ATOMIC STRUCTURE

Structure of matter always been an interesting area of research for physicists. Till 20th century it was assumed that matter consists of indivisible small tiny particles called "atoms". But with the study and research it was found that the atom is divisible and made of other small partiicles called electron, proton and Neutron. So may physicists tried to explain the structure of atom but finally it was Neils Bohr whose explanation about the structure was well accepted. For simplicity they have taken hydrogen atom and then it can be extended to other H-like atoms too. Some of the historical modles are also explained and them drawbacks.

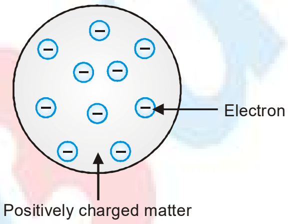
VARIOUS MODELS FOR STRUCTURE OF ATOM

Dalton's Theory

Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided. It can neither be created nor be destroyed. All atoms of same element are identical physically as well as chemically, whereas atoms of different elements are different in properties. The atoms of different elements are made up of hydrogen atoms. (The radius of the heaviest atom is about 10 times that of hydrogen atom and its mass is about 250 times that of hydrogen). The atom is stable and electrically neutral.

Thomson's Atom Model

The atom as a whole is electrically neutral because the positive charge present on the atom (sphere) is equal to the negative charge of electrons present in the sphere. Atom is a positively charged sphere of radius 10^{-10} m in which electrons are embedded in between. The positive charge and the whole mass of the atom is uniformly distributed throughout the sphere.



Shortcomings of Thomson's model

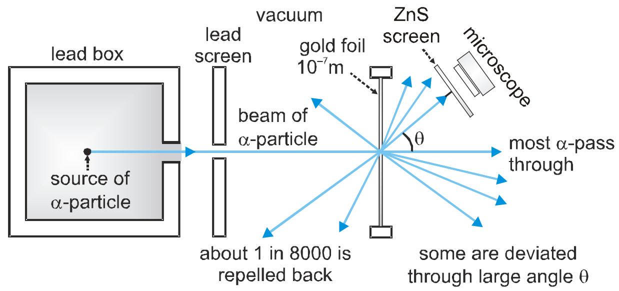
(i) The spectrum of atoms cannot be explained with the help of this model

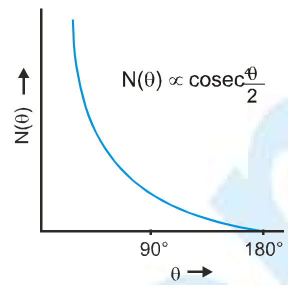
(ii) Scattering of α–particles cannot be explained with the help of this model

RUTHERFORD ATOM MODEL

Rutherford experiments on scattering of α – particles by thin gold foil

The experimental arrangement is shown in figure. α–particles are emitted by some radioactive material (polonium), kept inside a thick lead box. A very fine beam of α–particles passes through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil. While passing through the gold foil, α–particles are scattered through different angles. A zinc sulphide screen was placed out the other side of the gold foil. This screen was movable, so as to receive the α–particles, scattered from the gold foil at angles varying from 0 to 180°. When an α–particle strikes the screen, it produces a flash of light and it is observed by the microscope. It was found that :





(i) Most of the α – particles went straight through the gold foil and produced flashes on the screen as if there were nothing inside gold foil. Thus the atom is hollow.

(ii) Few particles collided with the atoms of the foil which have scattered or deflected through considerable large angles. Few particles even turned back towards source itself.

(iii) The entire positive charge and almost whole mass of the atom is concentrated in small centre called a nucleus.

(iv) The electrons could not deflected the path of a α – particles i.e. electrons are very light.

(v) Electrons revolve round the nucleus in circular orbits. So, Rutherford 1911, proposed a new type of model of the atom. According to this model, the positive charge of the atom, instead of being uniformly distributed throughout a sphere of atomic dimension is concentrated in a very small volume (Less than 10^{-13}m is diameter) at it centre. This central core, now called nucleus, is surrounded by clouds of electron makes.

The entire atom electrically neutral. According to Rutherford scattering formula, the number of α – particle scattered at an angle θ by a target are given by $N_\theta = \dfrac{N_0 nt(2Ze^2)^2}{16(4\pi\varepsilon_0)^2 r^2 (mv_0^2)^2} \times \dfrac{1}{\sin^4 \frac{\theta}{2}}$

Where N_0 = number of α – particles that strike the unit area of the scatter

n = Number of target atom per m^3

t = Thickness of target

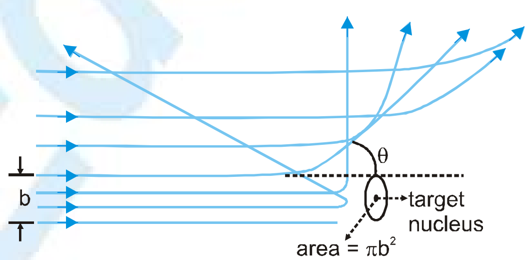
Ze = Charge on the target nucleus

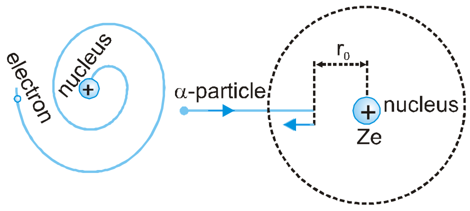
2e = Charge on α – particle

r = Distance of the screen from target

v_0 = Velocity of α – particles at nearer distance of approach the size of a nucleus or the distance of nearer approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0} \times \frac{(2Ze)^2}{\left[\frac{1}{2}mv_0^2\right]} = \frac{1}{4\pi\epsilon_0}\frac{(2Ze)^2}{E_K}$$ where E_K = K.E. of α–particle



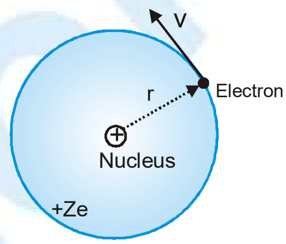


Bohr's Atomic Model

In 1913 Neils Bohr, a Danish Physicist, introduced a revolutionary concept i.e., the quantum concept to explain the stability of an atom. He made a simple but bold statement that "The old classical laws which are applicable to bigger bodies cannot be directly applied to the sub–atomic particles such as electrons or protons.

Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

1. The centripetal force required for an encircling electron is provided by the electrostatic attraction between the nucleus and the electron i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ ...(i)



ε_0 = Absolute permittivity of free space = $8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

m = Mass of electron

v = Velocity (linear) of electron

r = Radius of the orbit in which electron is revolving.

Z = Atomic number of hydrogen like atom.

2. Electrons can revolve only in those orbits in which angular momentum of electron about nucleus is an integral multiple of $\frac{h}{2\pi}$. i.e., $mvr = \frac{nh}{2\pi}$ ...(ii)

n = Principal quantum number of the orbit in which electron is revolving.

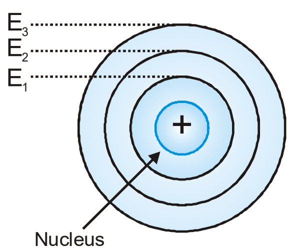
3. Electrons in an atom can revolve only in discrete circular orbits called stationary energy levels (shells). An electron in a shell is characterised by a definite energy, angular momentum and orbit number. While in any of these orbits, an electron does not radiate energy although it is accelerated.

4. Electrons in outer orbits have greater energy than those in inner orbits. The orbiting electron emits energy when it jumps from an outer orbit (higher energy states) to an inner orbit (lower energy states) and also absorbs energy when it jumps from an inner orbit to an outer orbit. $E_n - E_m = h\nu_{n,m}$

where, E_n = Outer energy state

E_m = Inner energy state

$\nu_{n,m}$ = Frequency of radiation



5. The energy absorbed or released is always in the form of electromagnetic radiations.

MATHEMATICAL ANALYSIS OF BOHR'S THEORY

From above equation (i) and (ii) i.e., $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ and $mvr = \frac{nh}{2\pi}$ ...(ii)

We get the following results.

1. **Velocity of Electron in nth Orbit :** By putting the value of mvr in equation (i) from (ii) we get

$$\frac{1}{4\pi\varepsilon_0}Ze^2 = \left(\frac{nh}{2\pi}\right)\times v \Rightarrow v = \frac{Z}{n}\left[\frac{e^2}{2\varepsilon_0 h}\right] = \frac{Z}{n}v_0 \qquad ....(iii)$$

Where, $v_0 = \frac{(1.6\times10^{-19})^2}{2\times8.85\times10^{-12}\times6.625\times10^{-34}} = 2.189\times10^6\ ms^{-1} = \frac{c}{137} = 2.2\times10^6$ m/s

where $c = 3\times10^8$ m/s = speed of light in vacuum

2. Radius of the n^{th} orbit : From equation **(iii)**, putting the value of v in equation **(ii)**, we get

$$m\left(\frac{Z}{n}\times\frac{e^2}{2\varepsilon_0 h}\right)r = \frac{nh}{2\pi} \Rightarrow r = \frac{n^2}{Z}\left[\frac{\varepsilon_0 h^2}{\pi m e^2}\right] = \frac{n^2}{Z}r_0 \quad ...(iv)$$

where $$r_0 = \frac{8.85\times10^{-12}\times(6.625\times10^{-34})^2}{3.14\times9.11\times10^{-31}\times(1.6\times10^{-19})^2} = 0.529\times10^{-10}\,m \approx 0.53\,Å$$

3. Total energy of electron in n^{th} orbit :

From equation **(i)** $KE = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$ and $PE = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r} = -2K.E.$ $\therefore |PE| = 2\,KE$

Total energy of the system $E = KE + PE = -2KE + KE = -KE = \frac{-Ze^2}{8\pi\varepsilon_0 r}$

By putting the value of r from the equation **(iv)**, we get $E = \frac{Z^2}{n^2}\left(-\frac{me^4}{8\varepsilon_0^2 h^2}\right) = \frac{Z^2}{n^2}\cdot E_0$ ...**(v)**

where $$E_0 = \frac{-(9.11\times10^{-3})(1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2\times(6.625\times10^{-34})^2} = -13.6\ eV$$

4. Time period of revolution of electron in n^{th} orbit : $T = \frac{2\pi r}{v}$

By putting the values of r and v, from **(iii)** and **(iv)** $T = \frac{n^3}{Z^2}\times\left(\frac{4\,\varepsilon_0^2\,h^3}{me^4}\right) = \frac{n^3}{Z^2}\cdot T_0$

where, $$T_0 = \frac{4\times(8.85\times10^{-12})^2\times(6.625\times10^{-34})^3}{9.11\times10^{-31}\times(1.6\times10^{-19})^4} = 1.51\times10^{-16}\ \text{second}$$

5. Frequency of revolution in n^{th} orbit :

$f = \frac{1}{T} = \frac{Z^2}{n^3}\times\frac{me^4}{4\varepsilon_0^2 h^3} = \frac{Z^2}{n^3}\cdot f_0$ where, $f_0 = \frac{9.11\times10^{-31}\times(1.6\times10^{-19})^4}{4\times(8.85\times10^{-12})^2(6.625\times10^{-34})^3} = 6.6\times10^{15}\,Hz$

6. Wavelength of photon

$$\Delta E = E_{n_2} - E_{n_1} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2 = 13.6\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2 \Rightarrow \Delta E = \frac{hc}{\lambda} \Rightarrow \frac{1}{\lambda} = \bar{\nu} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2$$

$= R_\infty\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2$ where, $\bar{\nu}$ is called wave number.

$R_\infty = R_H$ = Rydberg constant

$$= \frac{9.11\times10^{-31}\times(1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2(6.625\times10^{-34})^3\times3\times10^8} = 1.097\times10^7\,m^{-1} = 1.097\times10^{-3}\,Å^{-1}$$ (for stationary nucleus).

If nucleus is not stationary (i.e., mass of nucleus is not much greater than the mass of the revolving particle like electron), then $R = \frac{R_\infty}{1+m/M}$ where, m = mass of revolving particle and M = mass of nucleus

HYDROGEN LIKE ATOMS

The Bohr model of hydrogen can be extended to hydrogen like atoms, i.e., one electron atoms, the nuclear charge is +ze, where z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 every where by ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{nmze^2} = \frac{n^2}{z} a_0 \quad \text{or} \quad r_n \propto \frac{n^2}{z} \qquad \text{....(i)}$$

where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{ze^2}{2\varepsilon_0 nh} = \frac{z}{n} v_1 \quad \text{or} \quad v_n \propto \frac{z}{n} \qquad \text{....(ii)}$$

where $v_1 = 2.19 \times 10^6$ m/s (speed of electron in first orbit of H)

$$E_n = -\frac{mz^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{z^2}{n^2} E_1 \quad \text{or} \quad E_n \propto \frac{z^2}{n^2} \qquad \text{....(iii)}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)

Definations Valid for Single Electron System

(1) Ground state : Lowest energy state of any atom or ion is called ground state of the atom.

Ground state energy of H atom = –13.6 eV

Ground state energy of He^+ Ion = –54.4 eV

Ground state energy of Li^{++} Ion = –122.4 eV

(2) Excited State : State of atom other than the ground state are called its excited states.

n = 2	first excited state
n = 3	second excited state
n = 4	third excited state
$n = n_0 + 1$	n_0^{th} excited state

(3) Ionisation energy (I.E.) : Minimum energy required to move an electron from ground state to $n = \infty$ is called ionisation energy of the atom or ion

Ionisation energy of H atom = 13.6 eV

Ionisation energy of He^+ Ion = 54.4 eV

Ionisation energy of Li^{++} Ion = 122.4 eV

(4) Ionisation potential (I.P.) : Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionisation energy of the atom is called ionisation potential of the atom.

I.P of H atom = 13.6 V

I.P. of He^+ Ion = 54.4 V

(5) Excitation energy : Energy required to move an electron from ground state of the atom to any other exited state of the atom is called excitation energy of that state.

Energy in ground state of H atom = –13.6 eV

Energy in first excited state of H-atom = –3.4 eV

I^{st} excitation energy = 10.2 eV.

(6) Excitation Potential : Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.

I^{st} excitation energy = 10.2 eV.

I^{st} excitation potential = 10.2 V.

(7) **Binding energy or Seperation energy :** Energy required to move an electron from any state to $n = \infty$ is called binding energy of that state. or energy released during formation of an H-like atom/ion from $n = \infty$ to some particular n is called binding energy of that state.

Binding energy of ground state of H-atom = 13.6 eV

Ex. First excitation potential of a hypothetical hydrogen like atom is 15 volt. Find third excitation potential of the atom.

Sol. Let energy of ground state = E_0

$$E_0 = -13.6\,Z^2\,\text{eV} \quad \text{and} \quad E_n = \frac{E_0}{n^2}$$

$$n = 2,\ E_2 = \frac{E_0}{4}$$

$$\text{given} \quad \frac{E_0}{4} - E_0 = 15 \quad \Rightarrow \quad -\frac{3E_0}{4} = 15$$

$$\text{for} \quad n = 4,\ E_4 = \frac{E_0}{16}$$

$$\text{third exicitation energy} = \frac{E_0}{16} - E_0 = -\frac{15}{16}E_0 = -\frac{15}{16}\cdot\left(\frac{-4\times 15}{3}\right) = \frac{75}{4}\ \text{eV}$$

$\therefore$ third excitation potential is $\frac{75}{4}$ V

SPECTRAL SERIES OF HYDROGEN ATOM

It has been shown that the energy of the outer orbit is greater than the energy of the inner ones. When the Hydrogen atom is subjected to external energy, the electron jumps from lower energy state i.e. the hydrogen atom is excited. The excited state is unstable hence the electron return to its ground state in about 10^{-8} sec. The excess of energy is now radiated in the form of radiations of different wavelength. The different wavelength constitute spectral series. Which are characteristic of atom emitting, then the wave length of different members of series can be found from the following relations

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

This relation explains the complete spectrum of hydrogen. A detailed account of the important radiations are listed below.

