ formed between distance f and 2f.

The image is real inverted and diminished (smaller in size than the object)

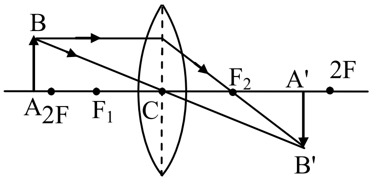


Fig. Convex lens : object beyond 2f, image between f and 2f.

Case 3. Object at distance twice the Focal Lengths

Real object AB has its image A′B′ formed at distance 2f.

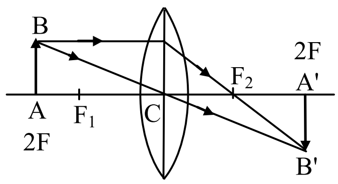


Fig. Convex lens : object at distance 2f, image at distance 2f.

The image is real, inverted and has same size as the object.

Case 4 : Object at distance more than Focal Length and less than twice is Focal Length

Real object AB has its image A′B′ formed beyond distance 2f.

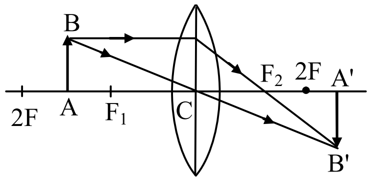


Fig. Convex lens : object at distance between f and 2f, image beyond 2f.

The image is real inverted and enlarged (bigger in size than the object).

Case 5 : Object at Focus

Real object AB has its image formed at infinity.

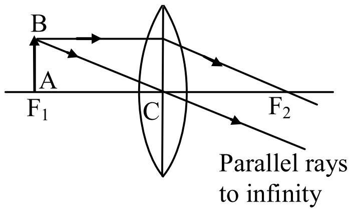
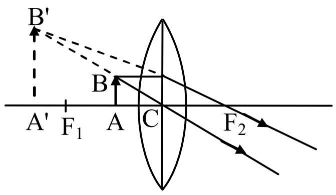


Fig. Convex lens : object at focus, image at infinity.

The image is imaginary inverted (refracted rays to downward) and must have very large size.

Case 6 : Object between Focus and Optical Centre

Real object AB has its image A′B′ formed in front of the lens.



◈ CONCAVE LENS IN DIFFERENT CASES

Case 1 : Object at infinity

- ◆ A point object lying on the principal axis.

  Rays come parallel to the principal axis and after refraction from the lens, appears to come from the second principal focus F_2.

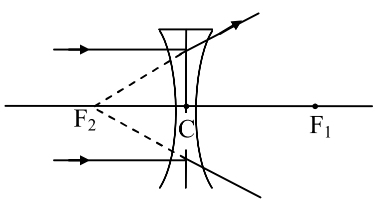


Fig. Concave lens point object at infinity, image at focus.

The image is formed at focus F_2. It is virtual and point sized (fig.)

◆ A big size object with its foot on the principal axis.

Parallel rays come inclined to the principal axis. Image of foot is formed at focus.

The image is formed at the second principal focus F_2.

It is virtual–erect and diminished (fig.)

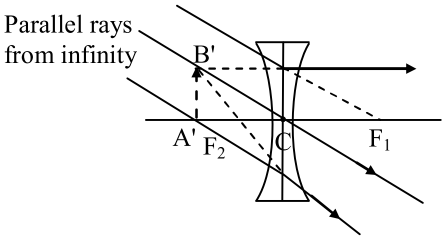


Fig. Concave lens : big size object at infinity image at focus.

Case 2 : Object at a Finite Distance

Real object AB has its image A′B′ formed between second principal focus F_2 optical centre C.

The image is virtual–erect and diminished.

NUMERICAL METHOD IN LENS

(A) LENS FORMULA

◆ **Definition :** The equation relating the object distance (u), the image distance (v) and the focal length (f) of the lens is called the lens formula.

◆ **Assumptions made :**

1. The lens is **thin.**
2. The lens has a **small aperture.**
3. The object lies **close** to principal axis.
4. The incident rays make **small angles** with the lens surface or the principal axis.

***Lens Formula**:* $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

(B) LINEAR MAGNIFICATION FOR LENS

◈ **LINEAR MAGNIFICATION**

◆ **Definition :** The ratio of the size of the image formed by refraction from the lens to the size of the object, is called linear magnification produced by the lens. It is represented by the symbol m.

