SIMPLIFICATION AND APPROXIMATION

Helping Hands :

1. Digits - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

2. Types of numbers.

(i) Natural numbers. = {1, 2, 3, 4,.....}

(ii) Whole numbers = {0, 1, 2, 3, 4.....}

(iii) Integers = {....., -3, -2, -1, 0, 1, 2, 3,.....}

(iv) Real numbers = {----, 2.8, -2, -10, 1, 1.9,--2, 3, 3.12, 3.13-----}

(v) Even numbers = {2, 4, 6,---------}

(vi) Odd numbers = {1, 3, 5, 7----------}

(vii) Prime numbers = {2, 3, 5, 7, 11, 13, 17, 19,------}

3. $(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

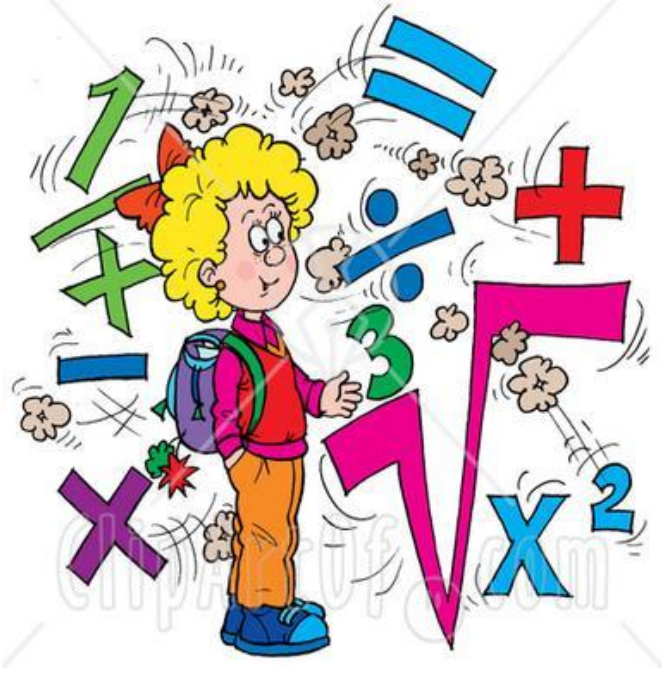
$a^2 - b^2 = (a+b).(a-b)$

$(a+b)^3 = a^3 + b^3 + 3ab\,(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab\,(a-b)$

$a^3 + b^3 = (a+b)\,(a^2 + b^2 - ab)$

$a^3 - b^3 = (a-b)\,(a^2 + b^2 + ab)$



(I) ADDITION & SUBTRACTION

Ex-1. ? = 8+88+888+8888+88888

? = 8(1+11+111+1111+11111)

? = 8(12345) ⇒ 98760

Ex-2. ? = 0.8 + 0.88 + 0.888 + 0.8888 + 0.88888

? = 8(0.1+0.11+0.111+0.1111+0.11111)

? = 8(0.54321) ⇒ 4.34568

Ex-3. $8.\underline{8} + 8.\underline{88} + 8.\underline{888} + 8.\underline{8888} + 8.\underline{88888} = ?$

First we can calculate decimal number and then whole no.

From Ex-2.

8(0.54321) = 4.34568

and 8+8+8+8+8 i.e. 8×5=40

Therefore, 40+4.34568 = 44.34568

Ex-4. 8456+3891+4560 = ?

Ans = 16907

Ex-5. 3.981+14.34+12.5= ?

First we can balance the number of decimal digits and then use elimination method.

i.e. 3.981 +14.340 + 12.500 = ?

Ans = 30.821

Ex-6. 5638 - 4131 + 8112 - 6888 = ?