(i) **Lyman Series** : The series consist of wavelength which are emitted when electron jumps from an outer orbits to the first orbit i. e., the electronic jumps to K orbit give rise to lyman series. Here $n_1 = 1$ & $n_2 = 2, 3, 4, ...\infty$

The wavelengths of different members of Lyman series are :

(ii) **First member** :In this case $n_1 = 1$ and $n_2 = 2$ hence $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$

$$\Rightarrow \lambda = \frac{4}{3R} \Rightarrow \lambda = \frac{4}{3\times 10.97\times 10^6} = 1216\times 10^{-10}\,\text{m} = 1216\text{Å}$$

(iii) **Second member** : In this case $n_1 = 1$ and $n_2 = 3$ hence $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9}$

$$\Rightarrow \lambda = \frac{9}{8R} \Rightarrow \lambda = \frac{9}{8\times 10.97\times 10^6} = 1026\times 10^{-10}\,\text{m} = 1026\text{Å}$$

Similarly the wavelength of the other members can be calculated.

(iv) **Limiting members** : In this case $n_1 = 1$ and $n_2 = \infty$, hence $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R$

$$\Rightarrow \lambda = \frac{1}{R} \Rightarrow \lambda = \frac{1}{10.97 \times 10^6} = 912 \times 10^{-10} m = 912 Å$$

This series lies in ultraviolet region.

(v) **Balmer Series :** This series is consist of all wave lengths which are emitted when an electron jumps from an outer orbit to the second orbit i. e., the electron jumps to L orbit give rise to Balmer series.
Here $n_1 = 2$ and $n_2 = 3, 4, 5 ....\infty$ The wavelength of different members of Balmer series.

(vi) **First member** : In this case $n_1 = 2$ and $n_2 = 3$, hence $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$

$$\Rightarrow \lambda = \frac{36}{5R} \Rightarrow \lambda = \frac{36}{5 \times 10.97 \times 10^6} = 6563 \times 10^{-10} m = 6563 Å$$

(vii) **Second member** : In this case $n_1 = 2$ and $n_2 = 4$, hence $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16}$

$$\Rightarrow \lambda = \frac{16}{3R} \Rightarrow \lambda = \frac{16}{3 \times 10.97 \times 10^6} = 4861 \times 10^{-10} m = 4861 Å$$

(viii) **Limiting members**: In this case $n_1 = 2$ and $n_2 = \infty$, hence $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4} \Rightarrow \lambda = \frac{4}{R} = 3646 Å$

This series lies in visible and near ultraviolet region.

(ix) **Paschen Series :** This series consist of all wavelength are emitted when an electron jumps from an outer orbit to the third orbit i. e., the electron jumps to M orbit give rise to paschen series. Here $n_1 = 3$ & $n_2 = 4, 5, 6 .....\infty$.

The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$

where $n_2 = 4, 5, 6 ...\infty$
For the first member, the wavelengths is 18750Å. This series lies in infra–red region.

(x) **Bracket Series :** This series is consist of all wavelengths which are emitted when an electron jumps from an outer orbits to the fourth orbit i. e., the electron jumps to N orbit give rise to Brackett series.
Here $n_1 = 4$ & $n_2 = 5, 6, 7, ...\infty$.

The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$

where $n_2 = 5, 6, 7 ....\infty$
This series lies in infra–red region of spectrum.

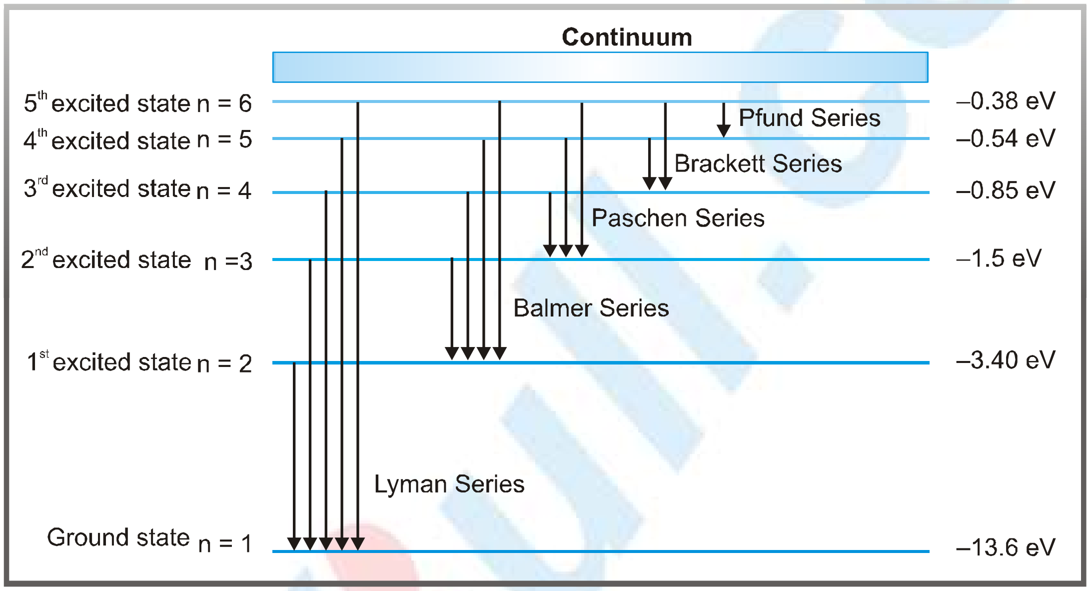
(xi) **Pfund series :** The series consist of all wavelengths which are emitted when an electron jumps from an outer orbit to the fifth orbit i. e., the electron jumps to O orbit give right to Pfund series.
Here $n_1 = 5$ and $n_2 = 6, 7, 8 ....\infty$.

The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$

where $n_2 = 6, 7, 8 ....\infty$
This series lies in infra–red region of spectrum.

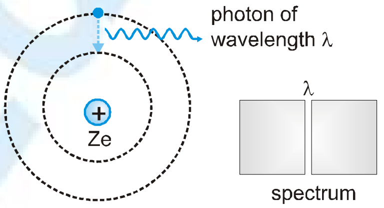
The result are tabulated below

S. No.	Series Observed	Value of n_1	Value of n_2	Position in the Spectrum
1.	Lyman Series	1	2,3,4...∞	Ultra Violet
2.	Balmer Series	2	3,4,5...∞	Visible
3.	Paschen Series	3	4,5,6....∞	Infra–red
4.	Brackett Series	4	5,6,7....∞	Infra–red
5.	Pfund Series	5	6,7,8....∞	Infra–red

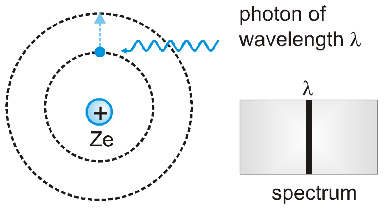


EXCITATION AND IONISATION OF ATOMS

Consider the case of simplest atom i. e., hydrogen atom, this has one electron in the innermost orbit i.e., (n = 1) and is said to be in the unexcited or normal state. If by some means, sufficient energy is supplied to the electron. It moves to higher energy states. When the atom is in a state of a high energy it is said to be excited. The process of raising or transferring the electron from lower energy state is called excitation. When by the process of excitation, the electron is completely removed from the atom. The atom is said to be ionized. Now the atom has left with a positive charge. Thus the process of raising the atom from the normal state to the ionized state is called ionisation. The process of excitation and ionisation both are absorption phenomena. The excited state is not stationary state and lasts in a very short interval of time (10^{-8} sec) because the electron under the attractive force of the nucleus jumps to the lower permitted orbit. This is accompanied by the emission of radiation according to BOHR'S frequency condition.



origin of emission spectra



origin of absorption spectra

The energy necessary to excite an atom can be supplied in a number of ways. The most commonly kinetic energy (Wholly or partly) of the electrons is transferred to the atom. The atom is now in a excited state. The minimum potential V required to accelerate the bombarding electrons to cause excitation from the ground state is called the resonance potential. The various values of potential to cause excitation of higher state called **excitation potential**. The potential necessary to accelerate the bombarding electrons to cause ionisation is called the **ionization potential**. The term critical potential is used to include the resonance, excitation and ionisation potential. We have seen that the energy required to excite the electron from first to second state is 13.6 – 3.4 = 10.2 eV from first to third state is 13.6 – 1.5 = 12.1 eV., and so on. The energy required to ionise hydrogen atom is 0 – (– 13.6) = 13.6 eV. Hence ionization potential of hydrogen atom is 13.6 volt.

Successes and Limitations

Bohr showed that Planck's quantum ideas were a necessary element of the atomic theory. He introduced the idea of quantized energy levels and explained the emission or absorption of radiations as being due to the transition of an electron from one level to another. As a model for even multielectron atoms, the Bohr picture is still useful. It leads to a good, simple, rational ordering of the electrons in larger atoms and quantitatively helps to predict a greater deal about chemical behavior and spectral detail.

Bohr's theory is unable to explain the following facts :

(i) The spectral lines of hydrogen atom are not single lines but each one is a collection of several closely spaced lines.

(ii) The structure of multielectron atoms is not explained.

(iii) No explanation for using the principles of quantization of angular momentum.

(iv) No explanation for Zeeman effect. If a substance which gives a line emission spectrum is placed in a magnetic field, the lines of the spectrum get splitted up into a number of closely spaced lines. This phenomenon is known as Zeeman effect.

Ex. A hydrogen like atom of atomic number Z is in an excited state of quantum number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de–excitation. Ground state energy of hydrogen atom is –13.6 eV.

Sol. The energy released during de–excitation in hydrogen like atoms is given by : $E_{n_2} - E_{n_1} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2$

Energy released in de–excitation will be maximum if transition takes place from nth energy level to ground state i.e.,

$$E_{2n} - E_1 = 13.6\left[\frac{1}{1^2} - \frac{1}{(2n)^2}\right]Z^2 = 204\text{ eV} \quad ...(i) \quad \& \text{ also } E_{2n} - E_n = 13.6\left[\frac{1}{n^2} - \frac{1}{(2n)^2}\right]Z^2 = 40.8\text{ eV} \quad ...(ii)$$

Taking ratio of (i) to (ii), we will get $\frac{4n^2 - 1}{3} = 5 \Rightarrow n^2 = 4 \Rightarrow n = 2$

Putting n=2 in equation (i) we get $Z^2 = \frac{204 \times 16}{13.6 \times 15} \Rightarrow Z = 4$

$\therefore E_n = -13.6\frac{Z^2}{n^2} \Rightarrow E_1 = -13.6 \times \frac{4^2}{1^2} = -217.6$ eV = ground state energy

ΔE is minimum if transition will be from 2n to 2n–1 i.e. between last two adjacent energy levels.

$$\therefore \Delta E_{min} = E_{2n} - E_{2n-1} = 13.6\left[\frac{1}{3^2} - \frac{1}{4^2}\right]4^2 = 10.57\text{ eV}$$

is the minimum amount of energy released during de–excitation.

Ex. A single electron orbits around a stationary nucleus of charge +Ze where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to third orbit. Find

(i) The value of Z.

(ii) The energy required to excite the electron from the third to the fourth Bohr orbit.

(iii) The wavelength of electronic radiation required to remove the electron from first Bohr orbit to infinity.

(iv) Find the K.E., P.E. and angular momentum of electron in the 1st Bohr orbit.

[The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} m,

Velocity of light = 3×10^{8} m/s, Planck's constant = 6.6×10^{-34} J–s]

Sol. The energy required to excite the electron from n_1 to n_2 orbit revolving around the nucleus with charge +Ze is given

by $E_{n_2} - E_{n_1} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2 \Rightarrow E_{n_2} - E_{n_1} = Z^2 \times (13.6)\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

(i) Since 47.2 eV energy is required to excite the electron from $n_1 = 2$ to $n_2 = 3$ orbit

$47.2 = Z^2 \times 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \Rightarrow Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 24.988 \approx 25 \Rightarrow Z=5$

(ii) The energy required to excite the electron from $n_1 = 3$ to $n_2 = 4$ is given by

$E_4 - E_3 = 13.6\,Z^2{=}5\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{25 \times 13.6 \times 7}{144} = 16.53$ eV

(iii) The energy required to remove the electron from the first Bohr orbit to infinity (∞) is given by

$E_\infty - E_3 = 13.6 \times Z^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 13.6 \times 25\,\text{eV} = 340\text{eV}$

In order to calculate the wavelength of radiation, we use Bohr's frequency relation

$hf = \frac{hc}{\lambda} = 13.6 \times 25 \times (1.6 \times 10^{-19})\text{J} \Rightarrow \lambda = \frac{(6.6 \times 10^{-34}) \times 10^8 \times 3}{13.6 \times 25 \times (1.6 \times 10^{-19})} = 36.397\text{Å}$

(iv) K.E. $= \frac{1}{2}mv_1^2 = \frac{1}{2} \times \frac{Ze^2}{4\pi g \in_0 r_1} = 543.4 \times 10^{-19}$ J P.E. $= -2 \times$ K.E. $= -1086.8 \times 10^{-19}$ J

Angular momentum $= mv_1r_1 = \frac{h}{2\pi} = 1.05 \times 10^{-34}$ Js

The radius r_1 of the first Bohr orbit is given by

$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2} \cdot \frac{1}{Z} = \frac{0.53 \times 10^{-10}}{5}\left(\because \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10}\,\text{m}\right) = 1.106 \times 10^{-10}\,\text{m} = 0.106\,\text{Å}$

Ex. An isolated hydrogen atom emits a photon of 10.2 eV.