If I be the size of the image and O be the size of the object, then

$$m = \frac{I}{O} = \frac{v}{u}$$

(C) POWER OF LENS

◆ **Definition :** It is the capacity or the ability of a lens to deviate (converge or diverge) the path of rays passing through it. A lens producing more converging or more diverging, is said to have more power

Power of lens (in diopter) $\propto \frac{1}{f(\text{in metre})}$

TOTAL INTERNAL REFLECTION

◆ **Definition :** When light travels from a denser medium to a rarer medium and is incident at an angle more than the critical angle for that medium, it is completely returned inwardly in the denser medium. This complete inward return of light is called **total** (complete) **internal** (inward) **reflection** (return).

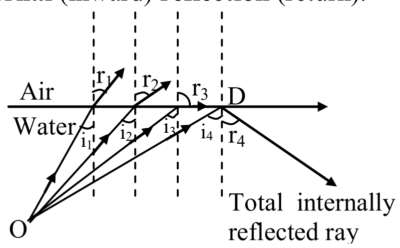


Fig. Total internal reflection.

◈ **CRITICAL ANGLE**

The angle of incidence in denser medium for which angle of refraction is 90°, is called the critical angle. It is represented by the symbol C.

⇒ $\mu_w \sin i_3 = \mu_a \sin 90°$

⇒ $\sin i_3 = \frac{1}{\mu_w}$

[Note. More is the value of μ, lesser will be angle C].

◆ **Condition**

(i) Light must travel from denser to rarer medium.

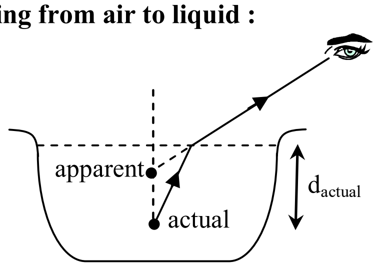
(ii) Light must be incident at an angle more than the critical angle for the denser medium.

Merit : In total internal reflection 100% light is reflected, hence images formed are more bright.

In ordinary reflection from mirrors, only 85% light is reflected, rest 15% is either absorbed by mirror glass or transmitted due to poor polish. Images formed by ordinary reflection are less bright.

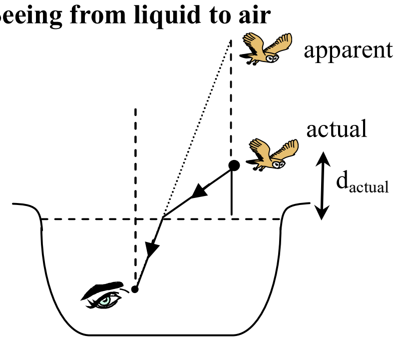
➤ REAL & APPARENT DEPTH & HEIGHT

(A) Seeing from air to liquid :



$$\text{apparent depth from surface} = \frac{d_{actual}}{\mu}$$

(B) Seeing from liquid to air



$$\text{apparent height from surface} = H_{actual} \times \mu$$

✦ SOLVED EXAMPLES ✦

Ex.1 Speed of light in water is 2.25×10^8 m/s. Calculate the refractive index of water.

Sol. Refractive index is given by

$$n = \frac{\text{speed of light in vaccum (c)}}{\text{speed of light in water } (v)}$$

$$= \frac{3\times10^8 \text{ m/s}}{2.25\times10^8 \text{ m/s}} = 1.33$$

Ex.2 Refractive index of diamond is 2.42. Calculate the speed of light in diamond.

Sol. We know that refractive index,

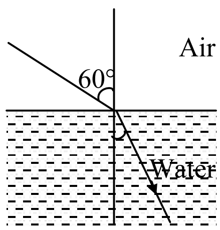
$$n = \frac{c}{v} = \frac{\text{speed of light in vaccum}}{\text{speed of light in diamond}}$$

or $\quad 2.42 = \dfrac{3\times10^8}{v}$

or $\quad v = \dfrac{3\times10^8}{2.42} = 1.24 \times 10^8$ m/s.

