MAGNETICS EFFECT OF CURRENT AND MAGNETISM

Magnetism Field

Magnets are familiar objects. The word magnetism is derived from the province of Magnesia where the ancient Greek mine magnetic, also known as lodestone, a mineral composed of iron oxide which attracts iron.

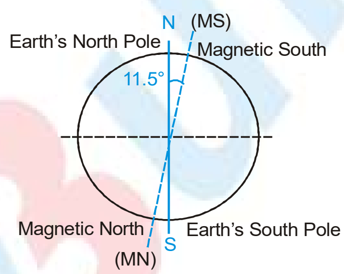
If you ask the average person what "magnetism" is, you will probably be told about the magnets those are used to hold notes on refrigerator door, or keeping paper clips in a holder or may be about lead stone (naturally occurring magnet).

Scholars still dispute about the origin of magnetism. It is believed that magnetism was originally used, not for navigation, but for geomancy ("foresight by earth") and fortune-telling by the Chinese. Chinese fortune tellers used lodestones to construct their fortune telling boards.

From Chinese text, it is known that magnetic compass (used for navigational purpose) is an old Chinese invention. An old Indian literature dates it to as back as 4th century. The compass was used in India was known as the matsya yantra, because of the placement of a metallic fish in a cup of oil.

Earth's Magnetic Field :

It is understood that a compass needle points along the horizontal component of Earth's magnetic field (a property called declination).



Earth's Geomagnetic North acts as a south pole of a magnet, while its Geomagnetic South acts as a North Pole of a magnet.

As shown in the diagram, the axis of the dipole makes an angle of about 11.5° with earth's rotational axis. The axis of dipole makes an angle of about 11.5° with the earth's rotational axis. And Earth's rotational axis makes an angle of 23.5° with the normal to the plane of earth's orbit about the sun.

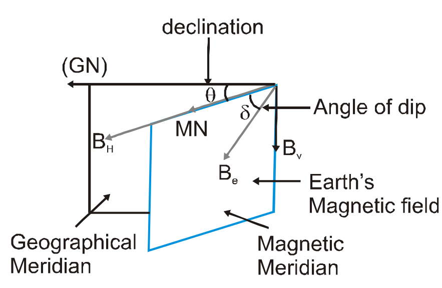
Elements of earth's magnetic field :

The earth's magnetic field is characterized by three quantities :

(a) Declination

(b) Inclination or dip

(c) Horizontal component of the field

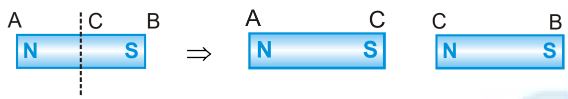


Pole Strength Magnetic Dipole and Magnetic Dipole Moment :

A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.

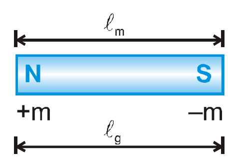
(i) (ii)

The poles of the same magnet do not come to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.



They are known as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" +m and –m respectively (just like we have charges +q and –q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges –q and +q). It is called MAGNETIC DIPOLE and it has a MAGNETIC DIPOLE MOMENT. It is represented by $\vec{M}$. It is a vector quantity. It's direction is from –m to +m that means from 'S' to 'N')



$M = m.\ell_m$ here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].

The units of m and M will be mentioned afterwards where you can remember and understand.

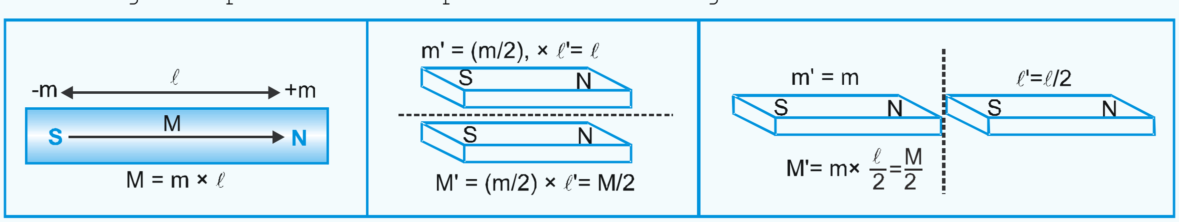
(i) **Inverse square law (Coulomb law) :** The magnetic force between two isolated magnetic poles of strength m_1 and m_2 lying at a distance 'r' is directly proportional to the product of pole strength and inversely proportional to the square of distance between their centres. The magnetic force between the poles can be attractive or repulsive according to the nature of the poles.

$$\left.\begin{aligned} F_m &\propto m_1 m_2 \\ F_m &\propto \frac{1}{r^2} \end{aligned}\right] \quad F_m = k\,\frac{m_1 m_2}{r^2} \qquad \text{where } k \begin{cases} \dfrac{\mu_0}{4\pi} & \text{(S.I.)} \\ 1 & \text{(C.G.S.)} \end{cases}$$

Inverse square law of Coulomb in magnetism is applicable only for two long bar magents becasue isolated poles cannot exist.

(ii) If a magnet is cut into two equal parts along the length then pole strength is reduced to half and length remains unchanged. New magnetic dipole moment $M' = m'(\ell) = \frac{m}{2} \times l = \frac{M}{2}$.

The new magnetic dipole moment of each part becomes half of original value.

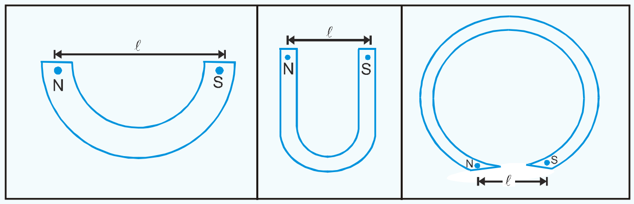


(iii) If a magnet is cut into two equal parts transverse to the length then pole strength remains unchanged and length is reduced to half. New magnetic dipole moment $M' = m\left(\frac{1}{2}\right) = \frac{M}{2}$.

The new magnetic dipole moment of each part becomes half of original value.

(iv) The magnetic dipole moment of a magnet is equal to product of pole strength and distance between poles.

M = m●



(v) As magnetic moment is a vector, in case of two magnets having magnetic moments M_1 and M_2 with angle θ between them, the resulting magnetic moment.

$$M = \left[M_1^2 + M_2^2 + 2M_1M_2\cos\theta\right]^{1/2} \quad \text{with} \quad \tan\phi = \left[\frac{M_2\sin\theta}{M_1 + M_2\cos\theta}\right]$$

Magnetic Field and Strength of Magnetic Field :

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is qualitatively represented by 'STRENGTH OF MAGNETIC FIELD' or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by $\vec{B}$. It is a vector quantity.

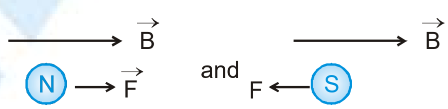
Definition of $\vec{B}$: The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \frac{\vec{F}}{m}$

Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{B}$ is **Tesla** or **Weber/m$_2$** (abbreviated as T and Wb/m^2).

We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$.

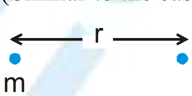


The field generated by sources does not depend on the test pole (for its any value and any sign).

(a) $\vec{B}$ due to various source

(i) Due to a single pole :

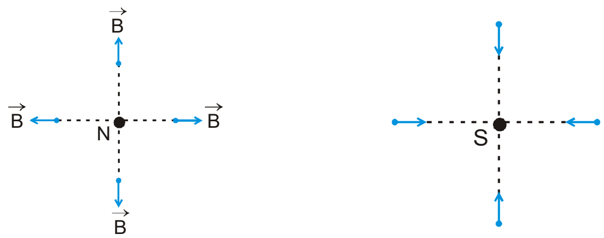
(Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}.$$

This is magnitude

Direction of B due to north pole and due to south poles are as shown

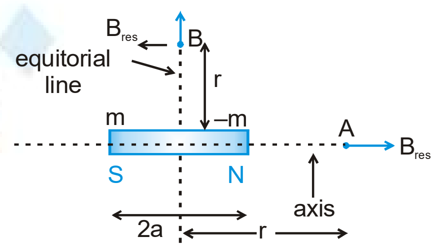


in vector form $\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$

here m is with sign and $\vec{r}$ = position vector of the test point with respect to the pole.

(ii) Due to a bar magnet :

(Same as the case of electric dipole in electrostatics) Independent case never found. Always 'N' and 'S' exist together as magnet.



at A (on the axis) $= 2\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for $a << r$

at B (on the equatorial) $= -\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for $a << r$

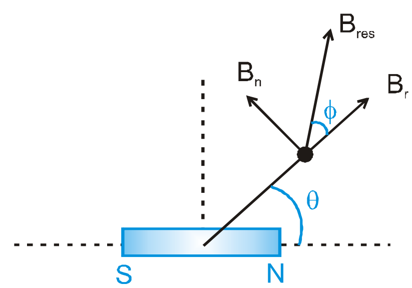
At General point :

$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$$

$$B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$



MAGNETIC FIELD LINES (By Michal Faraday)

In order to visualise a magnetic field graphically, Michal faraday introduced the concept of field lines.

Field lines of magnetic field are imaginary lines which represents direction of magnetic field continuously.

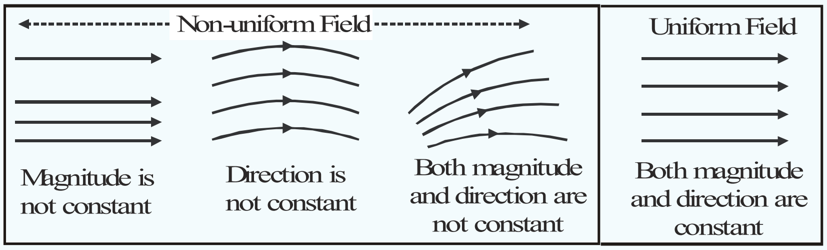
(i) Magnetic field lines are closed curves.

(ii) Tangent drawn at any point on field line represents direction of the field at that point.

(iii) Field lines never intersects to each other.

(iv) At any place crowded lines represent stronger field while distant lines represents weaker field.

(v) In any region, if field lines are **equidistant and straight** the field is uniform otherwise not.

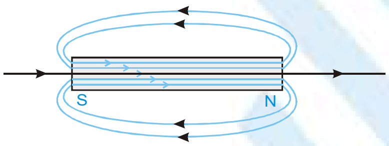


(vi) Magnetic field lines emanate from or enters in the surface of a magnetic material at any angle.

