MOTION

CONTENTS

INTRODUCTION

When a body does not change its position with time, we can say that the body is at **rest**, while if a body changes its position with time, it is said to be in **motion**.

- An object is said to be a **point object** if it changes its position by distances which are much greater than its size.
- A point or some stationary object with respect to which a body continuously changes its position in the state of motion is known as **origin** or **reference point**.

TYPES OF MOTION

- **According to Directions**
  - **One dimensional motion** is the motion of a particle moving along a straight line.
  - **Two dimensional motion** A particle moving along a curved path in a plane has 2-dimensional motion.
  - **Three dimensional motion** Particle moving randomly in space has 3-dimensional motion.
- **According to state of motion**

Uniform Motion

- A body is said to be in a state of uniform motion if it travels equal distances in equal intervals of time.
- If the time distance graph is a straight line the motion is said to be uniform motion.

Non-uniform motion

- A body has a non-uniform motion if it travels unequal distances in equal intervals of time. **Ex.** a freely falling body.
- Time - distance graph for a body with non-uniform motion is a curved line.

TERMS USED TO DEFINE MOTION

(i) Distance and displacement
(ii) Speed and velocity
(iii) Acceleration

(i) Distance & Displacement

- The path length between the initial and final positions of the particle gives the **distance** covered by the particle.
- The minimum distance between the initial and final positions of a body during that time interval is called **displacement**
- Distance and displacement both are measured in *meter* in m.k.s. system.

Difference between distance and displacement

- Distance travelled is a scalar quantity while displacement is a vector quantity.
- When a body continuously moves in the same straight line and in the same direction then displacement will be equal to the distance travelled. But if the body changes its direction while moving, then the displacement is smaller than the distance travelled.

Displacement ≤ Distance

- Displacement in any interval of time may be zero, positive or negative where as distance cannot be negative..

Ex.1 A person travels a distance of 5 m towards east, then 4 m towards north and then 2 m towards west.

(i) Calculate the total distance travelled.

(ii) Calculate the resultant displacement.

Sol. (i) Total distance travelled by the person

= 5 m + 4 m + 2 m = 11 m

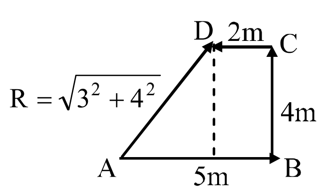
(ii) To calculate the resultant displacement, we choose a convenient scale, where 1 cm represents 1 m. We draw a 5 cm long line AB towards east and then 4 cm long line BC towards north. Finally, a 2 cm long line CD towards west. The resultant displacement is calculated by joining the initial position A to the final position D. We measure AB = 5 cm.

Since 1 cm = 1 m

∴ 5 cm = 5 m

Hence, the displacement of the person

= 5m towards AD.

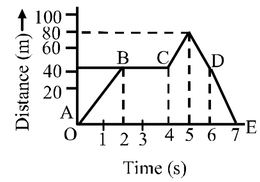


Ex.2 A body is moving in a straight line. Its distances from origin are shown with time in Fig. A, B, C, D and E represent different parts of its motion. Find the following :

(i) Displacement of the body in first 2 seconds.

(ii) Total distance travelled in 7 seconds.

(iii) Displacement in 7 seconds



Displacement-time graph

Sol. (i) Displacement of the body in first 2s = 40m

(ii) From t = 0 to t = 7 s, the body has moved a distance of 80 m from origin and it has again come back to origin. Therefore, the total distance covered = 80 × 2 = 160 m

(iii) Since the body has come back to its initial position, the displacement is zero.

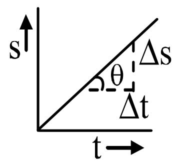
(ii) Speed and Velocity

- The 'distance' travelled by a body in unit time interval is called its **speed**. When the position of a body changes in particular direction, then speed is denoted by 'velocity'. i.e. the rate of change of displacement of a body is called its **Velocity.**
- Speed is a scalar quantity while velocity is a vector quantity.
- $\text{Speed} = \dfrac{\text{distance}}{\text{time}}$
- $\text{Velocity} = \dfrac{\text{displacement}}{\text{time}}$

- Unit : In M.K.S. system = ms^{-1}

  In C.G.S. system = cm/s
- If time distance graph is given then speed can be given by the slope of the line, at given time

  $$v = \frac{\Delta s}{\Delta t} = \text{slope}$$



- The area of velocity time graph gives displacement travelled.