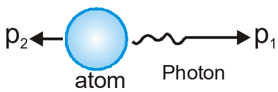
(i) Determine the momentum of photon emitted

(ii) Calculate the recoil momentum of the atom

(iii) Find the kinetic energy of the recoil atom [Mass of proton= $m_p = 1.67 \times 10^{-27}$ kg]

Sol. **(i)** Momentum of the photon is $p_1 = \dfrac{E}{c} = \dfrac{10.2 \times 1.6 \times 10^{-19}}{3 \times 10^{8}} = 5.44 \times 10^{-27}$ kg–m/s

(ii) Applying the momentum conservation $p_2 = p_1 = 5.44 \times 10^{-27}$ kg–m/s



(iii) $K = \dfrac{1}{2} mv^2$ (v = recoil speed of atom, m = mass of hydrogen atom) $K = \dfrac{1}{2} m \left(\dfrac{p}{m}\right)^2 = \dfrac{p^2}{2m}$

Substituting the value of the momentum of atom, we get $K = \dfrac{\left(5.44 \times 10^{-27}\right)^2}{2 \times 1.67 \times 10^{-27}} = 8.86 \times 10^{-27}$J

Physical quantity	Formula	Ratio Formulae of hydrogen atom
Radius of Bohr orbit (r_n)	$r_n = \dfrac{n^2h^2}{4\pi^2 mkZe^2}$; $r_n = 0.53 \dfrac{n^2}{Z}$ Å	$r_1 : r_2 : r_3 ... r_n = 1:4:9...n^2$
Velocity of electron in n^{th} Bohr orbit (v_n)	$v_n = \dfrac{2\pi kZe^2}{nh}$; $v_n = 2.2 \times 10^6 \dfrac{Z}{n}$	$v_1 : v_2 : v_3 ... v_n = 1 : \dfrac{1}{2} : \dfrac{1}{3} ... \dfrac{1}{n}$
Momentum of electron (p_n)	$p_n = \dfrac{2\pi mke^2 z}{nh}$; $p_n \propto \dfrac{Z}{n}$	$p_1 : p_2 : p_3 ... p_n = 1 : \dfrac{1}{2} : \dfrac{1}{3} ... \dfrac{1}{n}$
Angular velocity of electron(ω_n)	$\omega_n = \dfrac{8\pi^3k^2Z^2mc^4}{n^3h^3}$; $\omega_n \propto \dfrac{Z^2}{n^3}$	$\omega_1 : \omega_2 : \omega_3 ... \omega_n = 1 : \dfrac{1}{8} : \dfrac{1}{27} ... \dfrac{1}{n^3}$
Time Period of electron (T_n)	$T_n = \dfrac{n^3h^3}{4\pi^2k^2Z^2me^4}$; $T_n \propto \dfrac{n^3}{Z^2}$	$T_1 : T_2 : T_3 ... T_n = 1:8:27: ... :n^3$
Frequency (f_n)	$f_n = \dfrac{4\pi^2k^2Z^2e^4m}{n^3h^3}$; $f_n \propto \dfrac{Z^2}{n^3}$	$f_1 : f_2 : f_3 ... f_n = 1 : \dfrac{1}{8} : \dfrac{1}{27} ... \dfrac{1}{n^3}$
Orbital current (I_n)	$I_n = \dfrac{4\pi^2k^2Z^2me^5}{n^3h^3}$; $I_n \propto \dfrac{Z^2}{n^3}$	$I_1 : I_2 : I_3 ... I_n = 1 : \dfrac{1}{8} : \dfrac{1}{27} ... \dfrac{1}{n^3}$
Angular momentum (J_n)	$J_n = \dfrac{nh}{2\pi}$; $J_n \propto n$	$J_1 : J_2 : J_3 ... J_n = 1 : 2 : 3 ... n$
Centripetal acceleration (a_n)	$a_n = \dfrac{16\pi^4k^3Z^3me^6}{n^4h^4}$; $a_n \propto \dfrac{Z^3}{n^4}$	$a_1 : a_2 : a_3 ... a_n = 1 : \dfrac{1}{16} : \dfrac{1}{81} ... \dfrac{1}{n^4}$
Kinetic energy (E_{k_n})	$E_{K_n} = \dfrac{RchZ^2}{n^2}$; $E_{K_n} \propto \dfrac{Z^2}{n^2}$	$E_{K_1} : E_{K_2} ... E_{K_n} = 1 : \dfrac{1}{4} : \dfrac{1}{9} ... \dfrac{1}{n^2}$
Potential energy (U_n)	$U_n = \dfrac{-2RchZ^2}{n^2}$; $U_n \propto \dfrac{Z^2}{n^2}$	$U_1 : U_2 : U_3 ... U_n = 1 : \dfrac{1}{4} : \dfrac{1}{9} ... \dfrac{1}{n^2}$
Total energy (E_n)	$E_n = \dfrac{-RchZ^2}{n^2}$; $E_n \propto \dfrac{Z^2}{n^2}$	$E_1 : E_2 : E_3 ... E_n = 1 : \dfrac{1}{4} : \dfrac{1}{9} ... \dfrac{1}{n^2}$

Ex. Calculate **(a)** the wavelength and **(b)** the frequency of the H_β line of the Balmer series for hydrogen.

Sol. **(a)** H_β line of Balmer series corresponds to the transition from n = 4 to n = 2 level. The corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 0.2056 \times 10^7 \qquad \therefore\ \lambda = 4.9 \times 10^{-7}\ \text{m} \quad \textbf{Ans.}$$

(b) $$\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14}\ \text{Hz} \quad \textbf{Ans.}$$

Ex. Find the largest and shortest wavelengths in the Lymen series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

Sol. The transition equation for Lymen series is given by,

$$\frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right] \qquad n = 2, 3, ...$$

for largest wavelength, $n = 2$

$$\frac{1}{\lambda_{max}} = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{4}\right) = 0.823 \times 10^7$$

$$\therefore \qquad \lambda_{max} = 1.2154 \times 10^{-7}\ \text{m} = 1215\ \text{Å} \qquad \textbf{Ans.}$$

The shortest wavelength corresponds to $n = \infty$

$$\therefore \qquad \frac{1}{\lambda_{max}} = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{\infty}\right)$$

or $$\lambda_{min} = 0.911 \times 10^{-7}\ \text{m} = 911\ \text{Å} \qquad \textbf{Ans.}$$

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

Ex. How may different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n ?

Sol. From the nth state, the atom may go to (n – 1)th state, ..., 2nd state or 1st state. So there are (n – 1) possible transitions starting from the nth state. The atoms reaching (n – 1)th state may make (n – 2) different transitions. Similarly for other lower states. The total number of possible transitions is

$$(n-1) + (n-2) + (n-3) + ...2 + 1$$

$$= \frac{n(n-1)}{2} \qquad \textbf{(Remember)}$$

Ex. **(a)** Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

(b) How many spectral linea are observed in the emission spectrum of the above excited system?

Sol. **(a)** The energy in the first orbit = $E_1 = Z^2 E_0$ where $E_0 = -13.6$ eV is the energy of a hydrogen atom in ground state thus for Li^{++},

$$E_1 = 9E_0 = 9 \times (-13.6\ \text{eV}) = -122.4\ \text{eV}$$

The energy in the third orbit is

$$E_3 = \frac{E_1}{n^2} = \frac{E_1}{9} = -13.6\ \text{eV}$$

Thus, $$E_3 - E_1 = 8 \times 13.6\ \text{eV} = 108.8\ \text{eV}.$$

Energy required to excite Li^{++} from the first orbit to the third orbit is given by

$$E_3 - E_1 = 8 \times 13.6 \text{ eV} = 108.8 \text{ eV}.$$

The wavelength of radiation required to excite Li^{++} from the first orbit to the third orbit is given by

$$\frac{hc}{\lambda} = E_3 - E_1 \quad \text{or} \quad \lambda = \frac{hc}{E_3 - E_1} = \frac{1240 \text{ eV} - \text{nm}}{108.8 \text{ eV}} \approx 11.4 \text{ nm}$$

(b) The spectral lines emitted are due to the transitions $n = 3 \to n = 2$, $n = 3 \to n = 1$ and $n = 2 \to n = 1$. Thus, there will be three spectral lines in the spectrum.

Ex. Find the kinetic energy potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Sol. $E_1 = -13.60 \text{ eV}$ $\quad K_1 = -E_1 = 13.60 \text{ eV}$ $\quad U_1 = 2E_1 = -27.20 \text{ eV}$

$E_2 = \frac{E_1}{(2)^2} = -3.40 \text{ eV}$ $\quad K_2 = 3.40 \text{ eV}$ and $\quad U_2 = -6.80 \text{ eV}$

Now $U_1 = 0$, i.e., potential energy has been increased by 27.20 eV while kinetic energy will remain unchanged. So values of kinetic energy, potential energy and total energy in first orbit are 13.60 eV, 0, 13.60 respectively and for second orbit these values are 3.40 eV, 20.40 eV and 23.80 eV.

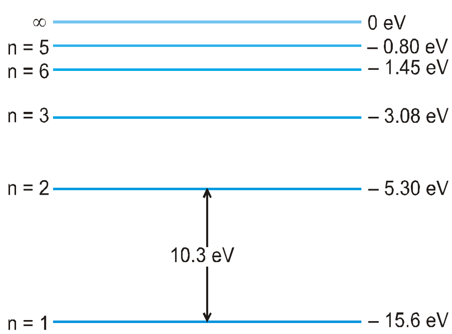
Ex. A lithium atom has three electrons, Assume the following simple picture of the atom. Two electrons move close to the nucleus making up a spherical cloud around it and the third moves outside this cloud in a circular orbit. Bohr's model can be used for the motion of this third electron but n = 1 states are not available to it. Calculate the ionization energy of lithium in ground state using the above picture.

Sol. In this picture, the third electron moves in the field of a total charge + 3e – 2e = + e. Thus, the energies are the same as that of hydrogen atoms. The lowest energy is :

$$E_2 = \frac{E_1}{4} = \frac{-13.6 \text{ eV}}{4} = -3.4 \text{ eV}$$

Thus, the ionization energy of the atom in this picture is 3.4 eV.

Ex. The energy levels of a hypothetical one electron atom are shown in the figure.



(a) Find the ionization potential of this atom.

(b) Find the short wavelength limit of the series terminating at n = 2

(c) Find the excitation potential for the state n = 3.

(d) Find wave number of the photon emitted for the transition n = 3 to n = 1.

(e) What is the minimum energy that an electron will have after interacting with this atom in the ground state if the initial kinetic energy of the electron is

(i) 6 eV **(ii)** 11 eV

Sol. **(a)** Ionization potential = 15.6 V

(b) $\lambda_{min} = \frac{12400}{5.3} = 2340 \text{ Å}$

(c) $\Delta E_{31} = -3.08 - (-15.6) = 12.52 \text{ eV}$

Therefore, excitation potential for state n = 3 is 12.52 volt.

(d) $\frac{1}{\lambda_{31}} = \frac{\Delta E_{31}}{12400} \quad Å^{-1} = \frac{12.52}{12400} Å^{-1} \approx 1.01 \times 10^7 \, m^{-1}$

(e) **(i)** $E_2 - E_1 = 10.3 \, eV > 6 \, eV$.

Hence electron cannot excite the atoms. So, $K_{min} = 6$ eV.

(ii) $E_2 - E_1 = 10.3 \, eV < 11 \, eV$.

Hence electron can excite the atoms. So, $K_{min} = (11 - 10.3) = 0.7$ eV.

Ex. A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

Sol. The force at a distance r is,

$$F = -\frac{dU}{dr} = -2ar$$

Suppose r be the radius of nth orbit. The necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ar$$

Further, the quantization of angular momentum gives,

$$mvr = \frac{nh}{2\pi}$$

Solving Eqs. **(i)** and **(ii)** for r, we get

$$r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$$ **Ans.**

Ex. An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge + 4e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to the system.

(a) Derive and expression for the radius of n^{th} Bohr orbit.

(b) Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to the second.

Sol. **(a)** We have $\frac{m_p v^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{ze^2}{r_n^2}$ .....**(i)**

The quantization of angular momentum gives,

$$m_p v r_n = \frac{nh}{2\pi} \quad ......\textbf{(ii)}$$

Solving Eqs. **(i)** and **(ii)**, we get

$$r = \frac{n^2h^2\varepsilon_0}{z\pi m_p e^2}$$

Substituting $m_p = 100\,m$

where m = mass of electron and z = 4

we get, $r_n = \frac{n^2h^2\varepsilon_0}{400\,\pi m e^2}$ **Ans.**

(b) As we know,

Energy of hydrogen atom in ground state = – 13.60 eV

and $$E_n \propto \left(\frac{z^2}{n^2}\right) m$$

For the given particle, $E_4 = \dfrac{(-13.60)(4)^2}{(4)^2} \times 100 = -1360$ eV

and $$E_2 = \frac{(-13.60)(4)^2}{(2)^2} \times 100 = -5440 \text{ eV}$$

$$DE = E_4 - E_2 = 4080 \text{ eV}$$

$\therefore$ $\lambda \text{ (in Å)} = \dfrac{12400}{4080} = 3.0$ Å **Ans.**

Ex. A particle known as μ-meason, has a charge equal to that of an electron and mass 208 times the mass of the electron. It moves in a circular orbit around a nucleus of charge +3e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system, **(a)** derive an expression for the radius of the nth Bohr orbit, **(b)** find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom and **(c)** find the wavelength of the radiation emitted when the μ–meson jumps from the third orbit to the first orbit.

Sol. **(a)** We have,

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

or, $$v^2 r == \frac{Ze^2}{4\pi\varepsilon_0 m} \quad ...(i)$$

The quantization rule is $vr = \dfrac{nh}{2\pi m}$

The radius is $r = \dfrac{(vr)^2}{v^2 r} = \dfrac{4\pi\varepsilon_0 m}{Ze^2}$

$$= \frac{n^2 h^2 \varepsilon_0}{Z\pi m e^2} \quad ....(ii)$$

For the given system, Z = 3 and m = 208 m_e.

Thus $$r_\mu = \frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2}$$

(b) From **(ii)**, the radius of the first Bohr orbit for the hydrogen atom is

$$r_h = \frac{h^2 \varepsilon_0}{\pi m_e e^2}$$

For $r_\mu = r_h$, $$\frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2} = \frac{h^2 \varepsilon_0}{\pi m_e e^2}$$

or, $n^2 = 624$

or, $n = 25$

(c) From **(i)**, the kinetic energy of the atom is

$$\frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

and the potential energy is $-\dfrac{Ze^2}{4\pi\varepsilon_0 r}$

The total energy is $E_n = \dfrac{Ze^2}{8\pi\varepsilon_0 r}$

Using **(ii)**, $E_n = -\dfrac{Z^2\pi me^4}{8\pi\varepsilon_0^2 n^2 h^2} = -\dfrac{9\times 208 m_e{}^4}{8\varepsilon_0^2 n^2 h^2} = \dfrac{1872}{n^2}\left(-\dfrac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$

But $\left(-\dfrac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to – 13.6 eV.

From **(iii)**, $E_n = -\dfrac{1872}{n^2}\times 13.6\,\text{eV} = \dfrac{-25459.2\,\text{eV}}{n^2}$

Thus, $E_1 = -25459.2$ eV and $E_3 = \dfrac{E_1}{9} = -2828.8$ eV. The energy difference is $E_3 - E_1 = 22630.4$eV.

The wavelength emitted is

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}-\text{nm}}{22630.4\,\text{eV}} = 55\text{ pm.}$$

Ex. A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelength. All the wavelengths are equal or smaller than that of the absorbed photon.

(a) Determine the initial state of the gas atoms.

(b) Identify the gas atoms.

(c) Find the minimum wavelength of the emitted radiations.

(d) Find the ionization energy and the respective wavelength for the gas atoms.

Sol. **(a)** $\dfrac{n(n-1)}{2} = 3$

$\therefore$ $n = 3$

i.e., after excitation atom jumps to second excited state.

Hence $n_f = 3$. So n_i can be 1 or 2

If $n_i = 1$ then energy emitted is either equal to, greater than or less than the energy absorbed.

Hence the emitted wavelength is either equal to, less than or greater than the absorbed wavelength.

Hence $n_i \neq 1$.

If $n_i = 2$, then $E_e \geq E_a$. Hence $\lambda_e \leq \lambda_0$

(b) $E_3 - E_2 = 68$ eV

$\therefore$ $(13.6)(Z^2)\left(\dfrac{1}{4}-\dfrac{1}{9}\right) = 68$

$\therefore$ $Z = 6$

(c) $\lambda_{min} = \frac{12400}{E_3 - E_1} = \frac{12400}{(13.6)(6)^2\left(1-\frac{1}{9}\right)} = \frac{12400}{435.2} = 28.49$ **Ans.**

(d) Ionization energy $= (13.6)(6)^2 = 489.6$ eV **Ans.**

$$\lambda = \frac{12400}{489.6} = 25.33 \text{ Å}$$ **Ans.**

Ex. An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find

(a) the allowed values of the radius 'r' of the orbit.