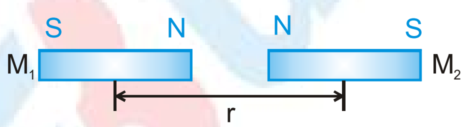
(vii) Magnetic field lines exist inside every magnetised material.

(viii) Magnetic field lines can be mapped by using iron dust or using compass needle.

Magnetic Lines of Force of a Bar Magnet :

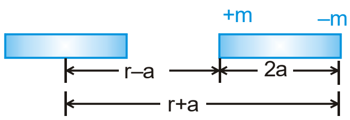


Ex. Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



Sol. To find the magnetic force we will use the formula of 'B' due to a magnet. We will also assume m and –m as pole strengths of 'N' and 'S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at +m and –m respectively. They experience magnetic forces F_1 and F_2 as shown.

$$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m$$



and $$F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$$

$\therefore$ $$F_{res} = F_1 - F_2 = 2\left(\frac{\mu_0}{4\pi}\right)M_1 m\left[\left(\frac{1}{(r-a)^3}\right)-\left(\frac{1}{(r+a)^3}\right)\right]$$

=

By using acceleration, Binomial expansion, and neglecting terms of high power we get

$$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4} = 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$$

Direction of F_{res} is towards right.

Alternative Method :

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2M_1}{r^3} \quad \Rightarrow \quad \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$$

$$F = -M_2 \times \frac{dB}{dr} \quad \Rightarrow \quad F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1M_2}{r^4}$$

Ex. A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Sol. The pole strength is m = 120 CGS units = 12A-m.

Magnetic length is 2● = 10 cm or ● = 0.05 m.

Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\lambda^2)^2} = \frac{\mu_0}{4\pi}\frac{4m\lambda d}{(d^2-\lambda^2)^2}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12A-m)\times(0.05m)\times(0.2m)}{[(0.2m)^2-(0.05m)^2]^2} = 3.4\times10^{-5}\ T.$$

Ex. Find the magnetic field due to a dipole of magnetic moment 1.2 A-m² at a point 1 m away from it in a direction making an angle of 60º with the dipole-axis.

Sol. The magnitude of the field is

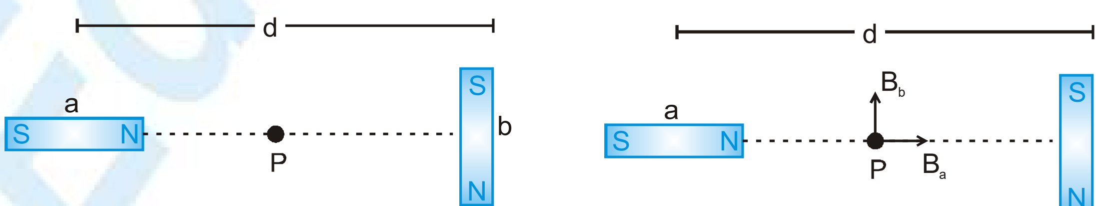
$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{1.2A-m^2}{1\,m^3}\sqrt{1+3\cos^2 60^\circ} = 1.6\times10^{-7}\ T.$$

The direction of the field makes an angle α with the radial line where

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$

Ex. Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



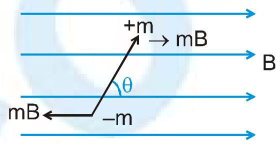
(a) (b)

Sol. The point P is in end-on position for the dipole **(a)** and in broadside-on position for the dipole **(b)**. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore.

$$B = \sqrt{B_a^2 + B_b^2} = \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2+2^2} = \frac{2\sqrt{5}\mu_0 M}{\pi d^3}$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

Magnet in an External Uniform Magnetic Field



(same as case of electric dipole)

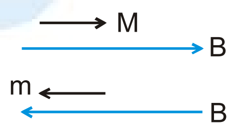
$F_{res} = 0$ (for any angle)

$\tau = MB \sin\theta$

*here θ is angle between $\vec{B}$ and $\vec{M}$

Note : **(i)** $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.

(ii) $\vec{\tau}$ is same about every point of the dipole it's potential energy is



$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$

$\theta = 0^\circ$ is stable equilibrium

$\theta = \pi$ is unstable equilibrium

for small 'θ' the dipole performs SHM about $\theta = 0^\circ$ position

$\tau = -MB\sin\theta$; $I\alpha = -MB\sin\theta$

for small θ, $\sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{I}{MB}}$$

Here $I = I_{cm}$ if the dipole is free to rotate $= I_{hinge}$ if the dipole is hinged

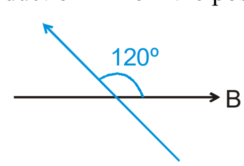
Ex. A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60º from the field.

Sol. The work done by the external agent = change in potential energy

$= (-MB\cos\theta_2) - (-MB\cos\theta_1) = -MB(\cos 60^\circ - \cos 0^\circ)$

$= \frac{1}{2}MB = \frac{1}{2} \times (1.0 \times 10^4 \text{ J/T})(4 \times 10^{-5}\text{ T}) = 0.2$ J

Ex. A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure. Find :



(i) Its kinetic energy at θ = 90º

(ii) its maximum kinetic energy during the motion.

(iii) will it perform SHM? oscillation? Periodic motion? What is its amplitude?

Sol. **(i)** Apply energy conservation at $\theta = 120^\circ$ and $\theta = 90^\circ$

$- MB\cos 120^\circ + 0$

$= - MB\cos 90^\circ + (K.E.)$

$KE = \frac{MB}{2}$ **Ans.**

(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0^\circ$. Now apply energy conservation between $\theta = 120^\circ$ and $\theta = 0^\circ$.

$- mB\cos 120^\circ + 0$

$= -mB\cos 0^\circ + (KE)_{max}$

$(KE)_{max} = \frac{3}{2}MB$ **Ans.**

The K.E. is max at $\theta = 0^\circ$ can also be proved by torque method. From $\theta = 120^\circ$ to $\theta = 0^\circ$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0^\circ$. Beyond that τ reverses its direction and then K.E. starts decreasing

$\therefore$ $\theta = 0^\circ$ is the orientation of M to here the maximum K.E.

(iii) Since 'θ' is not small.

$\therefore$ the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120°.

Ex. A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of $25\mu T$.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Sol. **(a)** The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2) = \frac{100\times10^{-3}}{12}[(7\times10^{-2})^2 + (1\times10^{-2})^2]\ \text{kg-m}^2. = \frac{25}{6}\times10^{-5}\ \text{kg -m}^2.$$

We have, $T = 2\pi\sqrt{\frac{I}{MB}}$ or, $M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2\times25\times10^{-5}\,kg/m^2}{6\times(25\times10^{-6}T)\times\frac{\pi^2}{4}s^2} = 27\ \text{A-m}^2.$

(b) In this case the moment of inertia becomes

$I' = \frac{m'}{12}(L^2 + b^2)$ where $b' = 0.5$ cm.

The time period would be

$$T' = \sqrt{\frac{I'}{MB}} \qquad \text{.... } \textbf{(ii)}$$

Dividing by equation **(i)**,

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2+b'^2)}}{\sqrt{\frac{m'}{12}(L^2+b^2)}} = \frac{\sqrt{(7\,cm)^2+(0.5\,cm)^2}}{\sqrt{(7\,cm)^2+(1.0\,cm)^2}} = 0.992 \quad \text{or,} \quad T' = \frac{0.992\times\pi}{2}\,s = 0.496\pi\ s.$$

Magnet in an External Non-uniform Magnetic Field :

No special formulae are applied is such problems. Instead see the force on individual poles and calculate the resistant force torque on the dipole.

Ex. Find the torque on M_1 due to M_2 in Que. 1

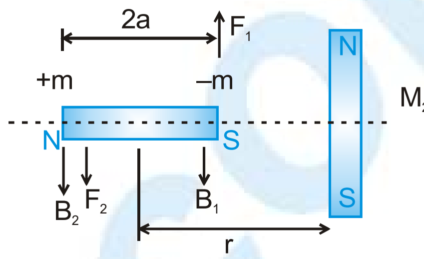
Sol. Due to M_2, magnetic fields at 'S' and 'N' of M_1 are B_1 and B_2 respectively. The forces on $-m$ and $+m$ are F_1 and F_2 as shown in the figure. The torque (about the centre of the dipole m_1) will be

$$= F_1 a + F_2 a = (F_1 + F_2)a$$

$$= \left[\left(\frac{\mu_0}{4\pi}\right)\frac{M_2}{(r-a)}m + \frac{\mu_0}{4\pi}\frac{M_2}{(r+a)}m\right]a$$

$$\cong \frac{\mu_0}{4\pi} M_2 m \left(\frac{1}{r^3} + \frac{1}{r^3}\right)a \quad \because a << r$$

$$= \frac{\mu_0 M_2 m}{4\pi}\,\frac{2}{r^3}a = \frac{\mu_0 M_1 M_2}{4\pi r^3} \qquad \textbf{Ans.}$$



Magnetism due to Electricity

Hans Christian Oersted was a professor of science at Copenhagen University. In 1819 he arranged in his home a science demonstration to friends and students. He planned to demonstrate the heating of a wire by an electric current, and also to carry out demonstrations of magnetism, for which he provided a compass needle mounted on a wooden stand.

While performing his electric demonstration, Oersted noted to his surprise that every time the electric current was switched on, the compass needle moved. He kept quiet and finished the demonstrations, but in the months that followed worked hard trying to make sense out of the new phenomenon. And this is what we are going to study now.

We have seen that currents (fundamentally moving charges) are the source of magnetism. This can be readily demonstrated by placing compass needles near wire. As shown in Figure, all compass needles point in the same direction in the absence of current (in the direction of earth's magnetic field). However, when a strong current passes through (so that earth's magnetic field becomes negligible), the needles will be deflected along the tangential direction of the circular path (Figure).