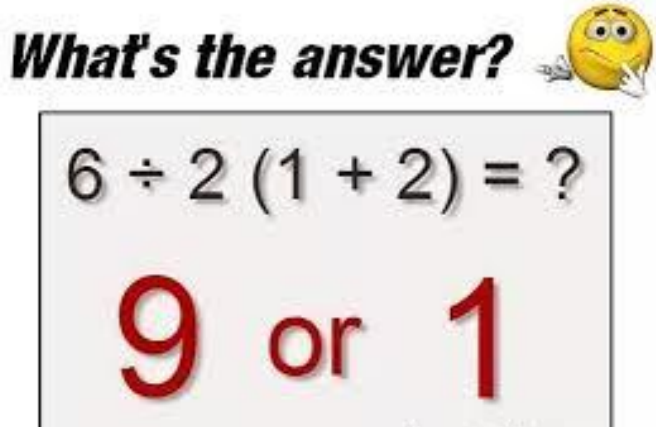
Ans = 2731

Ex-7. 8123-5688 + 3471 -(?)=4314

First we transfer the ? from left to right hand side and number 4314 from right to left hand side and then use elimination method.

8123-5688 +3471- 4314 = ?

Ans = 1592

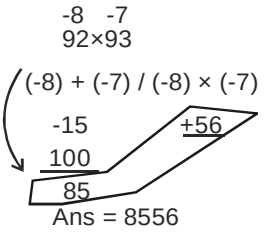


(II) MULTIPLICATION

(A) 100-Base (Near about 100)

Type-I. When both numbers are less than 100.

(1).

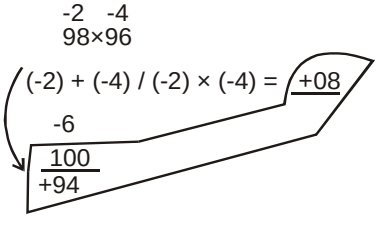


First we check and compare to 100 we find that first number is less by 8.

i.e. (-8) and 2nd number is also less by 7 (i.e. -7).

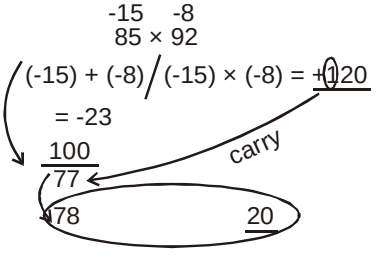
Rule - The right hand side part should be always positive and two digits number.

II.



Ans = 9408

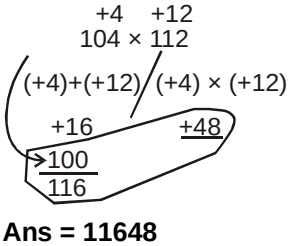
III.



Ans = 7820

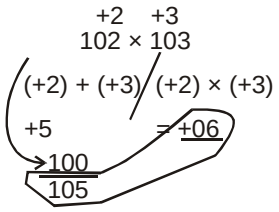
Type-II.(When both numbers are more than 100)

I.



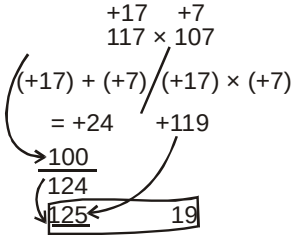
Ans = 11648

II.



Ans. = 10506

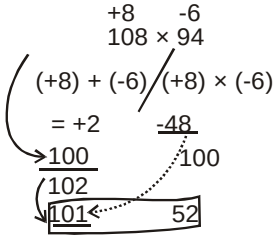
III.



Ans. 12519

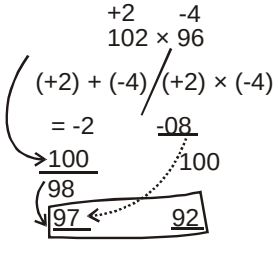
Type-III.(When first number is more than 100 and other one is less than 100)

I.



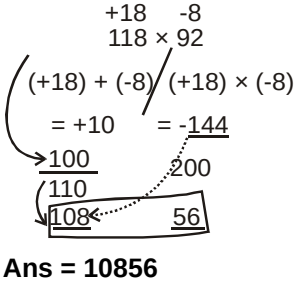
Ans. = 10152

II.



Ans. 9792

III.



Ans = 10856

200 Base (near about 200)

Type-I. When both numbers are less than 200.

I.

-4 -8
196 × 192
(-4) + (-8) / (-4) × (-8)
= -12 = +32
200
+188
×2
376

Ans. 37632

Type-II. When both numbers are more than 200.

I.

+8 +11
208 × 211
(+8) + (+11) / (+8) × (+11)
= +19 = +88
200
219
×2
438

Ans. 43888

_____Same as.....

Type-III. When one number is more than and other one is less than 200

I.

