Mechanical Properties of Solids

SOLIDS

The materials having a definite shape and volume are known as solids. All solids have the property of elasticity by virtue of which solids behave as incompressible substances and exhibit rigidity and the mechanical strength. Solids are classified into two categories namely Crystalline solids and amorphous solids (or glassy solids).

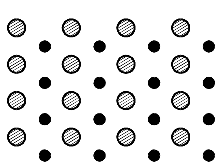


Figure (a)

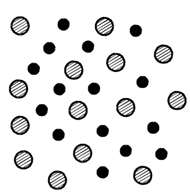


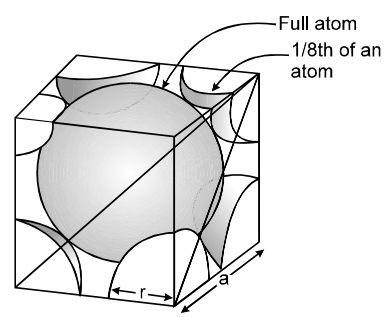
Figure (b)

Crystalline Solids : A solid in which atoms or molecules are arranged in a regular three dimensional pattern is known as crystalline solid shown in figure (a) For example : quartz, mica, sugar, copper sulphate, sodium chloride, potassium iodide, cesium chloride, carbon etc.

Amorphous Solids : A solid in which atoms or molecules are not arranged in a regular manner is known as amorphous solid shown in figure (b) For example : talc powder, glass, rubber, plastics etc.

UNIT CELL AND CRYSTAL LATTICE

Unit cell is the building block of a crystal. It is defined as the smallest pattern of atoms in a lattice, the repetition of which in three dimensions forms a crystal lattice.



Crystal lattice : It is defined as a regular arrangement of large number of points in space, each point representing the position of an atom or a group of atoms in a crystal. The crystal lattice is shown in a figure.

ELASTICITY AND PLASTICITY

The property of a material body by virtue of which it regains its original configuration (i.e. shape and size) when the external deforming force is removed is called elasticity. The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Deforming force : An external force applied to a body which changes its size or shape or both is called deforming force.

Perfectly Elastic body : A body is said to be perfectly elastic if it completely regains its original form when the deforming force is removed. Since no material can regain completely its original form so the concept of perfectly elastic body is only an ideal concept. A quartz fiber is the nearest approach to the perfectly elastic body.

Perfectly Plastic body : A body is said to be perfectly plastic if it does not regain its original form even slightly when the deforming force is removed. Since every material partially regain its original form on the removal of deforming force, so the concept of perfectly plastic body is only an ideal concept. Paraffin wax, wet clay are the nearest approach to a perfectly plastic bodies.

Cause of Elasticity : In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e. body) is in its equilibrium position and the inter molecular forces between the molecules of the solid are maximum.

On applying the deforming force on the body, the molecules either come closer or go far apart from each other. As a result of this, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

STRESS

When deforming force is applied on the body then the equal restoring force in opposite direction is developed inside the body. The restoring forces per unit area of the body is called stress.

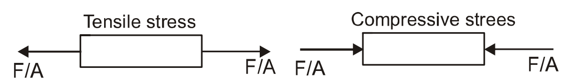
$$\text{stress} = \frac{\text{restoring force}}{\text{Area of the body}} = \frac{F}{A}$$

The unit of stress is N/m^2 or Nm^{-2}. There are three types of stress

1. Longitudinal or Normal stress

When object is one dimensional then force acting per unit area is called longitudinal stress.

It is of two types : (a) compressive stress (b) tensile stress



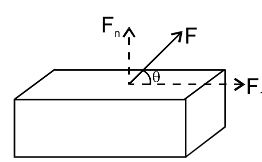
Examples :

(i) Consider a block of solid as shown in figure. Let a force F be applied to the face which has area A.

Resolve $\vec{F}$ into two components :

$F_n = F \sin\theta$ called normal force and $F_t = F \cos\theta$ called tangential force.

$$\therefore \text{ Normal (tensile) stress} = \frac{F_n}{A} = \frac{F\sin\theta}{A}$$



2. Tangential or shear stress

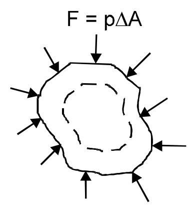
It is defined as the restoring force acting per unit area tangential to the surface of the body. Refer to shown in figure above

Tangential (shear) stress $= \dfrac{F_t}{A} = \dfrac{F\cos\theta}{A}$

The effect of stress is to produce distortion or a change in size, volume and shape

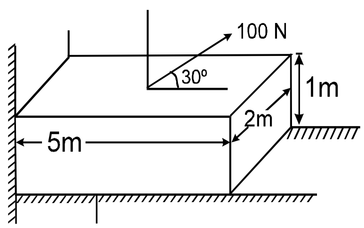
(i.e. configuration of the body).