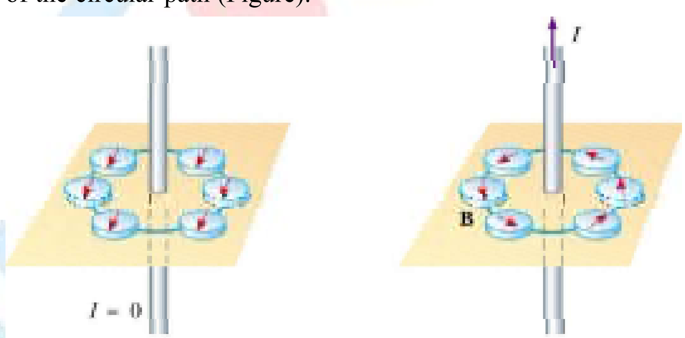


Figure : Deflection of compass needles near a current-carrying wire

Biot - Savart Law

Currents which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire and produce a current I, the magnetic field at any point P due to the current can be calculated by adding up the magnetic field contributions, $d\vec{B}$, from small segments of the wire $d\vec{s}$ (Figure).

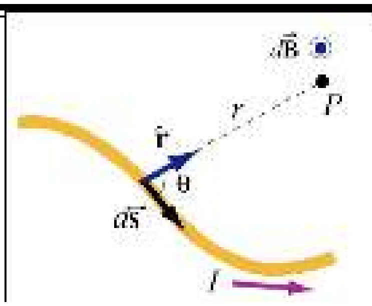


Figure : Magnetic field $d\vec{B}$ at point P due to a current-carrying element $I\,d\vec{s}$

The segments can be thought of as a vector quantity having a magnitude of the length of the segment and pointing in the direction of the current flow. The infinitesimal current source can then be written as $I\,d\vec{s}$.

Let r denote as the distance from the current source to the field point P and $\vec{r}$ the corresponding unit vector. The Biot–Savart law gives an expression for the magnetic field contribution, $d\vec{B}$, from the current source, $I\,d\vec{s}$,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times r}{r^2}$$

where μ_0 is a constant called the permeability of free space :

$\mu_0 = 4\pi \times 10^{-7}$ T.M/A here Tesla (T) is SI unit of $\vec{B}$

Notice that the expression is remarkably similar to the Coulomb's law for the electric field due to a charge element dq :

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

Adding up these contributions to find the magnetic field at the point P requires integrating over the current source.

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s} \times \hat{r}}{r^2}$$

The integral is a vector integral, which means that the expression for B is really three integrals, one for each component of $\vec{B}$. The vector nature of this integral appears in the cross product. Understanding how to evaluate this cross product and then perform the integral will be the key to learning how to use the Bio_Savart law.

Force on a Moving Charge in Magnetic Field

Consider a charge particle θ moving in a magnetic field B as shown in figure. At some instant its velocity is v which makes an angle θ with the direction of magnetic field.

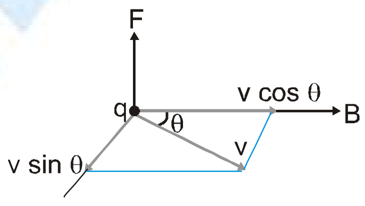


Figure 3.1

As we known that magnetic field exerts force on a moving charge particle. It is observed that the force acting on the charge is proportional to

(i) The magnitude of charge q on the particle

(ii) The magnitude B of the magnetic field

(iii) The component of velocity which is perpendicular to the magnetic field i.e. v sin θ

Combining the above three observations :

we have $F \propto qBv \sin\theta$

$\Rightarrow$ $F = kqBv \sin\theta$

where k is proportionally constant. Its value is chosen arbitrarily to be unity i.e. k = 1

$\Rightarrow$ $F = qBv \sin\theta$

Direction : The magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; that is, the magnetic force is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$. The direction of force can be calculated using right hand thumb rule.

These results show that the magnetic force on a particle is more complicated than the electric force. The magnetic force is distinctive because it depends on the velocity of the particle and because its direction is perpendicular to both $\vec{v}$ and $\vec{B}$. Despite this complicated behaviour, these observations can be summarized in a compact way by writing the magnetic force in the form

$$\vec{F} = q\vec{v} \times \vec{B}$$

where the direction of the magnetic force is that or $\vec{v} \times \vec{B}$ which, by definition of the cross product, is perpendicular to both $\vec{v}$ and $\vec{B}$.

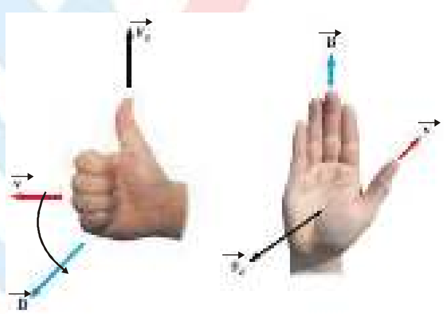
The S.I. unit of magnetic field is tesla (T), where 1 T = 1 N/Am.

Another unit is gauss (G). 1 T = 10^4 G.

1 T is a big unit. Earth's magnetic field is of the order or 10^{-5} T.

Right Hand Rules for Determining the Direction of the Magnetic Force Acting on a Moving Charged Particle

Figure reviews to right–hand rules for determining the direction of the cross product $\vec{v} \times \vec{B}$ and determining the direction of $\vec{F}_B$. The rule in Figure depends on our right-hand rule for the cross product. Point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$. The extended thumb, which is at a right angle to the fingers, points in the direction of $\vec{v} \times \vec{B}$. Then $\vec{F}$ is in the direction of your thumb if q is positive and opposite to the direction of your thumb if q is negative.



(a) (b)

Figure

An alternative rule is shown in figure **(b)**. Here the thumb points in the direction of $\vec{v}$ and the extended fingers in the direction of $\vec{B}$. Now, the force $\vec{F}_B$ on a positive charge extends outward from your palm. The advantage of this rule is that the force on the charge is in the direction that you would push on something with your hand outward from your palm. The force on a negative charge is in the opposite direction.

Note : If θ = 0° or θ = 180°, the F = 0. It means magnetic field does not extra a force on a charge particle moving parallel or antiparallel to the magnetic field.

F is maximum if θ = 90°, so force on the charge particle is maximum if velocity is perpendicular to the magnetic field.

If v = 0, then F = 0. A magnetic field does not exert force on a stationary charge particle.

There are important differences between electric and magnetic forces on charged particles :

(i) The electric force is always parallel or antiparallel to the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.

(ii) the electric force acts on a charged particle independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity.

(iii) The electric force does work in displacing a charged particle, whereas the magnetic force does no work when a charged particle is displaced.

This last statement is true because when a charge moves in a magnetic field, the magnetic force is always perpendicular to the velocity. Hence, the work done by the magnetic force on the particle is zero.

From the work–energy theorem, we conclude that the kinetic energy of a charged particle cannot be altered by a magnetic field alone. In their words, when a charge moves with a velocity, an applied magnetic field can alter the direction of the velocity vector, but it cannot change the speed of the particle.

Magnetic Field due to a Finite Straight Wire

A thin, straight wire carrying a current I is placed along the x-axis, as shown in Figure. Evaluate the magnetic field at point P due to the segment shown in figure.

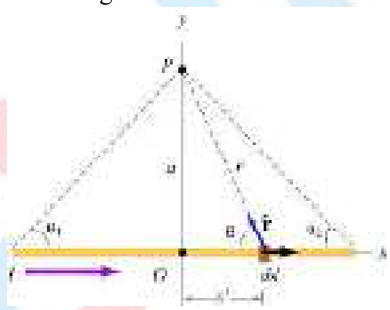


Figure : A thin straight wire carrying a current I

The contribution to the magnetic field due to $I\,d\vec{s}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{d \times \sin\theta}{r^2}\hat{k}$$

Which shows that the magnetic field at P will in the $+\hat{k}$ direction, or out of the page.

Simplify and carry out the integration.

The variables θ, x and r are not independent of each other. In order to complete the integration, let us rewrite the variables x and r in terms of θ. From Figure, we have

$$\begin{cases} r = a/\sin\theta = a\ \text{cosec}\ \theta \\ x = a\cot\theta \Rightarrow dx = -a\ \text{cosec}^2\theta\, d\theta \end{cases}$$

Upon substituting the above expressions, the differential contribution to the magnetic field is obtained as

$$dB = \frac{\mu_0 I}{4\pi}\frac{(-a\,\mathrm{cosec}^2\theta d\theta)\sin\theta}{(a\,\mathrm{cosec}\theta)^2} = \frac{\mu_0 I}{4\pi a}\sin\theta\, d\theta$$

Integrating over all angles subtended from $-\theta_1$ to θ_2 (a negative sign is needed for θ_1 in order to take into consideration the portion of the length extended in the negative x - axis from the origin), we obtain

$$dB = \frac{\mu_0 I}{4\pi a}\int_{-\theta_1}^{\theta_2}\sin\theta d\theta = \frac{\mu_0 I}{4\pi a}(\cos\theta_2 + \cos\theta_1)$$

The first term involving θ_2 accounts for the contribution from the portion along the +x axis, while the second term involving θ_1 contains the contribution from the portion along the –x axis. The two terms add.

Special Cases :

(i) Magnetic field on the perpendicular bisector of a finite straight wire of length 2L

In this case where $\theta_2 = \theta_1 = \theta$, the field point P is located along the perpendicular bisector. If the length of the rod is 2L, then $\cos\theta = L\sqrt{L^2 + a^2}$ and the magnetic field is

$$B = \frac{\mu_0 I}{2\pi a}\cos\theta = \frac{\mu_0 I}{2\pi a}\frac{L}{\sqrt{L^2 + a^2}}$$

(ii) Magnetic field due to semi infinite straight wire

Here $\theta_1 = 90°$, $\theta_2 = 0°$ or $\theta_1 = 0°$, $\theta_2 = 90°$

$$B = \frac{\mu_0 I}{4\pi a}$$

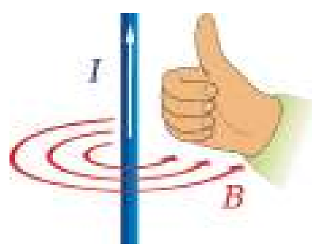
(iii) Magnetic field due to infinite straight wire

Here $\theta_1 = \theta_2 = 0°$

$$B = \frac{\mu_0 I}{2\pi a}$$

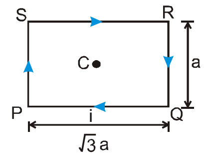
Direction of magnetic field of a straight wire

Note that in this limit, the system possesses cylindrical symmetry, and the magnetic field lines are circular, as shown in figure



In fact, the direction of the magnetic field due to a long straight wire can be determined by the right-hand rule (Figure). If you direct your right thumb along the direction of the current in the wire, then the fingers of your right hand curl in the direction of the magnetic field.