Types of speed

(a) Average and Instantaneous speed

Average speed :

It is obtained by dividing the total distance travelled by the total time interval. i.e.

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{total time}}$$

- Average speed is a scalar, while average velocity is a vector.
- For a moving body average speed can never be –ve or zero (unless $t \to \infty$), while average velocity can be i.e. $v_{av} > 0$ while $\vec{v}_{av} >= \text{or} < 0$
- In general average speed is not equal to magnitude of average velocity. However it can be so if the motion is along a straight line without change in direction
- If a particle travels distances L_1, L_2, L_3 at speeds v_1, v_2, v_3 etc respectively, then

  $$v_{av} = \frac{\Delta s}{\Delta t} = \frac{L_1 + L_2 + \ldots .. + L_n}{\frac{L_1}{v_1} + \frac{L_2}{v_2} + \ldots . + \frac{L_n}{v_n}} = \frac{\sum Li}{\sum \frac{L_i}{v_i}}$$
- If a particle travels at speeds v_1, v_2 etc for intervals t_1, t_2 etc respectively, then

  $$v_{av} = \frac{v_1 t_1 + v_2 t_2 + \ldots .}{t_1 + t_2 + \ldots .} = \frac{\sum v_1 t_1}{\sum t_1}$$

Instantaneous speed :

The speed of a body at a particular instant of time is called its instantaneous speed.

$$= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

(b) Uniform and Non uniform speed

Uniform speed :

If an object covers equal distance in equal interval of time, then time speed graph of an object is a straight line parallel to time axis then body is moving with a uniform speed.

Non-uniform speed :

If the speed of a body is changing with respect to time it is moving with a non-uniform speed.

Ex.3 The distance between two points A and B is 100 m. A person moves from A to B with a speed of 20 m/s and from B to A with a speed of 25 m/s. Calculate average speed and average velocity.

Sol. (i) Distance from A to B = 100 m

Distance from B to A = 100 m

Thus, total distance = 200 m

Time taken to move from A to B, is given by

$$t_1 = \frac{\text{distance}}{\text{velocity}} = \frac{100}{20} = 5 \text{ seconds}$$

Time taken from B to A, is given by

$$t_2 = \frac{\text{distance}}{\text{velocity}} = \frac{100}{25} = 4 \text{ seconds}$$

Total time taken = $t_1 + t_2 = 5 + 4 = 9$ sec.

$\therefore$ Average speed of the person

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{200}{9} \text{ m/s} = 22.2 \text{ m/s}$$

(ii) Since person comes back to initial position A, displacement will be zero, resulting zero average velocity.

Ex.4 A car moves with a speed of 40 km/hr for first hour, then with a speed of 60 km/hr for next half hour and finally with a speed of 30 km/hr for next $1\frac{1}{2}$ hours. Calculate the average speed of the car.

Sol. Distance travelled in first hour, is given by

s_1 = speed × time = 40 km/hr × 1 hr = 40 km

Distance travelled in next half an hour, is given by

s_2 = speed × time = 60 km/hr × $\frac{1}{2}$ hr = 30 km

Distance travelled in last $1\frac{1}{2}$ hours, is given by

s_3 = speed × time = 30 km/hr × $\frac{3}{2}$ hr = 45 km

Thus, total distance travelled = $s_1 + s_2 + s_3$

= 40 + 30 + 45 = 115 km

Total time taken = $1 + \frac{1}{2} + 1\frac{1}{2}$ = 3 hours

∴ Average speed =

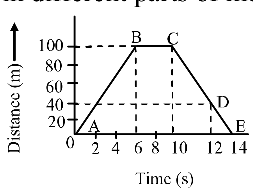
$\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{115\text{km}}{3\text{hrs}}$ =38.33 km/hr

Ex.5 Figure shows time distance graph of an object. Calculate the following :

(i) Which part of the graph shows that the body is at rest ?

(ii) Average speed in first 10 s.

(iii) Speeds in different parts of motion.



Distance-time graph

Sol. (i) The part BC shows that the body is at rest.