3. Bulk stress or All around stress or Pressure : When force is acting all along the surface normal to the area, then force acting per unit area is known as pressure. The effect of pressure is to produce volume change. The shape of the body may or may not change depending upon the homogeneity of body.



Solved Examples

Ex.1 Find out longitudinal stress and tangential stress on a fixed block

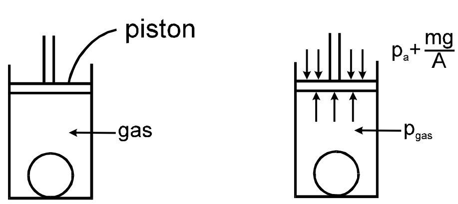


Sol. Longitudinal or normal stress

$\Rightarrow \quad \sigma_l = \dfrac{100\sin 30^\circ}{5 \times 2} = 5\ N/m^2$

Tangential stress $\Rightarrow \quad \sigma_t = \dfrac{100\cos 30^\circ}{5 \times 2} = 5\sqrt{3}\,N/m^2$

Ex.2 Find out Bulk stress on the spherical object of radius $\dfrac{10}{\pi}$ cm if area and mass of piston is 50 cm^2 and 50 kg respectively for a cylinder filled with gas.($g = 10\ m/s^2$)



Sol. $p_{gas} = \dfrac{mg}{A} + p_a = \dfrac{50 \times 10}{50 \times 10^{-4}} + 1 \times 10^5$

$= 2 \times 10^5\ N/m^2$

Bulk stress $= p_{gas} = 2 \times 10^5\ N/m^2$

STRAIN

The ratio of the change in configuration (i.e. shape, length or volume) to the original configuration of the body is called strain

i.e. Strain, $\in = \dfrac{\text{change in configuration}}{\text{original configuration}}$

It has no unit

Types of strain : There are three types of strain

(i) Longitudinal strain : This type of strain is produced when the deforming force causes a change in length of the body. It is defined as the ratio of the change in length to the original length of the body.

Consider a wire of length L **:** When the wire is stretched by a force F, then let the change in length of the wire is ΔL shown in the figure.

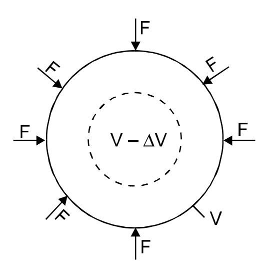
$\therefore$ Longitudinal strain , $\in_\ell = \dfrac{\text{change in length}}{\text{original length}}$

or Longitudinal strain $= \dfrac{\Delta L}{L}$

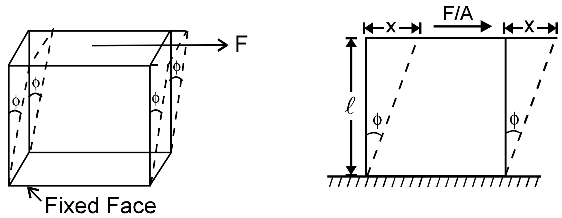
(ii) Volume strain : This type of strain is produced when the deforming force produces a change in volume of the body shown in the figure. It is defined as the ratio of the change in volume to the original volume of the body.

If ΔV = change in volume V = original volume

$\in_v$ = volume strain $= \dfrac{\Delta V}{V}$



(iii) Shear Strain : This type of strain is produced when the deforming force causes a change in the shape of the body. It is defined as the angle (θ) through which a face originally perpendicular to the fixed face is turned as shown in the figure.



$\tan\phi$ or $\phi = \frac{x}{\ell}$

HOOKE'S LAW AND MODULUS OF ELASTICITY

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain

or stress = constant × strain or $\frac{\text{stress}}{\text{strain}}$

= Modulus of Elasticity.

This constant is called modulus of elasticity.

Thus, modulus of elasticity is defined as the ratio of the stress to the strain.

Modulus of elasticity depends on the nature of the material of the body and is independent of its dimensions (i.e. length, volume etc.).

Unit : The Sl unit of modulus of elasticity is Nm^{-2} or Pascal (Pa).

TYPES OF MODULUS OF ELASTICITY

Corresponding to the three types of strain there are three types of modulus of elasticity.