(b) the kinetic energy of the electron in orbit

(c) The potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.

(d) The total energy of the allowed energy levels.

Sol. **(a)** radius of circular path

$$r = \frac{mv}{Be} \quad \text{....(i)}$$

$$mvr = \frac{nh}{2\pi} \quad \text{....(ii)}$$

Solving these two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}} \quad \text{and } v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

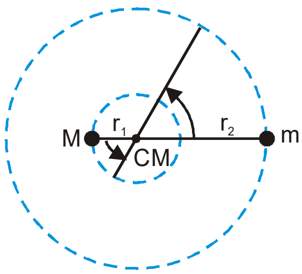
(b) $K = \frac{1}{2}mv^2 = \frac{nhBe}{4\pi m}$ **Ans.**

(c) $M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{evr}{2} = \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}} = \frac{nhe}{4\pi m}$

Now potential energy $U = -M \cdot B = \frac{nheB}{4\pi m}$

(d) $E = U + K = \frac{nheB}{2\pi m}$

EFFECT OF NUCLEUS MOTION ON ENERGY OF ATOM



Let both the nucleus of mass M, charge Ze and electron of mass m, and charge e revolve about their centre of mass (CM) with same angular velocity (ω) but different linear speeds. Let r_1 and r_2 be the distance of CM from nucleus and electron. Their angular velocity should be same then only their separation will remain unchanged in an energy level.

Let r be the distance between the nucleus and the electron. Then

$$Mr_1 = mr_2$$
$$r_1 + r_2 = r$$

$$\therefore \quad r_1 = \frac{mr}{M+m} \quad \text{and} \quad r_2 = \frac{Mr}{M+m}$$

Centripetal force to the electron is provided by the electrostatic force.

So, $$mr_2\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

or $$m\left(\frac{Mr}{M+m}\right)\omega^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r^2}$$ or $$\left(\frac{Mm}{M+m}\right)r^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$$

or $$\mu r^3\omega^2 = \frac{e^2}{4\pi\varepsilon_0}$$

where $$\frac{Mm}{M+m} = \mu$$

Moment of inertia of atom about CM,

$$I = Mr_1^2 + mr_2^2 = \left(\frac{Mm}{M+m}\right)r^2 = \mu r^2$$

According to Bohr's theory, $\frac{nh}{2\pi} = I\omega$ or $\mu r^2\omega = \frac{nh}{2\pi}$

Solving above equations for r, we get

$$r = \frac{\varepsilon_0 n^2h^2}{\pi\mu e^2 Z}$$ and $$\mathbf{r = (0.529\ \text{Å})}\ \frac{n^2}{n}\cdot\frac{n}{\mu}$$

Further electrical potential energy of the system,

$$U = \frac{-Ze^2}{4\pi\varepsilon_0 r}$$ $$U = \frac{-Z^2e^4\mu}{4\varepsilon_0^2n^2h^2}$$

and kinetic energy, $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\mu r^2\omega^2$ and $K = \frac{1}{2}\mu v^2$

v-speed of electron with respect to nucleus. ($v = r\omega$)

here $$\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0\,\mu r^3}$$

$\therefore$ $$K = \frac{Ze^2}{8\pi\varepsilon_0 r} = \frac{Z^2e^4\mu}{8\pi\varepsilon_0^2n^2h^2}$$

$\therefore$ Total energy of the system $E_n = K + U$ $\Rightarrow$ $E_n = -\frac{\mu e^4}{8\varepsilon_0^2n^2h^2}$

this expression can also be written as

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2}\cdot\left(\frac{\mu}{m}\right)$$

The expression for E_n without considering the motion of proton is $E_n = -\frac{me^4}{8\varepsilon_0^2n^2h^2}$, i.e., m is replaced by μ while considering the motion of nucleus.

Ex. A positronium 'atom' is a system that consists of a positron and an electron that orbit each other. Compare the wavelength of the spectral lines of positronium with those of ordinary hydrogen.

Sol. Here the two particle have the same mass m, so the reduced mass is

$$\mu = \frac{mM}{m+M} = \frac{m^2}{2m} = \frac{m}{2}$$

where m is the electron mass. We know that $E_n \propto m$

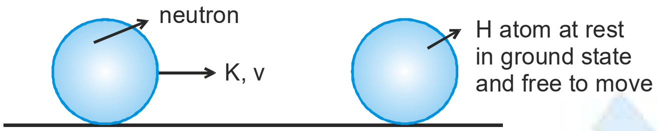
$\therefore \quad \frac{E'_n}{E_n} = \frac{\mu}{m} = \frac{1}{2}$ energy of each level is halved. $\therefore$ Their difference will also be halved.

Hence $\lambda'_n = 2\lambda_n$

ATOMIC COLLISION

In such collisions assume that the loss in the kinetic energy of system is possible only if it can excite or ionise.

Ex.



head on collision

What will be the type of collision, if K = 14eV, 20.4 eV, 22 eV, 24.18 eV

(elastic/inelastic/perectly inelastic)

Sol. Loss in energy (ΔE) during the collision will be used to excite the atom or electron from one level to another.

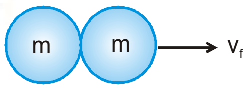
According to quantum Mechanics, for hydrogen atom.

$\Delta E = \{0, 10.2\text{ eV}, 12.09\text{ eV}, \ldots\ldots\ldots, 13.6\text{ eV}\}$

According to Newtonion mechanics

minimum loss = 0. (elastic collsion)

for maximum loss collision will be perfectly inelastic



if neutron collides perfectly inelastically

then, Applying momentum conservation

$$mv_0 = 2mv_f$$

$$v_f = \frac{v_0}{2}$$

Final $\quad \text{K.E.} = \frac{1}{2} \times 2m \times \frac{v_0^2}{4} = \frac{\frac{1}{2}mv_0^2}{2} = \frac{K}{2}$

maximum loss $= \frac{K}{2}$

According to classical mechanics $(\Delta E) = [0, \frac{K}{2}]$

(a) If $\quad K = 14\text{ eV}$, $\quad$ According to quantum mechanics

$(\Delta E) = \{0, 10.2\text{eV}, 12.09\text{ eV}\}$

According to classical mechanics

$\Delta E = [0, 7\text{ eV}]$

loss = 0, hence it is elastic collision speed of particle changes.

(b) If $K = 20.4$ eV

According to classical mechanics

loss = [0, 10.2 eV]

According to quantum mechanics

loss = {0, 10.2eV, 12.09eV,.........}

loss = 0 elastic collision.

loss = 10.2eV perfectly inelastic collision

(c) If $K = 22$ eV

Classical mechanics $\Delta E = [0, 11]$

Quantum mechanics $\Delta E = \{0, 10.2eV, 12.09eV, ...\}$

loss = 0 elastic collision

loss = 10.2 eV inelastic collsion

(d) If $K = 24.18$ eV

According to classical mechanics $\Delta E = [0, 12.09eV]$

According to quantum mechanics $\Delta E = \{0, 10.2eV, 12.09eV, ... 13.6eV\}$

loss = 0 elastic collision

loss = 10.2 eV inelastic collision

loss = 12.09 eV perfectly inelastic collision

Ex. A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particle.

Sol. Here the loss during the collision can only be used to excite the atoms or electrons.

So according to quantum mechanics

loss = {0, 40.8eV, 48.3eV, ..., 54.4eV} ....**(1)**

$$E_n = -13.6\,\frac{Z^2}{n^2}\text{ eV}$$

Now according to newtonion mechanics

Minimum loss = 0

maximum loss will be for perfectly inelastic collision.

let v_0 be the initial speed of neutron and v_f be the final common speed.

so by momentum conservation $mv_0 = mv_f + 4mv_f$ $\quad v_f = \frac{v_0}{5}$

where m = mass of Neutron

$\therefore$ mass of He^+ ion = 4m

So final kinetic energy of system

$$K.E. = \frac{1}{2}\,m\,v_f^2 + \frac{1}{2}\,4m\,v_f^2 = \frac{1}{2}.(5m).\frac{v_0^2}{25} = \frac{1}{5}.(\frac{1}{2}mv_0^2) = \frac{K}{5}$$

maximum loss $= K - \frac{K}{5} = \frac{4K}{5}$

So loss will be $\left[0, \frac{4K}{5}\right]$ ....(2)

For inelastic collision there should be at least one common value other than zero in set (1) and (2)

$\therefore \quad \frac{4K}{5} > 40.8 \text{ eV}$

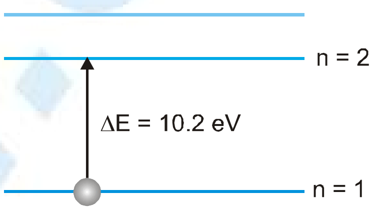
$K > 51 \text{ eV}$

minimum value of K = 51 eV.

Ex. A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.

Sol. Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.

From conservation of linear momentum,



$$p = p' \text{ or } \sqrt{2Km} = \sqrt{2K'(2m)}$$

or $K = 2K'$ ....(i)

From conservation of energy, $K = K' + \Delta E$ ....(ii)

Solving Eqs. (i) and (ii), we get $\Delta E = \frac{K}{2}$

Now minimum value of ΔE for hydrogen atom is 10.2 eV.

or $\Delta E \geq 10.2 \text{ eV}$

$\therefore \quad \frac{K}{2} \geq 10.2$

$\therefore \quad K \geq 20.4 \text{ eV}$

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV **Ans.**

Ex. A neutron moving with speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. Find the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron = mass of hydrogen = 1.67×10^{-27} kg.

Sol. Suppose the neutron and the hydrogen atom move at speed v_1 and v_2 after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy ΔE is used in this way. Using conservation of linear momentum and energy.

$$mv = mv_1 + mv_2 \quad ....(i)$$

and $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E$ ....(ii)

From (i), $v^2 = v_1^2 + v_2^2 + 2v_1v_2$,

From (ii), $v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$

Thus, $2v_1v_2 = \frac{2\Delta E}{m}$

Hence, $(v_1 - v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$

As $v_1 - v_2$ must be real, $v^2 - \frac{4\Delta E}{m} \geq 0$

or, $\frac{1}{2}mv^2 > 2\Delta E.$

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state is 10.2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is

$$\frac{1}{2}mv^2_{min} = 2 \times 10.2\,eV = 20.4\,eV$$

Ex. How many head-on, elastic collisions must a neutron have with deuterium nucleus to reduce its energy from 1 MeV to 0.025 eV.

Sol. Let mass of neutron = m and mass of deuterium = 2m

initial kinetic energy of neutron = K_0

Let after first collision kinetic energy of neutron and deuterium be K_1 and K_2.

Using C.O.L.M. along direction of motion

$$\sqrt{2mK_0} = \sqrt{2mK_1} + \sqrt{4mK_2}$$

velocity of seperation = velocity of approach

$$\frac{\sqrt{4mK_2}}{2m} - \frac{\sqrt{2mK_1}}{m} = \frac{\sqrt{2mK_0}}{m}$$

Solving equaiton **(i)** and **(ii)** we get

$$K_1 = \frac{K_0}{9}$$

Loss in kinetic eneryg after first collision

$$\Delta K_1 = K_0 - K_1$$

$$\Delta K_1 = \frac{8}{9}K_0 \qquad \text{.......} \textbf{(1)}$$

After second collision

$$\Delta K_2 = \frac{8}{9}K_1 = \frac{8}{9}\cdot\frac{K_0}{9}$$

$\therefore$ Total energy loss

$$\Delta K = \Delta K_1 + \Delta K_2 + ... + \Delta K_n$$

As, $\Delta K = \frac{8}{9}K_0 + \frac{8}{9^2}K_0 + \text{.........} + \frac{8}{9^n}K_0$

$$\Delta K = \frac{8}{9} K_0 (1 + \frac{1}{9} + ... + \frac{1}{9^{n-1}})$$

$$\frac{\Delta K}{K_0} = \frac{8}{9}\left[\frac{1-\frac{1}{9^n}}{1-\frac{1}{9}}\right] = 1 - \frac{1}{9^n}$$

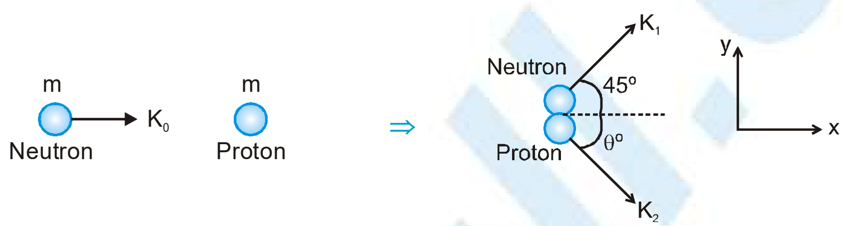
Here, $K_0 = 10^6$ eV, $\Delta K = (10^6 - 0.025)$ eV

$\therefore$ $\frac{1}{9^n} = \frac{K_0 - \Delta K}{K_0} = \frac{0.025}{10^6}$ or $9^n = 4 \times 10^7$

Taking log both sides and solving, we get $n = 8$

Ex. A neutron with an energy of 4.6 MeV collides with protons and is retarded. Assuming that upon each collision neutron is deflected by 45° find the number of collisions which will reduce its energy to 0.23 eV.

Sol. Mass of neutron $\approx$ mass of proton = m



From conservation of momentum in y-direction

$$\sqrt{2mK_1}\ \sin 45^\circ = \sqrt{2mK_2}\ \sin\theta \qquad ....(i)$$

In x-direction $$\sqrt{2mK_0} - \sqrt{2mK_1}\ \cos 45^\circ = \sqrt{2mK_2}\ \cos\theta \qquad ....(ii)$$

Squaring and adding equation **(i)** and **(ii)**, we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \qquad ....(iii)$$

From conservation of energy

$$K_2 = K_0 - K_1 \qquad ....(iv)$$

Solving equations **(iii)** and **(iv)**, we get

$$K_1 = \frac{K_0}{2}$$

i.e., after each collision energy remains half. Therefore, after n collisions,

$$K_n = K_0 \left(\frac{1}{2}\right)^n$$

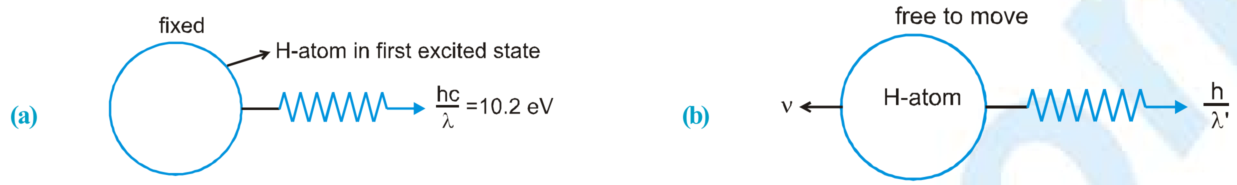
$\therefore$ $0.23 = (4.6 \times 10^6)\left(\frac{1}{2}\right)^n$ $\qquad 2^n = \frac{4.6 \times 10^6}{0.23}$

Taking log and solving, we get

$n \approx 24$ **Ans.**

Calculation of Recoil Speed of Atom on Emission of a Photon

$$\text{Momentum of Photon} = mc = \frac{h}{\lambda}$$



m - mass of atom

According to momentum conservation

$$mv = \frac{h}{\lambda'} \quad \text{.... (i)}$$

According to energy conservation

$$\frac{1}{2}mv^2 + \frac{hc}{\lambda'} = 10.2\text{ eV}$$

Since mass of atom is very large than photon

hence $\frac{1}{2}mv^2$ can be neglected

$$\frac{hc}{\lambda'} = 10.2\text{ eV} \quad \Rightarrow \quad \frac{h}{\lambda} = \frac{10.2}{c}\text{eV}$$

$$mv = \frac{10.2}{c}\text{eV} \quad \Rightarrow \quad v = \frac{10.2}{cm}$$

$$\text{recoil speed of atom} = \frac{10.2}{cm}$$

X-RAY

What are X - Rays

X-rays are electromagnetic radiation of very short wavelength 0.1 Å and 100 Å) and high energy which are emitted when fast moving electrons or cathode rays strike a target of high atomic mass.

