Ray Optics and Optical Instruments

9

REFLECTION

DEFINITION

When a light ray strikes the surface separating two media, a part of it gets reflected, i.e., returns back in the initial medium. It is known as **reflection.**

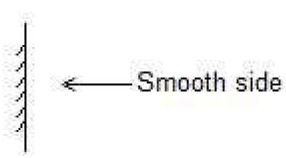
TERMS RELATED TO REFLECTION

1. RAY :

A ray of light is the straight line path of transfer of light energy. It is represented by a straight line with an arrow - head indicating the direction of propagation.

2. MIRROR :

It is a highly polished smooth surface from which most of the incident light gets reflected. It is represented by a line with hatches in the reverse side of the smooth surface.

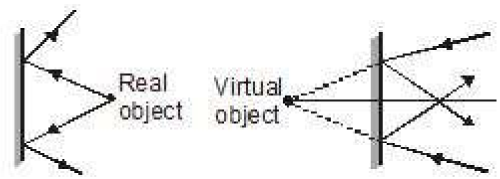


3. OBJECT :

(a) Point from which incident ray actually diverge is called real object. Or point at which incident rays appear to converge is called virtual object.

(b) Object is defined on the basis of incident ray.

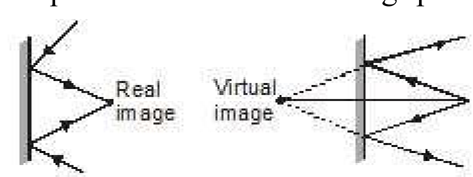
(c) Minimum two rays are required to show the position of object.



4. IMAGE :

(a) Point at which reflected or refracted rays actually converge is called real image. Or point from which reflected or refracted rays appear to diverge is called virtual image.

(b) Minimum two reflected or refracted rays are required to determine the image position.



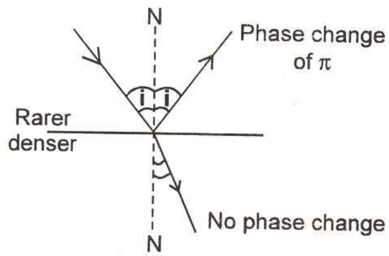
LAWS OF REFLECTION

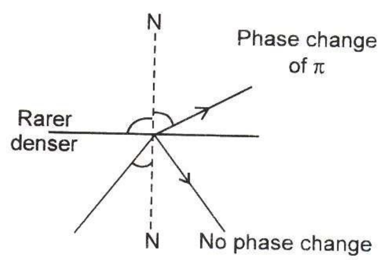
There are three laws of reflection:

[a] The angle of incidence is equal to the angle of refleciton.

[b] The incident ray, the normal and the reflected ray lie in the same plane.

[c] There is a phase change of π radians when light wave is reflected by denser medium surface but no phase change occurs if it is reflected by rarer medium surface.





REFLECTION BY PLANE MIRROR

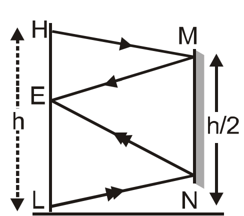
[a] The image formed by the plane mirror is always errect, of the same size and at the same distance as the object is.

[b] To see the full image in a plane mirror, its length is just half the height of the man.

[c] Image of an object formed by a plane mirror is laterally inverted and virtual in nature.

[d] Image due to a plane mirror is as far behind the mirror as is the object infront of it.

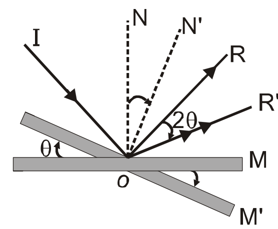
[e] The magnification produced by the plane mirror is 1 i.e. the size of the image is equal to the size of the object.



[f] Deviation δ is defined as the angle between directions of incident ray and emergent ray. So if light is incident at an angle of incidence i, $\delta = \pi - (i + r) = \pi - 2i$

[g] If keeping the incident ray fixed, the mirror is rotated by an angle θ, about an axis in the plane of mirror, the reflected ray is rotated through an angle 2θ.

[h] If object moves towards the plane mirror at speed v, the image moves towards the plane mirror at speed –v. So, the image speed w.r.t. object is $-v - v = -2v$



[i] If mirror is moved towards (or away from) the object with speed v the image will move towards (or away from) the object with a speed 2v.

[j] If real time of a watch is $A^{H}B'C''$ and $A^{H}B'C'' < 11^{H}59'60''$ then time as seen in the image of watch in a mirror will be $11^{H}59'60'' - A^{H}B'C''$

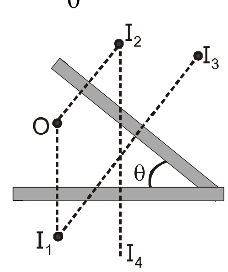
[k] If $A^{H}B'C'' > 11^{H}59'60''$ then time of image will be $23^{H}59'60'' - A^{H}B'C''$

IMAGES FORMED BY TWO MIRRORS

1. When two plane mirrors are inclined at an angle θ and an object is placed in between them due to multiple reflection more than one images are formed. This number of image n is either

$$\frac{360^0}{\theta} \quad \text{or} \quad \left(\frac{360^0}{\theta} - 1\right)$$

Accordingly as $\frac{360^0}{\theta}$ is odd or even respectively.



Again if $\frac{360^o}{\theta}$ is odd & object is place symmetrically between two mirrors, then final two images coincide and there by leaving $\left(\frac{360^0}{\theta} - 1\right)$ images.

$n=\frac{360^0}{\theta}$	Position of Object	Number of Images
even	anywhere	n –1
odd	symmetric	n –1
	assymetric	n

2. No. of images for some specific angles :

s.no.	θ in degrees	$n=\frac{360^0}{\theta}$	No.of images formed when the object is placed	
			Sysmmetrically	Asymmetrically
1.	0	∞	∞	∞
2.	0	12	11	11
3.	45	8	7	7
4.	60	6	5	5
5.	72	5	4	5
6.	90	4	3	3
7.	120	3	2	3

Solved Examples

Ex:1 Two mirrors are inclined at an angle of 50^0. Then what is the number of images formed for an object placed in between the mirros?

Sol: For the given $\theta = 50^0$,

$$n=\frac{360}{\theta}=\frac{360}{50}=7.2$$

The integer value of (7.2) is 7. Thus number of images formed is 7.

Ex:2 Two plane mirrors are inclined at an angle θ. A ray of light is incident on one mirror at an angle of incidence i. The ray is reflected from this mirror, falls on the second mirror from where it is reflected parallel to the first mirror. What is the value of i, the angle of incidence in terms of θ?

Sol: The situation is illustrated in figure. XA is the incident ray. BC is the final reflected ray. It is given that BC is parallel to mirror M_1. Look at the assignment of the angles carefully. Now N_2 is normal to mirror M_2. Therefore

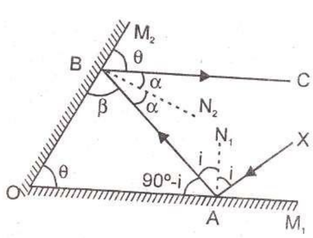
$\beta = \theta$

Then from ΔOAB

$\theta + \beta + 90^0 - i = 180^0$

or $\theta + \theta + 90^0 - i = 180^0$

or $i = 2\theta - 90^0$



Thus if the angle of incidence is $i = 2\theta - 90^0$, then the final reflected ray will be parallel to the first mirror.

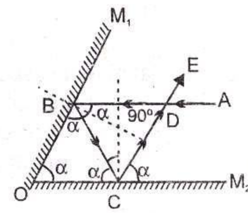
Ex:3 Two plane mirrors are placed at an angle θ so that a ray parallel to one mirror gets reflected parallel to the second mirror after two consecutive reflections. Find the value of θ.

Sol. As shown in figure, ray AB goes to mirror M_1, gets reflected and travels along BC and then gets reflected by M_2 and goes in CD direction. If the angle between M_1 and M_2 be α, then

In $\Delta OBC, \angle OBC$ and $\angle OCB$ are equal to α.

$\therefore 3\alpha = 180^o$

$\alpha = 60^o$ **Answer**



REFLECTION THROUGH SPHERICAL MIRRORS

Reflection through concave mirror:

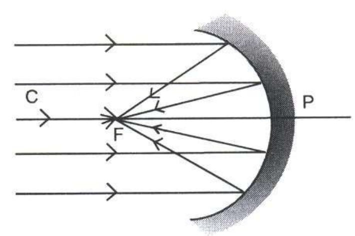
F $\rightarrow$ Principal focus

P $\rightarrow$ Pole of mirror

C $\rightarrow$ Centre of curvature

PC = Radius of curvature

PF = Focal length



CONCAVE MIRROR

Some Definitions

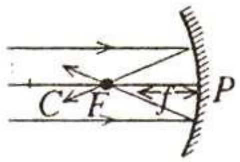
(a) Centre of curvature : The centre of the sphere of which the mirror is a part is called the centre of curvature.

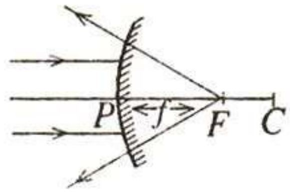
(b) Radius of curvature : The radius of the sphere of which the mirror is a part is called the radius of curvature. It is represented by C in the figure.

(c) Pole : The mid-point of circular aperture of the curved mirror is called pole. It is shown by point P in the mirror.

(d) Principal axis : The line drawn through the centre of curvature and the pole of mirror is called the principal axis.

(e) Focus : All rays parallel to the principal axis after reflection pass through a point F on the principal axis (or appear to be coming from F). This point F on the principal axis is called focus. It is represented by F in the figure.





(f) Focal length : The distance from the pole P to the focus is known as focal length of mirror.

When a narrow beam of light travelling parallel to the principal axis is incident on the reflecting surface of the concave mirror, the beam after reflection converge at a point on the principal axis.

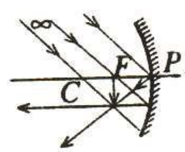
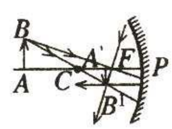
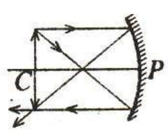
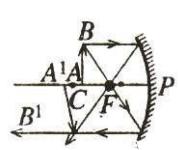
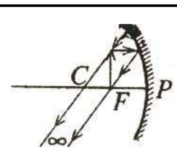
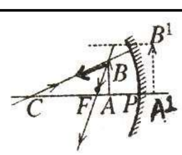
Note:

[i] A ray, passing through the centre of curvature falls normally on the mirror and is, therefore, reflected back along the same path.

[ii] A ray, parallel to the principal axis will after reflection, pass through the focus.

[iii] A ray, passing through the focus is reflected parallel to the principal axis.

IMAGE FORMED BY THE CONCAVE MIRROR

S.No.	Position of object	Ray diagram	Position of image	Nature of image	Size of image
1.	At infinity		at focus	real and inverted	very small
2.	Between infinity and centre of cruvature		between focus and centre of curvature	real and inverted	small
3.	At centre of curvature		at centre of curvature	real and inverted	equal to object size
4.	Between focus and centre of curvature		between centre of curvature and infinity	real and inverted	enlarged
5.	At focus		at infinity	real and inverted	very large
6.	Between pole and focus		between poles and focus	virtual and erect	enlarged

REFLECTION THROUGH CONVEX MIRROR

When a narrow beam of light travelling parallel to the principal axis is incident on the reflecting surface of the mirror, the beam after reflection appear to diverge from a point on the principal axis.

Note:

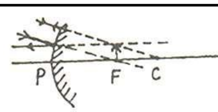
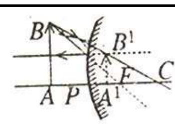
[i] A ray, passing through the centre of curvature falls normally on the mirror and is, therefore, reflected back along the same path.

[ii] A ray, parallel to the principal axis will, after reflection, appear to come from the focus.

[iii] A ray appearing to pass through the focus is reflected parallel to the principal axis.

IMAGE FORMED BY CONVEX MIRROR

A convex mirror forms only virtual images for all positions of the object. The image is always virtual, erect, smaller than the object and is located between the pole and the focus. The image becomes smaller and moves closer to the focus as the object is moved away from the mirror.

S.No.	Position of object	Ray diagram	Position of image	Nature of image	Size of image
1.	At infinity		at focus	virtual and erect	very small
2.	Between pole and infinity		between focus & pole	virtual and erect	small

Discussion

(1) The focal-length of a spherical mirror of radius R is given by $f = \left(\frac{R}{2}\right)$

In or conversion f(or R) is negative for concave or converging mirror and positive for convex or diverging mirror.

(2) The power of a mirror is defined as

$$P = \frac{1}{f\,(\text{in m})} = \frac{100}{f\,(\text{in cm})}$$

The unit of power is **diopter.**

(3) If a thin object linear size O situated vertically on the axis of a mirror at a distance u from the pole and its image of size I is formed at a distance v (from the pole) magnification (transverse) is defined as

$$m = \left[\frac{I}{O}\right] = -\left[\frac{v}{u}\right]$$

Here –ve magnification implies that image is inverted with respect to object while +ve magnification means that image is erect with respect to object.

(4) If an object is placed at a distance u from the pole of a mirror and its images is formed at a distance v (from the pole) then $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

In this formula to calculate unknown, known quantities are substituted with proper sign.

DERIVATION OF MIRROR FORMULA

Let P be the pole, F, the principal focus and C, the centre of curvature of a concave mirror of small aperture, (see fig.) Let PF = f be focal length and PC = R be radius of curvature of the mirror.

Depending on the position of the object, the image formed may be real or virtual.

(a) Real image :

When the object is held in front of the concave mirror beyond the principal focus F of the mirror, image formed is real. In fig. an object AB is held perpendicular to the principal axis of the mirror beyond C.

A ray of light starting from A and incident on the mirror along AD parallel to the principal axis, passes through F on reflection from the mirror. Another ray of light incident along AP is reflected along PA' such that $\angle APB = \angle i = \angle BPA' = \angle r$. The two reflected rays actually meet at A' which is real image of the point A. A third ray starting from A and incident on the mirror along AC falls normally on the mirror and retraces its path; meeting the two reflected rays at A'. From A' draw A'B' perpendicular on the principal axis. Therefore, A'B' is real, inverted image of AB formed by reflection from the concave mirror.

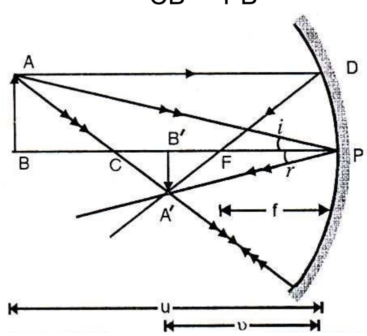
As Δs ABC and A'B'C are similar

$$\therefore \frac{AB}{A'B'} = \frac{CB}{CB'} \qquad \text{........(1)}$$

Again as Δs ABP and A'B'P are similar

$$\therefore \frac{AB}{A'B'} = \frac{PB}{PB'} \qquad \text{........(2)}$$

from (1) and (2), $\frac{CB}{CB'} = \frac{PB}{PB'}$ ....(3)



Measuring all distances from P, we have

CB = PB – PC

CB' = PC – PB'

$$\therefore \text{ from (3) } \frac{PB-PC}{PC-PB'} = \frac{PB}{PB'} \qquad \text{........(4)}$$

Using new cartesian sign conventions,

PB = –u, distance of object

PC = – R

PB' = –v, distance of image

we get from (4), $\frac{-u+R}{-R+v} = \frac{-u}{-v}$

or $+uR - uv = uv - vR$ or $uR + vR = 2uv$

Dividing both sides by uvR, we get $\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$

As R = 2f

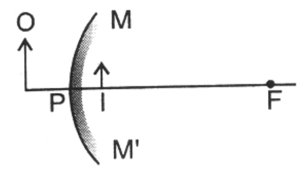
$$\therefore \boxed{\frac{1}{v} + \frac{1}{u} = \frac{2}{R} = \frac{2}{2f} = \frac{1}{f}}$$

which is the required mirror formula.