+12 -2
212 × 198
(+12) + (-2) / (+12) × (-2)
= +10 = -24
200
210
×2
420
419 76

Ans. 41976

_____Same as.....

Similarly we can use 300, 400, ...1000.....base multiplication.

Some Special Types

I. When sum of unit digit is 10 and remaining digit is same.

Exp-I. 43 × 47 = 4 × (4+1) / 3×7

_____= 4 × 5 / 21

_____= 20 / 21

Ans = 2021.

Exp-II. 72 × 78 = 7×8 / 2 × 8

= 56 / 16

Ans = 5616

Exp-III. 104 × 106 = 10 × (10 +1) / 4 × 6

= 10 × 11 / 24

= 110 / 24

Ans = 11024

2. When sum of ten's digit is 10 and unit digit is same

Exp-I. 46 × 66

= (4 × 6) + 6 / 6 × 6

= 24 + 6 / 36

_____=___30 /___36

_____**Ans = 3036**

Exp-II. 83 × 23

= (8 × 2) + 3 / 3 × 3

= 19 / 09

_____**Ans = 1909**

Exp-III. 92 × 12

= (9 × 1) + 2 / 2 × 2

= 11/04

_____**Ans = 1104**

3. When unit digit is 5 in both the numbers and difference between both number is 10.

Exp-I. 75 × 65

= 6 × (7 + 1) / 75

= 48 / 75

Ans = 4875

Exp-II. 45 × 35

= 3 × (4+1) / 75

_____=___15 / 75

_____**Ans = 1575**

Exp-III. 105 × 95

= 9 × (10 + 1) / 75

_____= 99 / 75

Ans = 9975

4.

$25 \rightarrow 5^2 \rightarrow \frac{100}{2^2} = \frac{100}{4}$

$125 \rightarrow 5^3 \rightarrow \frac{1000}{2^3} = \frac{1000}{8}$

$625 \rightarrow 5^4 \rightarrow \frac{10000}{2^4} = \frac{10000}{16}$

ie.

$25 \rightarrow = \frac{100}{4}$

$125 \rightarrow = \frac{1000}{8}$

$625 \rightarrow = \frac{10000}{16}$

Remember this

Exp-I. 64 × 125

=

Ans = 8000

Exp-II. 192 × 625

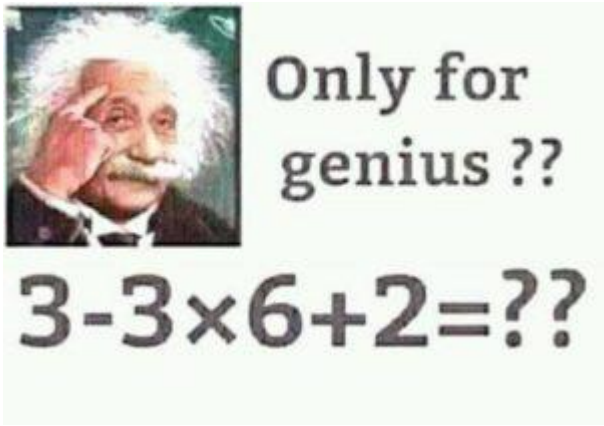
= 12 × 10000

Ans = 1,20,000

Exp-III. 84 × 25

=

= 2100



(III) SQUARE AND SQUARE ROOTS

Square of 1-50 numbers

1^2	=	1	26^2	=	676
2^2	=	4	27^2	=	729
3^2	=	9	28^2	=	784
4^2	=	16	29^2	=	841
5^2	=	25	30^2	=	900
6^2	=	36	31^2	=	961
7^2	=	49	32^2	=	1024
8^2	=	64	33^2	=	1089
9^2	=	81	34^2	=	1156
10^2	=	100	35^2	=	1225
11^2	=	121	36^2	=	1296
12^2	=	144	37^2	=	1369
13^2	=	169	38^2	=	1444
14^2	=	196	39^2	=	1521
15^2	=	225	40^2	=	1600
16^2	=	256	41^2	=	1681
17^2	=	289	42^2	=	1764
18^2	=	324	43^2	=	1849
19^2	=	361	44^2	=	1936
20^2	=	400	45^2	=	2025
21^2	=	441	46^2	=	2116
22^2	=	484	47^2	=	2209
23^2	=	529	48^2	=	2304
24^2	=	576	49^2	=	2401
25^2	=	625	50^2	=	2500