Ex. Find resultant magnetic field at 'C' in the figure shown.



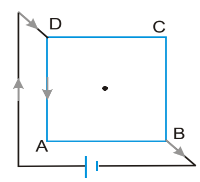
Sol. It is clear that 'B' at 'C' due all the wires is directed ⊗. Also B at 'C' due PQ and SR is same. Also due to QR and PS is same

$\therefore \qquad B_{res} = 2(B_{PQ} + B_{SP})$

$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}} (\sin 60^o + \sin 60^o), \qquad \Rightarrow \qquad B_{sp} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}} (\sin 30^o + \sin 30^o)$$

$$B_{res} = 2\left(\frac{\sqrt{3}\,\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

Ex. Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points B and D as shown in the figure

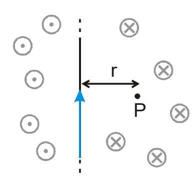


Sol.

The current will be equally divided at D. The fields at the centre due to the currents in the wires DA and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AB and CB will be zero. Hence, the net field at the centre will be zero.

Special case :

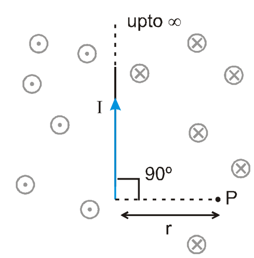
(i) If the wire is infinitely long then the magnetic field at 'P' (as shown in the figure) is given by (using $\theta_1 = \theta_2 = 90^o$ and the formula of 'B' due to straight wire)



$$B = \frac{\mu_0 I}{2\pi r} \qquad \Rightarrow \qquad B \propto \frac{I}{r}$$

The direction of $\vec{B}$ at various is as shown in the figure. The magnetic lines of force will be concentric circles around the wire (as shown earlier)

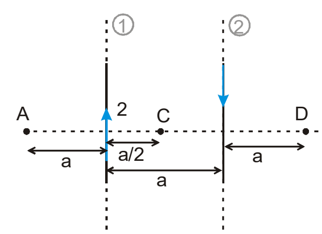
(ii) If the wire is infinitely long but 'P' is as shown in the figure. The direction of $\vec{B}$ at various points is as shown in the figure. At 'P'



$$B = \frac{\mu_0 I}{4\pi r}$$

Ex. In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents 2I and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find **(i)** $\vec{B}$ at A, C, D

(ii) position of point on line A C D where $\vec{B}$ is O.



Sol. **(i)** Let us call $\vec{B}$ due to **(2)** and **(2)** as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then

at A : $\vec{B}_1$ is $\odot$ and $\vec{B}_2$ is $\otimes$

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$\therefore$ $B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$ **Ans.**

at C : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ also $\otimes$

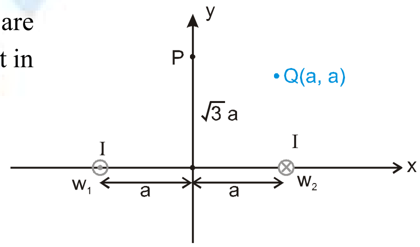
$\therefore$ $B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi\frac{a}{2}} + \frac{\mu_0 I}{2\pi\frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes$ **Ans.**

at D : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ is $\odot$ and both are equal in magnitude.

$\therefore$ $B_{res} = 0$ **Ans.**

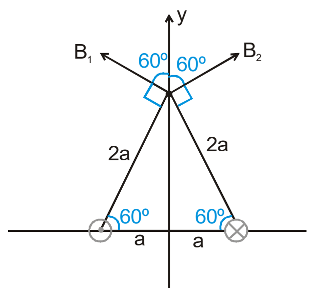
(ii) It is clear from the above solution that B = 0 at point 'D'.

Ex. In the figure shown two long wires W_1 and W_2 each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find the $\vec{B}$ at 'P' and 'Q'.



Sol. Let $\vec{B}$ due to W_1 be $\vec{B}_1$ and due to W_2 be $\vec{B}_2$.

By symmetry $|\vec{B}_1| = |\vec{B}_2| = B$



$$B_p = 2\,B \cos 60^o = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

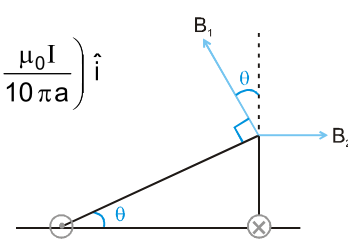
$\therefore$ $\vec{B}_p = \frac{\mu_0 I}{4\pi a}\hat{j}$ **Ans.**

For θ $B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}$, $\Rightarrow$ $B_2 = \frac{\mu_0 I}{2\pi a}$

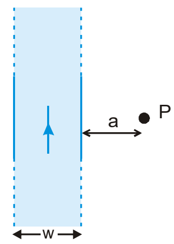
$\tan\theta = \frac{a}{2a} = \frac{1}{2}$ $\Rightarrow$ $\vec{B} = (B_1 \cos\theta\ \hat{j}) + (B_2 - B_1 \sin\theta)\,\hat{i}$

$\sin\theta = \frac{1}{\sqrt{5}}$ $\Rightarrow$ $\vec{B} = \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a}\right)\hat{i}$

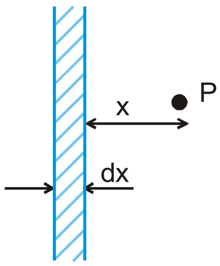
$\cos\theta = \frac{2}{\sqrt{5}}$ $\Rightarrow$ $\vec{B} = \frac{2\mu_0 I}{5\pi a}\hat{i} + \frac{\mu_0 I}{5\pi a}\hat{j}$



Ex. In the figure shown a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'. Find the magnetic field at point 'P' which lies in the plane of the sheet.



Sol. To find 'B' at 'P' the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'

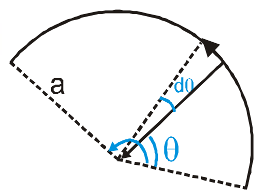


$$dB = \frac{\mu_0\left(\frac{I}{w}dx\right)}{2\pi x} \otimes$$

due to each such wire $\vec{B}$ will be directed inwards

$$\therefore\ B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}\int_{x=a}^{a+w}\frac{dx}{x} \int_{x=a}^{a+w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}\ .\ \ln\frac{a+w}{a}$$ **Ans.**

Magnetic Field due to a current carrying Arc at its centre

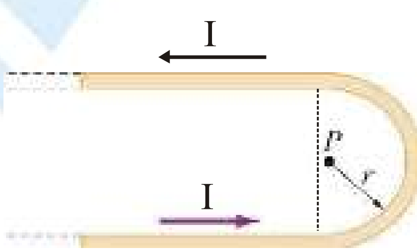


$$dl = ad\theta$$

$$dB = \frac{\mu_0}{4\pi}\frac{I(ad\theta)\sin 90°}{a^2} = \frac{\mu_0 I}{4\pi a}d\theta$$

$$B = \int dB = \frac{\mu_0 I}{4\pi a}\int_{\theta}^{\beta} d\theta \quad \Rightarrow \quad B = \frac{\mu_0 I}{4\pi R}(\beta)$$

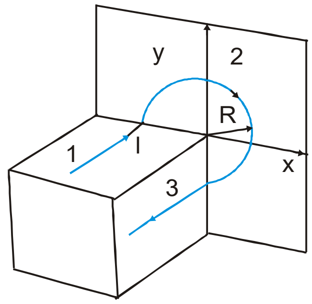
Ex. An infinitely long current-carrying wire is bent into a hairpin-like shape shown in figure. Find the magnetic field at the point P which lies at the centre of the half-circle.



Sol. The total magnitude of the magnetic field is

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = 2\vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4\pi r}\hat{k} + \frac{\mu_0 I}{4r}\hat{k} = \frac{\mu_0 I}{4\pi r}(2+\pi)\hat{k}$$

Ex. Find the magnetic induction at the point O if the wire carrying a current I A has the shape shown in Fig. The radius of the curved part of the wire is R, the linear parts of the wire are very long.



Sol. $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{4\pi r}(-\hat{j}) + \frac{\mu_0 I}{4\pi r}\left(\frac{3\pi}{2}\right)(\hat{k}) + \frac{\mu_0 I}{4\pi r}(-\hat{i})$

Ex. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle q at the centre. Magnetic field due to current at the centre of ring is

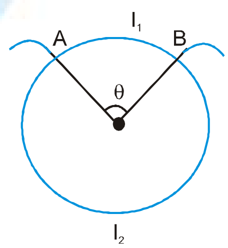
Sol. For a current flowing into a circular arc, magnetic induction in the centre

$$B = \frac{\mu_{0I}}{4\pi}\int \frac{dl \times r}{r^3} = \frac{\mu_{0I}}{4\pi}\int \frac{r^2 d\theta}{r^3} = \left(\frac{\mu_{0I}}{4\pi r}\right)\theta$$

The total current is divided into two arcs

$$I_1 = \frac{E}{R_1}$$

$$= \frac{E}{(R/2\pi r)I_1} = \frac{E}{(R/2\pi r)(r\theta)} = \frac{2\pi E}{R\theta}$$



Similarly $I_1\theta = \frac{2\pi E}{R} = \text{constant}$

$$I_2 = \frac{E}{R_2} = \frac{E}{(R/2\pi r)I_2}$$

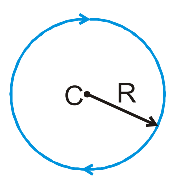
$$= \frac{E}{(R/2\pi r)\{r(2\pi - \theta)\}} = \frac{2\pi E}{R(2\pi - \theta)} = \text{constant}$$

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi r}\left(\frac{2\pi E}{R} - \frac{2\pi E}{R}\right) = \theta$$

$\vec{B}$ due to Circular Loop

At centre : Due to each $\vec{d\lambda}$ element of the loop $\vec{B}$ at 'c' is inwards (in this case).

$\therefore$ $\vec{B_{res}}$ at 'c' is $\otimes$.



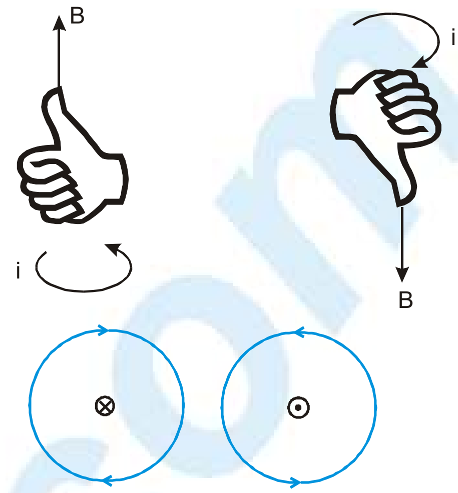
$B = \frac{\mu_0 NI}{2R}$,

N = No. of turns in the loop.