SPECIAL NOTE

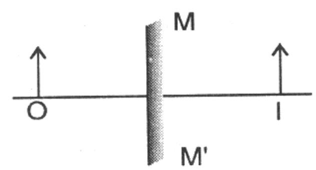
A. For real extended object if the imaged formed by a single mirror is erect it is always virtual (i.e m is +ve) and in this situation if the size of image is.

Smaller than object the mirror is convex



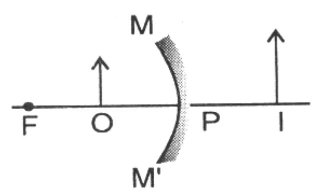
m < +1

Equal to object the mirror is plane



m = +1

Larger than object the mirror is concave

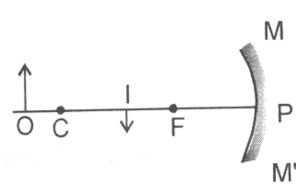


m > +1

So, by observing the size of erect image in a mirror we can decide the nature of the mirror i.e. whether it is convex concave or plane.

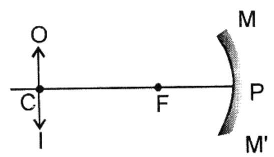
B. For real extended object if the image formed by a single mirror is inverted it is always real (i.e., m is –ve) and the mirror is concave. In this situation if the size of image is

Smaller than object is between the ∞ and C and image between F and C



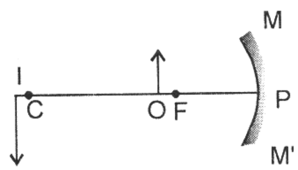
m < -1

Equal to object, Object is at C and image is at C



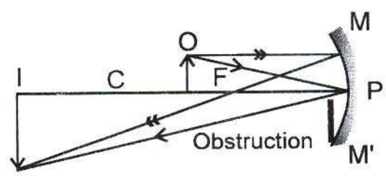
m = -1

Larger than object is between C and F and image betweeb C and ∞



m > +1

C. As every part of a mirror formed complete image, if a part of mirror (say half) is obstructed (say converted with black paper) full image will be formed but intensity will be reduced (to half)



D. If an object moved at constant speed towards a concave mirror from infinity to foucs, the image will move slower in the beginning and faster in the last, away from the mirror. This is becaue in the time object moves from ∞ to C the image will move from F to C and when object moves from C to F the image will move from C to ∞. At C the speed of object and image will be equal.

$v_i = -\left(\frac{v^2}{u^2}\right)v_0$ Where v_i = velocity of image, v_o = velocity of object

NEWTON'S FORMULA

$f^2 = x_1x_2$

where f = focal length

x_1 = distance between object and focus

x_2 = distance between image and focus

Solved Examples

Ex.4 A small linear object of length b is placed at a distance u from the pole of the concave mirror along the axis. The focal length of mirror is f. The length of image will be -

[1] $b\left(\frac{u-f}{f}\right)^{1/2}$ [2] $b\left(\frac{u-f}{f}\right)$

[3] $b\left(\frac{f}{f-u}\right)^{1/2}$ [4] $b\left(\frac{f}{u-f}\right)^2$

Sol. We know that $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ ....(1)

Differentiating (1),

$$-\frac{du}{u^2}-\frac{dv}{v^2}=0$$

$$\frac{dv}{du}=-\frac{v^2}{u^2} \quad ....(2)$$

From equation (1),

$$\frac{u}{f}=1+\frac{u}{v}$$

$$\frac{u}{v}=\frac{f}{u-f} = \frac{dv}{du}=\left(\frac{f}{u-f}\right)^2$$

$$dv=\left(\frac{f}{u-f}\right)^2 du$$

It is given du = b

$$\therefore \text{ length of image} = \left(\frac{f}{u-f}\right)^2 b$$

Hence correct answer is (4)

Ex.5 There is a convex mirror of radius 50 cm. The image of a ponit at a distance 50 cm from the pole of mirror on its axis will be formed at -

[1] infinity [2] pole

[3] focus [4] 16 cm behind the mirror

Sol. u = –50 cm , f = 25 cm

$$\frac{1}{25}=-\frac{1}{50}+\frac{1}{v}$$

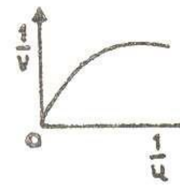
$$\frac{1}{v}=\frac{1}{25}+\frac{1}{50}$$

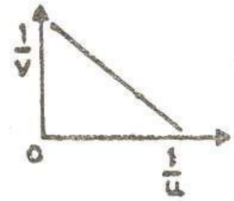
$$=\frac{2+1}{50}=\frac{3}{50}$$

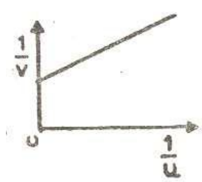
$$v = \frac{50}{3} = 15.3 \text{ cm}$$

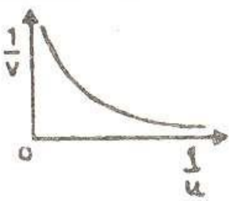
Hence correct answer is (4)

Ex.6 Which of the following graph is applicable between 1/v and 1/u for a spherical mirror ?

[1] 

[2] 

[3] 

[4] 

Sol. For spherical mirrors

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

If $\frac{1}{v}\equiv y$ and $\frac{1}{u}\equiv x$, then the above relation in x, y gives a straight line $\left(y=-x+\frac{1}{f}\right)$

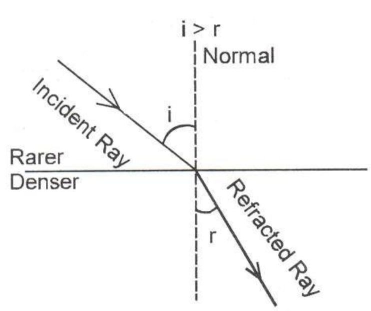
Hence correct answer is (2)

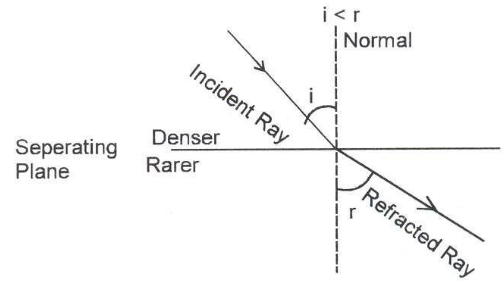
REFRACTION

REFRACTION OF LIGHT

The bending of the light ray from its path in passing from one medium to the other medium is called refraction of light.

If the refracted ray bends towards the normal relative to the incident ray, then the second medium is said to be denser than the first medium. But if the refracted ray bends away from the normal, then the second medium is said to be rarer than the first medium.





LAW OF REFRACTION

[a] For any two media and for light of a given wavelength, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

$\frac{\sin i}{\sin r}=\text{constant}$, where $i=$ incidence angle

$r=$ refraction angle.

[b] The incident ray, the refracted ray and the normal at the incident point all lie in the same plane.

Note :

[a] Refractive index of second medium w.r.t. first medium

$$n_{12}=\frac{n_2}{n_1}$$

[b] Refractive index of medium $(n \text{ or } \mu)$

$$\frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}=\frac{C}{V}=\frac{\sin i}{\sin r}=n=\mu$$

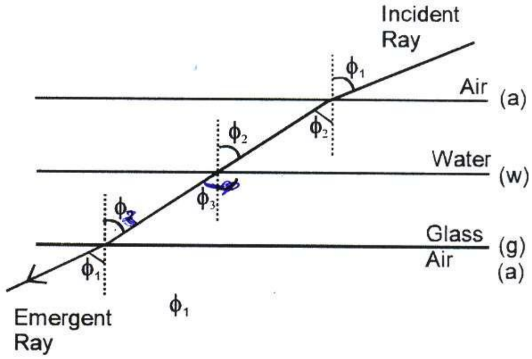
[c] Refractive index is the relative property of two media. If the first medium carrying the incident ray is a vacuum, then the ratio $\frac{\sin i}{\sin r}$ is called the 'absolute refractive index of the second medium'. The relative refractive index of any two media is equal to the ratio of their absolute refractive indices. Therefore, if the absolute refractive indices of media 1 and 2 be n_1 and n_2 respectively, then the refractive index of medium 2 with respect to medium 1 is

$${}_1n_2=n_{12}=\frac{n_2}{n_1}=\frac{\sin i}{\sin r} \qquad n_1\sin i=n_2\sin r$$

[d] $n_{12}=\frac{1}{n_{21}}$

[e] For three medium 1,2,3 due to successive refraction.

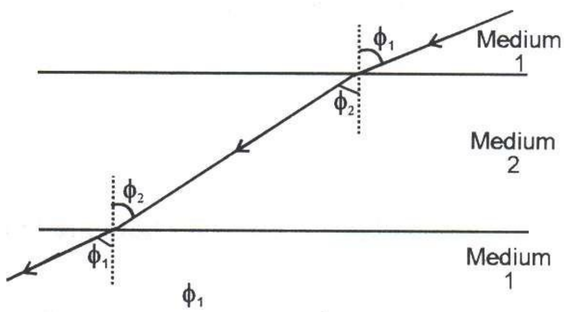
$$n_{12}\times n_{23}\times n_{31}=1 \qquad \frac{n_2}{n_1}\times\frac{n_3}{n_2}\times\frac{n_1}{n_3}=1$$



[f] $\frac{n_w}{n_a} \times \frac{n_g}{n_w} \times \frac{n_a}{n_g} = 1$

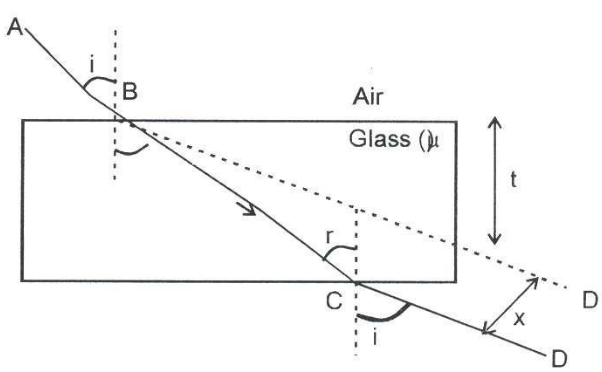
$\Rightarrow n_{aw} \times n_{wg} \times n_{ga} = 1$

[g] For two medium, n_1 and n_2 are refractive indices with respect to vacuum, the incident and emergent rays are parallel then $n_1 \sin\phi_1 = n_2 \sin\phi_2$



1. Refraction through slab :

Refractive index and thickness of glass slab is μ and t respectively. One light ray AB incidents on slab, displacement produced, in emergent ray due to refraction.

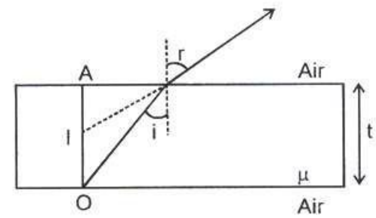


$$x = \frac{t\sin(i-r)}{\cos r} = t\sec r\sin(i-r)$$

Where $\mu = \frac{\sin i}{\sin r}$

[a] When object in denser medium and observer in rarer medium :

Thickness of denser medium is t, in which a object is at a point O. Due to refraction, image may be seen at a apoint I.



Refractive index $\mu = \frac{\text{Real depth}}{\text{Virtual depth}} = \frac{AO}{AI} = \frac{t}{AI}$

Virtual depth $= \frac{t}{\mu}$

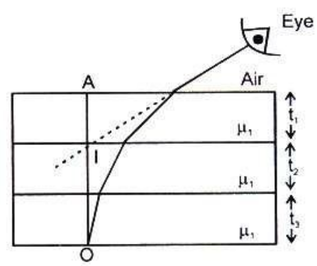
Virtual displacement (OI) $= OA - AI = t\left(1 - \frac{1}{\mu}\right)$

2. Refraction through successive slab of different thickness and refractive index.

Virtual depth (AI) $= \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \frac{t_3}{\mu_3} + \ldots.$

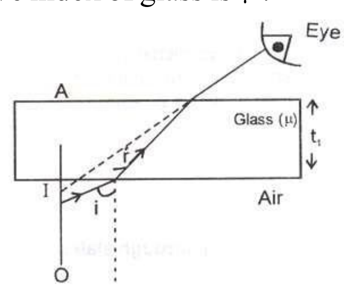
Virtual displacement (OI)

$$= t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) + t_3\left(1 - \frac{1}{\mu_3}\right) + \ldots.$$



3. When object and observer both are in rarer medium :

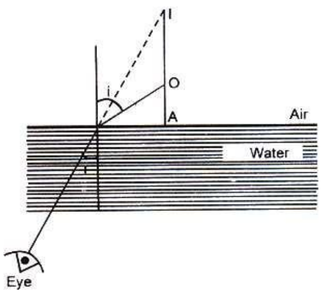
Let observer is in air and object is at a point O in air, as shown in figure. A glass is there in between observer and object. Images forms at point I refractive index of glass is μ.



Virtual displacement = OI = $\left(t - \frac{1}{\mu}\right)$

4. When object in rarer medium and observer in denser medium :

Refractive index of water is μ. Observer is in water, image may be seen at a point I when a object at a point O is viewed.

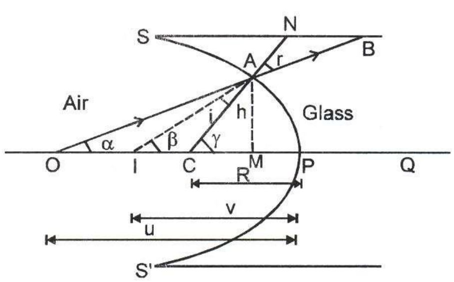


$$\frac{\text{Real height}}{\text{Virtual height}} = \frac{1}{\mu}$$

Virtual displacement (OI) = $AI - AO = (\mu - 1)AO$

REFRACTION THROUGH CONCAVE SPHERICAL SURFACE

Pole centre of curvature for spherical concave surface is P and C respectively. A object is placed at point O on principal aixs. An incident ray OA falls on point A of surface from point O. CAN is normal. Ray OA is refracted in direction AB. Second incident ray OP falls normally on the surface, therefore its refraction does not take place. Point is virtual image of point O.



Let incidence angle OAC = i
refraction angle BAN = r,

PO = –u, PI = –v, PC = –R,

in ΔOCA, $\gamma = \alpha + i$

$i = \gamma - \alpha$ ... (1)

in ΔICA, $\gamma = \beta + r$

$r = \gamma - \beta$ ... (2)

$\therefore$ we know that, $n = \frac{\sin i}{\sin r}$

Where n is refractive index of glass w.r.t. air.

If aperture of spherical surface is small, then point A will be near to pole P. Then the angle i and r will be small.

$\because$ sin i = i and sin r = r

$$\frac{i}{r} = n$$

i = nr ... (3)

Value of equation (1) and (2), on putting in (3)

$$(\gamma - \alpha) = n(\gamma - \beta)$$

$$(n\beta - \alpha) = (n-1)\gamma \quad \text{... (4)}$$

$\because$ It is supposed that aperture of surface is small, so angle α, β, γ will be small point A and M will be near to pole P.

Therefore,

$$\tan\alpha = \frac{AM}{MO} = \frac{AM}{PO} = \alpha = \frac{h}{-u} \quad [\because MO \cong PO]$$

$$\tan\beta = \frac{AM}{MI} = \frac{AM}{PI} = \beta = \frac{h}{-v} \quad [\because MI \cong PI]$$

$$\tan\gamma = \frac{AM}{MC} = \frac{AM}{PC} = \gamma = \frac{h}{-R} \quad [\because MC \cong PC]$$

Put these value in equation (4)

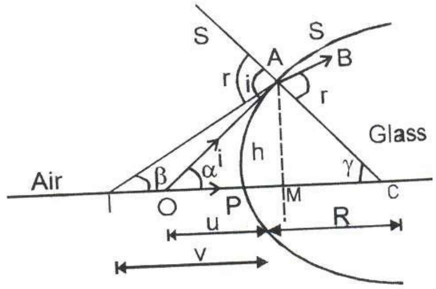
$$\frac{nh}{-v} - \frac{h}{-u} = (n-1)\frac{h}{-R}$$

$$\frac{n}{v} - \frac{1}{u} = \frac{n-1}{R}$$

Value of v does not depend on $\alpha, \because$ value of R is negative for concave surface, therefore for all negative value of u, value of v is negative. It means image is always formed in medium and it is virtual.