Type-I. Formula Method

We know that

$(a+b)^2 = a^2 + 2ab + b^2$ i.e. $(a/b)^2 = a^2 / 2ab / b^2$

Ex-1. $(56)^2$

$= (5/6)^2$

$= 5^2 / 2 \times 5 \times 6 / 6^2$

$= 25 / 60 / 36$ (6, 3)

$= 31 / 3 / 6$

$= 3136$

We break number in two parts i.e. 5 & 6 and follow the rule of $(a+b)^2 = a^2 /2ab/ b^2$

Ex-2. $(92)^2$

$= (9/2)^2$

$= 81/^3 6 /^0 4$

$= 8464$

Ex-3. Find the square of 112

I. Method :

(We break 112 in two parts like that 11/2)

$(11/ 2)^2 = 11^2 / 2 \times 11 \times 2 / 2^2$

$121 / 44 / 4$

= 125 / 4 / 4

Ans. = 12544

II. Method :

(We break 112 in two parts like that 1 / 12)

$(1/ 12)^2 = 1^2 / 2 \times 1 \times 12 / 12^2$

$= 1 / 24 / 144$

= 1 / 25 / 44

Ans = 12544

Ex-4. Find the square of 211

$(2 / 11)^2 = 2^2 / 2 \times 2 \times 11 / 11^2$

$= 4 / 44 / 121$

Ans. 44521

In $(998)^2$ — the above rule is failed

Now we can use Base-method.....

100-Base Method (When the number is near about 100).

Type-I :

(When number is less than 100).

Ex-1. $(96)^2$

In this example we can use above method but we also use 100 base method.

Let see in this example 96 is less than 100 by 4.

Find the square of 4

Subtract 4 from 96.

$96^2 = (96-4) / 4^2$

$96^2 = 92 / 16$

$96^2 = 9216$

Ex-2. $98^2 = (98-2) / 2^2$

= 96 / 04

$98^2 = 9604$

Ex-3. $(85)^2 = (85-15) / 15^2$

$= 70 / 25$

= 72 / 25

$85^2 = 7225$

Type-II :

(When the number is more than 100)

Ex-1. $(108)^2 = 108 + 8 / 8^2$

= 116 / 64

$(108)^2$ =11664

Since 108 is more than 100 by 8 we add 8 to 108.

Ex-2.

$(103)^2 = 103 + 3 / 3^2$

= 106 / 09

$(103)^2$ = 10609

Since base is 100 therefore right part always be in two digit.

$(112)^2 = 112 + 12 / 12^2$

Ex-3. $= 124 / 144$

$(112)^2 = \underline{125}\ \underline{44}$

200 - Base Method (Near about 200)

Type-III :

(When the number is less than 200)

Ex-1.

$= (192)^2 = 192-8 / 8^2$

= 184 / 64

×2

= 368 / 64

$(192)^2$ =368 64

Because of 200 base we multiply 184 × 2

Ex-2. $(198)^2 = 198-2 / 2^2$

= 196 / 04

= 392 / 04

$(198)^2$ = 39204

Ex-3. $(185)^2 = 185-15 / 15^2$

= 170 225

×2

= 340 / 225

= 342 / 25

= 34225

Similarly we can use 200, 300--------base method.

Type-IV :

1000 base (When the number is near about 1000)

Ex-1. $(998)^2 = 998-2 / 2^2$

= 996 / 004

$(998)^2$ = 996004

Since base is 1000 therefore Right hand side part one in three digit.

Ex-2. $(1012)^2 = 1012+12 \;/\; 12^2$

$= 1024 / 144$

$(1012)^2 = 1024144$

Ex-3. $(1008)^2 = 1008+8 \;/\; 8^2$

$= 1016 / 064$

$(1008)^2 = 1016064$

Special Type :

When unit place of the number is 5.