(ii) In first 10 seconds, distance travelled = 100m

∴ Thus, average speed = $\frac{\text{Distance covered}}{\text{Time taken}}$

$= \frac{100}{10} = 10$ m/s

(iii) Speed of the object in part AB is given by

slope = $\frac{100}{6}$ = 50/3 m/s

Speed of object in part BC = 0 m/s

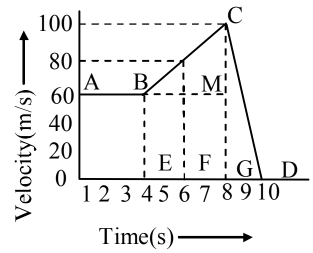
Speed of the object in part CD

$= \frac{100-40}{12-10} = \frac{60}{2} = 30$ m/s

Speed of object in part DE

$= \frac{40-0}{14-12} = \frac{40}{2} = 20$m/s

Ex.6 Time-velocity graph of a particle is shown in Figure. Calculate the distance travelled in first seconds.



Sol. Distance travelled in first 8s is given by area OABCG

= area of rectangle OAMG

+ area of triangle BMC

$= 8 \times 60 + \frac{1}{2} \times 4 \times 40 = 480 + 80 = 560$ m.

(iii) Acceleration

- Rate of change of velocity is called acceleration. It is a vector quantity

  i.e. $a = \frac{v-u}{t}$

- Unit of acceleration = m/s^2 or ms^{-2}

Types of acceleration

- **Uniform & Non uniform acceleration**

Uniform acceleration

If a body travels in a straight line and its velocity increases by equal amounts in equal intervals of time then it is said to be in state of uniform acceleration.

e.g. motion of a freely falling body.

Non uniform acceleration

A body has a non-uniform acceleration if its velocity increases by unequal amounts in equal intervals of time.

Instantaneous acceleration :

The acceleration of a body at any instant is called its instantaneous acceleration.

- If the velocity of a body decreases, then it will experience a negative acceleration which is called deceleration or retardation.

❖ Acceleration is determined by the slope of time-velocity graph.

$\tan\theta = \frac{dv}{dt}$

(i) If the time velocity graph is a straight line, acceleration remains constants.

(ii) If the slope of the straight line is positive, positive acceleration occurs.

(iii) If the slope of the straight line is negative, negative acceleration or retardation occurs.

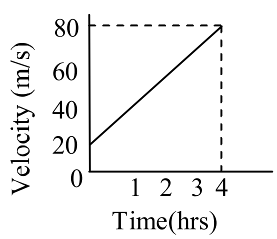
Ex.7 Time-velocity graph of a body is shown in the figure. Find its acceleration in m/s^2.

Sol. As it is clear from the figure,

At t = 0 s, v = 20 m/s

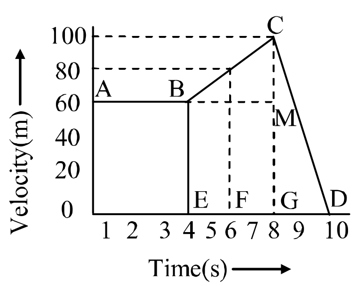
At t = 4 s, v = 80 m/s

∴ Acceleration, $a = \frac{\text{Change in velocity}}{\text{Time interval}}$



$= \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{(80-20)\,m/s}{(4-0)}$ $15 m/s^2$

Ex.8 Time-velocity graph of a particle is shown in figure. Find its instantaneous acceleration at following intervals :



(i) at t = 3s

(ii) at t = 6s

(iii) at t = 9s

Sol. (i) Instantaneous acceleration at t = 3s, is given by

a = slope of line AB = zero

(ii) Instantaneous acceleration at t = 6 s, is given by a = slope of line

$$BC = \frac{CM}{BM} = \frac{100-60}{8-4} = -10\ m/s^2$$

(iii) Instantaneous acceleration at t = 9 s, is given by a = slope of line $CD = \frac{0-100}{10-8} = -50\ m/s^2$

Ex.9 Starting from rest, Deepak paddles his bicycle to attain a velocity of 6 m/s in 30 seconds then he applies brakes so that the velocity of the bicycle comes down to 4 m/s in the next 5 seconds. Calculate the acceleration of the bicycle in both the cases.

Sol. (i) Initial velocity, u = 0, final velocity,

v = 6 m/s, time, t = 30 s

Using the equation v = u + at, we have

$$a = \frac{v-u}{t}$$

substituting the given values of u, v and t in the above equation, we get

$$a = \frac{6-0}{30} = 0.2\ m/s^2\ ;$$

which is positive acceleration.