REFRACTION THROUGH SPHERICAL CONVEX SURFACE

Pole and centre of curvature for spherical convex surface is P and C respectively. A object is placed at point O on principal axis. A incident ray OA falls on point A of surface from point O. CAN is normal. Ray OA is refracted in direction AB. Another incident ray OP falls normally on the surface, therefore its refraction does not take place. Point I is virtual of point O.



Let incidence angle $\angle OAN = i$,

Refraction angle $\angle BAC = r$

PO = –u, PI = –v and PC = + R

In ΔOCA, $i = \alpha + \gamma$ .... (1)

In ΔICA, $r = \beta + \gamma$ ... (2)

$\because$ We know that, $n = \dfrac{\sin i}{\sin r}$

Where n, is refractive index of glass relative to air. If aperture of spherical surface is small then point A will be near to pole P. Then angle $\angle i$ and $\angle r$ will small.

$\because \sin i = i$ and $\sin r = r$

$\therefore \dfrac{i}{r} = n \Rightarrow i = nr$ ... (3)

Putting value of equation (1) and (2) in equation (3)

$(a + \gamma) = n(\beta + \gamma)$

$-n\beta + \alpha = (n-1)\gamma$ ... (4)

$\therefore$ Aperture of surface is small, therefore angle α, β, γ will be small and point A and M will be near to point P therefore.

$$\tan\alpha = \frac{AM}{MO} = \frac{AM}{PO} = \alpha = \frac{h}{-u} \quad [\therefore MO \cong PO]$$

$$\tan\beta = \frac{AM}{PI} = \frac{AM}{PI} = \beta = \frac{h}{-v} \quad [\because MI \cong PI]$$

$$\tan\gamma = \frac{AM}{MC} = \frac{AM}{PC} = \gamma = \frac{h}{R} \quad [\because MC \cong PC]$$

Put these value in equation [4]

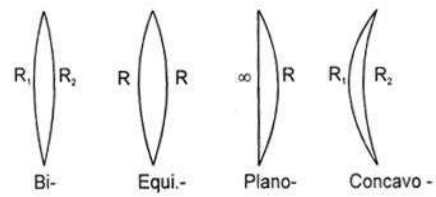
$$\frac{-nh}{-v} + \frac{h}{-u} = (n-1)\frac{h}{R} \qquad \frac{n}{v} - \frac{1}{u} = \frac{n-1}{R}$$

Value of v does not depend on α. $\therefore$ value of R for surface is positive, therefore if value of u exceeds than $\dfrac{R}{n-1}$ then value of v will be positive. In this condition image will form in another medium and it will be real.

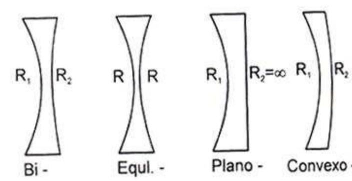
LENS THEORY

A lens is a piece of transparent material with two refracting surface such that least one is curved and refractive index of use material is different from that of the surroundings.

A thin spherical lens with refractive index greater than that of surrounding behaves a convergent or convex lens i.e. converges parallel rays its central (i.e. paraxial) portion is thicker than marginal one. However if the central portion of a lens (with $\mu_L > \mu_M$) is thinner than marginal, it diverges parallel rays and behaves as divergent or concave lens. This is how we classify and identify convergent and divergent lenses.



Convex lens
If $\mu_L > \mu_M$

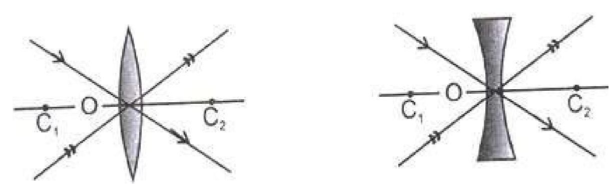


Convex lens
If $\mu_L > \mu_M$

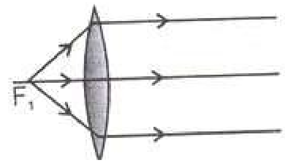
IN CASE OF THIN SPHERICAL LENS

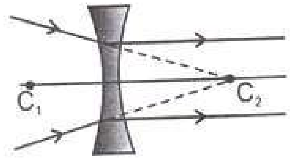
(1) **Optical Center - O** is a point for given lens through which any ray passes undeviated.

(2) **Principal-Axis** - C_1 C_2 is a line passing through optical and perpendicular to the lens. The centre of curvature of curved surface always lie on the principal axis (as in a sphere is always perpendicular to surface)

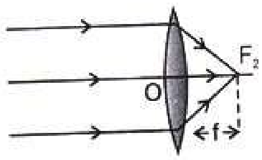


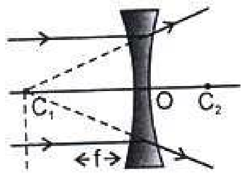
(3) **Principal-Focus** : A lens has two surface and hence two focal points first focal point is an object on the principal axis for which image is at infinite while





Second focal point is an image point on the principle axis for which object is at infinity.





(4) **Focal - Length (f) :** It is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appear to converge.

(5) **Aperture** : In reference to lens aperture means to effective diameter of its light transmitting area so that brightness ie., intensity of image formed by a lens which depends on the light passing through the lens will depends on the square of aperture i.e.

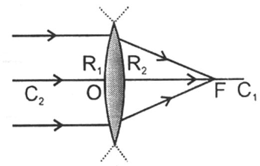
$I \propto (\text{Aperture})^2$

SIGN - CONVENTION

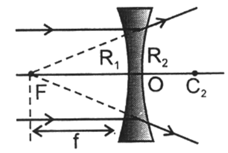
(1) Whenever and where possible, rays of light are taken to travel from left to right.

(2) Transverse distance measured from optical centre and are taken to be positive while those below it negative.

(3) Longitudinal distance are measured from optical centre and are taken to be positive if in the direction of light propagation and negative if opposite to it eg., according to our convention case of a.



$R_1(=OC_1)$=Positive
$R_2(=OC_2)$=Negative
$f(=OF)$=Positive



$R_1(=OC_1)$=Negative
$R_2(=OC_2)$=Positive
$f(=OF)$=Negative

While using the sign convention it must be kept in mind that -

(i) To calculate an unknown quantity the known quantities are substituted with sign in a given formula.

(ii) In the result sign must be interpreted as there are number of sign conventions and same sign has different meaning in different conventions.

RULES FOR IMAGE FORMATION

In order locate the image formed by a lens graphically following rules are adopted.

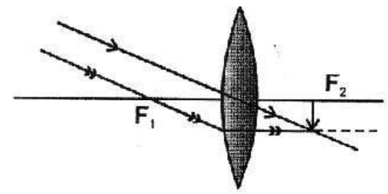
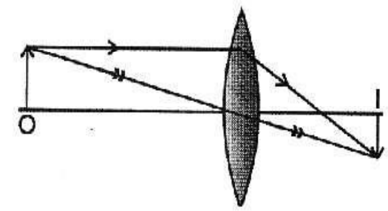
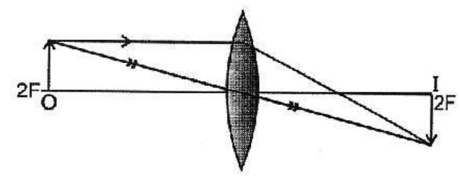
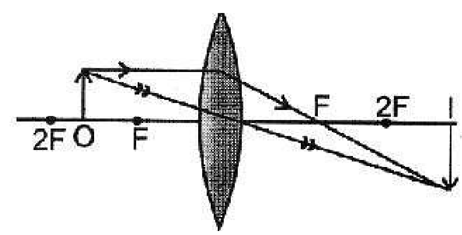
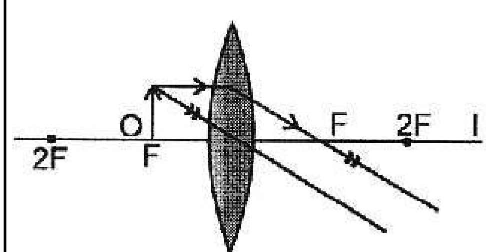
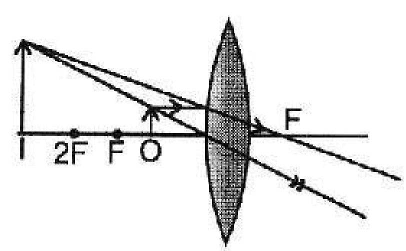
(1) A ray passing through optical centre proceeds undeviated through the lens. (by definition of optical centre).

(2) A ray passing through first focus or directed towards it, after refraction from the lens becomes parallel to the principle axis. (by definition of F_1)

(3) A ray passing parallel to the principle axis after refraction through the plens passes or appear to pass through F_2 (by definition of F_2)

(4) Only two rays from the same point of an object are needed for image formation and the point where the rays after refraction through the lens intersect or appear to intersect is the image of the object. If they actually intersect each other the image is real and if they appear to intersect the image is said to be virtual.

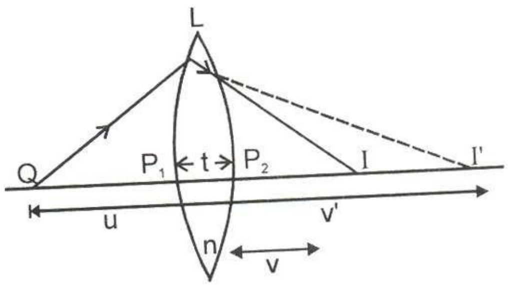
IMAGE FORMATION BY A LENS

Position of Object	Details of Image	Figure
At infinity	Real, Inverted Diminished ($m << -1$) At F	
Between ∞ and 2F	Real inverted Diminished ($m < -1$) between F and 2F	
At 2F	Real inverted equal $m = -1$ At 2F	
Between 2F and F	Real, inverted Enlarged ($m > -1$) between 2F and ∞	
At F	Real inverted Enlarged ($m >> -1$) At inifinity	
Between Focus & Pole	Virtual, erect enlarged ($m > +1$) between ∞ and Object on same side	

REFRACTION THROUGH A THIN LENS

Let radius of curvature of surfaces is R_1 and R_2 respectively. Let thickness of lens is t, and refractive index of lens is n w.r.t. air. Let a point object O be placed on the principle axis of the lens at a distance u from the pole P_1 of the first surface. This surface forms the image of O at I. Let the distance of I' from P_1 be v'. For the refraction at a spherical surface, we have.

$$\frac{n}{v'} - \frac{1}{u} = \frac{n-1}{R_1} \quad \ldots \quad (1)$$



Now I' acts as a virtual object for the second surface which forms final image I at a distance v from it. The distance of I' from the pole P_2 of the second surface is $(v' - t)$

By formula of refraction

$$\frac{\frac{1}{n}}{v} - \frac{1}{(v'-t)} = \frac{\frac{1}{n}-1}{R_2} \quad \ldots \quad (2)$$

$\because$ t is very small, therefore

$$\frac{1}{v} - \frac{n}{v'} = \frac{1-n}{R_2} = -\frac{n-1}{R_2} \quad \ldots \quad (3)$$

By equation (1) and (3)

$$\frac{1}{v} - \frac{1}{u} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots \quad (4)$$

If $u = \infty$, then v = f,

$$= \frac{1}{f} - \frac{1}{\infty} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

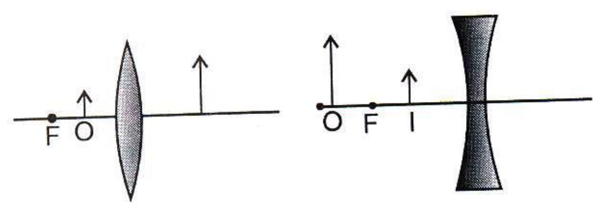
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots \quad (5)$$

By equation (4) and (5)

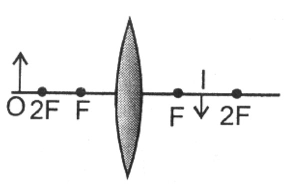
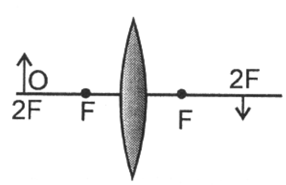
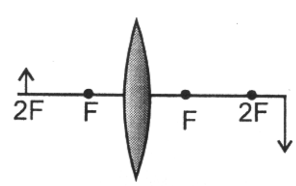
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

SPECIAL POINTS

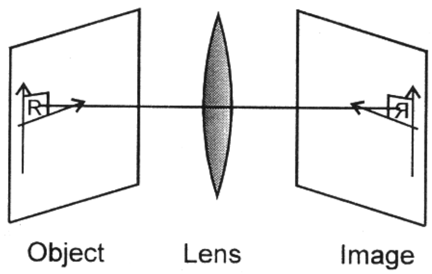
(a) For real extended objects if the image formed by a single lens is erect (i.e., m is positive) it is always virtual. In this situation if the image is enlarged the lens is converging (i.e. convex) with object between focus and optical centre and if diminished the lens is diverging (i.e. concave) with image between focus and optical centre.



(b) For real extended object, if the image formed by a single lens in inverted (i.e. m is negative) it is always real and the lens is convergent i.e., convex. In this situation if the size of image is -

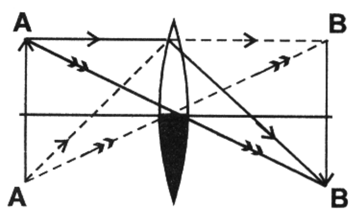
Smaller than object	Equal to object	Larger than object
Object between ∞ and 2F Image is between F and 2F  A	Object is at 2F Image is at 2F  B	Object between 2F and F Image is between 2F and ∞  C

(c) In case of inverted image formed by a lens the inversion is true i.e., left is turned right and up, down.



(A)

(d) As very part of a forms complete image if a portion (say lower half) is obstructed (say covered with black paper). Full image will be formed but brightness i.e., intensity will be reduced (to half). Also if a lens is painted with black strips and a donkey is seen through it, the donkey will not appear zebra but will remain donkey with reduced intensity.



(B)

(e) If L is the distance between a real image by a lens, then as

$$L = (|u| + |v|) = \left(\left(\sqrt{u} - \sqrt{v}\right)^2 + 2\sqrt{uv}\right)$$

So, L will be minimum when

$\left(\sqrt{u} - \sqrt{v}\right)^2 = \text{min} = 0$ i.e., u = v

On substanding u = –u and v = + u in lens formula, we get

$\frac{1}{u} - \frac{1}{-u} = \frac{1}{f}$ i.e., u = 2f

So that (L)min = 2f + 2f = 4f

[as for L min u = v]

i.e., the minimum distance between a real object and its real image formed by a single lens is 4f.