Ex-1. $75^2 = 7 \times 8 \;/\; 5^2$

$= 56 / 25$

$75^2 =$ **5625**

Ex-2. $85^2 = 8 \times 9 \;/\; 5^2$

$= 72 / 25$

$(85)^2 =$ **7225**

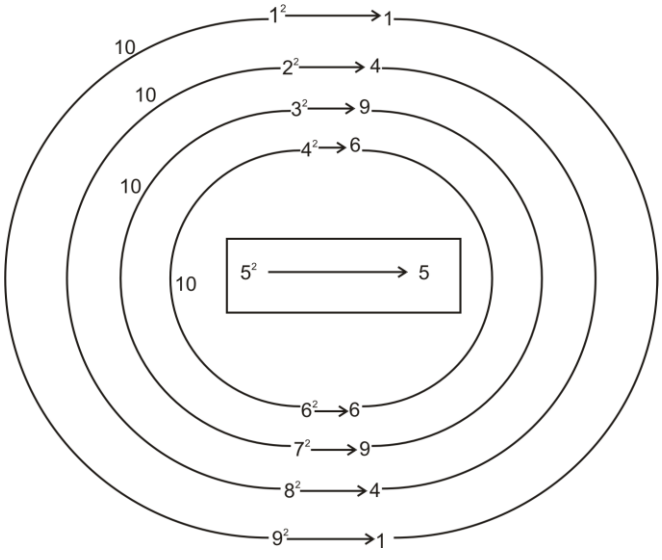
Ex-3. $105^2 = 10 \times 11 \;/\; 5^2$

$= 110 / 25$

$(105)^2 =$ **11025**

Square Roots

See the given table :



Last digit of 1^2 is 1

Last digit of 2^2 is 4

Last digit of 3^2 is 9

Last digit of 4^2 is 6

Last digit of 5^2 is 5

Last digit of 6^2 is 6

Last digit of 7^2 is 9

Last digit of 8^2 is 4

Last digit of 9^2 is 1

ie. We can find last digit 1 only when

$\overset{10}{\cdots}$ (Squaring 1 or 9)

We can find last digit 4 only when

$\overset{10}{\cdots}$ (Squaring 2 or 8)

We can find last digit 9 only when

$\overset{10}{\cdots}$ (Squaring 3 or 7)

We can find last digit 6 only when

$\overset{10}{\cdots}$ (Squaring 4 or 6)

We can find last digit 5 only when(Squaring 5 only)

Perfect square :

" If last digit of any number are 1, 4, 9, 6, 5 then it may be perfect square.

Ex-1. 25 — is a perfect square because it is a square of 5

i.e. $5^2 = 25$

36 — is a perfect square because it is a square of 6

i.e., $6^2 = 36$

64 — is a perfect square because it is a square of 8

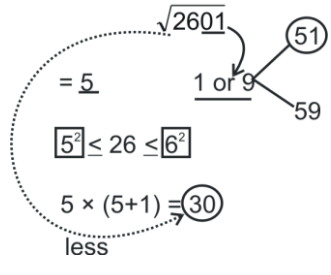
i.e., $8^2 = 64$

74 — is not a perfect square because 74 is not a square of a perfect number.

Non-perfect square :

If last digit of any number are 2, 3, 7, 8 then this number is called non-perfect square.

Ex-1. Find the $\sqrt{2601}$



Ans = 51

Rules :

- For unit place of the answer we see that last digit of the question i.e. 1 (see the above table)

For the ten's place of the ans. we always take smaller number whose square is most nearest to 26.

(i.e. 5)

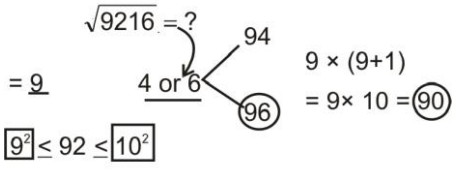
- In the above process we find two ans. i.e. 51 or 59
- We multiply ten digit number to next number

i.e. 5 × 6 = 30)

- now compare 26 to 30 (we find that 26 is less than 30).
- Therefore in both the ans. 51 and 59 the least ans is correct.

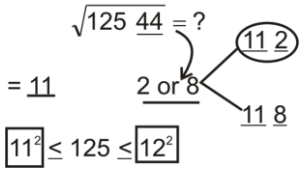
Other example.

Ex-2.



Since 92 is greater than therefore in both the ans. 96 is correct.