Ex.3 A ray of light travelling in air falls on the surface of water. The angle of incidence is 60° with the normal to the surface. The refractive index of water = 4/3. Calculate the angle of refraction.

Sol.



We know that $\dfrac{\sin i}{\sin r} = n$

Here, i = 60°, n = 4/3

$\therefore \quad \dfrac{\sin 60°}{\sin r} = \dfrac{4}{3}$

or $\quad \dfrac{\sqrt{3}/2}{\sin r} = \dfrac{4}{3}$

or $\quad \sin r = \dfrac{3\sqrt{3}}{8} = 0.65$

$\therefore \quad r = \sin^{-1} 0.65$

Ex.4 If the refractive index of water is 4/3 and that of glass is 3/2. Calculate the refractive index of glass with respect to water.

Sol. We known that

$$^{w}\mu_g = \frac{\mu_g}{\mu_w}$$

where $^{w}\mu_g$ = refractive index of glass with respect to water

μ_g = refractive index of glass = 3/2

μ_w = refractive index of water = 4/3

$$\therefore \quad ^{w}\mu_g = \frac{3/2}{4/3} = \frac{9}{8} = 1.1$$

Ex.5 A ray of light is incident on the plane surface of a transparent medium at an angle 60° with the normal. The angle of refraction is 30°. Calculate the refractive index of the transparent material.

Sol. Here,

Angle of incidence, $i = 60°$

Angle of refraction, $r = 30°$

Refractive index,

$$n = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Ex.6 A ray of light travelling in air falls on the surface of a glass slab at an angle 45° with the normal. The refractive index of glass is 1.5. Calculate the angle of refraction.

Sol. Angle of incidence = 45°

Refractive index of glass, $n = 1.5$

Since $n = \frac{\sin i}{\sin r}$

or $1.5 = \frac{\sin 45°}{\sin r}$

or $\sin r = \frac{\sin 45°}{1.5} = \frac{1/\sqrt{2}}{1.5}$

$= \frac{1}{\sqrt{2} \times 1.5}$

$= \frac{1}{1.41 \times 1.5} = \frac{1}{2.115} = 0.47$

$\therefore \quad r = \sin^{-1} 0.47$

Ex.7 The refractive index of diamond is 2.42 and that of carbon disulphide is 1.63. Calculate the refractive index of diamond with respect to carbon disulphide.

Sol. Refractive index of carbon disulphide, $n_1 = 1.63$

Refractive index of diamond, $n_2 = 2.42$

$\therefore$ Refractive index of diamond with respect for carbon disulphide,

$$^{1}n_2 = \frac{n_2}{n_1} = \frac{2.42}{1.63} = 1.48$$

Ex.8 A coin is placed in a tumbler, water is then filled in the tumbler to a height of 20 cm. If the refractive index of water is 4/3, calculate the apparent depth of the coin.

Sol. Here,

Real depth, $h = 20$ cm

Refractive index, $n = 4/3$

Now, $n = \frac{\text{real depth}}{\text{apparent depth}}$

or $\frac{4}{3} = \frac{20}{\text{apparent depth}}$

or Apparent depth $= \frac{20 \times 3}{4} = 15$ cm

Ex.9 There is a black spot on a table. A glass slab of thickness 6 cm is placed on the table over the spot. Refractive index of glass is 3/2. At what depth from the upper surface will the spot appear when viewed from above?

Sol. Real depth of the spot = 6 cm

Refractive index of glass, $n = \frac{3}{2}$

Now, $n = \frac{\text{real depth}}{\text{apparent depth}}$

or $\frac{3}{2} = \frac{6}{\text{apparent depth}}$

$\therefore$ Apparent depth $= \frac{6 \times 2}{3} = 4$ cm

Ex.10 Refractive index of diamond is 2.42 and that of glass is 1.5. Calculate the critical angle for diamond-glass surface.

Sol. Refractive index of diamond, $n_1 = 2.42$

Refractive index of glass, $n_2 = 1.5$

Now, $\sin i_C = \frac{n_2}{n_1}$

$$= \frac{1.5}{2.42} = 0.6198$$

$\therefore$ $i_C = \sin^{-1} 0.62$

Ex.11 Refractive index of glass is 3/2. A ray of light travelling in glass is incident on glass-water surface at an angle 30° with normal. Will it be able to come out into the water Refractive index of water = 4/3.