(ii) Initial velocity, u = 6 m/s, final velocity, v = 4 m/s, time, t = 5 s, then

$$a = \frac{v-u}{t} = \frac{4-6}{5} = -0.4\ m/s^2\ ;$$

which is retardation.

Note : The acceleration of the case (i) is positive and is negative in the case (ii).

EQUATIONS OF MOTION

Motion under uniform acceleration

(a) 1^{st} Equation of motion

Consider a body having initial velocity 'u'. Suppose it is subjected to a uniform acceleration 'a' so that after time 't' its final velocity becomes 'v'. Now we know,

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{Time}}$$

$$a = \frac{v-u}{t}$$

or $\quad \mathbf{v = u + at} \quad$ **…..(i)**

(b) 2^{nd} Equation of motion

Suppose a body has an initial velocity 'u' and uniform acceleration 'a' for time 't' so that its final velocity becomes 'v'. The distance travelled by moving body in time 't' is 's' then the average velocity = (v + u)/2.

Distance travelled = Average velocity × time

$$s = \left(\frac{u+v}{2}\right)t \Rightarrow s = \left(\frac{u+u+at}{2}\right)t \text{ (as } v = u + at)$$

$$s = \left(\frac{2u+at}{2}\right)t \Rightarrow s = \frac{2ut+at^2}{2}$$

$$\mathbf{s = ut + \frac{1}{2}at^2} \qquad \text{……(ii)}$$

(c) 3^{rd} Equation of motion

Distance travelled = Average velocity x time

$$s = \left(\frac{u+v}{2}\right)t \qquad \text{……(iii)}$$

from equation (i) $\quad t = \dfrac{v-u}{a}$

Substituting the value of t in equation (iii),

we get $s = \left(\dfrac{v-u}{a}\right)\left(\dfrac{v+u}{2}\right)$

$$s = \left(\frac{v^2-u^2}{2a}\right)$$

$\Rightarrow 2as = v^2 - u^2$ or

$\mathbf{v^2 = u^2 + 2as}$**….(iv)**

- The equations of motion under gravity can be obtained by replacing acceleration by acceleration due to gravity (g) and can be written as follows :
- When the body is coming towards the centre of earth

(a) $v = u + gt$ **(b)** $h = ut + \frac{1}{2}gt^2$

(c) $v^2 = u^2 + 2gh$

- When a body is thrown upwards with some initial velocity, then a retardation produced due to attraction of the earth. In equations of motion, a is replaced by (–g) and thus equations become.

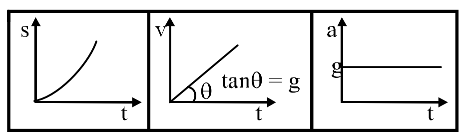
(a) $v = u - gt$ **(b)** $h = ut - \frac{1}{2}gt^2$

(c) $v^2 = u^2 - 2gh$

BODY FALLING FREELY UNDER GRAVITY

Assuming u = 0 for a freely falling body :

t is given	h is given	v is given
$v = gt$ $h = \frac{1}{2}gt^2$	$t = \sqrt{\frac{2h}{g}}$ $v = \sqrt{2gh}$	$t = \frac{v}{g}$ $h = \frac{v^2}{2g}$



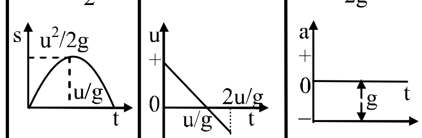
- Body is projected vertically up :

Taking initial position as origin and direction of motion (i.e. vertically up) as positive.

(a) At the highest point v = 0

(b) a = – g

t is given	h is given	u is given
$u = gt$ $h = \frac{1}{2}gt^2$	$t = \sqrt{2h/g}$ $u = \sqrt{2gh}$	$t = \frac{u}{g}$ $h = \frac{u^2}{2g}$



- It is clear that in case of motion under gravity
  (a) Time taken to go up is equal to the time taken to fall down through the same distance.
  (b) The speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
  (c) The body returns to the starting point with the same speed with which it was thrown.

Ex.10 A body starts moving with an initial velocity 50 m/s and acceleration 20 m/s^2. How much distance it will cover in 4s ? Also, calculate its average speed during this time interval.