Ex-3.



Now 11 × 12 = **132**

125 is less than 132

∴ 112 is correct answer.

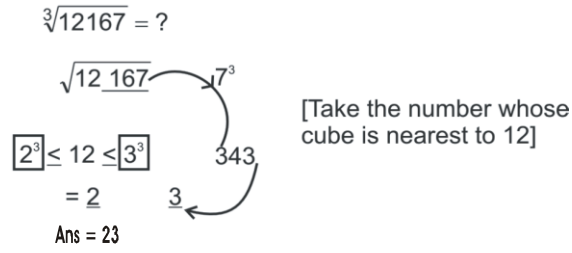
(IV) CUBE & CUBE ROOT

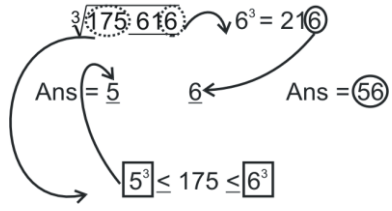
Special Note for cube : Learn cube of 1 to 25 only for the competitive exams.

Cube of the numbers:

1^3	= 1	14^3	=	2744
2^3	= 8	15^3	=	3375
3^3	= 27	16^3	=	4096
4^3	= 64	17^3	=	4913
5^3	= 125	18^3	=	5832
6^3	= 216	19^3	=	6859
7^3	= 343	20^3	=	8000
8^3	= 512	21^3	=	9261
9^3	= 729	22^3	=	10648
10^3	= 1000	23^3	=	12167
11^3	= 1331	24^3	=	13824
12^3	= 1728	25^3	=	15625
13^3	= 2197			

Ex-1. Cube root :





Ex2.

(V) FRACTIONS

is known as a fraction in which a is called numerator and b is called denominator.

Types of Fractions

I. **Proper Fraction :** If the numerator part of a fraction is less than denominator then the fraction is called proper fraction and proper fraction is always less than 1.

eg.

II. **Improper fraction :** If the numerator of a fraction is greater than denominator then the fraction is called improper fraction.

Improper fraction is always greater than 1.

$\frac{5}{4}, \frac{3}{2}, \frac{7}{5}, \frac{11}{8}$, ...etc.

III. **Mixed Fraction :**

(a) Mixed with proper fraction : When a proper fraction is mixed with a whole number known as mixed with proper fraction.

eg. $4\frac{3}{4}, 8\frac{1}{2}, 11\frac{5}{7}$ .....

(b) Mixed with Improper fraction :

When a improper fraction is mixed with a whole number, known as mixed with improper fraction.

eg. $8\frac{7}{2}, 4\frac{5}{3}, 8\frac{9}{8}$, ...etc.

Rules :

I. $8+\frac{3}{7}=8\frac{3}{7}$

II. $8+\frac{7}{3}=8+2\frac{1}{3}=10\frac{1}{3}$

III. $8-\frac{3}{7}=7+\left(1-\frac{3}{7}\right)=7+\frac{4}{7}=7\frac{4}{7}$

IV.

$$8-\frac{7}{3}=8-\left(2\frac{1}{3}\right)=6-\frac{1}{3}=5+\left(1-\frac{1}{3}\right)=5\frac{2}{3}$$

Ex-1.

$$4\frac{1}{2}-\frac{5}{8}+12\frac{1}{4}-3\frac{7}{8}=?$$

$$(4+12-3)+\left(\frac{1}{2}-\frac{5}{8}+\frac{1}{4}-\frac{7}{8}\right)=?$$

$$13+\frac{(4-5+2-7)}{8}=?$$

$$13+\left(\frac{-6}{8}\right)=?,\ 13-\frac{3}{4}=?\Rightarrow 12\frac{1}{4}=?$$

Ex-2.

$$12\frac{1}{3}-\frac{8}{9}+\frac{11}{3}-4\frac{14}{9}=?$$

$$12\frac{1}{3}-\frac{8}{9}+3\frac{2}{3}-5\frac{5}{9}=?$$

$$(12+3-5)+\left(\frac{1}{3}-\frac{8}{9}+\frac{2}{3}-\frac{5}{9}\right)=?$$

$$10+\left(\frac{3-8+6-5}{9}\right)=?$$

$$10-\frac{4}{9}=?$$

$$9\frac{5}{9}=?$$

Ex-3.