Sol. Refractive index of glass, $n_1 = 3/2$

Refractive index of water, $n_2 = 4/3$

Now, $\sin i_C = \frac{n_2}{n_1} = \frac{4/3}{3/2} = \frac{8}{9} = 0.88$

$\therefore$ $i_C = 62°$

Since, the angle of incidence (30°) is less than the critical angle, the ray will be refracted into the water.

Ex.12 The refractive index of dense flint glass is 1.65 and that of alcohol is 1.36, both with respect to air. What is the refractive index of flint glass with respect to alcohol ?

Sol. Refractive index of flint glass, $n_2 = 1.65$

Refractive index of alcohol, $n_1 = 1.36$

$\therefore$ Refractive index of flint glass with respect to alcohol is given by

$$^1n_2 = \frac{n_2}{n_1} = \frac{1.65}{1.36} = 1.21$$

Ex.13 An object is placed 36 cm from a convex lens. A real image is formed 24 cm from the lens. Calculate the focal length of the lens.

Sol. According to the sign convention the object is placed on the left-hand side of the lens. So object distance (u) is negative. Real image is formed on the other side of the lens. So the image distance (v) is positive. Thus, u = –36 cm, v = +24 cm, f = ?

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{+24} - \frac{1}{-36} = \frac{1}{f}$$

or $$\frac{1}{f} = \frac{1}{24} + \frac{1}{36} = \frac{5}{72}$$

$\therefore$ $$f = \frac{72}{5} = 14.4 \text{ cm}$$

Ex.14 A 2 cm long pin is placed perpendicular to the principal axis of a lens of focal length 15 cm at distance of 25 cm from the lens. Find the position of image and its size.

Sol. Here, u = –25 cm, f = +15

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

or $$\frac{1}{v} - \frac{1}{-25} = \frac{1}{+15}$$

or $$\frac{1}{v} = \frac{1}{15} - \frac{1}{25} = \frac{2}{75}$$

or $$v = \frac{75}{2} = 37.5 \text{ cm}$$

The positive sign shows that the image is formed on the right-hand side of the lens.

Magnification is given by

$$m = \frac{h'}{h} = \frac{v}{u}$$

or $$= \frac{h'}{h} = \frac{37.5}{-25} = -1.5$$

$\therefore$ h = – 1.5 × h = –1.5 × 2 cm

= –3 cm

The image of the pin is 3 cm long. The negative sign shows that it is formed below the principal axis, i.e. the image is inverted.

Ex.15 A point object is placed at a distance of 18 cm from a convex lens on its principal axis. Its image is formed on the other side of the lens at 27 cm. Calculate the focal length of the lens.

Sol. According to the sign convention, the object is placed on the left-hand side of the lens, therefore object-distance is negative,

i.e. u = –18 cm. Since the image is formed on the other side, the image-distance is positive, i.e., v = +27 cm. Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \text{ we have}$$

$$\frac{1}{+27} - \frac{1}{-18} = \frac{1}{f}$$

or $$\frac{1}{27} + \frac{1}{18} = \frac{5}{54} = \frac{1}{f}$$

or $$f = \frac{54}{5} = 10.8 \text{ cm}$$

Ex.16 A convex lens forms an image of the same size as the object at a distance of 30 cm from the lens. Find the focal length of the lens. Also find power of the lens. What is the distance of the object from the lens ?

Sol. A convex lens forms the image of the same size as the object only when the object is placed at a distance 2f from the lens. In this case the image is also equal to 2f from the lens.

Hence, $2f = 30$ cm

or $f = 15 \text{ cm} = 0.15$ m

Power of the lens, $P = \frac{1}{f} = \frac{1}{0.15} D = 6.6D$

The distance of the object from the lens is also $2f = 30$ cm.

Ex.17 A 3 cm high object is placed at a distance of 80 cm from a concave lens of focal length 20 cm. Find the position and size of the image.