Sol. Given : u = 50 m/s, a = 20 m/s^2,

t = 4s, s = ?

$$s = ut + \frac{1}{2}at^2 = 50 \times 4 + \frac{1}{2} \times 20 \times (4)^2$$

= 200 + 160 = 360 m

Average speed during this interval,

$$\overline{V} = \frac{\text{distance travelled}}{\text{time interval}} = \frac{360}{4} = 90 \text{ m/s}$$

Ex.11 A body is moving with a speed of 20 m/s. When certain force is applied, an acceleration of 4 m/s^2 is produced. After how much time its velocity will be 80 m/s ?

Sol. Given : u = 20 m/s, a = 4 m/s^2,

v = 80 m/s, t = ?

Using equation, v = u + at, we get

$80 = 20 + 4 \times t$

or $4t = 80 - 20 = 60$

or $t = 15$ s

Therefore, after 15 seconds, the velocity of the body will be 80 m/s.

Ex.12 A body starts from rest and moves with a constant acceleration. It travels a distance s_1 in first 10 s, and a distance s_2 in next 10 s. Find the relation between s_2 and s_1.

Sol. Given : u = 0, t_1 = 10 s

$\therefore$ Distance travelled in first 10 seconds, is given by

$$s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times a \times (10)^2$$

$$= 50a \qquad ...(1)$$

To calculate the distance travelled in next 10s, we first calculate distance travelled in 20 s and then subtract distance travelled in first 10 s.

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times a \times (20)^2$$

$$= 200a \qquad ...(2)$$

$\therefore$ Distance travelled in 10th second interval,

$$s_2 = s - s_1 = 200a - 50a \qquad ...(3)$$

or $s_2 = 150a$

Now, $\frac{s_2}{s_1} = \frac{150a}{50a} = \frac{3}{1}$

or $s_2 = 3s_1$

Ex.13 A train is moving with a velocity 400 m/s. With the application of brakes a retardation of 10 m/s^2 is produced. Calculate the following :

(i) After how much time it will stop ?

(ii) How much distance will it travel before it stops?

Sol. (i) Given: u = 400 m/s, a = –10 m/s^2, v = 0, t = ?

Using equation, v = u + at, we get

$0 = 400 + (-10) \times t$

or $t = 40$ s

(ii) For calculating the distance travelled, we use equation,

$v^2 = u^2 + 2as$, we get

$(0)^2 = (400)^2 + 2 \times (-10) \times s$

or $20s = 400 \times 400$

or $s = 8000$ m = 8 km

Ex.14 A body is thrown vertically upwards with an initial velocity of 19.6 m/s. If g = –9.8 m/s^2. Calculate the following :

(i) The maximum height attained by the body.

(ii) After how much time will it come back to the ground ?

Sol.(i) Given: u = 19.6 m/s, g = –9.8 m/s^2, v = 0, h = ?

Using equation $v^2 = u^2 + 2gh$, we get

$$(0)^2 = (19.6)^2 + 2(-9.8) \times h$$

or $$h = \frac{19.6 \times 19.6}{2 \times 9.8} = 19.6 \text{ m}$$

(ii) Time taken to reach the maximum height can be calculated by the equation,

$$v = u + gt$$

or $$0 = 19.6 + (-9.8) \times t$$

or $$t = 2s$$

In the same time, it will come back to its original position.

∴ Total time = 2 × 2 = 4s

Ex.15 From the top of a tower of height 490 m, a shell is fired horizontally with a velocity 100 m/s. At what distance from the bottom of the tower, the shell will hit the ground ?

Sol. We know that the horizontal motion and the vertical motion are independent of each other. Now for vertical motion, we have u = 0, h = 490 m, g = 9.8 m/s^2, t = ?

Using equation, $h = ut + \frac{1}{2} gt^2$, we get

$$490 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

or $$t^2 = \frac{490}{4.9} = 100$$

or $$t = 10 \text{ s}$$

∴ It takes 10 seconds to reach the ground.

Now, horizontal distance

= horizontal velocity × time

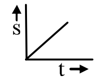
= 100 m/s × 10 s = 1000 m

∴ The shell will strike the ground at a distance of 100 m from the bottom of the tower.

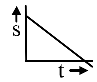
➢ VARIOUS GRAPHS RELATED TO MOTION

◈ Displacement- time graph :

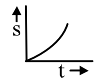
◆ The straight line inclined to time axis in s-t graph represents constant velocity.