(f) If an object is moved at constant towards a convex lens from infinity to Focus, the image will move slower in the begining and faster later- on, away from the lens. This is because in the time object moves from infinity to 2F, the image will move from F to 2F and when the object moves from 2F to F, the image will move from 2F to infinity. At 2F the speed of object and image will be equal.

$$v_i = v_0\left[\frac{f}{u+f}\right]^2$$

Where v_0 is the speed of object (u and f are to be substituted with proper sign)

(g) In case of sun-goggles, the radii of curvature of two surface are equal with centre on same side i.e., $R_1 = R_2 + R$

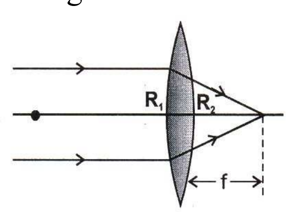
So $\frac{1}{f} = (\mu - 1)\left[\frac{1}{+R} - \frac{1}{+R}\right] = 0$

i.e., $f = \infty$ and $P = \left(\frac{1}{f}\right) = 0$

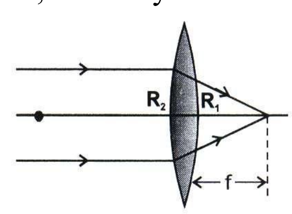
This is why sun-goggles have no power or infinite focal length. Same is true for transparent sheet with the difference that here

$R_1 = R_2 = \infty$

(h) If the two radii of curvatures of a thin lens are not equal, the focal length remains unchanged whether the light is incident on first face or the other. This is because if we substitute R_1 and R_2 with proper sign in lens-makers formula, we always have



(A)



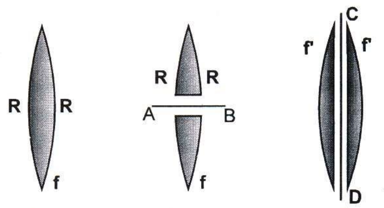
(A)

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

(i) If an equiconcave lens of focal length f is cut into equal parts by a horizontal plane AB then as none of μ, R_1 and R_2 will change the focal length of each part will be equal to that of initial lens i.e.,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{2(\mu - 1)}{R}$$

However in this situation as light transmitting area of each part becomes half of initial so intensity will reduce to half and aperture to $\left[\frac{1}{\sqrt{2}}\right]$ time of its initial value (as $\propto (\text{Aperture})^2$)



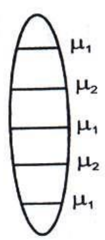
(A) (B) (C)

However if the same lens in cut into equal parts by a vertical plane CD the focal length of each part will become

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R}-\frac{1}{\infty}\right]=\frac{\mu-1}{R}-\frac{1}{2f}$$

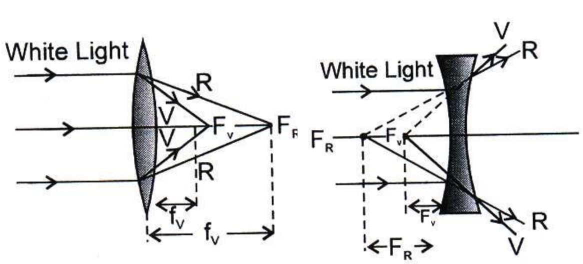
i.e., focal length of each part will be double of initial value. In this situation as the light transmitting area of each part of lens of remains equal to initial intensity and aperture will not change.

(j) If a lens is made of number of layers of different refractive indices as shown in figure for a given wavelength of light it will have as many length or will have as many lengths or will form as many image as there are μ's as



$$\boxed{\frac{1}{f}\propto(\mu-1)}$$

(k) As focal length of a lens depends on μ i.e., $\left(\frac{1}{f}\right)\propto(\mu-1)$ the focal length of given lens is different for different wavelengths and is maximum for red and minimum for violet whatever be the nature of



(A) (B)

(l) If a lens of glass $\left(\mu=\frac{3}{2}\right)$ is shifted from air $(\mu=1)$ to water $\left(\mu=\frac{4}{3}\right)$ then as.

$$\frac{1}{f_a}=K\left[\frac{3/2}{1}-1\right] \quad \text{and}$$

$$\frac{1}{f_w}=K\left[\frac{(3/2)}{(4/3)}-1\right]$$

With $\quad K=\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

$\frac{F_w}{f_A}=\left[\frac{8}{K}\right]\times\left[\frac{K}{2}\right]$ i.e., $f_w = 4f_A$

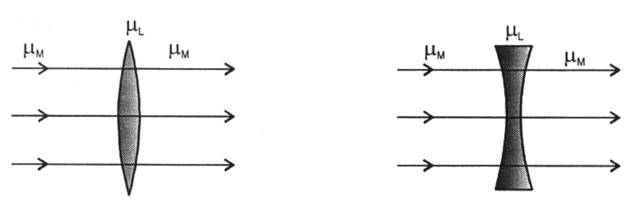
i.e., focal length of a lens in water becomes four times of its value in air and so power one fourth [as p = (1/f)]

(m) If a lens is shifted from one medium to the other depending on the refractive index of the lens and medium following three situation are possible.

(a) $\mu_m < \mu_L$ but μ_m increase : In this situation $\mu=\left(\frac{\mu_L}{\mu_m}\right)$ will remain greater than unity but will decrease and $\left(\frac{1}{f}\right)\propto(\mu-1)\left(\frac{1}{f}\right)$ will decrease i.e., f will increase (without change in nature of lens) as explained in previous point.

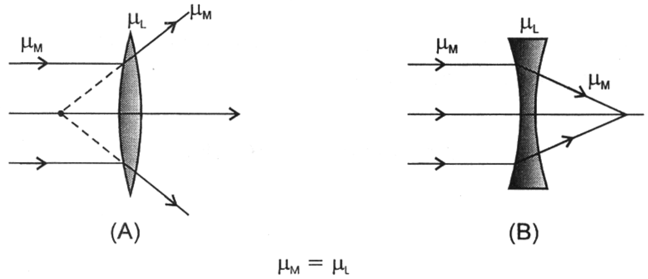
(b) $\mu_m=\mu_L$: In this situation $\mu=\left(\frac{\mu_L}{\mu_M}\right)=1$, so that $\left(\frac{1}{f}\right)\propto(\mu-1)=0$

i.e., $f=\infty$ i.e., lens will neither converge nor diverge but will behaves as a plane glass plate.



(A) $\mu_L=\mu_M$ (B)

(c) $\mu_M > \mu_L$: In this situation $\mu=\left(\frac{\mu_L}{\mu_M}\right)<1$ so in lens-maker's formula sign of f and hence nature of lens will change i.e, ... a convergent lens will behave as divergent and vice-versa.



Solved Examples

Ex.7 An object is situated at a distance of f/2 from a convex mirror of focal length f. Distance of image will be

[1] $+\left(\frac{f}{2}\right)$ [2] $+\left(\frac{f}{3}\right)$

[3] $+\left(\frac{f}{4}\right)$ [4] + f

Sol. For a spherical mirror $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

For convex mirror, $u=-\frac{f}{2}$ and f is +ve

$\therefore \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=+\frac{1}{f}+\frac{2}{f}=\frac{3}{f}$

$\therefore \quad v=\frac{f}{3}$

Ex.8 An object of length 1 cm is placed at a distance of 15 cm from a concave mirror of focal length 10 cm. The nature and size of the image are

[1] real, inverted, 1.0 cm

[2] real, inverted, 2.0 cm

[3] virtual, erect, 0.5 cm

[4] virtual, erect, 1.0 cm

Sol. Given u = –15 cm, f = –10 cm, O = 1 cm

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}-\frac{1}{-15}$

$\therefore$ v = – 30 cm

$\frac{I}{O}=-\frac{v}{u}=-\frac{-30}{-15}=-2$

$I=-2\times 1=-2\,\text{cm}$

Image is inverted and on the same side (real) of size 2 cm.

Ex.9 A biconvex lens whose both the surfaces have same radii of curvature has a power of 5D. The refractie index of material of lens is 1.5. The radius of curvature of each surface is

[1] 20 cm [2] 15 cm

[3] 10 cm [4] 5 cm

Sol. $P=\frac{1}{f}, \quad \therefore\ f=\frac{1}{P}=\frac{1}{5}\text{m}=20\,\text{cm}$

For an equiconvex lens

$\frac{1}{f}=\frac{2(\mu-1)}{R}$

$\therefore$ R = 2 (μ – 1)f

= 2 x 0.5 x 20 = 20 cm

Ex.10 A lens placed at a distance of 20 cm from an object produces a virtual image 2/3 the size of the object. Find the position of the image, find of lens and its focal length.

Sol. Virtual image means, I is positive and it is given that

$I=\frac{2}{3}$ O. Thus,

$m=+\frac{2}{3}$

Further because u = –20 cm (given), using

$m=\frac{f}{f+u}$

We get, $\frac{2}{3}=\frac{f}{f+(-20)}$

or $f=-40\,\text{cm}$

The f is negative, thus the lens is a concave lens. Again using

$m=\frac{v}{u}$

We get $\frac{2}{3}=\frac{v}{-20}$

or $v=-\frac{2}{3}$

$=-1.33\,\text{cm}$

The virtual image is on the same side of the object.

Ex.11 An object is placed at a distance of 1.50 m from a screen and a convex lens placed in between produces an image magnified 4 times on the screen. What is the focal length and the position of the lens.

Sol. The information given in the question ray diagram. It is given that

$$m = \frac{I}{O} = -4$$

Let lens is placed at a distance of x from the object. Then

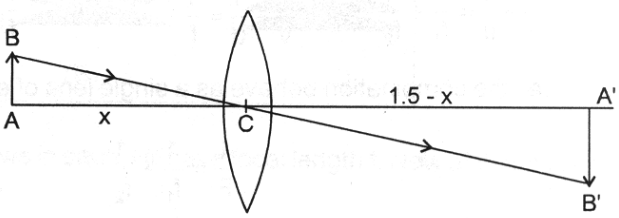
u = –x, and v = (1.5 – x)

Using $m = \frac{v}{u}$, we get $-4 = \frac{1.5 - x}{-x}$

or $4x = 1.5 - x$

or $5x = 1.5$

Thus x = 0.3 metre



The lens is placed at a distance of 0.3 m from the object (or 1.20 m from the screen)

For focal length, we may use

$$m = \frac{f}{f + u}$$

or $-4 = \frac{f}{f + (-0.3)}$

or -4f + 1.2 = f

or 5f = 1.2

Thus $f = \frac{1.5}{5} = 0.24$

The focal length is 0.24 m (or 24 cm)

COMBINATION OF LENSES AND MIRROR

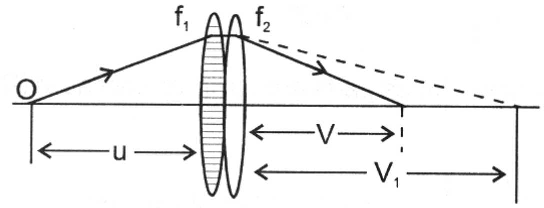
When several lenses of mirrors are used co-axially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as object for next lens or mirror the image formed by the second lens (or mirror) acts as object for the third and so on. The total magnification is such situtations will be given by

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times ......$$

i.e., $m = m_1 \times m_2 \times .....$

In case of two thin lens in contact if the first lens of focal length f_1 forms the image I_1 (of an object) at a distance v from it.

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \qquad .... (1)$$



now the image I_1 will act as object for second lense and if the second lens forms image. I at a distance v from it.

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_2} \qquad ... (2)$$

So, adding equation (1) and (2) we have

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$ or $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

i.e., the combination behave as a single lens of equivalent focal length f given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or $P = P_1 + P_2$ ... (3)

Note : If the two thin lens are separated by a distance d apart F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1 f_2}$$

So, $P = P_1 + P_2 - P_1 P_2 d$

Here is it worthy to note that

If two thin lens of equal focal length but of opposite nature (i.e., one convergent and other divergent) are put in contact, the resultant focal length of the combination be

$$\frac{1}{F} = \frac{1}{+f} + \frac{1}{-f} = 0$$

i.e., $F = \infty$ and P = 0

i.e., the system will behave as a plane glass plate.

If two thin lens of same nature are put in contact then as

$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ $\quad \frac{1}{F} > \frac{1}{f_1}$ and $\frac{1}{F} > \frac{1}{f_2}$

i.e., $F < f_1$ and $F < f_2$

i.e., the resultant focal length will be lesser than smallest individual.

If two thin lenses of opposite nature with different focal lengths are put in contact the resultant focal length will be of same nature as that of the lens of shorter focal length but its magnitude will be more than that of shorter focal length.

If a lens of focal length f is divided into two equal parts

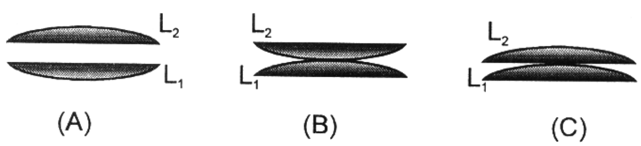
(A) each part has a focal length f ' then as

$\frac{1}{f}=\frac{1}{f'}+\frac{1}{f}$ $\quad$ f = 2f

i.e., each part have focal length f now if these parts are put in contact as in

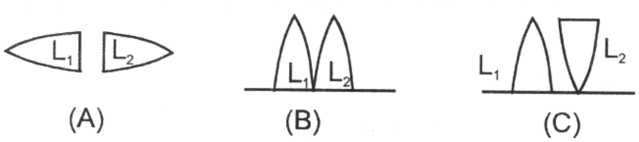
(B) or (C) the resultant focal length of the combination will be

$\frac{1}{F}=\frac{1}{2f}+\frac{1}{2f}$ $\qquad$ i.e., F = f (= initial value)



If a lens of focal length f is cut in two equal part as shows in each will have focal length f. Now of these parts are put in contact as shown in the resultant length will be

$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}$ i.e., $F=\left(\frac{f}{2}\right)$



However if the two parts are put in contact as shown in first will behave as convergent lens of focal length f while the other divergent of same focal length (being thinner near the axis) so in this situation.

$\frac{1}{F}=\frac{1}{+f}+\frac{1}{-f}$ $\qquad$ i.e., $-F=\infty$ or P = 0

Solved Examples

Ex.12 A convex lens of focal length 10.0 cm is placed in contact with a convex lens of 15.0 cm focal length. What is the focal length of the combination.

Sol. For combination of lenses

$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$ $\qquad$ $=\frac{1}{10}+\frac{1}{15}$ $\qquad$ $=\frac{25}{150}=\frac{1}{6}$

Therefore, f = 6 cm.

Ex.13 A 20 cm convex lens is placed in contact with a diverging lens of unknown focal length. The lens combination acts as a converging lens and has a focal length of 30 cm. What is the focal length of the diverging lens.

Sol. Let f_2 is the focal length of the diverging lens. Then,

$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

It is given that f_1 = + 20 cm, f = 30 cm

$\frac{1}{30}=\frac{1}{20}+\frac{1}{f_2}$

or $\frac{1}{f_2}=\frac{1}{30}-\frac{1}{20}$

$=\frac{2-3}{60}=-\frac{1}{60}$

Thus $\qquad f_2$ = –60 cm

Comment : If we change the question and claim that the combination acts as a diverging lens and has a focal length of 30 cm, then

$f_1=+20\,cm,\ f_2=?,\ f=-30\,cm$

$-\frac{1}{30}=\frac{1}{20}+\frac{1}{f_2}$

$\therefore \frac{1}{f_2}=-\frac{1}{30}-\frac{1}{20}=-\frac{5}{60}$

or f_2 = – 12 cm

Ex.14 Ten identical converging thin lenses, each of foal length 10 cm, are in contact. What is the power of the combined lens.

Sol. For thin lenses in contact

$P=P_1+P_2+....$

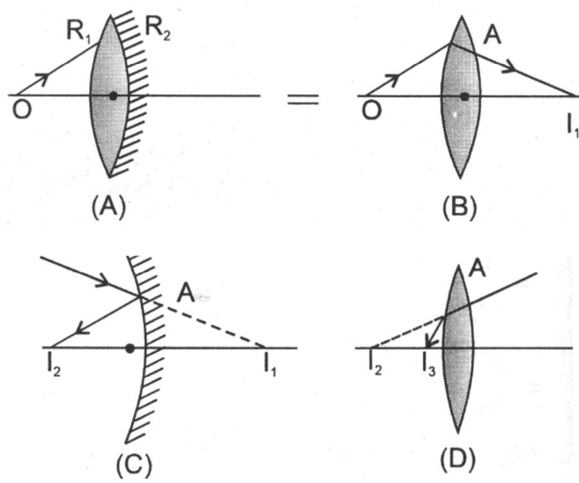
$=10P_1=\frac{10\times100}{10}$

$=100D$

LENS WITH ONE SILVERED SURFACE

If the back surface of a lens is silvered and an object is placed in front of it then :

(a) First, light will pass through the lens and it will form the image I_1.