1. Young's modulus of elasticity (Y)
2. Bulk modulus of elasticity (K)
3. Modulus of rigidity (η).

1. Young's modulus of elasticity

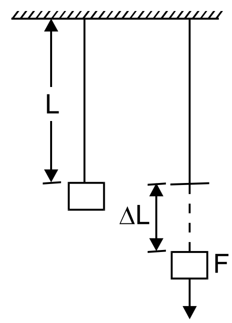
It is defined as the ratio of the normal stress to the longitudinal strain.

i.e. Young's modulus (Y) $= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

Normal stress = F/A,

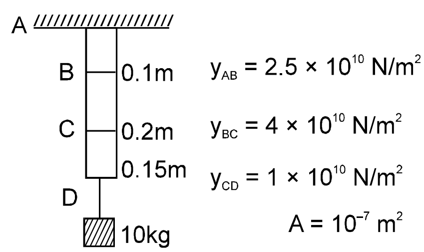
Longitudinal strain = $\Delta L/L$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$



Solved Examples

Ex.3 Find out the shift in point B, C and D



Sol. $\Delta L_B = \Delta L_{AB} = \frac{FL}{AY} = \frac{MgL}{AY} = \frac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}}$

$= 4 \times 10^{-3}$ m = 4mm

$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \frac{100 \times 0.2}{10^{-7} \times 4 \times 10^{10}}$

$= 4 \times 10^{-3} + 5 \times 10^{-3}$ = 9mm

$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \frac{100 \times 0.15}{10^{-7} \times 1 \times 10^{10}}$

$= 9 \times 10^{-3} + 15 \times 10^{-3}$ = 24 mm

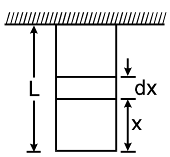
ELONGATION OF ROD UNDER IT'S SELF WEIGHT

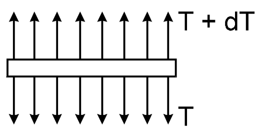
Let rod is having self weight 'W', area of cross-section 'A" and length 'L'. Considering on element at a distance 'x' from bottom.

then $T = \frac{W}{L}x$

elongation in 'dx' element $= \frac{T.dx}{Ay}$

Total elongation $s = \int_0^L \frac{T\,dx}{Ay} = \int_0^L \frac{W\,x\,dy}{LAy} = \frac{WL}{2Ay}$



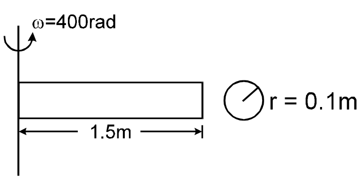


Note : One can do directly by considering total weight at C.M. and using effective length $\ell/2$.

Solved Examples

Ex.4 Given y = 2 × 10^{11} N/m^2 ⇒ $\rho = 10^4$ kg/m^3

Find out elongation in rod.

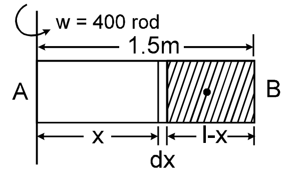


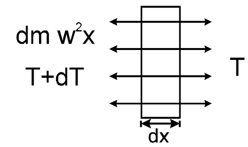
Sol. mass of shaded portion

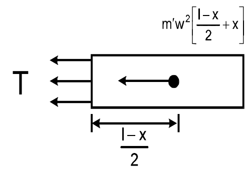
$m' = \frac{m}{\ell}(\ell - x)$ [where m = total mass = $\rho A l$]

$T = m' w^2\left[\frac{\ell - x}{2} + x\right] \Rightarrow T = \frac{m}{\ell}(l - x)\, w^2\left(\frac{\ell + x}{2}\right)$

$T = \frac{mw^2}{2\ell}(\ell^2 - x^2)$







this tension will be maximum at A $\left(\frac{mw^2\ell}{2}\right)$ and minimum at 'B' (zero), elongation in element of width 'dx' $= \frac{Tdx}{Ay}$

Total elengation

$$\delta = \int \frac{Tdx}{Ay} = \int_0^{\ell} \frac{mw^2(\ell^2 - x^2)}{2\ell\, Ay}\, dx$$

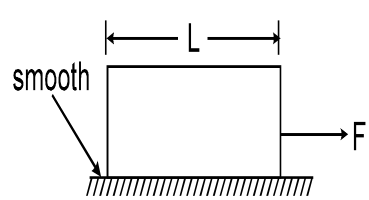
$$\delta = \frac{mw^2}{2\ell Ay}\left[\ell^2 x - \frac{x^3}{3}\right]_0^{\ell}$$

$$= \frac{mw^2 \times 2\ell^3}{2\ell Ay \times 3} = \frac{mw^2\ell^2}{3Ay} = \frac{\rho A\ell w^2\ell^2}{3Ay}$$