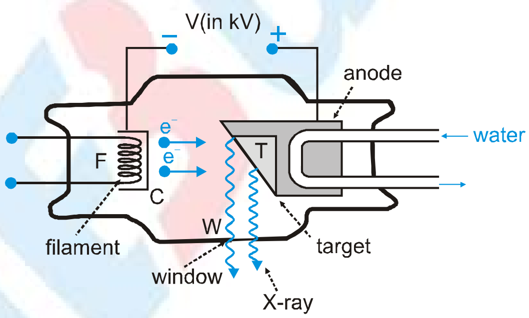
The X-Rays extends from the ultraviolet band to gamma rays band. Although their is very thin line between the X-ray and gamma rays but they are distinguished by the source of their generation. In the older days the X-Rays were distinguished from gamma rays by the wavelength and frequency but with time it has been observed that their exists X-Rays and gamma rays overlapping band and hence it is then understood that the distinction between them is source of generation.

Historical background

On Nov. 1895, Wilhelm Conrad Rontgen (accidentally) discovered an image cast from his cathode ray generator, projected far beyond the possible range of the cathode rays (now known as an electron beam). Further investigation showed that the rays were generated at the point of contact of the cathode ray beam on the interior of the vacuum tube, that they were not deflected by magnetic fields, and they penetrated many kinds of matter. Rontgen named the new form of radiation X-radiation (X standing for "Unknown").

COOLIDGE METHOD

Coolidge developed thermoionic vacuum X–ray tube in which electron are produced by thermoionic emission method. Due to high potential difference electrons (emitted due to thermoionic method) move towards the target and strike from the atom of target due to which X–ray are produced. Experimentally it is observed that only 1% or 2% kinetic energy of electron beam is used to produce X–ray. Rest of energy is wasted in form of heat.



Characteristics of Target

(a) Must have high atomic number to produce hard X–rays.

(b) High melting point to withstand high temperature produced.

(c) High thermal conductivity to remove the heat produced

(d) Tantalum, platinum, molybdenum and tungsten serve as target materials

Control of intensity : The intensity of X–ray depends on number of electrons striking the target and number of electron depend on temperature of filament which can be controlled by filament current.

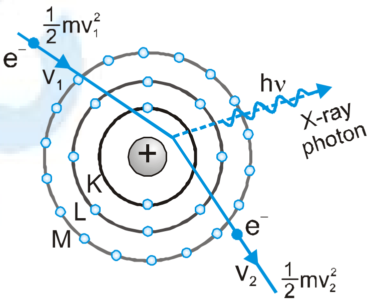
Thus intensity of X–ray depends on current flowing through filament.

Control of Penetrating Power: The Penetrating power of X–ray depends on the energy of incident electron. The energy of electron can be controlled by applied potential difference. Thus penetrating power of X–ray depend on applied potential difference. Thus the intensity of X–ray depends on current flowing through filament while penetrating power depends on applied potential difference

	Soft X–ray	Hard X–ray
Wavelength	10 Å to 100 Å	0.1 Å – 10 Å
Energy	$\frac{12400}{\lambda}$ eV–Å	$\frac{12400}{\lambda}$ eV–Å
Penetrating power	Less	More
Use	Radio photography	Radio therapy

Continuous spectrum of X–ray :

When high speed electron collides from the atom of target and passes close to the nucleus. There is coulomb attractive force due to this electron is deaccelerated i.e. energy is decreased. The loss of energy during deacceleration is emitted in the form of X–rays. X–ray produced in this way are called Braking or Bremstralung radiation and form continuous spectrum. In continuous spectrum of X–ray all the wavelength of X–ray are present but below a minimum value of wavelength there is no X–ray. It is called cut off or threshold or minimum wavelength of X–ray. The minimum wavelength depends on applied potential.

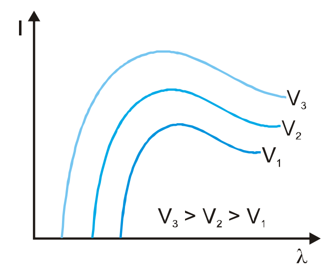


Loss in Kinetic Energy

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = h\nu + \text{heat energy} \quad \text{if } v_2 = 0,\ v_1 = v \text{ (In first collision, heat = 0)}$$

$$\frac{1}{2}mv^2 = h\nu_{max} \qquad ...(i)$$

$$\frac{1}{2}mv^2 = eV \qquad ...(ii) \qquad [\text{here } V \text{ is applied potential}]$$

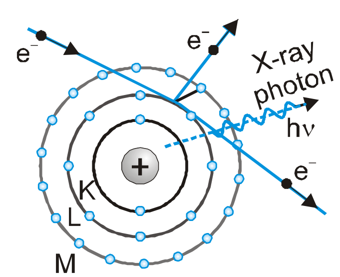


from (i) and (ii) $h\nu_{max} = eV \Rightarrow \frac{hc}{\lambda_{min}} = eV \Rightarrow \lambda_{min} = \frac{12400}{V} \times \text{ volt} = \frac{12400}{V} \times 10^{-10}\text{ m} \times \text{ volt}$

Continuous X–rays also known as white X–ray. Minimum wavelength of these spectrum only depends on applied potential and doesn' t depend on atomic number.

Characteristic Spectrum of X–ray

When the target of X–ray tube is collide by energetic electron it emits two type of X–ray radiation. One of them has a continuous spectrum whose wavelength depend on applied potential while other consists of spectral lines whose wavelength depend on nature of target. The radiation forming the line spectrum is called characteristic X–rays. When highly accelerated electron strikes with the atom of target then it knockout the electron of orbit, due to this a vacancy is created. To fill this vacancy electron jumps from higher energy level and electromagnetic radiation are emitted which form characteristic spectrum of X–ray. Whose wavelength depends on nature of target and not on applied potential.



From Bohr Model

$n_1 = 1, \quad n_2 = 2, 3, 4.......$K series

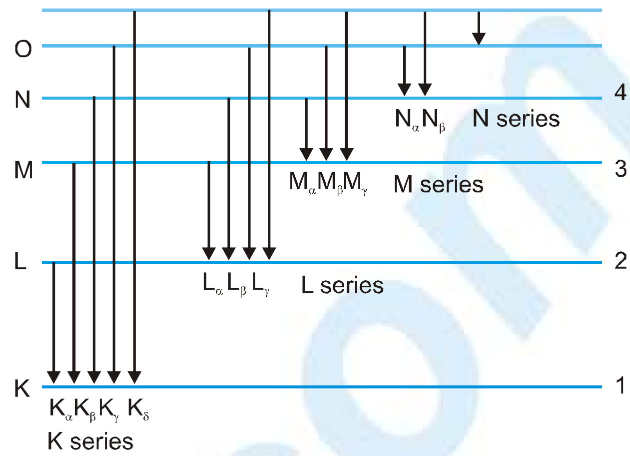
$n_1 = 2, \quad n_2 = 3, 4, 5.......$L series

$n_1 = 3, \quad n_2 = 4, 5, 6.......$M series

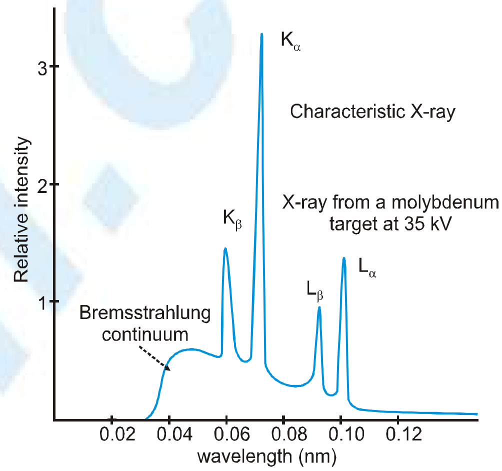
First line of series = α

Second line of series = β

Third line of series = γ



Transition	Wave–length	Energy difference	Energy	Wavelength
$L \to K$	$\lambda_{K\alpha}$	$h\nu_{K\alpha}$	$-(E_K - E_L)$	$\lambda_{K\alpha} = \frac{hc}{(E_K - E_L)}$
$(2 \to 1)$		$= h\nu_{K\alpha}$		$= \frac{12400}{(E_K - E_L)}$ eVÅ
$M \to K$	$\lambda_{K\beta}$	$h\nu_{K\beta}$	$-(E_K - E_M)$	$\lambda_{K\beta} = \frac{hc}{(E_K - E_M)}$
$(3 \to 1)$		$= h\nu_{K\beta}$		$= \frac{12400}{(E_K - E_M)}$ eVÅ
$M \to L$	$\lambda_{L\alpha}$	$h\nu_{L\alpha}$	$-(E_L - E_M)$	$\lambda_{L\alpha} = \frac{hc}{(E_L - E_M)}$
$(3 \to 2)$		$= h\nu_{L\alpha}$		$= \frac{12400}{(E_L - E_M)}$ eVÅ



MOSELEY'S LAW

Moseley studied the characteristic spectrum of number of many elements and observed that the square root of the frequency of a K– line is closely proportional to atomic number of the element. This is called Moseley's law.

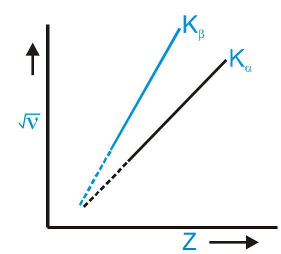
$$\sqrt{\nu} \propto (Z - b) \Rightarrow \nu \propto (Z - b)^2 \Rightarrow \nu = a(Z - b)^2 \quad ...(i)$$

Z = atomic number of target

ν = frequency of characteristic spectrum

b = screening constant (for K– series b=1, L series b=7.4)

a = proportionality constant



From Bohr Model $\quad \nu = RcZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad ...(ii)$

Comparing (i) and (ii) $\quad a = Rc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

❖ Thus proportionality constant 'a' does not depend on the nature of target but depend on transition.

	Bohr model		Moseley's correction
1.	For single electron species	1.	For many electron species
2.	$\Delta E = 13.6Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]eV$	2.	$\Delta E = 13.6\,(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]eV$
3.	$\nu = RcZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	3.	$\nu = Rc(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$
4.	$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	4.	$\frac{1}{\lambda} = R\,(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

❖ For X–ray production, Moseley formulae are used because heavy metal are used.

When target is same $\lambda \propto \dfrac{1}{\frac{1}{n_1^2}-\frac{1}{n_2^2}}$ When transition is same $\lambda \propto \dfrac{1}{(Z-b)^2}$

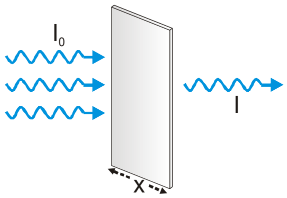
ABSORPTION OF X–RAY

When X–ray passes through x thickness then its intensity $I = I_0 e^{-\mu x}$

I_0 = Intensity of incident X–ray

I = Intensity of X–ray after passing through x distance

μ = absorption coefficient of material



❖ Intensity of X–ray decrease exponentially.

❖ Maximum absorption of X–ray $\longrightarrow$ Lead

❖ Minimum absorption of X–ray $\longrightarrow$ Air

HALF THICKNESS ($x_{1/2}$)

The distance travelled by X–ray when intensity become half the original value $x_{1/2} = \dfrac{ln_2}{\mu}$

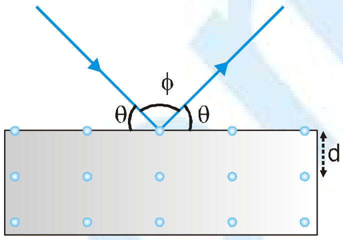
Ex. When X–rays of wavelength 0.5Å pass through 10 mm thick Al sheet then their intensity is reduced to one sixth. Find the absorption coefficient for Aluminium .

Sol. $\mu = \dfrac{2.303}{x}\log\left(\dfrac{I_o}{I}\right) = \dfrac{2.303}{10}\log_{10} 6 = \dfrac{2.303 \times 0.7781}{10} = 0.1752 / \text{ mm}$

DIFFRACTION OF X–RAY

Diffraction of X–ray is possible by crystals because the interatomic spacing in a crystal lattice is order of wavelength of X–rays it was first verified by Lauve.

Diffraction of X–ray take place according to Bragg's law $2d \sin\theta = n\lambda$



d = spacing of crystal plane or lattice constant or distance between adjacent atomic plane

θ = Bragg's angle or glancing angle

ϕ = Diffracting angle n = 1, 2, 3 ....

For Maximum Wavelength

$\sin\theta = 1, n = 1 \Rightarrow \lambda_{max} = 2d$

so if $\lambda > 2d$ diffraction is not possible i.e. solution of Bragg's equation is not possible.

PROPERTIES OF X–RAY

(i) X–ray always travel with the velocity of light in straight line because X–rays are em waves

(ii) X–ray is electromagnetic radiation it show particle and wave both nature

(iii) In reflection, diffraction, interference, refraction X–ray shows wave nature while in photoelectric effect it shows particle nature.

(iv) There is no charge on X–ray thus these are not deflected by electric field and magnetic field.

(v) X–ray are invisible.

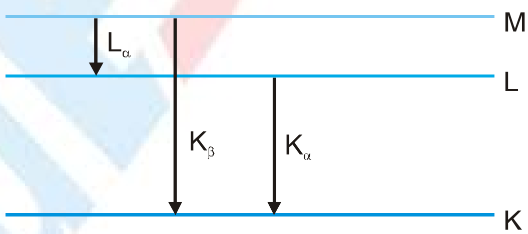
(vi) X–ray affects the photographic plate

(vii) When X–ray incidents on the surface of substance it exerts force and pressure and transfer energy and momentum

(viii) Characteristic X–ray can not obtained from hydrogen because the difference of energy level in hydrogen is very small.

Ex. Show that the frequency of K_β X–ray of a material is equal to the sum of frequencies of K_α and L_α X–rays of the same material.

Sol. The energy level diagram of an atom with one electron knocked out is shown above.



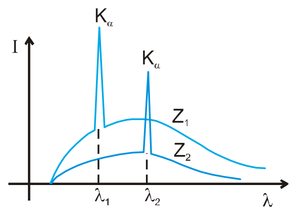
Energy of K_α X–ray is $E_{K\alpha} = E_L - E_K$

and of K_β X–ray is $E_{K\beta} = E_M - E_K$

and of L_α X–rays is $E_{L\alpha} = E_M - E_L$

thus $E_{K\beta} = E_{K\alpha} + E_{L\alpha}$ or $\nu_{K\beta} = \nu_{K\alpha} + \nu_{L\alpha}$

Ex.