$= \frac{\lambda}{2\pi R}$; ● = length of the loop.

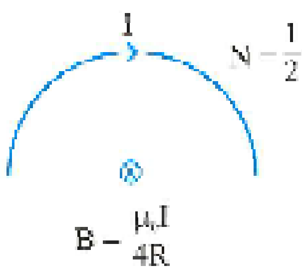
N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} \text{ etc.}\right)$ or integer.

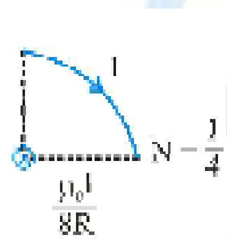
Direction of $\vec{B}$: The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure).



Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.

Semicircular and Quarter of a circle :





Magnetic Field due to a Circular Current Loop at a point on its axis

Consider a circular loop of radius a carrying a current i. We have to find the magnetic field at a Point P on the axis of the loop at a distance d from its centre O. In figure

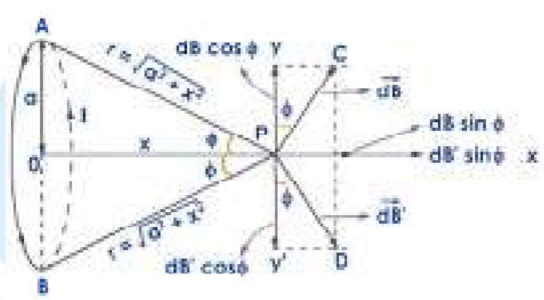


Figure shows the geometry for calculating the magnetic field at a point on the axis of a circular current loop a distance x from its center. We first consider the current element at the top of the loop. Here, as everywhere around the loop, $Id\vec{l}$ is tangent to the loop and perpendicular to the vector $\vec{r}$ from the current element to the field point P. The magnetic field $d\vec{B}$ due to this element is in the direction shown in the figure, perpendicular to $\vec{r}$ and also perpendicular to $Id\vec{l}$. The magnitude of $d\vec{B}$ is

$$dB = \frac{\mu_0}{4\pi r}\frac{Idl\sin 90}{r^2}$$

When we sum around all the current elements in the loop, the components of $d\vec{B}$ perpendicular to the axis of the loop, such as dB_y in Figure sum to zero, leaving only the components dB_x that are parallel to From Figure, we have

$$B_x = \int dB \sin\phi = \int \sin\phi \frac{\mu_0 I}{4\pi r^2} dl = \frac{\mu_0 I}{4\pi r^2} \sin\phi \int dl = \frac{\mu_0 I}{4\pi r^2} \frac{a}{r}(2\pi a) = \frac{\mu_0 I (2\pi a^2)}{4\pi r^2}$$

using the facts that

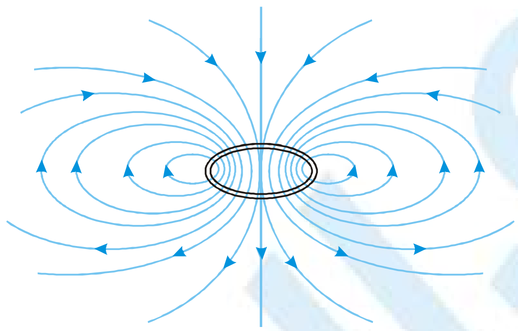
$r^2 + x^2 + a^2$

we get

$$B = \frac{\mu_0 I (2\pi a^2)}{4\pi (a^2 + x^2)^{3/2}}$$

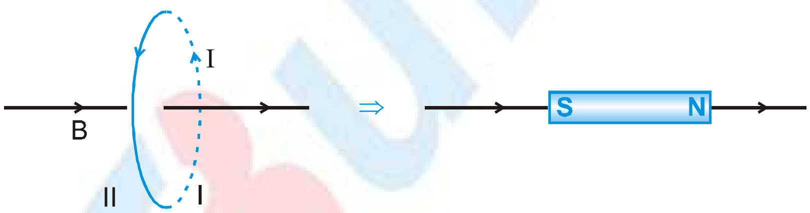
Note : If loop has N close turns then field becomes N times the field die to one turn.

Magnetic field lines due to a circular current

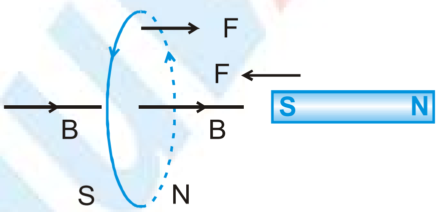


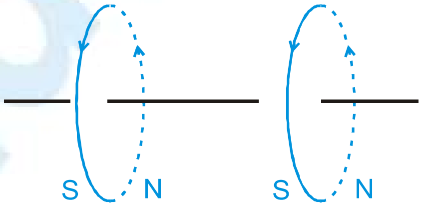
A loop as a magnet :

The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



The side 'I' (the side from which the $\vec{B}$ emerges out) of the loop acts as 'NORTH POLE' and side II (the side in which the $\vec{B}$ enters) acts as the 'SOUTH POLE'. It can be verified by studying force on one loop due to a magnet or a loop.





Mathematically

$$B_{axis} = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}} \cong \frac{\mu_0 NIR^2}{2x^3} \text{ for } x >> R \quad = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IN\pi R^2}{x^3}\right)$$

it is similar to B_{axis} due to magnet $= 2\left(\frac{\mu_0}{4\pi}\right)\frac{m}{x^3}$

Magnetic dipole moment of the loop

$M = IN\pi R^2$

$M = INA$ for any other shaped loop.

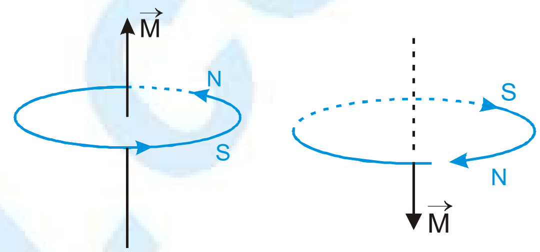
Unit of M is Amp. m^2.

Unit of m (pole strength) = Amp. m {$\because$ in magnet $M = m\ell$}

$\vec{M} = IN\vec{A}$,

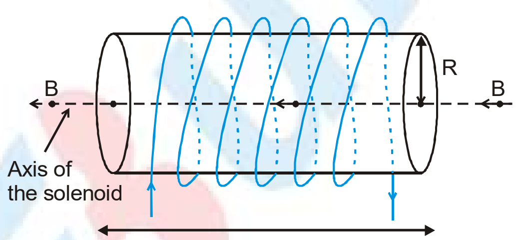
$\vec{A}$ = unit normal vector for the loop.

To be determined by right hand rule which is also used to determine direction of $\vec{B}$ on the axis. It is also from 'S' side to 'N' side of the loop.



Solenoid

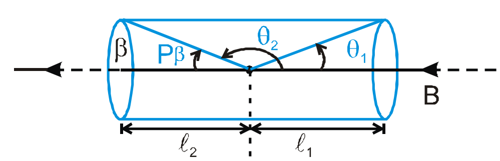
(i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)



(ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) $\vec{B}$ on axis (turns should be very close to each others).

$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$



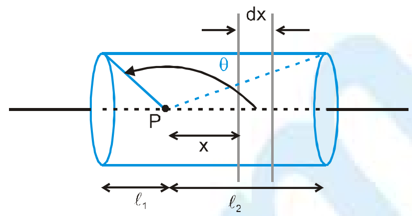
where n : number of turns per unit length.

$$\cos\theta_1 = \frac{\lambda_1}{\sqrt{\lambda_1^2+R^2}} \ ; \quad \cos\beta = \frac{\lambda_2}{\sqrt{\lambda_2^2+R^2}} \quad = -\cos\theta_2$$

$$B = \frac{\mu_0 ni}{2}\left[\frac{\lambda_1}{\sqrt{\lambda_1^2+R^2}} + \frac{\lambda_2}{\sqrt{\lambda_2^2+R^2}}\right] = \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$$

Note : Use right hand rule for direction (same as the direction due to loop).

Derivation



Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element dn = ndx where n : number of turns per unit length.

$$dB = \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}\,(ndx)$$

$$B = \int dB = \int_{-\lambda_1}^{\lambda_2} \frac{\mu_0 i R^2\, ndx}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 ni}{2}\left[\frac{\lambda_1}{\sqrt{\lambda_1^2+R^2}} + \frac{\lambda_2}{\sqrt{\lambda_2^2+R^2}}\right] = \frac{\mu_0 ni}{2}[\cos\theta_1 - \cos\theta_2]$$

(iv) For 'Ideal Solenoid' :

***Inside** (at the mid point)

●>> R or length is infinite

$\theta_1 \to 0$

$\theta_2 \to \pi$

$$B = \frac{\mu_0 ni}{2}\,[1-(-1)]$$

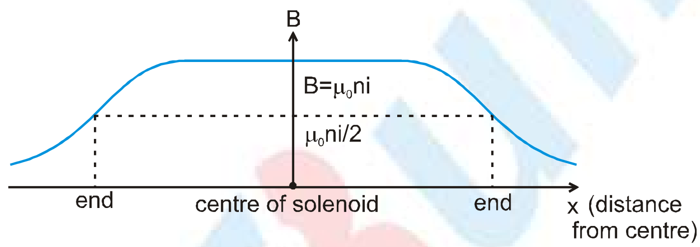
$B = \mu_0 ni$

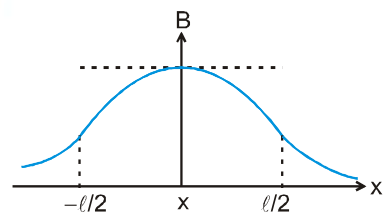
If material of the solid cylinder has relative permeability 'μ_r' then $B = \mu_0\mu_r ni$

At the ends $B = \dfrac{\mu_0 ni}{2}$

(v) Comparison between ideal and real solenoid :

(a) **Ideal Solenoid** **Real Solenoid**





Ex. A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0×10^{-3} ampere. Find the magnetic field on the axis at the middle and at the ends of the solenoid.