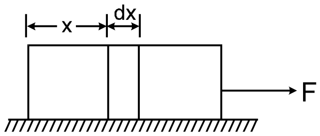
$$\delta = \frac{\rho w^2\ell^3}{3y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}}$$

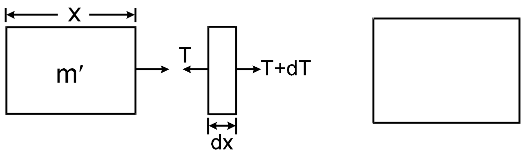
$= 9 \times 10^{-3}$ m = 9mm

Ex.5 Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and y respectively.



Sol.





Acceleration, $a = \dfrac{F}{m}$

then $T = m'a$ where $\Rightarrow$ $m' = \dfrac{m}{\ell}x$

$$T = \frac{m}{\ell}x\,\frac{F}{m} = \frac{Fx}{\ell}$$

Elongation in element 'dx' = $\dfrac{Tdx}{Ay}$

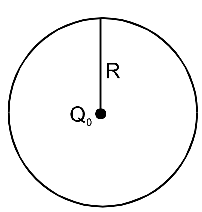
total elongation, $\delta = \displaystyle\int_0^{\ell}\frac{Tdx}{Ay}$ $d = \displaystyle\int_0^{\ell}\frac{Fxdx}{A\ell y} = \frac{F\ell}{2Ay}$

Note : - Try this problem, if friction is given between block and surface (μ = friction coefficient), and

Case : (I) $F < \mu mg$ (II) $F > \mu mg$

Ans. In both cases answer will be $\dfrac{F\ell}{2Ay}$

Ex.6 In a ring having linear charge density 'λ', made up of wire of cross-section area 'a', young modulus y, a charge Q_0 is placed at it's centre. If initial radius is 'R', then find out change in radius



Sol. Considering an element of angular width '2θ' -

$dq = \lambda R'\,.\,2\theta$ $\Rightarrow$ $F = 2T\sin\theta$

$$\frac{kdq.Q_0}{R'^2} = 2T\sin\theta \quad\Rightarrow\quad \frac{k\lambda R' \times 2\theta.Q_0}{R'^2} = 2T\sin\theta$$

if θ is small, then $\sin\theta = \theta$

further, $\dfrac{k\lambda Q_0}{R'} = T$

But $y = \dfrac{\text{stress}}{\text{strain}}$, strain $= \dfrac{2\pi(R'-R)}{2\pi R} = \dfrac{R'-R}{R}$

$$y = \frac{T}{a(R'-R)} \quad\Rightarrow\quad R' - R = \frac{k\lambda Q_0}{R'ay}$$

$$R'^2 - RR' - \alpha = 0 \quad\Rightarrow\quad \left[\alpha = \frac{k\lambda Q_0}{ay}\right]$$

$$R' = \frac{R \pm \sqrt{R^2 + 4\alpha}}{2}$$

[(–) is only when charge is of opposite nature]

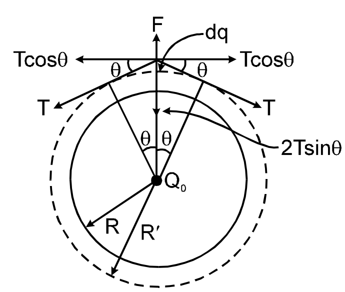
then $R' = \dfrac{R}{2}\left[1 + \left[1 + \dfrac{4\alpha}{R^2}\right]^{1/2}\right]$

$\Rightarrow$ $R' = \left[1 + 1 + \dfrac{2\alpha}{R^2}\right]\dfrac{R}{2}$ [neglecting higher term.]

$$R' = R + \frac{\alpha}{R}$$

change in radius $\Delta R = R' - R = \dfrac{\alpha}{R} = \dfrac{k\lambda Q_0}{ayR}$

$$= \frac{\lambda Q_0}{4\pi \epsilon_0\, ayR}$$



2. Bulk modulus :

It is defined as the ratio of the normal stress to the volume strain

i.e. $K = \dfrac{\text{Pressure}}{\text{Volume strain}}$

The stress being the normal force applied per unit area and is equal to the pressure applied (p).

$$K = \frac{p}{\frac{-\Delta V}{V}} = -\frac{pV}{\Delta V}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

Compressibility : The reciprocal of bulk modulus of elasticity is called compressibility. Unit of compressibility in Sl is $N^{-1}\,m^2$ or pascal^{-1} (Pa^{-1}).