Find in Z_1 and Z_2 which one is greater.

Sol. ✈ $\sqrt{\nu} \equiv \sqrt{cR\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}\ .(Z-b)$

If Z is greater then ν will be greater, λ will be less

∴ $\lambda_1 < \lambda_2$ ∴ $Z_1 > Z_2$.

Ex. A cobalt target is bombarded with electrons and the wavelength of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

Sol. Using Moseley's law and putting c/λ for ν (and assuming b = 1), we obtain

$$\sqrt{\frac{c}{\lambda_{c_0}}} = aZ_{c_0} - a \quad \text{and} \quad \sqrt{\frac{c}{\lambda_x}} = aZ_x - a$$

Dividing yields $$\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$$

Substituting gives us $$\sqrt{\frac{178.9\text{pm}}{143.5\text{pm}}} = \frac{Z_x - 1}{27 - 1}.$$

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

Ex. Find the constants a and b in Moseley's equation $\sqrt{v} = a(Z-b)$ from the following data.

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178.5 pm

Sol. Moseley's equation is

$$\sqrt{v} = a(Z-b)$$

Thus, $$\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b) \quad ....(i) \quad \text{and} \quad \sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b) \quad ....(ii)$$

From **(i)** and **(ii)** $$\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2) \quad \text{or,} \quad a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$$

$$= \frac{(3\times10^8 m/s)^{1/2}}{42-27}\left[\frac{1}{(71\times10^{-12}m)^{1/2}} - \frac{1}{(178.5\times10^{-12}m)^{1/2}}\right] = 5.0\times10^7\,(\text{Hz})^{1/2}$$

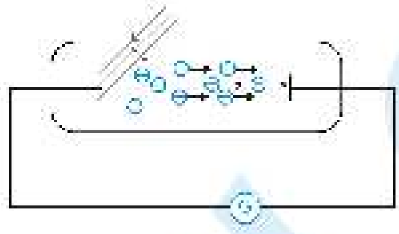
Dividing **(i)** by **(ii)**,

$$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b} \quad \text{or,} \quad \sqrt{\frac{178.5}{71}} = \frac{42-b}{27-b} \quad \text{or,} \quad b = 1.37$$

PHOTOELECTRIC EFFECT

INTRODUCTION

The photoelectric effect or photoemission is the production of electrons or other free carriers when light is shone onto a material. Electrons emitted in this manner can be called photoelectrons. It was discovered by manner can be called photoelectrons. It was discovered by Hertz in 1887. He found that when negative plate of an electronic discharge tube was illuminated with ultraviolet light. The electric discharge took place more readily. Further experiments carried out by Hallwachs confirmed that certain negatively charge particles are emitted, when a Zn plate is illuminated with ultraviolet light. These particles were identified as electrons.



1. **Photoelectron :** The electron emitted in photoelectric effect is called photoelectron.

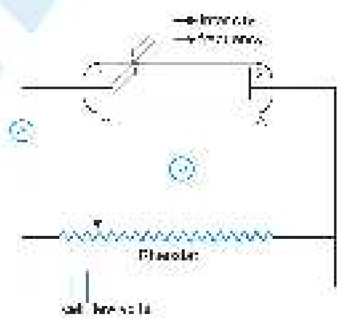
2. **Photoelectric current :** If current passes through the circuit in photoelectric effect then the current is called photoelectric current.

3. **Work function :** The minimum energy required to make an electron free from the metal is called work function. It is constant for a metal and denoted by ϕ or W. It is the minimum for Cesium. It is relatively less for alkali metals.

WORK FUNCTIONS OF SOME PHOTOSENSITIVE METALS

Metal	Work function (eV)	Metal	Work function (eV)
Cesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	[illegible]	Platinum	5.6

To produce photo electric effect only metal and light is necessary but for observing it, the circuit is completed. Figure shows an arrangement used to study the photoelectric effect.



Here the plate (1) is called emitter or cathode and other plate (2) is called collector or anode.

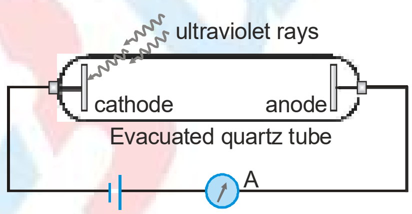
1. **Saturation current :** When all the photo electrons emitted by cathode reach the anode then current flowing in the circuit at that instant is known as saturated current, this is the maximum value of photoelectric current.
2. **Stopping potential :** Minimum magnitude of negative potential of anode with respect to cathode for which current is zero is called stopping potential. This is also known as cutoff voltage. This voltage is independent of intensity.
3. **Retarding potential :** Negative potential of anode with respect to cathode which is less than stopping potential is called retarding potential.

EXPLANATION OF PHOTOELECTRIC EFFECT

1. **On the basis of wave theory :** According to wave theory, light is an electromagnetic radiation consisting of oscillating electric field vectors and magnetic field vectors. When electromagnetic radiations are incident on a metal surface, the free electrons [free electrons means the electrons which are loosely bound and free to move inside the metal] absorb energy from the radiation. This occurs by the oscillations of electron under the action of electric field vector of electromagnetic radiation. When an electron acquires sufficiently high energy so that it can overcome its binding energy, it comes out from the metal.
2. **On the basis of photon theory:** According to photon theory of light, light consists of particles (called photons). Each particle carries a certain amount of energy with it. This energy is given by $E=h\nu$, where h is the Plank's constant and ν is the frequency. When the photons are incident on a metal surface, they collide with electrons. In some of the collisions, a photon is absorbed by an electron. Thus an electron gets energy $h\nu$. If this energy is greater than the binding energy of the electron, it comes out of the metal surface. The extra energy given to the electron becomes its kinetic energy.

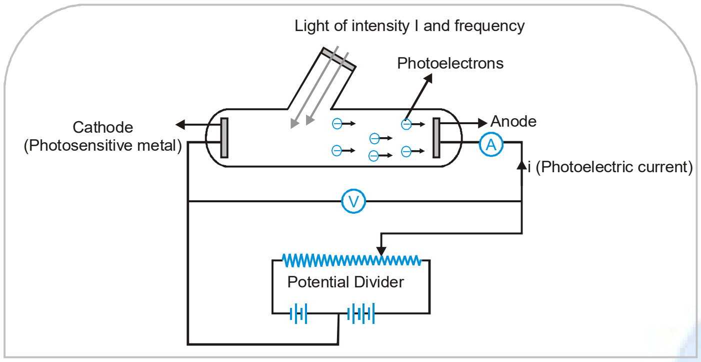
EXPERIMENTS

1. **Hertz Experiment :** Hertz observed that when ultraviolet rays are incident on negative plate of electric discharge tube then conduction takes place easily in the tube.

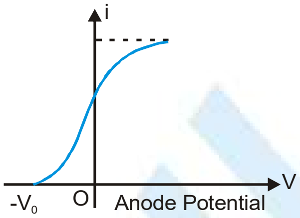


2. **Hallwach Experiment :** Hallwach observed that if negatively charged Zn plate is illuminated by U.V. light, its negative charge decreases and it becomes neutral and after some time it gains positive charge. It means in the effect of light, some negative charged particles are emitted from the metal.
3. **Lenard Explanation :** He told that when ultraviolet rays are incident on cathode, electrons are ejected. These electrons are attracted by anode and circuit is completed due to flow of electrons and current flows. When U.V. rays incident on anode, electrons are ejected but current does not flow.

   For the photo electric effect the light of short wavelength (or high frequency) is more effective than the light of long wavelength (or low frequency)
4. **Experimental study of photoelectric Effect :** When light of frequency ν and intensity I falls on the cathode, electrons are emitted from it. The electrons are collected by the anode and a current flows in the circuit. This current is called photoelectric current. This experiment is used to study the variation of photoelectric current with different factors like intensity, frequency and the potential difference between the anode and cathode.



(i) Variation of photoelectric current with potential difference : With the help of the above experimental setup, a graph is obtained between current and potential difference. The potential difference is varied with the help of a potential divider. The graph obtained is shown below.



The main points of observation are :

(a) At zero anode potential, a current exists. It means that electrons are emitted from cathode with some kinetic energy.

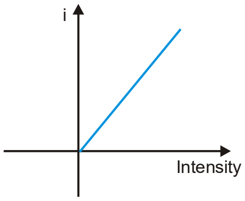
(b) As anode potential is increased, current increases. This implies that different electrons are emitted with different kinetic energies.

(c) After a certain anode potential, current acquires a constant value called saturation current. Current acquires a saturation value because the number of electrons emitted 7 per second from the cathode are fixed.

(d) At a certain negative potential, the photoelectric current becomes zero. This is called stopping potential (V_0). Stopping potential is a measure of maximum kinetic energy of the emitted electrons. Let KE_{max} be the maximum kinetic energy of an emitted electron, then $KE_{max} = eV_0$.

(ii) Variation of current with intensity

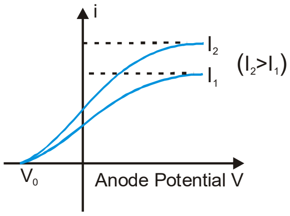
The photoelectric current is found to be directly proportional to intensity of incident radiation.



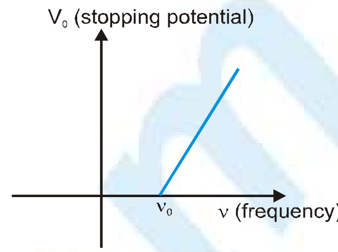
(iii) Effect of intensity on saturation current and stopping potential

(a) Saturation current increases with increase in intensity.

(b) Stopping potential (and therefore maximum kinetic energy) is independent of intensity.



(iv) Effect of frequency



(a) Stopping potential is found to vary with frequency of incident light linearly. Greater the frequency of incident light, greater the stopping potential.

(b) There exists a certain minimum frequency ν_0 below which no stopping potential is required as no emission of electrons takes place. This frequency is called threshold frequency. For photoelectric emission to take place, $\nu > \nu_0$.

(i) Photo electric effect is an instantaneous process, as soon as light is incident on the metal, photo electrons are emitted.

(ii) Stopping potential does not depend on the distance between cathode and anode.

(iii) The work function represented the energy needed to remove the least tightly bounded electrons from the surface. It depends only on nature of the metal and independent of any other factors.

1. Failure of wave theory of light

(i) According to wave theory when light incident on a surface, energy is distributed continuously over the surface. So that electron has to wait to gain sufficient energy to come out. But in experiment there is no time lag. Emission of electrons takes place in less than 10^{-9} s. This means, electron does not absorb energy. They get all the energy once.

(ii) When intensity is increased, more energetic electrons should be emitted. So that stopping potential should be intensity dependent. But it is not observed.

(iii) According to wave theory, if intensity is sufficient then, at each frequency, electron emission is possible. It means there should not be existence of threshold frequency.

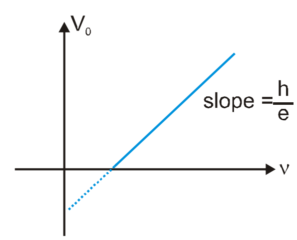
2. Einstein's Explanation of Photoelectric Effect

Einstein explained photoelectric effect on the basis of photon–electron interaction. The energy transfer takes place due to collisions between an electrons and a photon. The electrons within the target material are held there by electric force. The electron needs a certain minimum energy to escape from this pull. This minimum energy is the property of target material and it is called the work function. When a photon of energy E=hν collides with and transfers its energy to an electron, and this energy is greater than the work function, the electron can escape through the surface.

3. Einstein's Photoelectric Equation $h\nu = \phi + KE_{max}$

Here hν is the energy transferred to the electron. Out of this, ϕ is the energy needed to escape.

The remaining energy appears as kinetic energy of the electron.



Now $KE_{max} = eV_0$ (where V_0 is stopping potential)

$\therefore h\nu = \phi + eV_0 \Rightarrow V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$

Thus, the stopping potential varies linearly with the frequency of incident radiation.

Slope of the graph obtained is $\frac{h}{e}$. This graph helps in determination of Planck's constant.

(i) Einstein's Photo Electric equation is based on conservation of energy.

(ii) Einstein explained P.E.E. on the basis of quantum theory, for which he was awarded noble prize.

(iii) According to Einstein one photon can eject one e^- only. But here the energy of incident photon should greater or equal to work function.

(iv) In photoelectric effect all photoelectrons do not have same kinetic energy. Their KE range from zero to E_{max} which depends on frequency of incident radiation and nature of cathode.

(v) The photo electric effect takes place only when photons strike bound electrons because for free electrons energy and momentum conservations do not hold together.

Ex. In an experiment on photo electric emission, following observations were made;

(i) Wavelength of the incident light = 1.98×10^{-7} m;

(ii) Stopping potential = 2.5 volt.

Find : **(a)** Kinetic energy of photoelectrons with maximum speed.

(b) Work function and

(c) Threshold frequency;

Sol. **(a)** Since $v_s = 2.5$ V, $K_{max} = eV_s$ So, $K_{max} = 2.5$ eV

(b) Energy of incident photon

$$E = \frac{12400}{1980} eV = 6.26 \text{ eV} \qquad W = E - K_{max} = 3.76 \text{ eV}$$

(c) $h\nu_{th} = W = 3.76 \times 1.6 \times 10^{-19}$ J $\therefore$ $\nu_{th} = \frac{3.76 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \approx 9.1 \times 10^{14}$ Hz

Ex. A beam of light consists of four wavelength 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity 1.5×10^{-3} Wm^{-2}. The beam falls normally on an area 10^{-4} m^2 of a clean metallic surface of work function 1.9 eV. Assuming no loss of light energy (i.e. each capable photon emits one electron) calculate the number of photoelectrons liberated per second.

Sol. $E_1 = \frac{12400}{4000} = 3.1$ eV, $E_2 = \frac{12400}{4800} = 2.58$ eV $E_3 = \frac{12400}{6000} = 2.06$ eV

and $E_4 = \frac{12400}{7000} = 1.77$ eV

Therefore, light of wavelengths 4000 Å, 4800 Å and 6000 Å can only emit photoelectrons.

$\therefore$ Number of photoelectrons emitted per second = No. of photons incident per second)

$$= \frac{I_1A_1}{E_1} + \frac{I_2A_2}{E_2} + \frac{I_3A_3}{E_3} = IA\left(\frac{1}{E_1} + \frac{1}{E_2} + \frac{1}{E_3}\right)$$

$$= \frac{(1.5 \times 10^{-3})(10^{-4})}{1.6 \times 10^{-19}} \left(\frac{1}{3.1} + \frac{1}{2.58} + \frac{1}{2.06}\right)$$

Ex. Calculate the possible velocity of a photoelectron if the work function of the target material is 1.24 eV and wavelength of light is 4.36 × 10^{-7} m. What retarding potential is necessary to stop the emission of electrons?