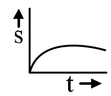
◆ In s-t graph the straight line inclined to time axis at angle greater than 90º shows negative velocity



◆ Body with accelerated motion

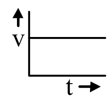


◆ Body with decelerated motion

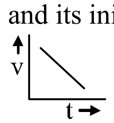


◈ Velocity -time graph :

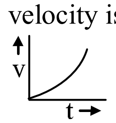
◆ For the body having constant velocity or zero acceleration.



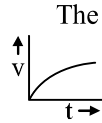
◆ The body is moving with constant retardation and its initial velocity is not zero.



◆ The body is accelerated and the initial velocity is zero.

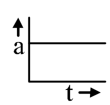


◆ The body is decelerated

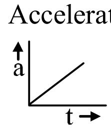


◈ Acceleration-time graph :

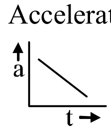
◆ Acceleration is constant



◆ Acceleration is increasing and is +ve



◆ Acceleration is decreasing and is –ve

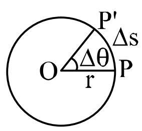


CIRCULAR MOTION

When a body moves in such a way that its distance from a fixed point always remains constant, then its motion is said to be the circular motion.

Uniform circular motion :

- If the radius vector sweeps out equal angles in equal times, then its motion is said to be uniform circular motion.



- In uniform circular motion speed remains const.
- Linear velocity, being a vector quantity, its direction changes continuously.
- The direction of velocity is along the tangent at every point.

Angular velocity :

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- A vector quantity
- Direction is perpendicular to plane of rotation

  Note : If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane downwards. Whereas in case of anticlockwise direction, the direction will be upwards.

- Unit is Radian/sec.
- In uniform circular motion the direction of angular velocity is along the axis of rotation which is constant throughout.
- Angular velocity remains constant in magnitude as well as in direction.
- $v = r\omega$ where r = radius of the circle.

Centripetal acceleration

- In uniform circular motion the particle experiences an acceleration called the centripetal acceleration.
- $a_c = \frac{v^2}{r}$
- The direction of centripetal acceleration is along the radius towards the centre.

Centripetal force :

- Always acts towards centre.
- Centripetal force is required to move a particle in a circle.
- Because F_c is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.

Note :

- Circular motion in horizontal plane is usually uniform circular motion.
- Remember that equations of motion are not applicable for circular motion.

Time period :

- It is the time taken to complete one complete revolution.
- In one revolution, angle subtended is 2π and if T is time period, then the angular velocity is given by

$$\omega = \frac{2\pi}{T} \quad \text{or} \quad T = \frac{2\pi}{\omega}$$

Frequency :

- Frequency is defined as the number of revolutions per second.

i.e. $n = \frac{1}{T} = \frac{\omega}{2\pi}$

Ex.16 A particle moves in a circle of radius 2 m and completes 5 revolutions in 10 seconds. Calculate the following :

(i) Angular velocity and

(ii) Linear velocity.

Sol. Since, it completes 5 revolutions in 10 seconds.

$\therefore$ Time period = $\frac{10}{5} = 2s$

(i) Now angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s

(ii) Linear velocity is given by

$v = r\omega = 2\pi$

$\therefore$ $v = 2\pi$ m/s

Ex.17 The length of second's needle in a watch is 1.2 cm. Calculate the following :

(i) Angular velocity and

(ii) Linear velocity of the tip of the needle.

Sol. (i) We know that the second's needle in a watch completes one revolution in 60 seconds.

$\therefore$ Time period, T = 60 s

Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/s}$$

(ii) Length of the needle = 1.2 cm = Radius of the circle

Linear velocity of the tip of the needle is given by

$$v = r\omega = 1.2 \times \frac{\pi}{30} = \frac{\pi}{25}$$

or $v = \frac{\pi}{2s} = 1.266 \times 10^{-1}$ cm/sec.

Ex.18 Earth revolves around the sun in 365 days. Calculate its angular velocity.

Sol. Time period,

T = 365 days

= 365 × 24 × 60 × 60 seconds

$\therefore$ Angular velocity, $\omega = \frac{2\pi}{T}$

$$= \frac{2\pi}{365 \times 24 \times 60 \times 60} \text{ rad/s} = 1.99 \times 10^{-7} \text{ rad/s.}$$