Bulk modulus of solids is about fifty times that of liquids, and for gases it is 10^{-8} times of solids.

$K_{solids} > K_{liquids} > K_{gases}$

Isothermal modulus of elasticity of gas K = P (pressure of gas)

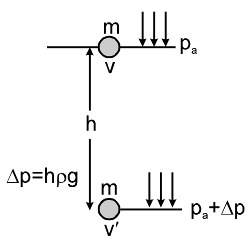
Adiabatic modulus of elasticity of gas $K = \gamma \times P$

where $\gamma = \frac{C_p}{C_v}$.

Solved Examples

Ex.7 Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibily $k = 50 \times 10^{-6}$ /atm.[assuming ρ = const.]

Sol. $B = \frac{\Delta p}{-\frac{\Delta V}{V}}$ $\quad \frac{\Delta V}{V} = -\frac{\Delta p}{B}$



We know $p = p_{atm} + h\rho g$

or $\quad m = \rho V = $ const.

$d\rho.\, v + dv.\, \rho = 0$

$$d\rho\, V + dV\,.\,\rho = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$$

i.e. $\quad \frac{\Delta\rho}{\rho} = \frac{\Delta p}{B} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{1}{100}$

$\frac{1}{100} = \frac{h\rho g}{B}$ $\quad$ [assuming ρ = const.]

$$h\,\rho\,g = \frac{B}{100} = \frac{1}{100\,k} \Rightarrow$$

$$h\,\rho\,g = \frac{1\times1\times10^5}{100\times50\times10^{-6}}$$

$$h = \frac{10^5}{5000\times10^{-6}\times1000\times10}$$

$$= \frac{100\times10^3}{50} = 2\text{km}$$ **Ans.**

3. Modulus of Rigidity :

It is defined as the ratio of the tangential stress to the shear strain. Let us consider a cube whose lower face is fixed and a tangential force F acts on the upper face whose area is A.

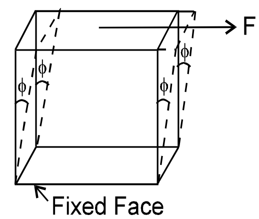
$\therefore$ Tangential stress = F/A.

Let the vertical sides of the cube shifts through an angle θ, called shear strain

$\therefore$ Modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}}$$

or $\eta = \frac{F/A}{\phi} = \frac{F}{A\phi}$



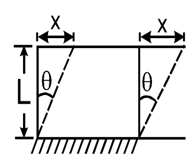
Solved Examples

Ex.8 A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m^2.

Sol. $L = 5 \times 10^{-2}$ m $\quad \Rightarrow \quad \frac{F}{A} = \eta\frac{x}{L}$

strain $\theta = \frac{F}{A\eta} = \frac{1800}{25\times10^{-4}\times2.4\times10^6}$

$= \frac{180}{25\times24} = \frac{3}{10} = 0.3$ radian

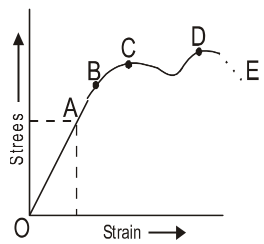


$\frac{x}{L} = 0.3 \Rightarrow x = 0.3 \times 5 \times 10^{-2}$

$= 1.5 \times 10^{-2}$ m = 1.5 mm **Ans.**

VARIATION OF STRAIN WITH STRESS

When a wire is stretched by a load, it is seen that for small value of load, the extension produced in the wire is proportional to the load. On removing the load, the wire returns to its original length. The wire regains its original dimensions only when load applied is less or equal to a certain limit. This limit is called elastic limit. Thus, elastic limit is the maximum stress on whose removal, the bodies regain their original dimensions. In shown figure, this type of behavior is represented by OB portion of the graph. Till A the stress is proportional to strain and from A to B if deforming forces are removed then the wire comes to its original length but here stress is not proportional to strain.



OA → Limit of Proportionality
OB → Elastic limit
C → Yield Point
CD → Plastic behaviour
D → Ultimate point
DE → Fracture

As we go beyond the point B, then even for a very small increase in stress, the strain produced is very large. This type of behaviour is observed around point C and at this stage the wire begins to flow like a viscous fluid. The point C is called yield point. If the stress is further increased, then the wire breaks off at a point D called the breaking point. The stress corresponding to this point is called breaking stress or tensile strength of the material of the wire. A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as elastic limit is crossed.