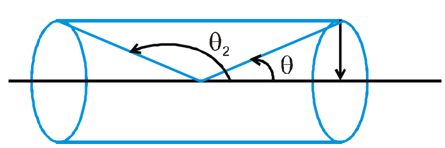
(Given $\mu_0 = 4\pi \times 10^{-7}\ \dfrac{V-s}{A-m}$).

Sol. $B = \dfrac{1}{2}\mu_0 ni\,[\cos\theta_1 - \cos\theta_2]$

$\Rightarrow$ $n = \dfrac{1000}{0.4} = 2500$ per meter

$i = 5 \times 10^{-3}$ A.

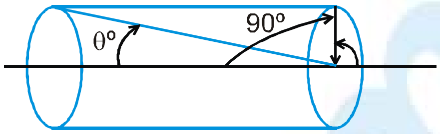
(i) $\cos\theta_1 = \dfrac{0.2}{\sqrt{(0.3)^2+(0.2)^2}} = \dfrac{0.2}{\sqrt{0.13}}$



$\cos\theta_2 = \dfrac{-0.2}{\sqrt{0.13}}$

$$\Rightarrow \quad B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3}\ \frac{2\times 0.2}{\sqrt{0.13}} = \frac{\pi \times 10^{-5}}{\sqrt{13}}\,T$$

(ii) At the end



$$\cos\theta_1 = \frac{0.4}{\sqrt{(0.3)^2+(0.4)^2}} = 0.8$$

$$\cos\theta_2 = \cos 90^\circ = 0$$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$$

$\Rightarrow$ $B = 2\pi \times 10^{-6}\ \text{Wb/m}^2$

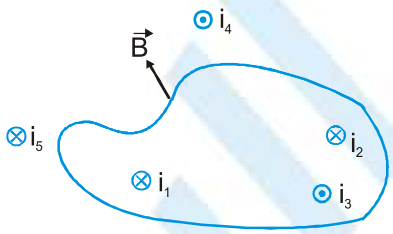
Ampere's Law :

Like Gauss's law in electrostatics, this law provides us a simple method to find magnetic fields in cases of symmetry.

Ampere's law gives another method to calculate the magnetic field due to a given current distribution.

Statement : The circulation $\oint \vec{B}.\vec{dl}$ of the resultant magnetic field (of a closed circuit or an infinite wire containing steady current) along a closed path (called amperian path) is equal to μ_0 times total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant.

Thus, $$\oint \vec{B}.\vec{dl} = \mu_0 i$$



In figure, the positive side is going into the plane of the diagram so that i_1 and i_2 are positive and i_3 is negative. Thus, the total current crossing the area is $i_1 + i_2 - i_3$. Any current outside the area is not included in writing the right - hand side of equation. The magnetic field $\vec{B}$ on the left-hand side is the resultant field due to all the currents existing anywhere.

Ampere's law may be derived from the Biot-Savart law may be derived from the Ampere's law. Thus, the two are equivalent in scientific content. However, Ampere's law is useful under contain symmetrical conditions.

Justification of Ampere's law

Let us consider a long straight wire carrying a current I in upward direction. Now take a circular path of radius symmetric to the wire. Let us now divide a circular path of radius r into a large number of small length vectors $\Delta\vec{s} = \Delta s\hat{\phi}$, where $\hat{\phi}$ point along the tangential direction with magnitude Δs (Figure)

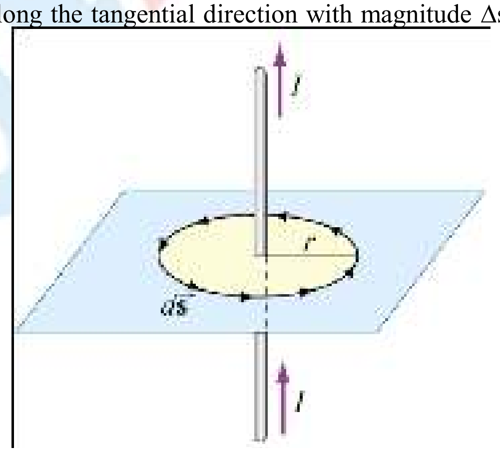


Figure : Amperian loop

In the limit $\Delta\vec{s} \to \vec{0}$, we obtain

$$\oint \vec{B}.d\vec{s} = \vec{B}\oint ds = \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r) = \mu_0 I$$

The result above is obtained by choosing a closed path, or an "Amperian loop" that follows one particular magnetic field line. Let's consider a slightly more complicated Amperian loop, as that shown in Figure.

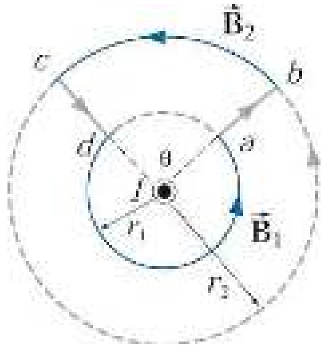


Figure : An Amperian loop involving two field lines

The line integral of the magnetic field around the contour abcda is

$$\oint_{abcda} \vec{B}.d\vec{s} = \oint_{ab} \vec{B}.d\vec{s} + \oint_{bc} \vec{B}.d\vec{s} + \oint_{cd} \vec{B}.d\vec{s} + \oint_{da} \vec{B}.d\vec{s} = 0 + B_2(r_2\theta) + 0 + B_1[r_1(2\pi - \theta)]$$

where the length of arc bc is $r_2\theta$ and $r_1(2\pi - \theta)$ for arc da. The first and the third integrals vanish since the magnetic field is perpendicular to the paths of integration. With

$B_1 = \frac{\mu_0 I}{2\pi r_1}$ and $B_2 = \frac{\mu_0 I}{2\pi r_2}$ the above expression become

$$\oint_{abcda} \vec{B}.d\vec{s} = \frac{\mu_0 I}{2\pi r^2}(r_2\theta) + \frac{\mu_0 I}{2\pi r_1}[r_1(2\pi - \theta)] = \frac{\mu_0 I}{2\pi}\theta + \frac{\mu_0 I}{2\pi}(2\pi - \theta) = \mu_0 I$$

We see that the same result is obtained whether the closed path involves one or two magnetic field lines. As shown above example, in polar coordinates (r,φ) with current flowing in the +z axis, the magnetic field is given by $\vec{B} = (\mu_0 I / 2\pi r)\hat{\varphi}$. An arbitrary length element in the polar coordinates can be written as

$$d\vec{s} = dr\,\hat{r} + r d\phi\;\hat{\phi}$$

Which implies

$$\oint_{\text{closed path}} \vec{B}\,.\,d\vec{s} = \oint_{\text{closed path}} \left(\frac{\mu_0 I}{2\pi r^2}\right) r d\varphi = \frac{\mu_0 I}{2\pi}\oint_{\text{closed path}} d\varphi = \frac{\mu_0 I}{2\pi}(2\pi) = \mu_0 I$$

In other words, the line integral of $\oint \vec{B}\,.\,d\vec{s}$ around any closed Amperian loop is proportional to I_{enc}, the current encircled by the loop.

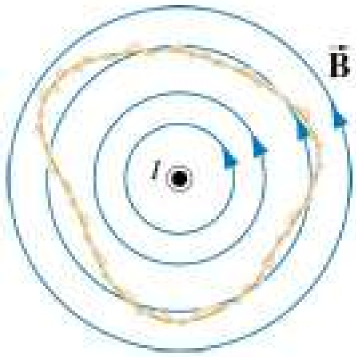


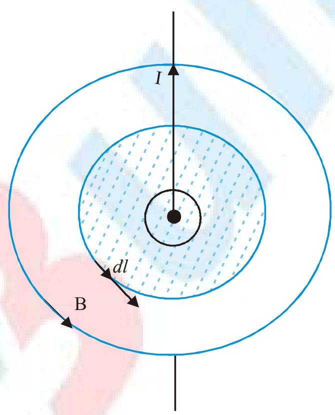
Figure : an Amperian loop of arbitary shape

The generalization to any closed loop of arbitary shape (see for example, Figure) that involves many magnetic field lines is known as Ampere's law :

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

Calculation of magnetic field due to long straight wire

Figure shows a long straight current i. we have to calculate the magnetic field at a point P which is at a distance r from the wire. Figure shows the situation in the plane perpendicular to the wire and passing through P. The current is perpendicular to the plane of the diagram and is coming out of it.



Let us draw a circle passing through the point and with the axis as wire. We put an arrow to show the positive sense of the circle. The radius of the circle is r. The magnetic field due to the long, straight current at any point on the circle is along the tangent as shown in the figure. Same is the direction of the length-element dI there. By symmetry, all points of the circle are equivalent and hence the magnitude of the magnetic field should be the same at all these points. The circulation of magnetic field along the circle is

$$\oint \vec{B} \cdot \vec{dl} = \oint B dl = B \oint dl = B(2\pi r)$$

The current crossing the area bounded by the circle is

$$\sum I_{en} = +I$$

Thus, from Ampere's law,

$$B(2\pi r) = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field due to a current carrying thin long pipe

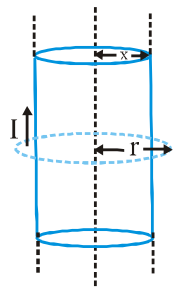
Case I : r > R

$$\oint \vec{B}.\vec{dl} = \oint B.dl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = +I$$

$\Rightarrow$ $B(2\pi r) = \mu_0 I$

$\Rightarrow$ $B = \dfrac{\mu_0 I}{2\pi r}$



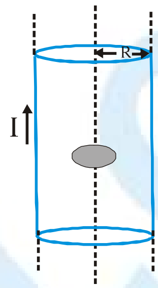
Case II : r < R

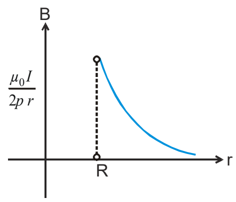
$$\oint \vec{B}.\vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = 0$$

$\Rightarrow$ $B(2\pi r) = \mu_0 (0)$

$\Rightarrow$ $B = 0$





Magnetic field due to a current carrying rod having uniform current density

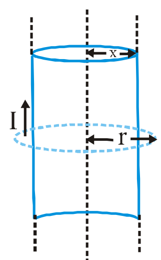
Case I : r > R

$$\oint \vec{B}.\vec{dl} = \oint B.dl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = +I$$