Sol. As $KE_{max} = h\nu - \phi \Rightarrow \frac{1}{2}mv_{max}^2 = h\nu - \phi = \frac{hc}{\lambda} - \phi$

$$v_{max} = \sqrt{\frac{2\left(\frac{hc}{\lambda} - \phi\right)}{m}} = \sqrt{\frac{2\left(\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4.36 \times 10^{-7}} - 1.24 \times 1.6 \times 10^{-19}\right)}{9.11 \times 10^{-31}}} = 7.523 \times 10^5 \text{ m/s}$$

$\therefore$ The speed of a photoelectron can be any value between 0 and 7.43 × 10^5 m/s

If V_0 is the stopping potential, then $eV_0 = \frac{1}{2}mv_{max}^2$

$$\Rightarrow V_0 = \frac{1}{2}\frac{mv_{max}^2}{e} = \frac{hc}{e\lambda} - \frac{\phi}{e} = \frac{12400}{4360} - 1.24 = 1.60 \text{ V} \quad \left[Q\ \frac{hc}{e} = 12400 \times 10^{-10} \text{ V} - \text{m}\right]$$

Ex. The surface of a metal of work function ϕ is illuminated by light whose electric field component varies with time as $E = E_0 [1 + \cos \omega t] \sin\omega_0 t$. Find the maximum kinetic energy of photoelectrons emitted from the surface.

Sol. The given electric field component is $E = E_0 \sin \omega_0 t + E_0 \sin \omega_0 t \cos \omega t = E_0 \sin \omega_0 t + \frac{E_0}{2}[\sin(\omega_0 + \omega)t + \sin(\omega_0 - \omega)t]$

$\therefore$ The given light comprises three different frequencies viz. $\omega, \omega_0 + \omega, \omega_0 - \omega$

The maximum kinetic energy will be due to most energetic photon.

$$\therefore KE_{max} = h\nu - \phi = \frac{h(\omega + \omega_0)}{2\pi} - \phi \quad \left(Q\ \omega = 2\pi\nu \text{ or } \nu = \frac{\omega}{2\pi}\right)$$

Ex. When light of wavelength λ is incident on a metal surface, stopping potential is found to be x. When light of wavelength $n\lambda$ is incident on the same metal surface, stopping potential is found to be $\frac{x}{n+1}$. Find the threshold wavelength of the metal.

Sol. Let λ_0 is the threshold wavelength. The work function is $\phi = \frac{hc}{\lambda_0}$.

Now, by photoelectric equation $\quad ex = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ ...**(i)** $\quad \frac{ex}{n+1} = \frac{hc}{n\lambda} - \frac{hc}{\lambda_0}$ ...**(ii)**

From **(i)** and **(ii)** $\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = (n+1)\frac{hc}{n\lambda} - (n+1)\frac{hc}{\lambda_0} \Rightarrow \frac{nhc}{\lambda_0} = \frac{hc}{n\lambda} \Rightarrow \lambda_0 = n^2\lambda$

PHOTON THEORY OF LIGHT

According to Plank's Quantum Theory light consist of photon as energy packets having following properties :

(i) Each photon i.e. of energy

$E = h\nu = hc/\lambda$ where h is plank's constant

(ii) A photon is a particle of light moving with speed 299792458 m/s in vacuum.

(iii) The speed of a photon is independent of frame of reference. This is the basic postulate of theory of relativity.

(iv) The rest mass of a photon is zero. i.e. photons do not exist at rest.

(v) Effective mass of photon $m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ i.e. $m \propto \frac{1}{\lambda}$

So mass of violet light photon is greater than the mass of red light photon. $(\because \lambda_R > \lambda_V)$

(vi) According to Planck the energy of a photon is directly proportional to the frequency of the radiation.

$E \propto \nu$ or $E = h\nu$

$E = \frac{hc}{\lambda}$ joule $(\because c = \nu\lambda)$ or $E = \frac{hc}{\lambda e} = \frac{12400}{\lambda}$ eV – Å $\left[\because \frac{hc}{e} = 12400(\overset{o}{A} - eV)\right]$

Here E = energy of photon, c = speed of light, h = Planck's constant, e = charge of electron

$h = 6.62 \times 10^{-34}$ J–s, ν = frequency of photon, λ = wavelength of photon

(vii) Linear momentum of photon $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

(viii) A photon can collide with material particles like electron. During these collisions, the total energy and total momentum remain constant.

(ix) Energy of light passing through per unit area per unit time is known as intensity of light.

Intensity of light $I = \frac{E}{At} = \frac{P}{A}$ ...(i)

Here P = power of source, A = Area, t = time taken

E = energy incident in t time = $Nh\nu$, N = number of photon incident in t time

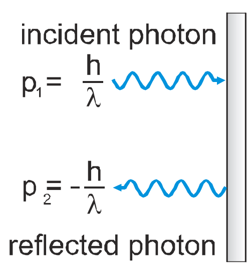
Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ...(ii) $\left[\because n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

From equation (i) and (ii), $\frac{P}{A} = \frac{n(h\nu)}{A} \Rightarrow n = \frac{P}{h\nu} = \frac{P\lambda}{hc} = 5 \times 10^{24} J^{-1} m^{-1} \times P \times \lambda$

(x) When photons fall on a surface, they exert a force and pressure on the surface. This pressure is called radiation pressure.

(xi) Force exerted on perfectly reflecting surface

Let 'N' photons are there in time t,



Momentum before striking the surface $(p_1) = \frac{Nh}{\lambda}$

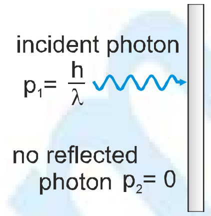
Momentum after striking the surface $(p_2) = -\frac{Nh}{\lambda}$

Change in momentum of photons $= p_2 - p_1 = \frac{-2Nh}{\lambda}$

But change in momentum of surface $= \Delta p = \frac{2Nh}{\lambda}$; So that force on surface $F = \frac{2Nh}{t\lambda} = n\left[\frac{2h}{\lambda}\right]$ but $n = \frac{P\lambda}{hc}$

$\therefore F = \frac{2h}{\lambda} \times \frac{P\lambda}{hc} = \frac{2P}{c}$ and Pressure $= \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}$ $\left[\because I = \frac{P}{A}\right]$

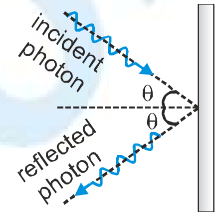
(xii) Force exerted on perfectly absorbing surface



$$F = \frac{p_1 - p_2}{t} = \frac{\frac{Nh}{\lambda} - 0}{t} = \frac{Nh}{t\lambda} = n\frac{h}{\lambda}\ ;\ F = \frac{P}{c}\left(\because\ n = \frac{P\lambda}{hc}\right)$$

$$\text{and Pressure} = \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$$

(xiii) When a beam of light is incident at angle θ on perfectly reflector surface



$$F = \frac{2P}{c}\cos\theta = n\left[\frac{2h}{\lambda}\right]\cos\theta = \frac{2IA\cos\theta}{c}\ ;\ \text{Pressure} = \frac{F}{A} = \frac{2I\cos\theta}{c}$$

Ex. Violet light $(\lambda = 4000\ \text{Å})$ of entensity 4 watt/m^2 falls normally on a surface of area 10 cm × 20 cm.

Find

(a) the energy received by the surface per second.

(b) the number of photons hitting the surface per second.

(c) If surface is tiled such that plane of the surface makes an angle 30° with light beam, find the number of photons hitting the surface per second.

Sol. **(a)** Energy received per second per unit area

$E = IA\cos\theta = 4 \times 0.02\ J \times \cos 0° = 0.08\ J$

(b) $n h (c/\lambda) = E$

$$\Rightarrow\ n = \frac{0.08 \times 4000 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} = \frac{32 \times 10^{17}}{19.89} = 1.699 \times 10^{17}$$

(c) $$n = \frac{IA\cos 60° \times \lambda}{hc} = \frac{1}{2} \times \frac{32 \times 10^{17}}{19.89} = 0.805 \times 10^{17}$$

Ex. The intensity of sunlight on the surface of earth is 1400 W/m^2. Assuming the mean wavelength of sunlight to be 6000 Å, calculate:–

(a) The photon flux arriving at 1 m^2 area on earth perpendicular to light radiations and

(b) The number of photons emitted from the sun per second (Assuming the average radius of Earth's orbit to be 1.49×10^{11} m)

Sol. **(a)** Energy of a photon $E = \frac{hc}{\lambda} = \frac{12400}{6000} = 2.06\ eV = 3.3 \times 10^{-19}\ J$

$$\text{Photon flux} = \frac{IA}{E} = \frac{1400 \times 1}{3.3 \times 10^{-19}} = 4.22 \times 10^{21}\ \text{photons/sec.}$$

(b) Number of photons emitted per second $n = \frac{P}{E} = \frac{IA}{E} = \frac{1400 \times 4\pi \times (1.49 \times 10^{11})^2}{3.3 \times 10^{-19}} = 1.18 \times 10^{45}$

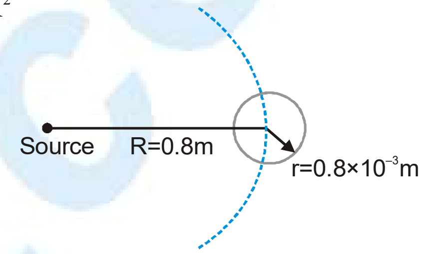
Ex. In a photoelectric setup, a point source of light of power 3.2×10^{-3} W emits monochromatic photons of energy 5.0 eV. The source is located at a distance 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and radius 8×10^{-3} m. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assuming that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission, Find **(i)** the number of photoelectrons emitted per second. **(ii)** the time t after light source is switched on, at which photoelectron emission stops.

Sol. Energy of a single photon E=5.0 eV = 8×10^{-19} J

Power of source P = 3.2×10^{-3} W

$\therefore$ number of photons emitted per second $n = \frac{P}{E} = \frac{3.2 \times 10^{-3}}{8 \times 10^{-19}} = 4\times 10^{15}$/s

The number of photons incident per second on metal surface is $n_0 = \frac{n}{4\pi R^2} \times \pi r^2$



$$n_0 = \frac{4 \times 10^{15}}{4\pi(0.8)^2} \times \pi\left(8 \times 10^{-3}\right)^2 = 1.0 \times 10^{11} \text{ photon/s}$$

Number of electrons emitted $= \frac{1.0 \times 10^{11}}{10^6} = 10^5$ /s

$$KE_{max} = h\nu - \phi = 5.0 - 3.0 = 2.0 \text{ eV}$$

The photoelectron emission stops, when the metallic sphere acquires stopping potential.

$$\text{As } KE_{max} = 2.0 \text{ eV} \Rightarrow \text{Stopping potential } V_0 = 2V \Rightarrow 2 = \frac{q}{4\pi\varepsilon_0 r} \Rightarrow q = 1.78 \times 10^{-12} C$$

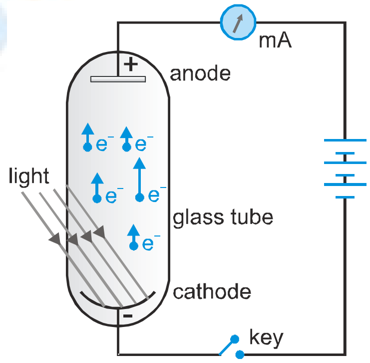
Now charge q = (number of electrons/second) × t× e $\Rightarrow t = \frac{1.78 \times 10^{-12}}{10^5 \times 1.6 \times 10^{-19}} = 111$s

PHOTO CELL

A photo cell is a practical application of the phenomenon of photo electric effect, with the help of photo cell light energy is converted into electrical energy.

1. **Construction :** A photo cell consists of an evacuated sealed glass tube containing anode and a concave cathode of suitable emitting material such as Cesium (Cs).

2. **Working:** When light of frequency greater than the threshold frequency of cathode material falls on the cathode, photoelectrons emitted are collected by the anode and an electric current starts flowing in the external circuit. The current increase with the increase in the intensity of light. The current would stop, if the light does not fall on the cathode.



3. Application

(i) In television camera.

(ii) In automatic door

(iii) Burglar's alarm

(iv) Automatic switching of street light and traffic signals.

1, 2, 3..... represents the I, II & III line of Lymen, Balmer, Paschen series.

The hydrogen spectrum (some selected lines)

(c) $\Delta E_{31} = -3.08 - (-15.6) = 12.52$ eV

Therefore, excitation potential for state n = 3 is 12.52 volt.

(d) $$\frac{1}{\lambda_{31}} = \frac{\Delta E_{31}}{12400}\ \text{Å}^{-1} = \frac{12.52}{12400}\ \text{Å}^{-1}$$

$$\approx 1.01 \times 10^{7}\ \text{m}^{-1}$$

(e) (i) $E_2 - E_1 = 10.3\text{ eV} > 6\text{ eV}$.

Hence electron cannot excite the atoms. So, $K_{min} = 6$ eV.

(ii) $E_2 - E_1 = 10.3\text{ eV} < 11\text{ eV}$.

Hence electron can excite the atoms. So, $K_{min} = (11 - 10.3) = 0.7$ eV.

MATTER WAVES

DUAL NATURE OF LIGHT

Experimental phenomena of light reflection, refraction, interference, diffraction are explained only on the basis of wave theory of light. These phenomena verify the wave nature of light. Experimental phenomena of light photoelectric effect and Crompton effect, pair production and positron inhalational can be explained only on the basis of the particle nature of light. These phenomena verify the particle nature of light.

It is inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon. This is known as the dual nature of light. The wave nature and particle nature both can not be possible simultaneously.

De-Broglie HYPOTHESIS

De Broglie imagined that as light possess both wave and particle nature, similarly matter must also posses both nature, particle as well as wave. De Broglie imagined that despite particle nature of matter, waves must also be associated with material particles. Wave associated with material particles, are defined as matter waves.

(i) De Broglie wavelength associated with moving particles

If a particle of mass m moving with velocity v

Kinetic energy of the particle $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ momentum of particle $p = mv = \sqrt{2mE}$ the wave length associated with the particles is $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$ $\lambda \propto \frac{1}{p} \Rightarrow \lambda \propto \frac{1}{v} \Rightarrow \lambda \propto \frac{1}{\sqrt{E}}$

The order of magnitude of wave lengths associated with macroscopic particles is 10^{-24} Å.

The smallest wavelength whose measurement is possible is that of γ – rays ($\lambda \cong 10^{-5}$ Å). This is the reason why the wave nature of macroscopic particles is not observable.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α – particle, atom, molecule etc. is of the order of 10^{-10} m, it is equal to the wavelength of X–rays, which is within the limit of measurement. Hence the wave nature of these particles is observable.

(ii) De Broglie wavelength associated with the charged particles

Let a charged particle having charge q is accelerated by potential difference V.