Sol. Here, $u = -80$ cm, $f = -20$ cm

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

or $\frac{1}{v} - \frac{1}{-80} = \frac{1}{-20}$

or $\frac{1}{v} = -\frac{1}{20} - \frac{1}{80} = \frac{-5}{80} = -\frac{1}{16}$

or $v = -16$ cm

Magnification, $m = \frac{h'}{h} = \frac{v}{u} = \frac{-16}{-80} = \frac{1}{5}$

or $h' = \frac{h}{5} = \frac{3.0}{5} = 0.6$ cm

Length of image is 0.6 cm. Positive sign shows that the image is erect.

Ex.18 An object is placed on the principal axis of a concave lens at a distance of 40 cm from it. If the focal length of the lens is also 40 cm, find the location of the image and the magnification.

Sol. For a concave lens focal length f is negative, i.e. $f = -40$ cm. Since by convention, object is placed on the left of the lens, so $u = -40$ cm.

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

or $\frac{1}{v} - \frac{1}{-40} = \frac{1}{-40}$

or $\frac{1}{v} = -\frac{1}{40} - \frac{1}{40} = -\frac{1}{20}$

or $v = -20$ cm

The image is formed 20 cm from the lens. Minus sign shows that the image is formed on the same side of the lens as the object.

Now, magnification, $m = \frac{h'}{h} = \frac{v}{u} = \frac{-20}{-40} = \frac{1}{2}$

Positive sign shows that the image is erect.

Ex.19 A beam of light travelling parallel to the principal axis of a concave lens appears to diverge from a point 25 cm behind the lens after refraction. Calculate the power of the lens.

Sol. When a parallel beam after refraction through the lens is incident on a concave lens, it appears to diverge from the focus of the lens. Hence, the focal length of the lens is 25 cm. According to sign convention, focal length of a concave lens is negative.

$\therefore$ $f = -25 \text{ cm} = -0.25$ m

$\therefore$ Power, $P = \frac{1}{f} = \frac{1}{-0.25} = -4D$

Ex.20 A convex lens of power 5D is placed at a distance of 30 cm from a screen. At what distance from the lens should the screen be placed so that its image is formed on the screen?

Sol. Power of the lens, $P = +5D$

$\therefore$ Focal length, $f = \frac{1}{5D} = \frac{1}{5} = 0.20 \text{ m} = 20$ cm

Here, the screen is placed 30 cm from the lens.

$\therefore$ $v = +30$ cm, $f = +20$ cm, $u = ?$

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$\frac{1}{30} - \frac{1}{u} = \frac{1}{20}$

or $\frac{1}{u} = \frac{1}{30} - \frac{1}{20} = -\frac{1}{60}$

or $u = -60$ cm

Therefore, the screen should be placed at 60 cm from the lens.

Ex.21 A pin 3 cm long is placed at a distance of 24 cm from a convex lens of focal length 18 cm. The pin is placed perpendicular to the principal axis. Find the position, size and nature of the image.

Sol. Here, $u = -24$ cm, $f = +18$ cm, $v = ?$

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$\frac{1}{v} - \frac{1}{-24} = \frac{1}{+18}$

or $\frac{1}{v} = \frac{1}{18} - \frac{1}{24} = \frac{1}{72}$

or $v = 72$ cm

The image is formed 72 cm from the lens on the other side. So the image is real.

Magnification, $m = \frac{h'}{h} = \frac{v}{u} = \frac{72}{-24} = -3$

or $h' = -3 \times h = -3 \times 3.0 = -9$ cm

The image is 9 cm in size. Negative sign shows that the image is inverted.

Ex.22 A convex lens of focal length 40 cm and a concave lens of focal length 25 cm are placed in contact in such a way that they have the common principal axis. Find the power of the combination.

Sol. Focal length of the convex lens,

$f_1 = 40$ cm $= +0.4$ m

$\therefore$ Power of the convex lens,

$$P_1 = \frac{1}{+0.40} = +2.5D$$

Focal length of the concave lens,

$$f_2 = -25 \text{ cm} = -0.25 \text{ m}$$

$\therefore$ Power of the concave lens,

$$P_2 = \frac{1}{-0.25} = -4D$$

Power of the combination,

$$P = P_1 + P_2 = 2.5 - 4D = -1.5D$$

Ex.23 A concave lens has a focal length of 15 cm. At what distance should the object be from the lens placed so that it forms an image 10 cm from the lens ? Also find the magnification.

Sol. A concave lens always forms a virtual, erect image on the same side as the object.