Important points

- Breaking stress = Breaking force/area of cross section.
- Breaking stress is constant for a material
- Breaking force depends upon the area of the section of the wire of a given material.
- The working stress is always kept lower than that of a breaking stress so that safety factor = breaking stress/working stress may have a large value.
- Breaking strain = elongation or compression / original dimension.
- Breaking strain is constant for material.

Elastic after effect

We know that some material bodies take some time to regain their original configuration when the deforming force is removed. The delay in regaining the original configuration by the bodies on the removal of deforming force is called elastic after effect. The elastic after effect is negligibly small for quartz fibre and phosphor bronze. For this reason, the suspensions made from quartz and phosphor-bronze are used in galvanometers and electrometers.

For glass fibre elastic after effect is very large. It takes hours for glass fibre to return to its original state on removal of deforming force.

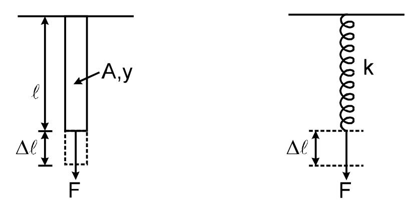
Elastic Fatigue

The, the loss of strength of the material due to repeated strains on the material is called elastic fatigue. That is why bridges are declared unsafe after a longtime of their use.

Analogy of Rod as a spring

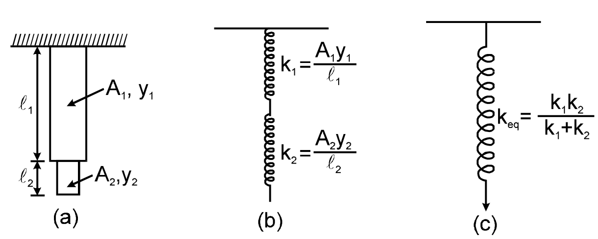
$$y = \frac{\text{stress}}{\text{strain}} \Rightarrow y = \frac{F\ell}{A\Delta\ell}$$

or $\quad F = \frac{Ay}{\ell}\Delta\ell$



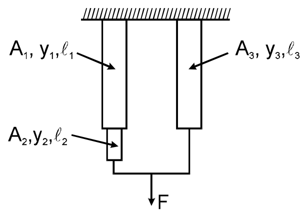
$\frac{Ay}{\ell}$ = constant, depends on type of material and geometry of rod. $F = k\Delta\ell$

where $k = \frac{Ay}{\ell}$ = equivalent spring constant.

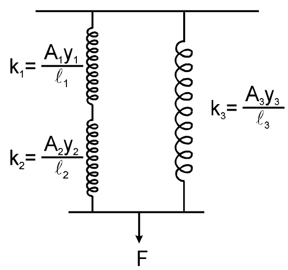


for the system of rods shown in figure (a), the replaced spring system is shown in figure (b) two spring in series]. Figure (c) represents equivalent spring system.

Figure (d) represents another combination of rods and their replaced spring system.

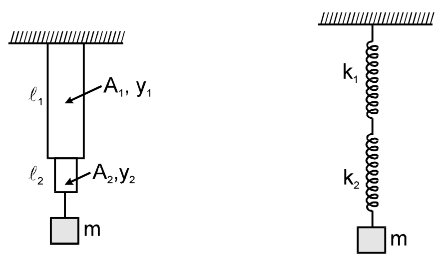


(d)



Solved Examples

Ex.9 A mass 'm' is attached with rods as shown in figure. This mass is slightly stretched and released whether the motion of mass is S.H.M., if yes then find out the time period.



Sol. $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$ $\qquad T = 2\pi\sqrt{\dfrac{m}{k_{eq}}} = 2\pi\sqrt{\dfrac{m(k_1+k_2)}{k_1 k_2}}$

where $k_1 = \dfrac{A_1 y_1}{\ell_1}$ and $k_2 = \dfrac{A_2 y_2}{\ell_2}$

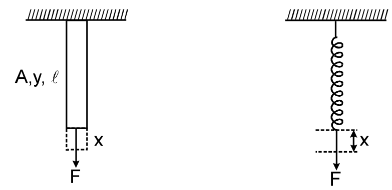
ELASTIC POTENTIAL ENERGY STORED IN A STRETCHED WIRE OR IN A ROD