(b) The image I_1 will act as an object for silvered surface which acts as curved mirror and forms an image I_2 of object I_1.

(c) The light after reflection from silvered surface will again pass through the lens and lens will form final image I_3 of object I_2.

This all is shown in figure. In such situation power* of the silvered lens will be

$$P = P_L + P_M + P_L$$

with $P_L = \frac{1}{f_L}$ where $\frac{1}{f_L} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

and $P_M = -\frac{1}{f_M}$ where $f_M = \frac{R_2}{2}$

So, the system will behave as a curved mirror of focal length F given by

$$F = -\frac{1}{P}$$

To make this all clear we now consider the case of a silvered plano-convex lens under following circumstances:

(d) When the plane surface is silvered and the object is in front of curved surface: In this situation,

$$\frac{1}{f_L} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] = \frac{(\mu - 1)}{R} \quad \text{and} \quad F_M = \frac{\infty}{2} = \infty$$

So, $P_L = \frac{1}{f_L} = \frac{(\mu - 1)}{R}$ and $P_M = -\frac{1}{f_M} = \frac{1}{\infty} = 0$

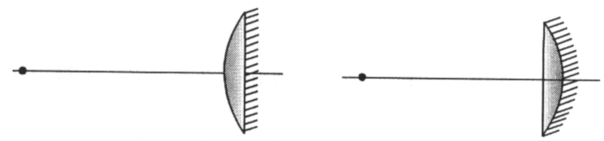
And hence power of system

$$P = P_L + P_M + P_L = 2P_L + P_M$$

i.e., $P = 2\frac{(\mu - 1)}{R} + 0 = \frac{2(\mu - 1)}{R}$ .... (1)

So, $F = -\frac{1}{P} = -\frac{R}{2(\mu - 1)}$ ... (2)

i.e., the lens will behave as a concave mirror of focal length $\left[\frac{R}{2}(\mu - 1)\right]$



(B) When the curved surface is silvered and the object is in front of plane surface: In this situation

$$\frac{1}{f_L} = (\mu - 1)\left[\frac{1}{\infty} - \frac{1}{-R}\right] = \frac{(\mu - 1)}{R} \quad \text{and} \quad f_M = \frac{(-R)}{2}$$

So, $P_L = \frac{1}{f_L} = \frac{(\mu - 1)}{R}$ and $P_M = -\frac{1}{f_M} = \frac{2}{R}$

And hence power of system

$$P = P_L + P_M + P_L = 2P_L + P_M$$

i.e., $P = \frac{2(\mu - 1)}{R} + \frac{2}{R} = \frac{2\mu}{R}$ ... (3)

So, $F = -\frac{1}{P} = -\frac{R}{2\mu}$ ... (4)

i.e., the lens equivalent to a converging mirror of focal length $\left(\frac{R}{2\mu}\right)$

Solved Examples

Ex.15 The radius of curvature of the convex face of a plano–convex lens is 12 cm and its refractive index is 1.5. (a) Find the focal length of this lens. The plane surface of the lens is now silvered. (b) At what distance from the lens will parallel rays incident on the convex face converge? (c) Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens. (d) Calculate the image distance when the object is placed as in (c).

Sol.

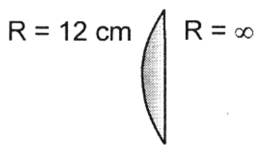
(a) As for a lens, by lens–maker's formula.

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

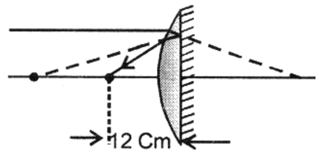
Here, $\mu=1.5; R_1=12\text{cm}$ and $R_2=\infty$

So, $\frac{1}{f}=(1.5-1)\left[\frac{1}{12}-\frac{1}{-\infty}\right]$ or f = 24 cm

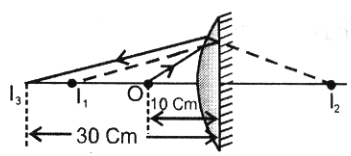
i.e., the lens as convergent with focal length 24 cm.



(A)



(B)



(C)

(b) As light after passing through the lens will be incident on the mirror which will reflect it back through the lens agian, so

$$P = P_L + P_M + P_L = 2P_L + P_M$$

But $P_L=\frac{1}{f_L}=\frac{1}{0.24}$ and $P_M=-\frac{1}{\infty}=0$

$$\left[\text{as } f_M=\frac{R}{2}=\infty\right]$$

So $P=2\times\frac{1}{0.24}+0=\frac{1}{0.12}D$

The system is equilivalent to a concave mirror of focal length F,

$P=-\frac{1}{F}$ i.e., $F=-\frac{1}{P}$ = – 0.12 m = – 12 cm

i.e., the rays will behave as a concave mirror of focal length 12 cm. So as for parallel incident rays u = – ∞, from mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we have

$\frac{1}{v}+\frac{1}{-\infty}=\frac{1}{-12}$ i.e., v = – 12 cm

i.e., parallel incident rays will focus will at a distance of 12 cm in front of the lens as shown in Fig.

(c) and (d) When object is at 20 cm in front of the given silvered lens which behaves as a concave mirror of focal length 12 cm, from mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we have

$\frac{1}{v}+\frac{1}{-20}=\frac{1}{-12}$ i.e., v = – 30 cm,

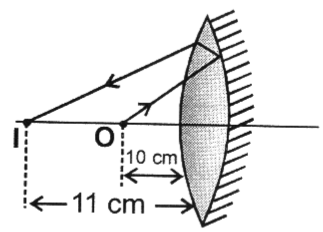
i.e., the silvered lens will form image at a distance of 30 cm if front of it as shown in Fig(C)

Ex.16 A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made of material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual ?

Sol. As radius of curvature of silvered surface is 22 cm,

so $f_M=\frac{R}{2}=\frac{-22}{2}=-11\text{ cm}=-0.11\text{ m}$

and hence $P_M=-\frac{1}{f_M}=-\frac{1}{-0.11}=\frac{1}{0.11}D$



Further as the focal length of lens is 20 cm, i.e., 0.20 m, its power will be given by :

$$P_L=\frac{1}{f_L}=\frac{1}{0.20}D$$

Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again

$$P = P_L + P_M + P_L = 2P_L + P_M$$

i.e., $P=\frac{1}{0.20}+\frac{1}{0.11}=\frac{210}{11}D$

So the focal length of equivalent mirror

$$F=-\frac{1}{P}=-\frac{11}{210}m=-\frac{110}{21}cm$$

i.e., the silvered lens behave as a concave mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it.

$\frac{1}{v}+\frac{1}{-10}=-\frac{21}{110}$ i.e., v = – 11 cm

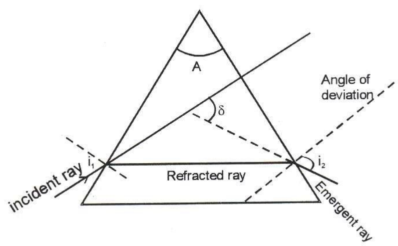
i.e, image will be 11 cm in front of the silvered lens and will be real as shown in Fig.(A)

PRISM & T.I.R.

REFRACTION BY A PRISM

1. PRISM DEFINITION

Prism is a homogeneous, transparent medium enclosed by two plane surfaces inclined at an angle. These surfaces are called the refracting surfaces and the angle between them is called the refracting angle or the angle of prism.



2. ANGLE OF DEVIATION

Angle between the incident rays and emergent ray is called angle of deviation.

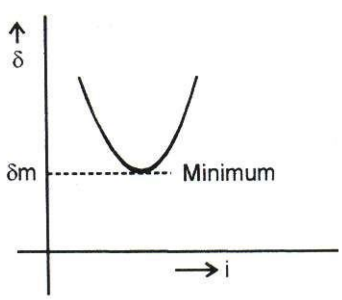
3. MINIMUM DEVIATION

For a given prism, the angle of deviation depends upon the angle of incidence of the light-ray falling on the prism. It is seen from the curve that as the angle of incidence i increases, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then again increases. Thus, for one and only one, particular angle of incidence the prism produce minimum deviation.

4. FORMULA FOR THE REFRACTIVE INDEX OF THE PRISM

$$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Where A is prism angle and δ_m is minimum deviation.



Note:

[a] When prism is thin, then value of A will be small $(\le 10^0)$.

$\delta_m = (n-1)A$

[b] Condition for maximum deviation i_1 or $i_2 = 90^0$

5. ANGULAR DISPERSION FOR PRISM

White light splits into its constituent colours, on passing through prism. This is known as dispersion. The angle between the emergent rays of any two colours is called angular dispersion between those colours. If deviation angle for violet and red are δ_v & δ_R respectively, then angular dispersion.

$\theta = \delta_v - \delta_R$

$\delta_R = (n_R - 1)A,$

$\delta_v = (n_v - 1)A$

$\theta = (n_v - 1)A - (n_R - 1)A = (n_v - n_R)A$

6. DISPERSIVE POWER OF PRISM

$$\omega = \frac{\delta_V - \delta_R}{\delta_Y} = \frac{(n_V - n_R)A}{(n_Y - 1)A}$$

Where δ_Y is deviation angle for yellow colour.

$$\omega = \frac{n_V - n_R}{n_Y - 1}$$

7. DEVIATION LESS DISPERSION

Yellow (mean ray) emergent through the combination of prism of different material will be parallel to the incident ray, but other emerging rays of different colours are not parallel to each other.

$$\frac{\text{Prism angle}(2)}{\text{Prism angle}(1)} = \frac{A_2}{A_1} = \frac{(n_1 - 1)}{(n_2 - 1)}$$

8. DISPERSION LESS DEVIATION

Emerging rays of different colour through the combination of prism of different material will be parallel to each other but not parallel to incident ray. Ray of different colour are parallel, therefore angle between them is zero. It means angular dispersion is zero. Although rays of different colour are parallel but these are not coincident.

$$\frac{\text{Prism angle }(2)}{\text{Prism angle}(1)} = \frac{A_2}{A_1} = \frac{n_{1v} - n_{1r}}{n_{2v} - n_{2r}}$$

Solved Examples

Ex.17 Prism angle of a prism is 10^0. Their refractive index for red and violet colour is 1.51 and 1.52 respectively. Then dispersive power will be

(1) 0.5 (2) 0.15

(3) 0.019 (4) 0.032

Sol. Dispersive power of prism

$$\omega = \left(\frac{\mu_v - \mu_r}{\mu_y - 1}\right)$$

but $\mu_y = \frac{\mu_v + \mu_r}{2} = \frac{1.52 + 1.51}{2} = 1.515$

Therefore $\omega = \frac{1.52 - 1.51}{1.515 - 1} = \frac{0.01}{0.515} = 0.019$

Ex.18 Prism angle and refractive index for a prism for a 60^0 and 1.414. Angle of minimum deviation will be

(1) 15^0 (2) 30^0 (3) 45^0 (4) 60^0

Sol. $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin 30^0}$

$$\Rightarrow 1.414 = \frac{\sin\left(\dfrac{60 + \delta_m}{2}\right)}{\sin 30^0}$$

$$\Rightarrow \sin\left(\frac{60^0 + \delta_m}{2}\right) = 0.707 = \sin 45^0$$

$$\Rightarrow \frac{60 + \delta_m}{2} = 45^0 \Rightarrow \delta_m = 30^0$$

Ex.19 The refracting angle of the prism is 60^0. What is the angle of incidence for minimum deviation? The refractive index of material of prism is $\sqrt{2}$.

Sol. For minimum deviation

$$r = \frac{A}{2} = \frac{60}{2} = 30^0$$

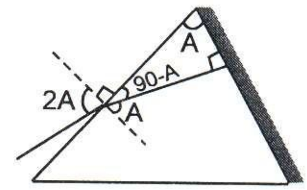
From Snell's law $\frac{\sin i}{\sin r} = \mu$ or $\sqrt{2} = \frac{\sin i}{\sin 30^0}$

$\therefore \sin i = \frac{1}{2} \times \sqrt{2} = \frac{1}{\sqrt{2}} = \sin 45^0$ or $i = 45^0$

Ex.20 Angle of a prism is A and its one surface is silvered. Light ray falling at an angle of incidence 2A on first surface return back through the same path after suffering reflection at second silvered surface. Refractive index of material is

Sol. Given i = 2A

From figure r = A



$$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin A}$$

$$= \frac{2\sin A \cos A}{\sin A}$$

$$= 2\cos A$$

9. COMBINATION OF PRISMS

Two prisms of refracting angles A_1 and A_2 and dispersive power ω_1 and ω_2 are placed symmetrically (as shown in the figure) for a particular ray refractive indices of the two prisms are μ_1 and μ_2 respectively.

Thus deviation produced by two prisms are,

$$\delta_1 = (\mu_1 - 1)A_1 \quad \text{and} \quad \delta_2 = (\mu_2 - 1)A_2$$

As the two deviations are opposite to each other,

Net deviation, $\delta = \delta_1 - \delta_2$

$$= (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2$$

If white light passes through the combination then,

$$\delta_v = (\mu_{1v} - 1)A_1 - (\mu_{2v} - 1_1)A_2$$

and $\delta_r = (\mu_{1r} - 1)A_1 - (\mu_{2r} - 1)A_2$

Thus, the angular dispersion produced by the combination is

$$\delta_v - \delta_r = (\mu_{1v} - \mu_{1r})A_1 - (\mu_{2v} - \mu_{2r})A_2$$

The dispersive powers are

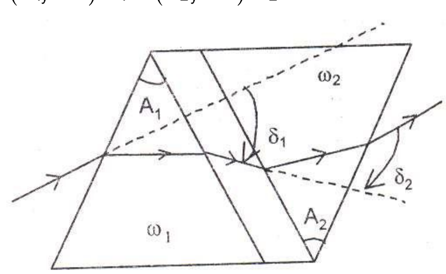
$$\omega_1 = \frac{\mu_{1v} - \mu_{1r}}{\mu_{1y} - 1} \text{ and } \omega_2 = \frac{\mu_{2v} - \mu_{2r}}{\mu_{2y} - 1}$$

So, net angular dispersion

$$\delta_v - \delta_r = (\mu_{1y} - 1)\omega_1 A_1 - (\mu_{2v} - 1)\omega_2 A_2$$

Thus the average deviation (or net deviation for yellow ray)

$$\delta_y = (\mu_{1y} - 1)A_1 - (\mu_{2y} - 1)A_2$$



9.1 DISPERSION WITHOUT AVERAGE DEVIATION :

If $\delta_y = 0$

$$(\mu_{1y} - 1)A_1 = (\mu_{2y} - 1)A_2$$

$$\therefore \delta_v - \delta_r = (\mu_{1y} - 1)A_1(\omega_1 - \omega_2) = \delta_1(\omega_1 - \omega_2)$$

Note: By choosing ω_1 and ω_2 different and the refracting angles we can get dispersion without average deviation.