Image distance, $v = -10$ cm

Focal length $f = -15$ cm

Object distance, $u = ?$

Using, the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

or $\frac{1}{-10} - \frac{1}{u} = \frac{1}{-15}$

or $-\frac{1}{10} = \frac{2-3}{30} = -\frac{1}{30}$

or $u = -30$ cm

Thus, the object should be placed 30 cm on the lens.

Magnification, $m = \frac{v}{u} = \frac{-10}{-30} = \frac{1}{3} = 0.33$

The positive sign shows that the image is erect and virtual. The size of the image is one-third of that of the object.

Ex.24 A 2 cm tall object is placed perpendicular to the principal axis of a convex lens of focal length 10 cm. The distance of object from the lens is 15 cm. Find the position, nature and size of the image. Calculate the magnification of the lens.

Sol. Object distance, $u = -15$ cm

Focal length, $f = +10$ cm

Object height, $h = +2$ cm

Image distance, $v = ?$

Image height, $h' = ?$

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} = \frac{1}{-15} = \frac{1}{+10}$$

or $\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{+1}{30}$

or $v = +30$ cm

Positive sign of v shows that the image is formed at a distance of 30 cm on the right side of the lens. Therefore the image is real and inverted

Magnification, $m = \frac{h'}{h} = \frac{v}{u}$

$$\frac{h'}{2.0} = \frac{+30}{-15} = -2$$

or $h' = -2 \times 2 = -4$ cm

Magnification, $m = \frac{v}{u} = \frac{30}{-15} = -2$

Negative sign with the magnification and height of the image shows that the image is inverted and real. Thus, a real image of height 4 cm is formed at a distance of 30 cm on the right side of the lens. Image is inverted and twice the size of the object.

◈ IMPORTANT POINTS TO BE REMEMBER

- ◆ **Refraction :** The bending of a ray of light as it passes from one medium to another is called refraction.

(a) A ray of light travelling from a rarer medium to denser medium (say, water or glass) bends towards the normal.
(b) A ray of light travelling from a denser to a rarer medium bends (or refractive) away from the normal.

- **Laws of refraction :**

(a) The ratio of sine of the angle of incidence to the sine of the angle of refraction for a particular pair of media is constant, i.e. sin i / sin r = constant is equal to the refractive index of the medium into which the light is entering.
(b) The incident ray, the refracted ray and the normal all lie in the same plane.

- **Lateral displacement :** The perpendicular distance of separation between the emergent ray and the original path of the incident ray is called lateral displacement.
- **Lens :** A piece of any transparent material bound by two curved surfaces is called a lens. A lens which is thicker in the middle and thinner at the edges is called a convex lens. A convex lens is also called converging lens.
  A lens which is thicker at the edges and thinner at the centre is called a concave lens.
  A concave lens is also called a diverging lens.
- **Optical centre of a lens :** The centre point of a lens is called its optical centre. A ray of light passing through the optical centre does not suffer any deviation.
- **Image formed by the lenses :** A convex lens forms real and inverted images for all the positions of an object at and outside the focus (F). However, when the object is placed between F and O, the image formed by a convex lens is virtual and erect.
  A concave lens always forms a virtual, erect and a diminished image whatever may be the distance of the object from the lens.
- **Lens formula :**

  The lens formula is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$.

- **Power of a lens :** Reciprocal of the focal length of a lens measured in metres is called its power. Power of a lens is described in dioptre (D) unit.

- **Refractive indices of various substances relative to vacuum with light of wavelength 589 mm**

- **Solids (at 20ºC)**

Substance	Refractive index
Diamond	2.42
Ruby	1.71
Sapphire	1.77
Quartz (fused)	1.46
Canada Balsam	1.53
Rock salt	1.54
Glass (crown)	1.52
Glass (flint)	1.66
Ice	1.31

- **Liquids (at 20ºC)**

Substance	Refractive index
Water	1.33
Ethyl alcohol	1.36
Kerosene	1.44
Turpentine oil	1.47
Glycerine	1.47
Benzene	1.5
Carbon disulphide	1.63

- **Gases (at 0ºC, 1 atm)**

Substance	Refractive index
Air	1.00029
Carbon dioxide	1.00045