Strain energy stored in equivalent spring

$$U = \frac{1}{2} k x^2$$

where $x = \dfrac{F\ell}{Ay}$, $\qquad k = \dfrac{Ay}{\ell}$

$$U = \frac{1}{2}\,\frac{Ay}{\ell}\,\frac{F^2\ell^2}{A^2y^2} = \frac{1}{2}\,\frac{F^2\ell}{Ay}.$$



equation can be re-arranged

$$U = \frac{1}{2}\,\frac{F^2}{A^2} \times \frac{\ell A}{y}$$

[ℓA = volume of rod, F/A = stress]

$$U = \frac{1}{2}\,\frac{(\text{stress})^2}{y} \times \text{volume}$$

again, $U = \dfrac{1}{2}\,\dfrac{F}{A} \times \dfrac{F}{Ay} \times A\,\ell$

$$\left[\text{Strain} = \frac{F}{Ay}\right]$$

$$U = \frac{1}{2}\ \text{stress} \times \text{strain} \times \text{volume}$$

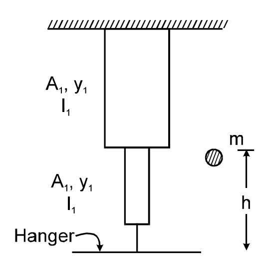
again, $U = \dfrac{1}{2}\,\dfrac{F^2}{A^2y^2}\,A\ell y$

$$U = \frac{1}{2}\, y\,(\text{strain})^2 \times \text{volume}$$

strain energy density $= \dfrac{\text{strain energy}}{\text{volume}}$

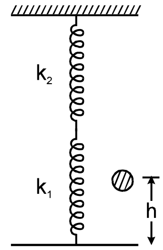
$$= \frac{1}{2}\,\frac{(\text{stress})^2}{y} = \frac{1}{2}\,y(\text{strain})^2 = \frac{1}{2}\ \text{stress} \times \text{strain}$$

Ex.10 Hanger is mass-less A ball of mass 'm' drops from a height 'h', which sticks to hanger after striking. Neglect over turning, find out the maximum extension in rod. Asumming rod is massless.



Sol. Applying energy conservation

$$mg\,(h + x) = \frac{1}{2}\,\frac{k_1 k_2}{k_1 + k_2}\,x^2$$



where $k_1 = \frac{A_1 y_1}{\ell_1}$ $\quad k_2 = \frac{A_2 y_2}{\ell_2}$

& $K_{eq} = \frac{A_1 A_2 y_1 y_2}{A_1 y_1 \ell_2 + A_2 y_1 \ell_1}$

$$k_{eq} x^2 - 2mgx - 2mgh = 0$$

$$x = \frac{2mg \pm \sqrt{4m^2 g^2 + 8mghk_{eq}}}{2k_{eq}}$$

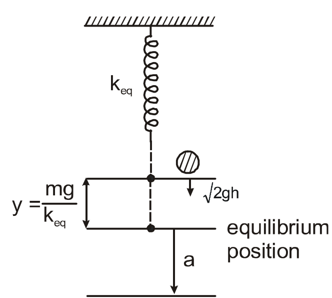
$$x_{max} = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2 g^2}{k_{eq}^2} + \frac{2mgh}{k_{eq}}}$$

OTHERWAY BY S.H.M.

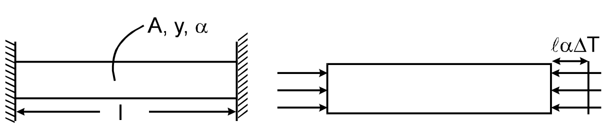
$$w = \sqrt{\frac{k_{eq}}{m}} \qquad v = \omega\sqrt{a^2 - y^2}$$

$$\sqrt{2gh} = \sqrt{\frac{keq}{m}}\sqrt{a^2 - y^2} \;\Rightarrow\; \sqrt{\frac{2mgh}{k_{eq}} + \frac{m^2 g^2}{k_{eq}^2}} = a$$

$$\text{max}^m \text{ extension } = a + y = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2 g^2}{k_{eq}} + \frac{2mgh}{k_{eq}}}$$



THERMAL STRESS :



If temp of rod is increased by ΔT, then change in length

$$\Delta\ell = \ell\,\alpha\,\Delta T \qquad \text{strain} = \frac{\Delta\ell}{\ell} = \alpha\,\Delta T$$

But due to rigid support, there is no strain. Supports provide force on stresses to keep the length of rod same

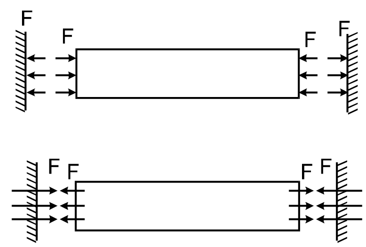
$$y = \frac{\text{stress}}{\text{strain}}$$