9.2 AVERAGE DEVIATION WITHOUT DISPERSION :

If, $\delta_v - \delta_r = 0$

$$(\mu_{1y} - 1)\omega_1 A_1 = (\mu_{2y} - 1)\omega_2 A_2$$

or, $(\mu_{1y} - \mu_{1r})A_1 = (\mu_{2y} - \mu_{2r})A_2$

So, net average deviation

$$\delta = (\mu_{1y} - 1)A_1 - (\mu_{2y} - 1)A_2$$

$$= (\mu_{1y} - 1)A_1\left[1 - \frac{\mu_{2y} - 1}{\mu_{1y} - 1}\frac{A_2}{A_1}\right]$$

$$= (\mu_{1y} - 1)A_1\left[1 - \frac{\omega_1}{\omega_2}\right] - \delta_1\left(1 - \frac{\omega_1}{\omega_2}\right)$$

Solved Examples

Ex.21 A crown glass prism of angle 5° is to be combined with a flint glass prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.514, 1.517 and 1.523 respectively for crown glass and 1.613, 1.620 and 1.632 for flint glass. – 0.0348°

Sol. The deviation produced by the crown prism is

$$\delta = (\mu - 1)A$$

and by the flint prism is

$$\delta' = (\mu' - 1)A'$$

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A' \quad ..... (1)$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'$$

or, $$A' = \frac{(\mu - 1)}{(\mu' - 1)}A \qquad = \frac{1.517 - 1}{1.620 - 1} \times 5^0 = 4.2^0$$

(b) The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is

$$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A'$$

The net angular dispersion is,

$$\delta = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

$$= (1.523 - 1.514) \times 5° - (1.632 - 1.613) \times 4.2°$$

$$= -\ 0.0348°$$

The angular dispersion has magnitude 0.0348^0

Ex.22 Calculate the dispersive power for crown glass from the given data $\mu_v = 1.5230, \mu_r = 1.5145$.

Sol. Mean refractive index,

$$\mu = \frac{\mu_v + \mu_r}{2} = \frac{1.5230 + 1.5145}{2}$$

$$\mu = 1.5187$$

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}$$

$$= \frac{1.5230 - 1.5145}{1.5187 - 1} = \frac{0.0085}{0.5187} = 0.0163$$

TOTAL INTERNAL REFLECTION

The phenomenone :

In case of refraction of light, from Snell's law we have

$$\mu_1 \sin i = \mu_2 \sin r$$

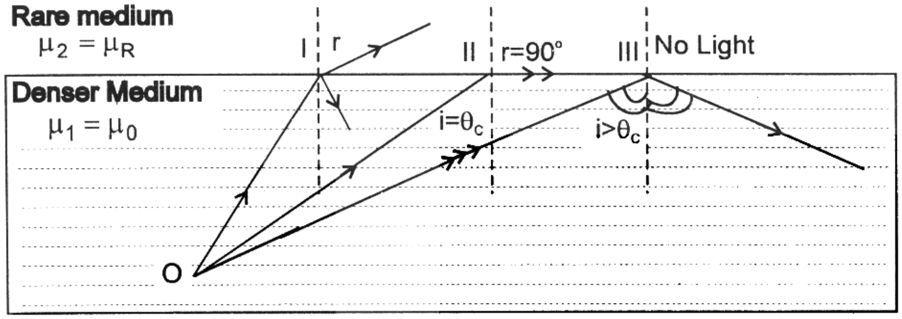
If light is passing from denser to rarer medium through a plane boundary then $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$ so with

$$\mu = \left(\frac{\mu_D}{\mu_R}\right)$$

$$\sin i = \frac{\mu_R}{\mu_D}\sin r \qquad .... (1)$$

i.e., $$\sin i = \frac{\sin r}{\mu} \qquad .... (2)$$

i.e., $\sin i \propto \sin r$ with $(\angle i) < (\angle r)$ (as u > 1)



So as angle of incident i increase angle of refraction r will aslo increase and for certain value of i $\left((\angle 90^0)\right)$ r will become 90^0. The value of angle of incidence for which $v = 90^0$ is called critical angle and is denoted by θc and in the light of equation (2) will be given by

$$\sin\theta_c = \frac{\sin 90}{\mu} \quad \text{i.e,} \quad \sin\theta_c = \frac{1}{\mu} \quad \text{... (3)}$$

And hence equation (2) in terms of critical angle can be written as

$\sin i = \sin r \times \sin\theta_c$ i.e.,

$$\sin r = \frac{\sin i}{\sin\theta_c} \quad \text{... (4)}$$

So, if $i < \theta_c$ $\sin r > 1$. This means that r is imaginary (as the value fo sin of any angle can never be greater than unity physically this situation implies that refracted ray does not exist. So the total light incident of the boundary will be reflected back in to the same medium from the boundary. This phenomena is called total internal reflection.

(1) For total internal reflection to take place light from air to water (or glass) and from water to glass total internal reflection can-never take place.

(2) When light is passing from denser to rare medium total internal reflection will take place only if angle of incidence is greater than a certain value called critical angle given by

$$\theta_c = \sin^{-1}\left[\frac{1}{\mu}\right] \text{ with } \mu = \frac{\mu_D}{\mu_R}$$

(3) In case of total internal reflection as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case reflection from mirror or refraction from lenses there is some loss of intensity as all light can never be reflected or refracted. This is why image formed by TIR are much brighter than formed by mirror or lenses.

1. Critical angle θ_C

In case of propagation of light from denser to rarer medium through a plane boundary critical angle is the angle of incidence for which angle of refraction is 90^0 and so from Snell's law $\mu \sin i = \mu_2 \sin r$

$$\mu_D \sin\theta_C = \mu_R \sin 90$$

ie., $\sin\theta_C = \frac{\mu_R}{\mu_D} = \left[\frac{1}{\mu}\right]$ with $\mu = \frac{\mu_R}{\mu_D}$

or $\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

(1) For a given pair of medium critical angle depends on wavelength of light used i.e, greater the wavelength of light lesser will be μ $\left[\text{as } \mu \propto \frac{1}{v} \propto \frac{1}{\lambda}\right]$ and so greater will be the critical angle. This is why critical angle is maximum for red and minimum for violet rays.

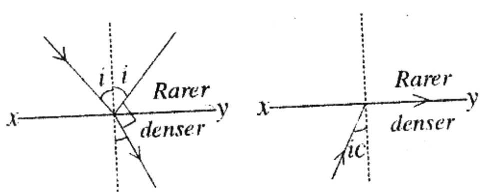
(2) For a given light is depends on nature of pair of medium lesser the μ greater will the critical angle and vice–versa. This is why in case of

Glass Air	**Water Air**	**Glass Water**
As $\mu g = \frac{3}{2}$ and $\mu_A = 1$	As $\mu_W = \frac{4}{3}$ and $\mu_A = 1$	As $\mu_G = \frac{3}{2}$ and $\mu_W = \frac{4}{3}$
i.e., $\mu = \frac{\mu_G}{\mu_A} = \frac{3}{2}$	i.e., $\mu = \frac{\mu_W}{\mu_A} = \frac{4}{3}$	i.e., $\mu = \frac{\mu_G}{\mu_W} = \frac{9}{8}$
So $(\theta_C)_{GA} = \sin^{-1}\left[\frac{2}{3}\right] = 42^0$	So $(\theta_C)_{WA} = \sin^{-1}\left[\frac{3}{4}\right] = 49^0$	So $(\theta_C)_{GW} = \sin^{-1}\left[\frac{8}{9}\right] \approx 63^0$

Solved Examples

Ex.23 A ray of light from a denser medium strikes a rarer medium at an angle of incidence i. If the reflected and refracted rays are mutually perpendicualr to each other, what is the value of critical angle?

Sol. If refractive index of rater medium with respect to denser medium is n



(1) $i + r = 90^o \quad \therefore \quad r = 90 - i$

$$\frac{\sin i}{\sin r} = n \quad \text{or} \quad \frac{\sin i}{\sin(90-i)} = n$$

$$\therefore n = \frac{\sin i}{\cos i} = \tan i$$

(2) $\frac{\sin i_C}{\sin 90} = \frac{1}{n}$

$$\Rightarrow \sin i_C = \frac{1}{n} = \cot i$$

$$\therefore i_C = \sin^{-1}[\cot i]$$

Ex.24 What will be the cirtical angle for diamond in contact with air and water. If refractive index of diamond is 2.42.

Sol.(1) In air $\frac{\sin i_c}{\sin 90} = \frac{1}{n}$

$$\Rightarrow \sin i_c = \frac{1}{2.42} = 0.413$$

$i_c = 24.\ 4^o$

(2) In water $\frac{\sin i_c}{\sin 90} = \frac{1}{{}_1n_2} = \frac{1}{2.42/1.33}$

$\sin i_c = 1.33 \times 0.431 = 0.55 \quad i_c = 33.4^o$

Note : As refracitve index of denser medium decrease i_c increase.

2. Some illustration of total internal reflection :

(1) Shining of air bubble

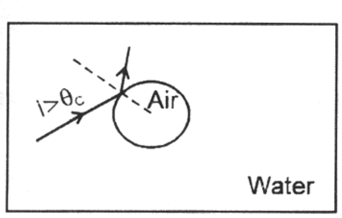
(2) Sparking of diamond

(3) Optical - fibre

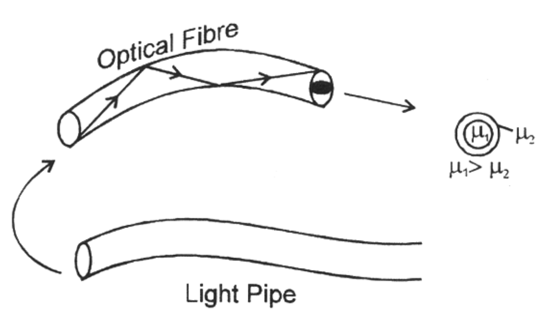
(4) Action of 'Porro' prism

(5) Duration of suns visibility

(6) Mirrage and looming



(A)



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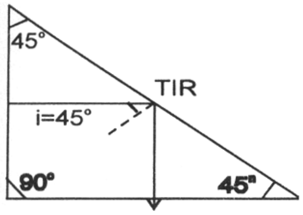
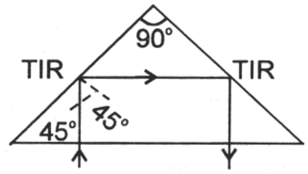


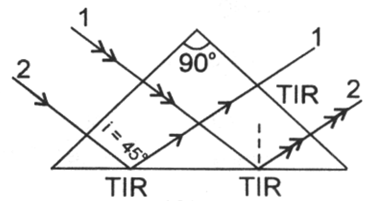
(A)

Bending of rays by 90°



(B)

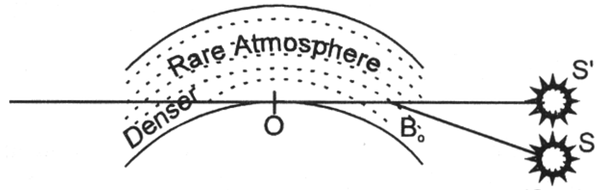
Bending of rays by 180°



(C)

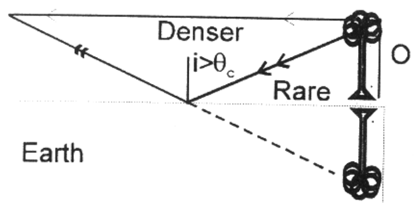
Bending of Image

(A)

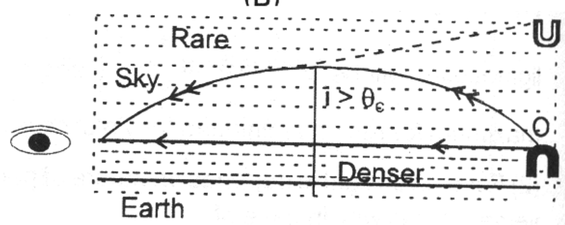


(B)

(Sun)

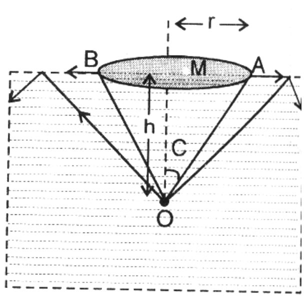


(A) Mirrage



(B) Looming

3. **Outside object seen by fish under water** :
A fish inside water cannot seen the entire surface of the pond. It sees only a circular patch of light because only those light rays which are incident within a cone of semi vertex angle C (critical angle) are refracted out of the water surface all other rays suffer total internal reflection from the figure in ΔAMO



$$\sin C = \frac{1}{\sqrt{r^2 + h^2}}$$

But $\sin C = \frac{1}{\mu}$

Where r is the radius of circular patch of light and h is depth of the fish.

$$\therefore \frac{1}{\mu} = \frac{r}{\sqrt{r^2 + h^2}} \qquad \therefore \qquad r = \frac{h}{\sqrt{\mu^2 - 1}}$$

Solved Examples

Ex.25 In a jar of radius 0.2 m, glycerine is filled upto height of 0.1 m (refracitve index $\mu = 1.4$). The area on teh surface of glycerine from which light emerges out, is 0.032 m^2

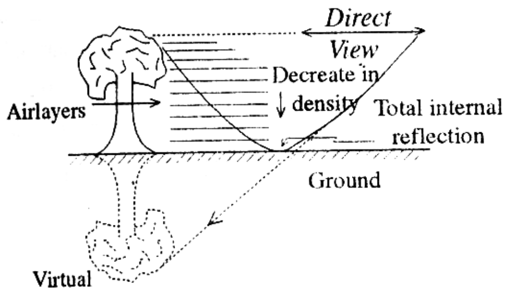
Sol. Given h = 0.1 m, $\mu = 1.4$

$$\therefore \quad r = \frac{h}{\sqrt{\mu^2 - 1}} = \frac{0.1}{\sqrt{(0.4)^2 - 1}} = \frac{0.1}{\sqrt{0.96}}$$

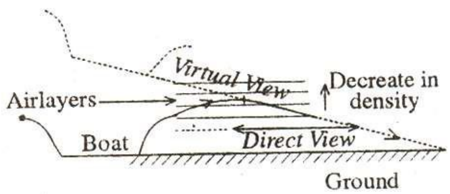
$$\therefore \quad \text{Area} = pr^2 = 3.14 \times \frac{(0.1)^2}{0.96} = 0.032 m^2$$

4. **Mirage**

In deserts, due to heating of earth, refractive index of air near the surface of earth become lesser than above it at a height. This makes light from top of tree to travel from denser to rare medium and thus gradually causing total internal reflection as shown in figure.

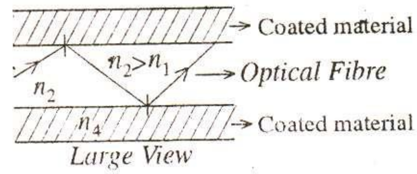


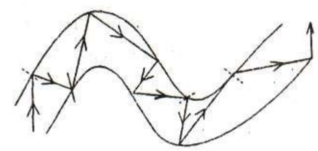
5. **Sun is seen for awhile after sunset and before sun rise. The mechanism is similar to that observed in appearance of mirage**



6. **Optical Fibre**

This is based on total internal reflection. It has fine quartz or plastic fibres spun in the form of rope. It is covered with a material having lower refractive index so that the condition of total internal reflection may be satisfied. Light





is propagated through this fibre by the process of total internal reflection. No power is wasted by way of transmission outside the fiber.

7. **Difference between total internal Reflection and reflection**

S.No.	Reflection	Total Internal Reflection
1.	It is possible in any medium	Possible only when light enters from denser to rarer medium
2.	At any angle of incidence it is possible	When angle of incidence is greater than critical angle.
3.	In this some part of light is absorbed while some is reflected	Total light intensity is reflected
4.	The surface will appear less bright	By total internal reflection surface appears more bright.

RAINBOW

(a) The coloured spectrum of white light from the sun in the form of bows which is seen immediately after the rain in the sky be an observer with his back towards the sun is called a rainbow.

(b) These are of two types :

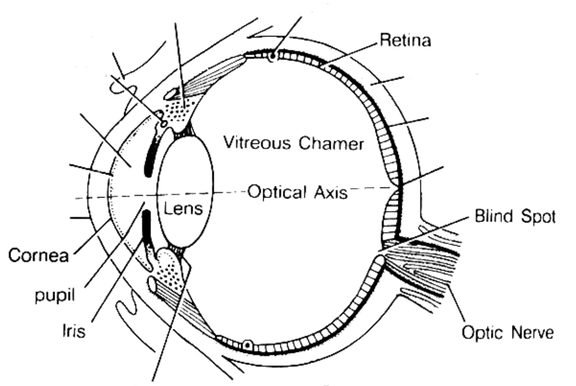
Primary rainbow	Secondary rainbow
Its order of colour is from red to violet Its intensity is more It is formed by first total internal reflection and then the dispersion.	Its order of colour is from violet to red Its intensity is less It is formed by two times internal reflection and then dispersion.