Kinetic energy of this particle $E = \frac{1}{2}mv^2 = qV$ Momentum of particle $p = mv = \sqrt{2mE} = \sqrt{2mqV}$

The De Broglie wavelength associated with charged particle $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

(iii) For an Electron $m_e = 9.1 \times 10^{-31}$ kg, $q = 1.6 \times 10^{-19}$ C, $h = 6.62 \times 10^{-34}$ J–s

De Broglie wavelength associated with electron $\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$

$$\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}} \text{ meter} = \frac{12.27}{\sqrt{V}} \text{ Å} \quad \text{So } \lambda \propto \frac{1}{\sqrt{V}}$$

Potential difference required to stop an electron of wavelength λ is $V = \frac{150.6}{\lambda^2}$ volt $(\text{Å})^2$

(iv) For Proton $m_p = 1.67 \times 10^{-27}$ kg

De Broglie wavelength associated with proton

$$\lambda_p = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}\,V}}\ ;\ \lambda_p = \frac{0.286\times10^{-10}}{\sqrt{V}}\text{ meter} = \frac{0.286}{\sqrt{V}}\text{ Å}$$

(v) For Deuteron $m_d = 2\times1.67\times10^{-27}$ kg, $q_d = 1.6\times10^{-19}$ C

$$\lambda_d = \frac{6.62\times10^{-34}}{\sqrt{2\times2\times1.67\times10^{-27}\times1.6\times10^{-19}\,V}} = \frac{0.202}{\sqrt{V}}\text{ Å}$$

(vi) For α Particles $q_\alpha = 2\times1.6\times10^{-19}$ C, $m_\alpha = 4\times1.67\times10^{-27}$ kg

$$\therefore\quad \lambda_\alpha = \frac{6.62\times10^{-34}}{\sqrt{2\times4\times1.67\times10^{-27}\times2\times1.6\times10^{-19}\,V}} = \frac{0.101}{\sqrt{V}}\text{ Å}$$

De Broglie Wavelength Associated with Uncharged Particles

(i) Kinetic energy of uncharged particle $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

m = mass of particle, v = velocity of particle, p = momentum of particle.

(ii) Velocity of uncharged particle $v = \sqrt{\frac{2E}{m}}$

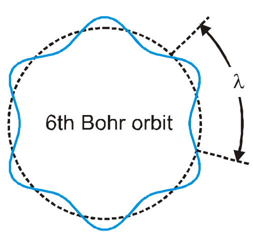
(iii) Momentum of particle $p = mv = \sqrt{2mE}$

wavelength associated with the particle $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

Kinetic energy of the particle in terms of its wavelength $E = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m\lambda^2\times1.6\times10^{-19}}$ eV

For a neutron $m_n = 1.67\times10^{-27}$ kg $\therefore\ \lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times E}} = \frac{0.286\times10^{-10}}{\sqrt{E}}\text{ meter}\sqrt{eV} = \frac{0.286}{\sqrt{E}}\text{ Å}\sqrt{eV}$

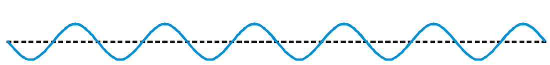
EXPLANATION OF BOHR QUANTIZATION CONDITION



According to De Broglie electron revolves round the nucleus in the form of stationary waves (i. e. wave packet) in the similar fashion as stationary waves in a vibrating string. Electron can stay in those circular orbits whose circumference is an integral multiple of De–Broglie wavelength associated with the electron, $2\pi r = n\lambda$

$\Rightarrow\ \lambda = \frac{h}{mv}$ and $2\pi r = n\lambda$ $\qquad \therefore\ mvr = \frac{nh}{2\pi}$

This is the Bohr quantizations condition.



equivalent straightened orbit

Ex. Find the initial momentum of electron if the momentum of electron is changed by p_m and the De Broglie wavelength associated with it changes by 0.50 %

Sol. $\frac{d\lambda}{\lambda}\times 100 = 0.5 \Rightarrow \frac{d\lambda}{\lambda} = \frac{0.5}{100} = \frac{1}{200}$ and $\Delta p = p_m$

$p = \frac{h}{\lambda}$, differentiating $\frac{dp}{d\lambda} = -\frac{h}{\lambda^2} = -\frac{h}{\lambda}\times\frac{1}{\lambda} = -\frac{p}{\lambda} \Rightarrow \frac{|dp|}{p} = \frac{d\lambda}{\lambda}$ $\therefore \frac{p_m}{p} = \frac{1}{200} \Rightarrow p = 200\,p_m$

Ex. An α–particle moves in circular path of radius 0.83 cm in the presence of a magnetic field of 0.25 Wb/m^2. Find the De Broglie wavelength associated with the particle.

Sol. $\lambda = \frac{h}{p} = \frac{h}{qBr} = \frac{6.62\times10^{-34}}{2\times1.6\times10^{-19}\times0.25\times83\times10^{-4}}$ meter $= 0.01$ Å $\left[\because \frac{mv^2}{r} = qvB\right]$

Ex. A proton and an α–particle are accelerated through same potential difference. Find the ratio of their de- Broglie wavelength.

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$ $[\because E = qV]$ For proton $m_p = m$, $q = e$

For α–particle $m_\alpha = 4$ m, $q = 2e$, $\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}} = \frac{1}{2\sqrt{2}}$

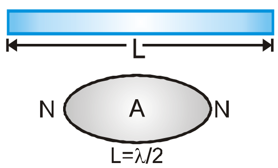
Ex. A particle of mass m is confined to a narrow tube of length L.

(a) Find the wavelengths of the de–Broglie wave which will resonate in the tube.

(b) Calculate the corresponding particle momenta, and

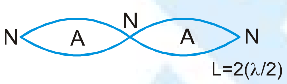
(c) Calculate the corresponding energies.

Sol. **(a)** The de–Broglie waves will resonate with a node at each end of the tube.



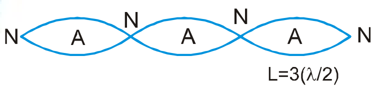
Few of the possible resonance forms are as follows : $\lambda_n = \frac{2L}{n}$, n=1,2,3.....

(b)



Since de–Broglie wavelengths are $\lambda_n = \frac{h}{p_n}$

$p_n = \frac{h}{\lambda_n} = \frac{nh}{2L}$, n = 1,2,3....



(c) The kinetic energy of the particles are $K_n = \frac{p_n^2}{2m} = \frac{n^2h^2}{8L^2m}$, n = 1, 2, 3,....

1. Atomic Models :

(a) Thomson model : (Plum pudding model)

(1) Most of the mass and all the positive charge of an atom is uniformly distributed over the full size of atom (10^{-10} m).

(2) Electrons are studded in this uniform distribution.

(3) Failed to explain the large angle scattering α-particle scattered by thin foils of matter.

(b) Rutherford model : (Nuclear Model)

(1) The most of the mass and all the positive charge is concentrated within a size of 10-14 m inside the atom. This concentrated is called the atomic nucleus.

(2) The electron revolves around the nucleus under electric interaction between them in circular orbits.

(3) An accelerating charge radiates the nucleus spiralling inward and finally fall into the nucleus, which does not happen in an atom. This could not be explained by this model.

(c) Bohr atomic model : Bohr adopted Rutherford model of the atom & added some arbitrary conditions. These conditions are known as his postulates

(1) The electron in a stable orbit does not radiate energy.

(2) A stable orbit is that in which the angular momentum of the electron about nucleus is an integral (n) multiple of $\frac{h}{2\pi}$ i.e., $mvr = n\frac{h}{2\pi}$; $n = 1, 2, 3, ..(n \neq 0)$.

(3) The electron can absorb or radiate energy only if the electron jumps from a lower to a higher orbit or falls from a higher to lower orbit.

(4) The energy emitted or absorbed is a light photon of frequency ν and of energy. $E = h\nu$

For hydrogen atom : (z = atomic number = 1)

L_n = angular momentum in the n^{th} orbit $= n\frac{h}{2\pi}$

r_n = radius of n^{th} circular orbit $= (0.529\ \mathring{A})\, n^2 \Rightarrow r_n \propto n^2$

E_n = Energy of the electron in the n^{th} orbit $= \frac{-13.6\,eV}{n^2} \Rightarrow E_n \propto \frac{1}{n^2}$

Note : Total energy of the electron in an atom is negative, indicating that it is bound.

Binding Energy $(BE)_n = E_n = \frac{13.6\,eV}{n^2}$

(5) $E_{n2} - E_{n1}$ = Energy emitted when an electron jumps from n_2^{th} orbit to n_1^{th} orbit $(n_2 > n_1)$

$$\Delta E = (13.6\,eV)\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$\Delta E = h\nu$; ν = frequency of spectral line emitted.

$\frac{1}{\lambda}$ = wave no, [no. of waves in unit length (1m)] $= R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

where R = Rydberg's constant, for hydrogen $= 1.097 \times 10^7\ m^{-1}$

(6) For hydrogen like atom/species of atomic number Z :

$$r_{nz} = \frac{\text{Bohr radius}}{Z} n^2 = \left(0.529\,\mathring{A}\right)\frac{n^2}{Z}; E_{nz} = (-13.6)\frac{Z^2}{n^2}\text{eV}$$

$R_z = RZ^2$; Rydberg's constant for element of atomic no. Z.

Note : If motion of the nucleus is also considered, then μ is replaced by μ. Where μ = reduced mass of electron - nucleus system = mM/(m + M)

In this case $E_n = (-13.6\text{ eV})\ \frac{Z^2}{n^2}\cdot\frac{\mu}{m_e}$

2. Spectral series :

(a) Lyman Series : (Landing orbit n = 1)

Ultraviolet region $\bar{v} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]; n_2 > 1$

(b) Balmer Series : (Landing orbit n = 2)

Visible region $\bar{v} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]; n_2 > 2$

(c) Paschan Series : (Landing orbit n = 3)

In the mid infrared region $\bar{v} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]; n_2 > 3$

(d) Bracket Series : (Landing orbit n = 4)

In the mid infrared region $\bar{v} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]; n_2 > 4$

(e) Pfund Series : (Landing orbit n = 5)

In far infrared region $\bar{v} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]; n_2 > 5$

In all these series n_2 = $n_1 + 1$ is the α line

= $n_1 + 2$ is the β line

= $n_1 + 3$ is the γ line ....etc

where n_1 = Landing orbit

3. Total emission spectral lines

From $n_1 = n$ to $n_2 = 1$ state $= \frac{n(n-1)}{2}$

From $n_1 = n$ to $n_2 = m$ state $= \left(\frac{(n-m)(n-m+1)}{2}\right)$

4. Excitation Potential of Atom :

Excitation potential for quantum jump from $n_1 \rightarrow n_2 = \dfrac{E_{n2} - E_{n1}}{\text{electron charge}}$

5. Ionization energy of hydrogen atom :

The energy required to remove an electron from an atom. The energy required to ionize hydrogen atom is $= 0 - (-13.6) = 13.6$ eV.

6. Ionization Potential :

Potential difference through which a free electron is moved to gain ionization energy $= \dfrac{-E_n}{\text{electron charge}}$

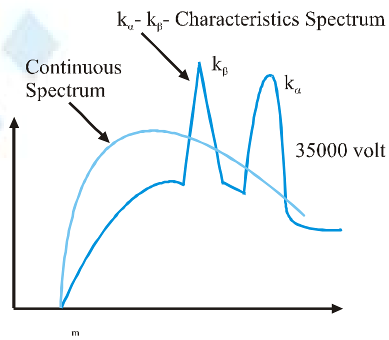
7. X - RAYS

(a) X-rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.

(b) Short wavelength (0.1 $\overset{o}{A}$ to 10 $\overset{o}{A}$) electromagnetic radiation.

(c) Are produced when a metal anode is bombarded by very high energy electrons

(d) Are not affected by electric and matgnetic field.



(e) They cause photoelectric emission.

Characteristic equation $eV = h\nu_m$

e = electron charge;

V = accelerating potential

ν_m = maximum frequency of X - radiation

(f) Intensity of X - rays depends on number of electrons hitting the target.

(g) Cut off wavelength or minimum wavelength, where V (in volts) is the p.d. applied to the tube $\lambda_{min} \cong \dfrac{12400}{V}$ Å

(h) Continuous spectrum due to retardation of electrons.

8. Characteristic X - rays

For $K_\alpha, \lambda = \dfrac{hc}{E_K - E_L}$ For $K_\beta, \lambda = \dfrac{hc}{E_L - E_M}$

9. Moseley's law for characteristic spectrum :

Frequency of characteristic line $\sqrt{\nu} = a(Z - b)$

Where a, b are constant, for K_α line b = 1

Z = atomic number of target

ν = frequency of characteristic spectrum

b = screening constant (for K -series b = 1, L series b = 7.4),

a = proportionality constant

	Bohr model		**Moseley's correction**
1.	For single electron species	**1.**	For many electron species
2.	$\Delta E = 13.6 Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] eV$	**2.**	$\Delta E = 13.6 (Z-b)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] eV$
3.	$\nu = RcZ^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	**3.**	$\nu = RcZ^2 (Z-b)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
4.	$\frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	**4.**	$\frac{1}{\lambda} = R(Z-b)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

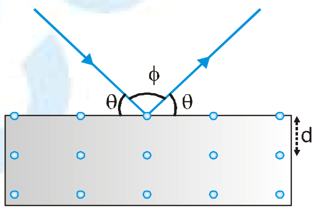
10. Diffraction of X - ray

Diffraction of X- ray take place according to Bragg's law $2d \sin\theta = n\lambda$

d = Spacing of crystal plane or lattice constant or distance between adjacent atomic plane

θ = Bragg's angle or glancing plane

ϕ = Diffracting angle n = 1, 2, 3.....



11. For Maximum Wavelength

$\sin\theta = 1, n = 1 \Rightarrow \lambda_{max} = 2d$

So if $\lambda > 2d$ diffraction is not possible i.e., solution of Bragg's equation is not possible.

12. Binding energy = – [Total Mechanical Energy]

13. Velocity of electron in n^{th} orbit for hydrogen atom $\cong \frac{c}{137n}$; c = speed of light.

14. Series limit means minimum wave length of that series.

15. Binding energy = – [Total Mechanical Energy]

16. Velocity of electron in n^{th} orbit for hydrogen atom $\cong \frac{c}{137n}$; c = speed of light.

17. Series limit means minimum wave length of that series.

18. Photo Electric Effect

The phenomenon of the emission of electrons, when metals are exposed to light (of a certain minimum frequency) is called photo electric effect.

19. Results :

(a) Can be explained only on the basis of the quantum theory (concept of photon)

(b) Electrons are emitted if the incident light has frequency $\nu \geq \nu_0$ (threshold frequency) emission of electrons is independent of intensity. The wave length corresponding to ν_0 is called threshold wave length λ_0

(c) ν_0 is different for different metals

(d) Number of electrons emitted per second depends on the intensity of the incident light.

20. Einsteins Photo Electric Equation :

Photon energy $= KE_{max}$ of electron + work function

$$h\nu = KE_{max} + \phi$$

ϕ = Work function = energy needed by the electron in freeing itself from the atoms of the metal $\phi = h\nu_0$

21. Stopping Potential or Cut Off Potential :

The minimum value of the retarding potential to prevent electron emission is

$$eV_{cut\,off} = (KE)_{max}$$

Note : The number of photons incident on a surface per unit time is called photon flux.

22. Wave Nature of Matter :

Beams of electrons and other forms of mattter exhibit wave properties including interference and diffraction with a de Broglie wave length given by $\lambda = \frac{h}{p}$ (wave length of a pratical)

23. De Broglie wavelength associated with moving particles

If a particle of mass m moving with velocity v.

Kinetic energy of the particle $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

momentum of particle $p = mv = \sqrt{2mE}$

the wave length associated with the praticles is $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

24. De Broglie wavelength associated with the charged particle

(a) For an Electron $\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}} m = \frac{0.286}{\sqrt{V}} \mathring{A}$ So $\lambda \propto \frac{1}{\sqrt{V}}$

(b) For Proton $\lambda_p = \frac{0.286 \times 10^{-10}}{\sqrt{V}} m = \frac{0.286}{\sqrt{V}} \mathring{A}$

(c) For Deuteron $\lambda_d = \frac{0.202}{\sqrt{V}} \mathring{A}$

(d) For α Particle $\therefore \lambda_\alpha = \frac{0.101}{\sqrt{V}} \mathring{A}$