If ΔT = (+) tive

thermal stress = y strain = $y\,\alpha\,\Delta T$

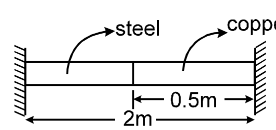
If ΔT = (–) tive

$$\frac{F}{A} = y\,\alpha\,\Delta T \quad F = Ay\,\alpha\,\Delta T$$



Solved Examples

Ex.11 When composite rod is free, then composite length increases to 2.002 m for temp 20ºC to 120ºC. When composite rod is fixed between the support, there is no change in component length find y and α of steel, if $y_{cu} = 1.5 \times 10^{13}$ N/m^2 $\alpha_{cu} = 1.6 \times 10^{-5}$/ºC.



Sol. $\Delta\ell = \ell_s\,\alpha_s\,\Delta T + \ell_c\,\alpha_c \Delta T$

$.002 = [1.5\,\alpha_s + 0.5 \times 1.6 \times 10^{-5}] \times 100$

$$\alpha_s = \frac{1.2 \times 10^{-5}}{1.5} = 8 \times 10^{-6}/^{\circ}C$$

there is no change in component length

For steel $x = \ell_s\,\alpha_s\,\Delta T - \frac{F\ell_s}{AY_s} = 0$

$$\frac{F}{AY_s} = \alpha_s\,\Delta T \qquad \text{....(A)}$$

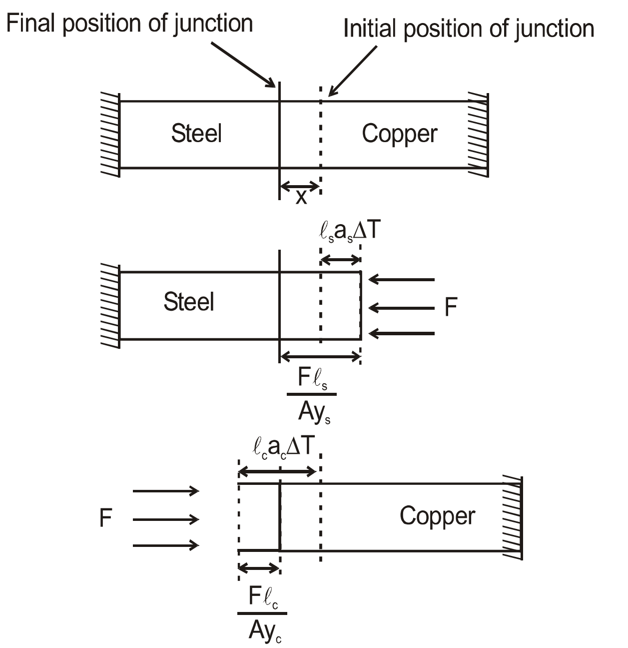
for copper

$$x = \frac{F\ell_c}{Ay_c} - \ell_c\,\alpha_c\,\Delta T = 0$$

$$\frac{F}{Ay_c} = \alpha \Delta T \qquad ...(B)$$

$$B/A \quad \Rightarrow \quad \frac{y_s}{y_c} = \frac{\alpha_c}{\alpha_s}$$

$$y_s = y_c \frac{\alpha_c}{\alpha_s} = \frac{1.5\times10^{13}\times16\times10^{-5}}{8\times10^{-6}} = 3\times10^{13}\ N/m^2$$



APPLICATIONS OF ELASTICITY

Some of the important applications of the elasticity of the materials are discussed as follows :

1. The material used in bridges lose its elastic strength with time bridges are declared unsafe after long use.

2. **To estimate the maximum height of a mountain :**
The pressure at the base of the mountain = $h\rho g$ = stress. The elastic limit of a typical rock is 3×10^8 N m^{-2}
The stress must be less than the elastic limits, otherwise the rock begins to flow.

$$h < \frac{3\times10^8}{\rho g} < \frac{3\times10^8}{3\times10^3\times10} < 10^4\ m\ (\because \rho = 3\times10^3$$

kg m^{-3} ; g = 10 ms^{-2}) or h = 10 km
It may be noted that the height of Mount Everest is nearly 9 km.

TORSION CONSTANT OF A WIRE

$C = \frac{\pi\eta r^4}{2\ell}$ Where η is modulus of rigidity r and ℓ is radius and length of wire respectively.

(a) Toque required for twisting by angle θ, $\tau = C\theta$.

(b) Work done in twisting by angle θ, $W = \frac{1}{2} C\theta^2$.