1. **Conditions required for rainbow**
   (i) Rainbow is formed at a place having latitude of 42º from surface of earth (the light ray which emerge from the droplet must make an angle 42º with the light rays coming from sun).
   (ii) The light rays must travel from denser (droplet) medium to rarer medium (air) so that conditions of total internal reflection are fully satisfied. Hence $i > i_c$ and also the dispersion should take place.

OPTICAL INSTRUMENT

STRUCTURE OF EYE

Fig. Shows the eye, light enters the eye through a curved front surface, the corner. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye-lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles. For example, when the muscle is released, the focal length is about 2.5 cm and (for a normal eye) objects at infinity are in sharp focus on the retinas. When the object is brought closer to the eye, in order to maintain the same image-lens distance ($\cong$ 2.5 cm), the focal length of the eye-lens becomes shorter by the action of the ciliary muscles. This property of the eye in called accommodation. If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred.



Human Eye

The closest distance for which the lens can focus light on the retina is called the least distance of distinct vision, or the near point. The standard value (for normal vision) taken here is 25 cm. (Often the near point is given the symbol D.)

DEFECTS OF VISION

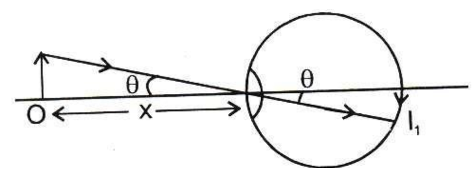
Regarding eye it is nothing that:

(1) In eye convex eye-lens forms real inverted and diminished image at the retina by changing its convexity (the distance between eye lens and retina is fixed)

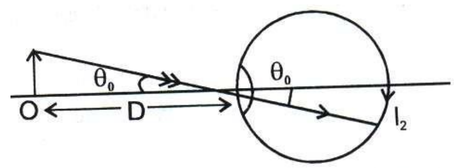
(2) The human eye is most sensitive to yellow green light having wavelength 5550 A^0 and least to violet (4000 A^0) and red (7000 A^0)

(3) The size of an object as perceived by eye depends on its visual-angle when object is distant its visual angle θ and hence image I_1 at retina is small and it will appear small and as it is brought near to the eye its visual angle θ_0 and hence size of image I_2 will increase.

(A)

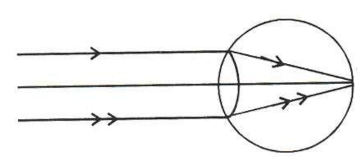


(B)



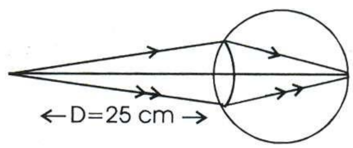
(4) The far and near point for normal eye are usually taken to be infinity and 25 cm an respectively ie., normal eye can see very distant object clearly but near objects only if they are a distance greater than 25 cm from the eye. The ability of eye to see objects from infinite distance to 25 cm from it is called Power of accommodation.

(5) If object is at infinity i.e., parallel beam of light enters the eye is least strained and said to be relaxed or unstrained. However, if the object is at least distance of dinstinct vision (L.D.D.V] i.e., D (=25 cm) eye is under maximum strain and visual angle is maximum.



Relaxed or normal eye

(A)



Maximum strained eye

(B)

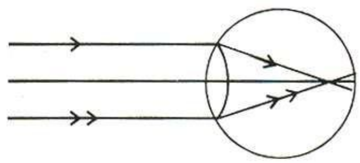
(6) The limit of resolution of eye is one minute ie., two object will not be visible dinstinctly to the eye if the angle substanded by them on the eye is lesser than one minute.

(7) The persistance of vision is (1/10) sec i.e., If time interval between two consecutive light pulses is lesser than 0.1 sec eye cannot distinguish them separately. This fact is taken into account in motion pictures.

In case of eye following are the common defects of vision.

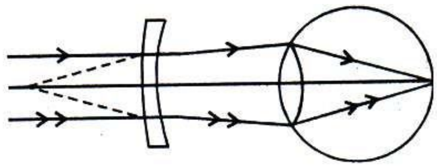
1. MYOPIA [or short-sightendness or near - sightendness]

In it distant objects are not clearly visible. i.e. Far Point is at a distance lesser than



Defective-eye

(A)



Corrected-eye

(B)

Infinity and hence image of distant object is formed before the retina.

This defect is (i.e., negative focal length or power) which forms the image of distant object at the far point of patient - eye [which is lesser than ∞] so that in this case from lens formula we have

$$\frac{1}{-F.P} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

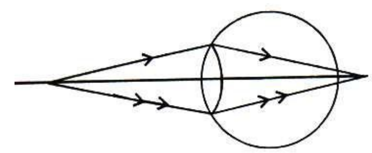
And if the object is at ∞

$$P = \frac{1}{f} = \frac{1}{-F.P.} \quad .... \quad (1)$$

2. HYPERMETROPIA [Or Long-sightendness or far-sightendness]

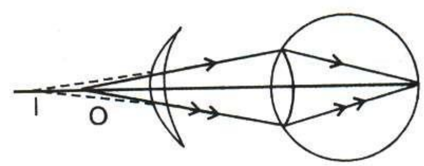
In it near object are clearly visible i.e., Near Point is at a distance greater than 25 cm and hence image of near object is formed behind the retina.

This defect is remedied by using spectacles having convergent lens (i.e., positive focal length of power) which the image of near objects at the Near Point of the



Defective-eye

(A)



Corrected-eye

(B)

Patient-eye (which is more than 25 cm). So that in this case from lens formula we have

$$\frac{1}{-N.P.} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

If object is placed at D = 25 cm = 0.25 m

$$P = \frac{1}{f} = \left[\frac{1}{0.25} - \frac{1}{N.P.}\right] \quad ......(2)$$

3. PRESBYOPIA

In this both near and far objects are not clearly visible i.e., far point is lesser than infinity and near point greater than 25 cm. This is removed either by using two separate spectacles one for lenses. It is an old age disease as at old age ciliary muscles lose their elasticity and so can not change the focal length of eye-lens effectively and hence eye loses its power of accommodation.

4. ASTIGMATISM

In it due to imperfect spherical nature of eye-lens, the focal length of eye lens is two orthogonal directions becomes different and so eye cannot see object in two orthogonal directions clearly simultaneously. This defect is directional and is remedied by using cylinderical lens in particular direction. If in the spectacle of a person suffering from astigmatism, the lens is slightly rotated the arrangement will get spoiled.

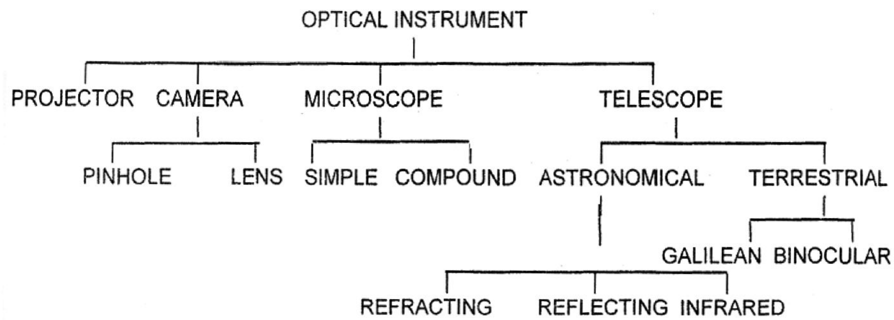
OPTICAL INSTRUMENTS

Definition :

Optical instruments are used primarily to assist the eye in viewing an object.

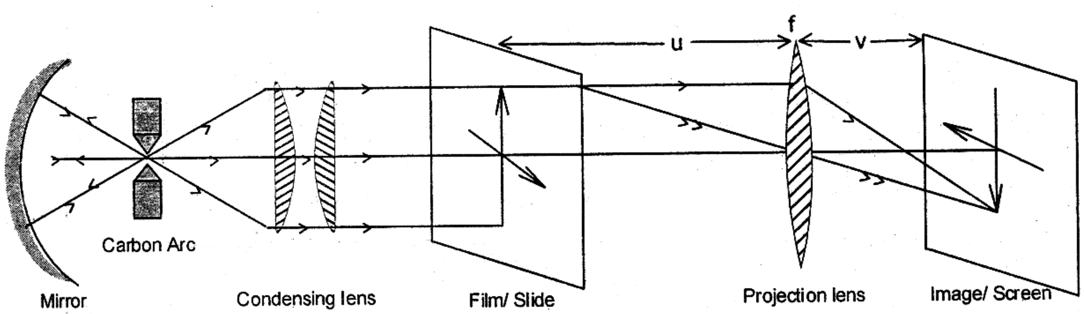
Types of Instruments :

Depending upon the use, optical instruments can be categorised in the following way :



1. FILM OR SLIDE PROJECTOR

It projects real, inverted and magnified image of an object, when the object is placed between F and 2F and screen between 2F and ∞



Magnification :

If the sides of slide are a and b and magnification of projection lens is m, each side will become m times so that there area of image formed.

$A_i = ma \times mb = m^2ab = m^2A_0$

Intensity :

As area of image becomes m^2 times that of object, the intensity of image will become $\frac{1}{m^2}$ times that of object, as

$IA_0 = I_iA_i = I_im^2A_0 \Rightarrow I_i\left(\frac{I}{m^2}\right)$

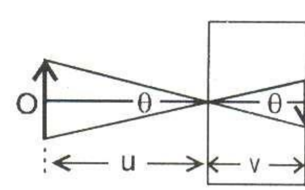
2. CAMERA

(A) PINHOLE CAMERA

It is bases on rectilinear propagation of light and forms the so called image on the screen which is real and inverted. If an object of size O is situated at a distance u from the pinhole and its image of size I is formed at a distance v from the pin hole-

$\theta = \frac{O}{u} = \frac{I}{v}$

i.e., $\frac{I}{O} = \frac{v}{u}$

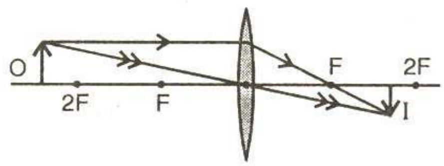


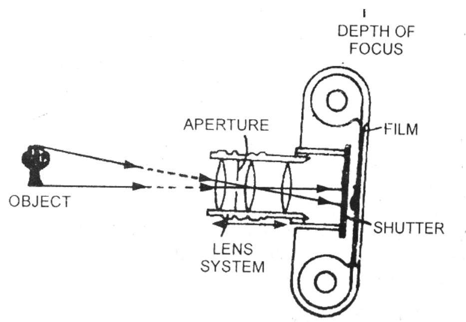
(B) LENS-CAMERA

In it a covering lens whose aperture and distance from the film can be adjusted, is used. Usually object is real and between ∞ and 2F, so the image is real, inverted diminished and between F and 2F as shown in figure.

Here lens formula

$\frac{1}{\upsilon}-\frac{1}{u}=\frac{1}{f}$ with $m=\frac{I}{O}=\frac{\upsilon}{u}$ is applicable.





In photographing an object, the image is first focused on the film by adjusting the distance between lens and film (called focusing). After focusing, aperture is set to a specific value (for desired effect) and then film is exposed to light for a given time through a shutter. For proper exposure of a particular film, a definite amount of light energy must be incident on the film. So if I is the intensity of light, S is the light transmitting area of lens and t is the exposure time, then for proper exposure,

$I\times S\times t=\text{constant}$

Light transmitting area of a lens is proportional to the square of its aperture D; so above expression reduces to $I\times D^2\times t=\text{constant}$

SPECIAL POINTS

If aperture is kept fixed, for proper exposure,

$I\times t=\text{constant}$ i.e., $I_1t_1=I_2t_2$

and if the source of light is a point

$\frac{L_1}{r_1^2}\times t_1=\frac{L_2}{r_2^2}\times t_2$ $\left[\text{as } I=\frac{L}{r^2}\right]$

If intensity is kept fixed, for proper exposure,

$D^2\times t=\text{constant}$

i.e., Time of exposure $\propto\frac{1}{(\text{Aperture})^2}$

The ratio of focal length to the aperture of lens is called f-number of the camera, i.e.,

$\text{f-number}=\frac{\text{Focal length}}{\text{Aperture}}$

If focal length = constant

$\text{Aperture}\propto\frac{1}{\text{f-number}}$

$\text{Time of expose}\propto(\text{f-number})^2$

Solved Examples

Ex.26 Photograph of the ground are taken from an aircraft, flying at an altitude of 2000 m, by a camera with a lens of focal length 50 cm. The size of the film in the camera is 18 cm x 18 cm. What area of the ground can be photographed by this camera at any one time.

Sol. As here u = –2000 m, f = 0.50 m, so from lens formula $\frac{1}{\upsilon}-\frac{1}{u}=\frac{1}{f}$, we have

$\frac{1}{\upsilon}-\frac{1}{(-2000)}=\frac{1}{0.5}$

i.e., $\frac{1}{\upsilon}=\frac{1}{0.5}-\frac{1}{2000}\cong\frac{1}{0.5}\left[\text{as }\frac{1}{0.5}>>\frac{1}{2000}\right]$

$\upsilon=0.5\,\text{m}=50\,\text{cm}=f$

Now as in case of a lens

$m=\frac{\upsilon}{u}=\frac{0.5}{-2000}=-\frac{1}{4}\times10^{-3}$

So $I_1=(ma)(mb)=m^2A$

$[\because A=ab]$

i.e, $A=\frac{I_1}{m^2}=\frac{18\,\text{cm}\times18\,\text{cm}}{\left[\left(\frac{1}{4}\right)\times10^{-3}\right]^2}=(720\,\text{m}\times720\,\text{m})$

Ex.27 The proper exposure time for a photographic print is 20 s at a distance of 0.6 m from a 40 candle power lamp. How long will you expose the same print at a distance of 1.2 m from a 20 candle power lamp?

Sol. In case of camera, for proper exposure

$$I_1D_1^2t_1 = I_2D_2^2t_2$$

As here D is constant and $I = \left(\frac{L}{r^2}\right)$

$$\frac{L_1}{r_1^2} \times t_1 = \frac{L_2}{r_2^2} \times t_2$$

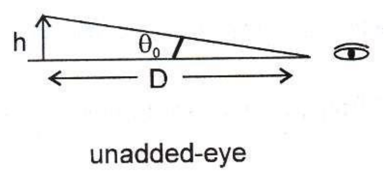
So, $\frac{40}{(0.6)^2} \times 20 = \frac{20}{(12)^2}t$ i.e., t = 160 sec

3 MICROSCOPE

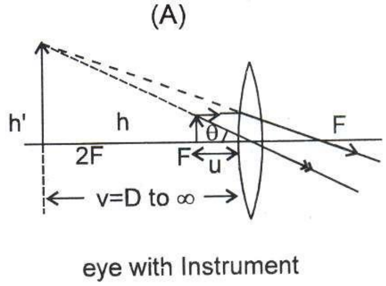
It is an optical instrument used to increase the visual angle of neat objects which are too small to be seen by naked eye.

3.1 SIMPLE MICROSCOPE

It is also known as magnifying glass or simply magnifier and consists of a convergent lens with object between its focus and optical centre and eye close to it. The image formed by it is erect, virtual enlarged and on same side of lens between object and infinity.