$\Rightarrow$ $B(2\pi r) = \mu_0 I$

$\Rightarrow$ $B = \dfrac{\mu_0 I}{2\pi r}$

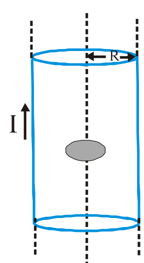


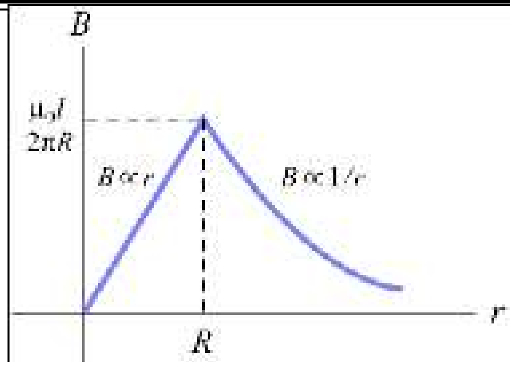
Case II : r < R

$$\oint \vec{B}.\vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = \frac{I}{R^2} r^2$$

$\Rightarrow$ $B(2\pi r) = \mu_0 \dfrac{1}{R^2} r^2$ $\Rightarrow$ $B = \dfrac{\mu_0 I}{2\pi R^2} r$





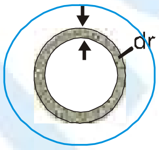
Magnetic Field Due to non uniform current density

Suppose that the current density in a wire of radius a varies with r according to $J = Kr^2$, where K is a constant and r is the distance from the axis of the wire. We have to find the magnetic field at a point distance r from the axis when **(a)** r < a and r > a

Choose a circular path centered on the axis of the conductor and apply Ampere's law

(a) To find the current passing through the area enclosed by the path integrate

$dI = JdA = (Kr^2)(2\pi rdr)$



$\Rightarrow \quad I = \int dI = K\int_a^r 2\pi r^3 dr = \frac{K\pi r^4}{2}$

Since $\oint \vec{B}.d\vec{l} = \mu_0 I$

$\Rightarrow \quad B2\pi r = \mu_0 . \frac{\pi K r^4}{2} \qquad \Rightarrow \quad B = \frac{\mu_0 K r^3}{4}$

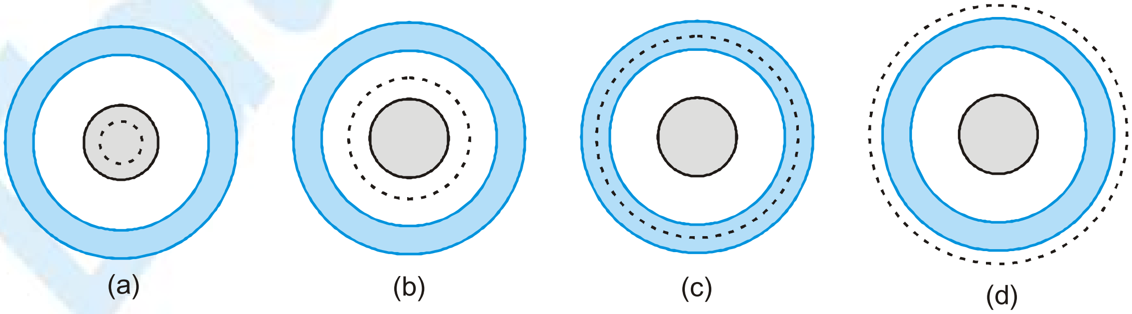
(b) If r > a, then net current through the Amperian loop is

$$I = \int_a^r Kr^2 2\pi rdr = \frac{\pi K a^4}{2}$$

$\Rightarrow \quad B = \frac{\mu_0 K a^4}{4r}$

Ex. Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in same direction. Find the magnetic field at a distance x from the axis where **(a)** x < a, **(b)** a < x < b **(c)** b < x < c and **(d)** x > c. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Sol.



A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B}.d\vec{\lambda} = B2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle in part b is i_0 so that

$$\frac{i_0}{\pi a^2}\,.\,\pi x^2 = \frac{i_0}{a^2}x^2.$$

Ampere's law

$\oint \vec{B}.d\vec{\lambda} = \mu_0 i$ gives

$$B.2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or, } B = \frac{\mu_0 i_0 x}{2\pi a^2}.$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is i_0 so that

$$B\,2\pi x = \mu_0\, i_0 \qquad \text{or,} \qquad B = \frac{\mu_0 i_0}{2\pi x}.$$

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$.

Thus, the current through this part is $\frac{i_0(x^2-b^2)}{(c^2-b^2)}$. This is in the same direction to the current i_0 in the inner wire.

Thus, the net current enclosed by the circle is

$$i_{net} = i_0 + \frac{i_0(x^2-b^2)}{c^2-b^2} = \frac{i_0(c^2+x^2-2b^2)}{c^2-b^2}.$$

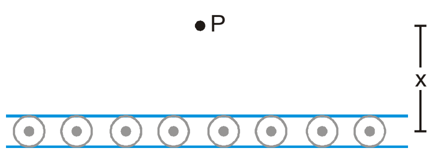
From Ampere's law,

$$B\,2\pi x = \frac{i_0(c^2+x^2-2b^2)}{c^2-b^2} \qquad \text{or,} \qquad B = \frac{\mu_0 i_0(c^2+x^2-2b^2)}{2\pi x(c^2-b^2)}$$

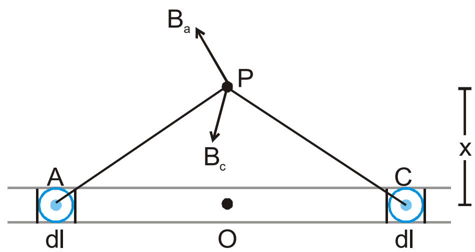
(d) The net current enclosed by the circle in part d of the figure is $2i_0$ and hence

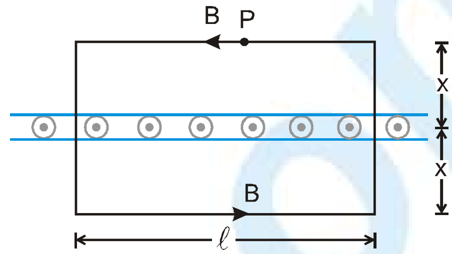
$$B\,2\pi x = \mu_0 2i_0 \qquad \text{or,} \qquad B = \frac{\mu_0 i_0}{\pi x}\text{-}.$$

Ex. Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is λdl where λ is a constant. Find the magnetic field at a point P at a distance y from the metal sheet.



Sol. Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is B_0 perpendicular to AP and that due to the strip C is B_C perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.



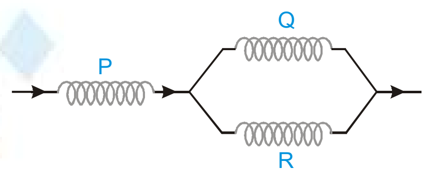


The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

$$2B\bullet = \mu_0 \lambda \bullet \qquad \text{or,} \qquad B = \frac{1}{2}\mu_0 \lambda.$$

Note that it is independent of y.

Ex. Three identical long solenoids P, Q and R are connected to each other as shown in figure. If the magnetic field at the centre of P is 4 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.



Sol. As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be 2T.

Magnetic Field due to long Solenoid :

A solenoid is a long coil of wire tightly wound in the helical form. Figure 9.4.1 shows the magnetic field lines of a solenoid carrying a steady current *I*. We see that if the turns are closely spaced, the resulting magnetic field inside the solenoid becomes fairly uniform, provided that the length of the solenoid is much greater than its diameter. For an "ideal" solenoid, which is infinitely long with turns tightly packed, the magnetic field inside the solenoid is uniform and parallel to the axis, and vanishes outside the solenoid.

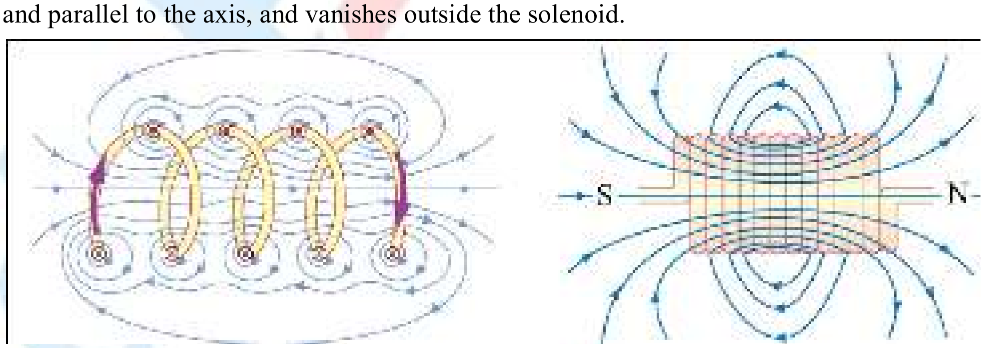


Figure : Magnetic field lines of a solenoid

We can use Ampere's law to calculate the magnetic field strength inside an ideal solenoid. The cross-sectional view of an ideal solenoid is shown in Figure. To compute $\vec{B}$, we consider a rectangular path of length *l* and width *w* and traverse the path in a counterclockwise manner. The line integral of $\vec{B}$ along this loop is

$$\oint \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s}$$

$$= 0 + 0 + Bl + 0$$

In the above, the contributions along sides 2 and 4 are zero because $\vec{B}$ is perpendicular to $d\vec{s}$. In addition, $\vec{B} = \vec{0}$ along side 1 because the magnetic field is non-zero only inside the solenoid. On the other hand, the total current enclosed by the Amperian loop is I_{enc} = n/I, where *n* is the total number of turns per unit length. Applying Ampere's law yields

$$\oint \vec{B}\cdot d\vec{s} = Bl = \mu_0 nl\,I \qquad\qquad B = \mu_0 nI$$

Magnetic Field due to Toroid

Consider a toroid that consists of *N* turns, as shown in Figure. Find the magnetic field everywhere.