$$6\frac{5}{6}\times 5\frac{1}{3}+17\frac{2}{3}\times 4\frac{1}{2}=?$$

$$\frac{41}{6}\times\frac{16}{3}+\frac{53}{3}\times\frac{9}{2}=?$$

$$\frac{328}{9}+\frac{159}{2}=\frac{656+1431}{18}=\frac{2087}{18}$$

$$115\frac{17}{18}=?$$

(VI) VBODMAS RULE

V → Vinculum means bar as (-)

B → Bracket- () {} and then [ ]

O → of

D → Division [÷]

M → Multiplication [×]

A → Addition [+]

S → Subtraction [-]

The word **'VBODMAS'** represents the order of calculation i.e. order of signs

B	O	D	M	A	S
Brackets	Orders	Divide	Multiply	Add	Subtract
( ) {} []	x^2 $\sqrt{x}$	÷ or	X	+ or	-

Ex-1. 35 ÷ 7 × 5 = ?

Sol. According to the order of **VBODMAS**, first we solve division and then multiplication

i.e. 35 ÷ 7 × 5 = ?

5 × 5 = ?

? = 25

Ex-2. 35 ÷ 5 of 7 = ?

Sol. According to the order of **VBODMAS**, first we solve 'of' and then division.

i.e. 35 ÷ 5 of 7 = ?

35 ÷ 35 = ?

? = 1

Ex-3 48 ÷ 12 of 2 + [3 + 17 × 2] = ?

Sol. 48 ÷ 24 + 37 = ?

2 + 37 = ?

? = 39

Ex-4. 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 = ?

Sol. $\frac{2}{2\times 2\times 2\times 2\times 2}=?$

$? = \frac{1}{16}$

Ex-5. $2 \div 2 \div 2 \div 2 \div 2$ of $2 = ?$

Sol. $2 \div 2 \div 2 \div 2 \div 4 = ?$

$$\frac{2}{2 \times 2 \times 2 \times 4} = ?$$

$$? = \frac{1}{16}$$

Ex-6. $2 \div 2 \div 2 \div 2 \div 2 \times 2 = ?$

Sol. $$\frac{2}{2 \times 2 \times 2 \times 2} \times 2 = ?$$

$$? = \frac{1}{4}$$

Ex-7. $5 \div 5 \div 5 + 5 \div 5 \div 5 = ?$

Sol. $$\frac{5}{5 \times 5} + \frac{5}{5 \times 5} = ?$$

$$\frac{1}{5} + \frac{1}{5} = ?$$

$$? = \frac{2}{5}$$

Some more formulae based on Indices.

I. $\left(a^m\right)^n = a^{mn}$

II. $a^m \div a^n = a^{m-n}$

III. $a^m \times a^n = a^{m+n}$

IV. $a^m = b^m \Rightarrow a = b$

V. $a^m = a^n \Rightarrow m = n$

VI. $a^0 = 1$

Ex-8. $8^{12} \div 16^2$ of $32^3 \times \sqrt{256} = 2^?$

Sol. $\left(2^3\right)^{12} \div \left(2^4\right)^2$ of $\left(2^5\right)^3 \times 16 = 2^? \ldots\ldots \left\{\left(a^m\right)^n = a^{mn}\right\}$

$2^{36} \div 2^8$ of $2^{15} \times 2^4 = 2^?$

$2^{36-23+4} = 2^?$

$2^{17} = 2^?$

$? = 17$

Ex-9. $108 \div 36$ of $\frac{1}{4} + \frac{2}{5} \times 3\frac{1}{4} = ?$

Sol. $108 \div 9 + \frac{2}{5} \times \frac{13}{4} = ?$

$12 + \frac{13}{10}$

$? = 13\frac{3}{10}$

Ex-10. $33\%\ \frac{1}{3}$ of $633 + 129 = 66\frac{2}{3}\%$ of $?$

Sol. $\frac{1}{3} \times 633 + 129 = \frac{2}{3} \times ?$

$(211 + 129) \times \frac{3}{2} = ?$

$? = 340 \times \frac{3}{2} = 170 \times 3 = 510$