(A)



(B)

The magnifying power (MP) or angular magnification of a simple microscope (or an optical instrument) is defined as the ratio of visual angle with instrument to the maximum visual angle for clear vision when eye is unadded (i.e., when the object is at least distance of distinct vision)

i.e.,

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max.visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$

If an object of size h is placed at a distance u (< D) from the lens and its image size h' is formed at a distance V ($\geq$ D) from the eye

$$\theta = \frac{h'}{v} = \frac{h}{u} \quad \text{with} \quad \theta_0 = \frac{h}{D}$$

So $MP = \frac{\theta}{\theta_0} = \frac{h}{u} \times \frac{D}{h} = \frac{D}{u}$ ...... (1)

Now there are two possibilities

(a_1) If their image is at infinity [Far point]

In this situation from lens formula -

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have $\frac{1}{\infty} - \frac{1}{-u} = \frac{1}{f}$ i.e., u = f

So $MP = \frac{D}{u} = \frac{D}{f}$ ...... (2)

As here u is maximum [as object is to be with in focus], MP is minimum and as in this situation parallel beam of light enters the eye, eye is least strained and is said to be normal, relaxed or unstrained.

(a_2) If the image is at D [Near point]

In this situation as v = D, from lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have $\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$

i.e., $\frac{D}{u} = 1 + \frac{D}{f}$

So $MP = \frac{D}{u} = \left[1 + \frac{D}{f}\right]$ ...... (3)

As the minimum value of v for clear vision is D, in this situation u is minimum and hence this is the maximum possible MP of a simple microscope and as in this situation final image is closest to eye, eye is under maximum strain.

SPECIAL POINTS

(1) Simple magnifier is an essential part of most optical instruments (such as microscope or telescope) in the form of eye piece or ocular.

(2) The magnifying power (MP) have no unit. It is different from power of a lens which is expressed in diopter (D) and is equal to the reciprocal of focal length in metre.

(3) With increase in wavelength of light used, focal length of magnifier will increase and hence its MP will decrease.

Solved Examples

Ex.28 A man with normal near point (25 cm) reads a book with small print using a magnifying a thin convex lens of focal length 5 cm. (a) What is the closest farest distance at which he can read the book when viewing through the magnifying glass? (b) What is the maximum and minimum MP possible using the above simple microscope?

Sol.

(a) As for normal eye far and near point are ∞ and 25 cm respectively, so for magnifier $\upsilon_{max} = -\infty$ and $\upsilon_{min} = -25\,cm$. However, for a lens as

$$\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{f} \quad \text{i.e.,} \quad u = \frac{f}{\left(\frac{f}{\upsilon}\right) - 1}$$

So u will be minimum when $\upsilon = min = -25\,cm$

i.e., $$(u)_{min} = \frac{5}{\left(\frac{-5}{25}\right) - 1} = -\frac{25}{6} = -4.17\,cm$$

And u will be maximum when $\upsilon = max = \infty$

So, the closest and farest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) An in case of simple magnifier $MP = \left(\frac{D}{u}\right)$. So MP will be minimum when u = max = 5 cm

i.e., $$(MP)_{min} = \frac{-25}{-5} = 5 \qquad \left[= \frac{D}{f}\right]$$

And MP will be maximum when $u = min = \left(\frac{25}{6}\right) cm$

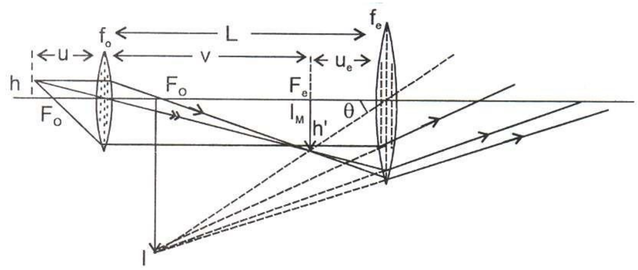
i.e., $$(MP)_{max} = \frac{-25}{-\left(\frac{25}{6}\right)} = 6\left[= 1 + \frac{D}{f}\right]$$

3.2 COMPOUND-MICROSCOPE CONSTRUCTION

It consists of two convergent lenses of short focal lengths and apertures arranged co-axially lens (of focal length f_0) facing the object is called objective or field lens while the lens (of focal length f_e) facing the eye, eye-piece or ocular. The objective has a smaller aperture and smaller focal length than eye-piece. The separation between objective and eye-piece can be varied.

IMAGE FORMATION :

The object is placed between F and 2F of objective so the image IM formed by objective (called intermediate image) is inverted, real enlarged and at a distance greater than f_0 on the otherside of the lens. This image IM acts as object for eye-piece and is with in its focus. So eye-piece forms final image I which is erect, virtual and enlarged with respect to intermediate image I_M. So the final image I with respect to object is inverted, virtual, enlarged and at a distance D to ∞ from eye on the same side of eye-piece as I_M. This all is shown in figure.



Magnifying power (MP)

Magnifying power of an optical instrument is defined as

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max.Visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$

If the size of object is h and least distance of distinct vision is D.

$$\theta_0 = \left[\frac{h}{u_e}\right] \times \left[\frac{D}{h}\right] = \left[\frac{h'}{h}\right]\left[\frac{D}{u_e}\right]$$

But for objective

$$m = \frac{I}{O} = \frac{v}{u} \text{ i.e., } \frac{h'}{h} = -\frac{v}{u} \text{ [as u is positive]}$$

So, $MP = -\frac{v}{u}\left[\frac{F}{u_e}\right]$ with length of tube

$L = v + u_e$, ...(1)

now there are two possibilities

(b_1) If the final image is at infinity (far point):

This situation is called normal adjustment as in this situation eye is least strained or relaxed. In this situation as for eye-piece $v = \infty$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{i.e.,} \quad u_e = f_e = \text{maximum}$$

Substitution this value of u_e in equation (1), we have

$$MP = -\frac{v}{u}\left[\frac{D}{f_e}\right] \text{ with } L = v + f_e \quad ..... (2)$$

A microscope is usually considered to operate in this mode unless state otherwise. In this mode as u_e is maximum MP is minimum for a given microscope.

(b_2) If the final image is at D (near point) :

In this situation as for eye-piece v = D

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \qquad \text{i.e.,} \qquad \frac{1}{u_e} = \frac{1}{D}\left[1 + \frac{D}{f_e}\right]$$

Subsitting this value of u_e in equation (1), we have

$$MP = -\frac{v}{u}\left[1 + \frac{D}{f_e}\right] \quad \text{with} \quad L = v + \frac{f_e D}{f_e + D} \quad ..... (3)$$

In this situation as u_e is minimum MP is maximum and eye is most strained.

Solved Examples

Ex.29 The focal length of the objective and eyepiece of a microscope are 2 cm and 5 cm respectively and the distance between them is 20 cm. Find the distance of object from the objective, when the final image seen by the eye is 25 cm from the eyepiece. Also find the magnifying power.

Sol. Given $f_0 = 2$ cm, $f_e = 5$ cm

$| v_o | + | u_e | = 20$ cm

$\therefore \quad v_e = -25$ cm

From lens formula $\frac{1}{f_e} = \frac{1}{v_o} - \frac{1}{u_e}$

$$\frac{1}{u} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$$

$$\therefore \quad u_e = -\frac{25}{6} \text{ cm}$$

Distance of real image from objective

$$v_o = 20 - | u_e | = 20 - \frac{25}{6}$$

$$= \frac{120 - 25}{6} = \frac{95}{6} \text{ cm}$$

Now $\quad \frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

given $\quad \frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{(95/6)} - \frac{1}{2}$

i.e., $\frac{1}{u_o} = \frac{6}{95} - \frac{1}{2} = \frac{12 - 95}{190} = -\frac{83}{190}$

$$\therefore \quad u_o = -\frac{190}{83} = -2.3 \text{ cm}$$

Magnifying power $M = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

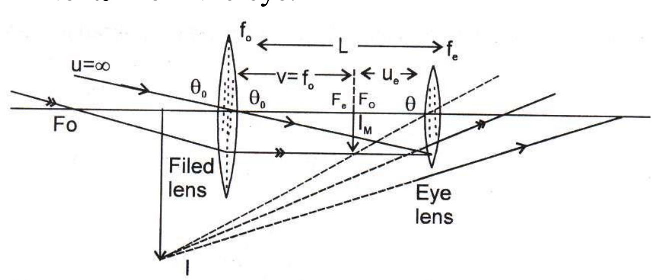
$$= -\frac{95/6}{(190/83)}\left(1 + \frac{25}{3}\right)$$

$= -41.5$

4. TELESCOPE

It is an optical instrument used to increase the visual angle of distant large objects such as a star a planet or a cliff etc. Astronomical telescope consists of two converging lens. The one facing the object is called objective or field-lens and has large focal length and aperture. The distance between the two lenses is adjustable.

As telescope is used to see distant objects, in it object is between ∞ and 2F of objective and hence image formed by objective is real, inverted, and diminished and is between F and 2F on the other side of it. This image is (called intermediate image) acts as object for eye-piece and shifting the position of eye-piece is brought with in its focus. So final image I, with respect to intermediate image is erect, virtual, enlarged and at a distance D to ∞ from the eye. This in turns implies that final image with respect to object is inverted, enlarged and at a distance D to ∞ from the eye.



Magnifying Power (MP)

Magnifying Power of a telescope is defined as

$$MP = \frac{\text{Visual angle with instrument}}{\text{Visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$

But from figure.

$$\theta_0 = \left(\frac{y}{f_0}\right) \text{ and } \theta = \left(\frac{y}{-u_e}\right)$$

So $MP = \frac{\theta}{\theta_0} = -\left[\frac{f_0}{u_e}\right]$ with length of tube

$L = (f_0 + u_e)$ .... (1)

Now there are two possibilities

(d_1) If the final image is at infinity (far point)

This situation is called normal adjustment as in this situation eye is least strained or relaxed. In this situation as for eye-piece v = ∞

$$\frac{1}{-\infty} - \frac{1}{u_e} = \frac{1}{f_e} \quad \text{i.e.,} \quad u_e = f_e$$

So, substituting this value of u_e in equation (1) we have

$$MP = -\left(\frac{f_0}{f_e}\right) \text{ and } L = (f_0 + f_e)$$

Usually telescope operates in this mode unless stated other wise. In this mode as u_e is maximum for a given telescope MP is minimum while length of tube maximum.

(d_2) If the final image is at D (near point)

In this situation as for eye-piece v = D

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{i.e.,} \quad \frac{1}{-u_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

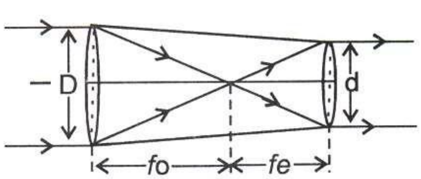
So substituting this value of u_e in Equation (1), we have

$$MP = \frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right] \quad \text{with} \quad L = f_0 + \frac{f_e D}{f_e + D} \quad \text{...... (3)}$$

In this situation u_e is minimum so for a given telescope MP is maximum while length of tube minimum and eye is most strained. In case of a telescope if object and final image are at infinity and total light entering the telescope leaves it parallel to its axis as shown in figure.

$$\frac{f_0}{f_e} = \frac{\text{Aperture of object}}{\text{Aperture of eye piece}}$$

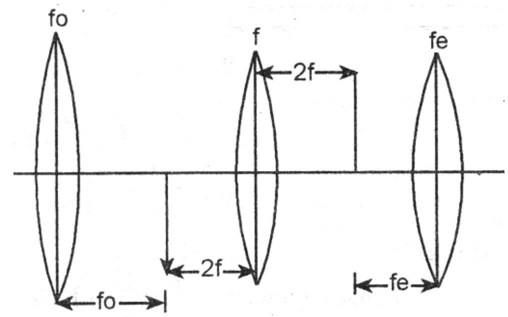
i.e., $MP = \frac{f_0}{f_e} = \frac{D}{d}$ ..... (4)



4.1. TERRESTRIAL TELESCOPE

Uses a thrd lens in between objective and eyepieces so as to form final image erect. This lens simply invert the image formed by objective without affecting the magnification.

Length of tube $L = f_0 + f_e + 4f$



4.2 GALILEO'S TELESCOPE

Convex lens as objective.

Concave lens as eyepiece.

Field of view is much smaller

$\because$ eyepiece lens in concave.

(i) $M = \frac{f_0}{f_e}\left[1 - \frac{f_e}{v_e}\right]$

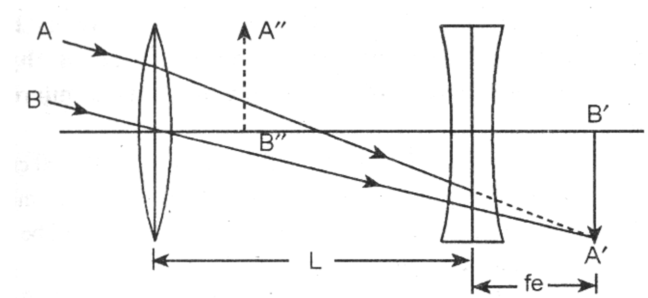
(ii) $M = \frac{f_0}{f_e}$

Final image is at α

$L = f_0 - f_e$

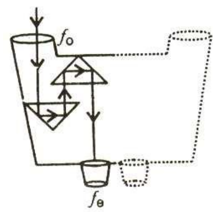
(iii) $M = \frac{f_0}{f_e}\left[1 - \frac{f_e}{D}\right]$

Final image is at D. $L = f_0 - u_e$



4.3 BINOCULAR

In this telescope as intermediate image is outside the tube, the telescope cannot be used for making measurements. If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called **'binocular'**. In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms (fig.)which provide intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time. Their superposition gives the perception of depth also along with length and breadth, i.e., binocular vision given proper three-dimensional (3-D) image.



Binocular

Solved Examples

Ex.30 A telescope consists of two convex lens of focal length 16 cm and 2 cm. What is angular magnification of telescope for relased eye? What is the separation between the lenses?

If object subtends an angle of 0.5° on the eye, what will be angle subtended by its image ?

Sol. Angular magnification

$$M = \frac{\alpha}{\beta} = \frac{F}{f} = \frac{16}{2} = 8 \text{ cm}$$

Separation between lenses

$= F + f = 16 + 2 = 18$ cm

Here $\alpha = 0.5^\circ$

$\therefore$ Angular subtended by image

$\beta = M\,\alpha = 8 \times 0.5^\circ = 4^\circ$

Ex.31 The magnifying power of the telescope if found to be 9 and the separation between the lenses is 20 cm for relased eye. What are the focal lengths of component lenses ?

Sol. Magnification $M = \frac{F}{f}$

Separaton between lenses

$d = F + f$

Given $\frac{F}{f} = 9$ i.e., $F = 9f$ ......(1)

and $F + f = 20$ ......(2)

Putting value of F from (1) in (2), we get

$9f + f = 20 \Rightarrow 10\,f = 20$

$\Rightarrow \frac{20}{10} = 2\text{cm}$

$\therefore$ $F = 9f = 9 \times 2 = 18$ cm

$\therefore$ $F = 18$ cm, $f = 2$ cm

5. COMPARISION BETWEEN COMPOUND - MICROSCOPE & ASTRONOMICAL - TELESCOPE

S.No.	Compound - Microscope	Astronomical - Telescope
1.	It is used to increase visual angle of near tiny object.	It is used to increase visual angle of distant large objects.
2.	In it field and eye lense both are convergent, of short focal lengh and aperture.	In it field lens is of large focal length and aperture while eye lens of short focal length and aperature and both are convergent.
3. 4.	Final image is inverted. virtual and enlarged and at a distance D to ∞ from the eye. MP does not change appreciably if field and	Final image in inverted, virtual and enlarged at a distance D to ∞ from the eye MP becomes $(1/m^2)$ times of its initial value if
 5.	eye lens are intercharged $[MP \sim (LD/f_0 f_e)]$ MP is increased by decreasing the focal length of both the lenses viz. find and eye lens.	field and eye-lenses are interchanged as MP $\sim[f_0/f_e]$ MP is increased by increasing the focal length of field of field lens (and decreasing the focal length of eye lens.)
6.	RP is increased by decreasing the wavelength of light used.	RP is increased by increasing the aperture of objective.