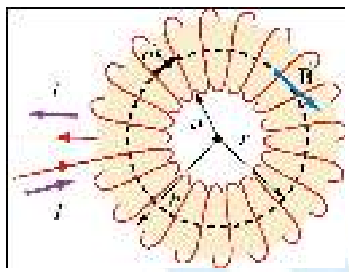


Figure : A toroid with *N* turns

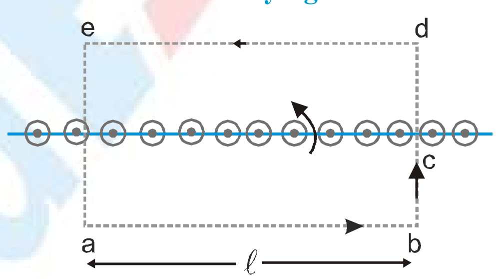
One can think of a toroid as a solenoid wrapped around with its ends connected. Thus, the magnetic field is completely confined inside the toroid and the field points in the azimuthal direction (clockwise due to the way the current flows, as shown in Figure.)

Applying Ampere's law, we obtain

$$\oint \vec{B}\cdot d\vec{s} = \oint B\,ds = B\oint ds = B(2\pi r) = \mu_0 nI \quad \text{or} \quad B = \frac{\mu_0 NI}{2\pi r}$$

where *r* is the distance measured from the center of the toroid. Unlike the magnetic field of a solenoid, the magnetic field inside the toroid is non-uniform and decreases as $1/r$.

Magnetic Field thin sheet of infinite dimension carrying a current of uniform linear current density i



$$\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^e \vec{B}\cdot d\vec{l} + \int_e^f \vec{B}\cdot d\vec{l} + \int_f^g \vec{B}\cdot d\vec{l}$$

$$= \int_a^b B.\cos\theta.dl + 0 + 0 + \int_d^e Bdl + 0 + 0 = \int_a^b Bdl + B\int_d^e dl = Bl + Bl = 2Bl$$

$$\sum I = il \quad \Rightarrow \quad \oint \vec{B}.\vec{dl} = \mu_0 \sum I_{en}$$

$$\Rightarrow 2BI = \mu_0 il \quad \Rightarrow \quad B = \frac{\mu_0 I}{2}$$

Magnetic Field of a Moving Point Charge :

Suppose we have an infinitesimal current element in the form of a cylinder of cross-sectional area A and length ds consisting of n charge carries per unit volume, all moving at a common velocity $\vec{v}$ along the axis of the cylinder. Let I be the current in the element, which we define as the amount of change passing through any cross-section of the cylinder per unit time. We see that the current I can be written as

$$nAq|\vec{v}| = I$$

The total number of charge carries in the current element is simply dN = nAds, so that the magnetic field $d\vec{B}$ due to the dN charge carries is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(nAq|\vec{v}|)d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(nAds)q\vec{v} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(dN)q\vec{v} \times \hat{r}}{r^2}$$

where r is the distance between the charge and the field point P as which the field is being measured, the unit vector $\hat{r} = \vec{r}/r$ points from the source of the field (the charge) to P. To differential length vector $d\vec{s}$ is defined to be parallel to $\vec{v}$. In case of a single charge, dN = 1, the above equation becomes

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

Magnetic Force on Moving Charge

When a charge q moves with velocity $\vec{V}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula :

$\vec{F} = q(\vec{V} \times \vec{B})$ Put q with sign.

$\vec{V}$: Instantaneous velocity

$\vec{B}$: Magnetic f

(i) $\vec{F} \perp \vec{V}$ and also $\vec{F} \perp \vec{B}$

(ii) $\because \vec{F} \perp \vec{V}$ $\therefore$ power due to magnetic force on a charged particle is zero. (use the formula of power $P = \vec{F}.\vec{V}$ for its proof).

(iii) Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.

(iv) On a stationary charged particle, magnetic force is zero.

(v) If $\vec{V} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

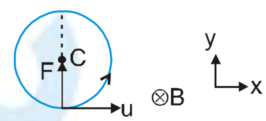
Ex. A charged particle of mass 5 mg and charge q = +2μC has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

Sol. $\vec{F} = q\vec{v} \times \vec{B} = 2 \times 10^{-6}(2\hat{i} - 3\hat{j} \times 4\hat{k}) \times (3\hat{j} - 2\hat{k}) = 2 \times 10^{-6}[-6\hat{i} + 4\hat{j} + 6\hat{k}]$ N

By Newton's Law $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{2 \times 10^{-6}}{5 \times 10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k}) = 0.8(-3\hat{i} + 2\hat{j} + 3\hat{k})$ m/s^2

Motion of charged particles under the effect of magnetic force

- Particle released if v = 0 then $f_m = 0$
  $\therefore$ particle will remain at rest
- $\vec{V} \parallel \vec{B}$ here θ = 0 or θ = 180º
  $\therefore F_m = 0$ $\therefore \vec{a} = 0$ $\therefore \vec{V}$ = const.
  $\therefore$ particle will move in a straight line with constant velocity
- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform



In this case ∵ B is in z direction so the magnetic force in z-direction will be zero $(\because \vec{F}_m \perp \vec{B})$.

Now there is no initial velocity in z-direction.

$\therefore$ particle will always move in xy plane.

$\therefore$ velocity vector is always $\perp \vec{B}$ $\therefore$ F_m = quB = constant

Now $quB = \dfrac{mu^2}{R} \Rightarrow R = \dfrac{mu}{qB}$ = constant.

The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle.

$\therefore$ path of the particle is circular.

$R = \dfrac{mu}{qB} = \dfrac{p}{qB} = \dfrac{\sqrt{2mk}}{qB}$

Here p = linear momentum ; k = kinetic energy

Now $v = \omega R \Rightarrow \omega = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f$

Time period T = 2πm/qB

frequency f = qB/2πm

Note : ω, f, T are independent of velocity.

Ex. A proton (p), α-particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

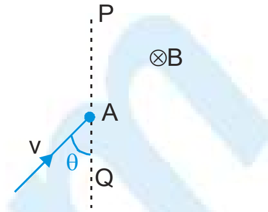
Sol. $R = \dfrac{\sqrt{2mK}}{qB}$

$\therefore$ $R_p : R_\alpha : R_D = \dfrac{\sqrt{2mK}}{qB} : \dfrac{\sqrt{2.4mK}}{2qB} : \dfrac{\sqrt{2.2mK}}{qB} = 1 : 1 : \sqrt{2}$

T = 2πm/qB

$\therefore$ $T_p : T_\alpha : T_D = \dfrac{2\pi m}{qB} : \dfrac{2\pi 4m}{2qB} : \dfrac{2\pi 2m}{qB} = 1 : 2 : 2$ **Ans.**

Ex. A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ.



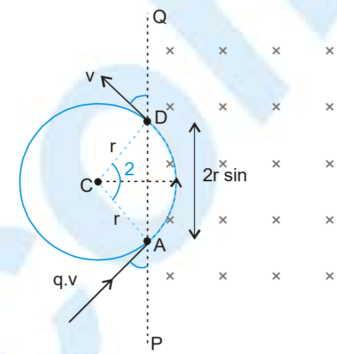
Find **(i)** time spent,

(ii) distance travelled in the magnetic field

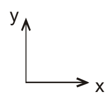
(iii) impulse of magnetic force.

Sol. The particle will move in the field as shown

Angle subtended by the arc at the centre = 2θ



(i) Time spent by the charge in magnetic field



$$\omega t = \theta \;\Rightarrow\; \frac{qB}{m}\,t = \theta \;\Rightarrow\; t = \frac{m\theta}{qB}$$

(ii) Distance travelled by the charge in magnetic field :

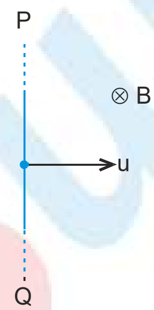
$$= r(2\theta) = \frac{mv}{qB}\,.\,2\theta$$

(iii) Impulse = change in momentum of the charge

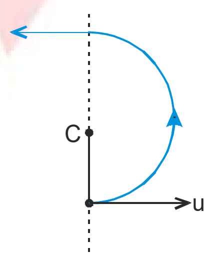
$$= (-mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}) - (mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j})$$

$$= -2mv\sin\theta\,\hat{i}$$

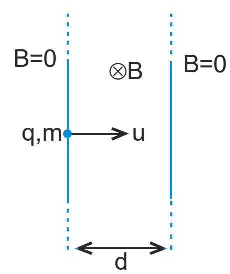
Ex. In the figure shown the magnetic field on the left on 'PQ' is zero and on the right of 'PQ' it is uniform. Find the time spent in the magnetic field.



Sol. The path will be semicircular time spent = T/2 = πm/qB



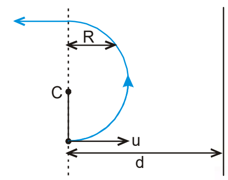
Ex. A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in the figure) into the magnetic field. Find the time spend by the particle in the magnetic field if



(i) $d > \frac{mu}{qB}$ **(ii)** $d < \frac{mu}{qB}$

Sol. **(i)** $d > \frac{mu}{qB}$ means $d > R$

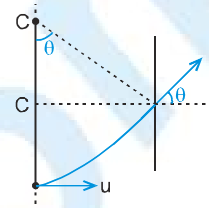
$\therefore \; t = \frac{T}{2} = \frac{\pi m}{qB}$



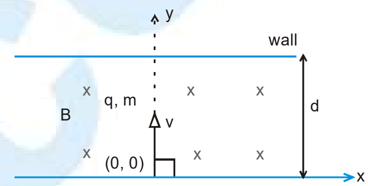
(ii) $\sin\theta = \frac{d}{R}$

$\theta = \sin^{-1}\left(\frac{d}{R}\right)$

$\omega t = \theta \Rightarrow t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$



Ex. What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.

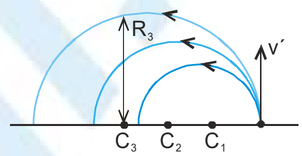


Sol. **(i)** The path of the particle will be circular larger the velocity, larger will be the radius.

For particle not to s

strike $R < d$

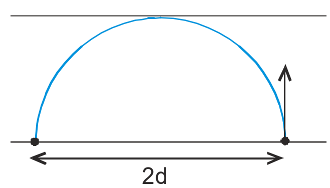
$\therefore \quad \frac{mv}{qB} < d \Rightarrow \quad v < \frac{qBd}{m}$.



(ii) for limiting case $v = \frac{qBd}{m}$

$R = d$

$\therefore$ coordinate = (–2d, 0, 0)

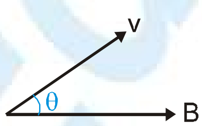


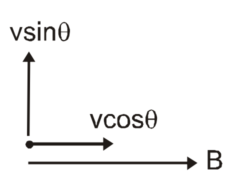
Helical path

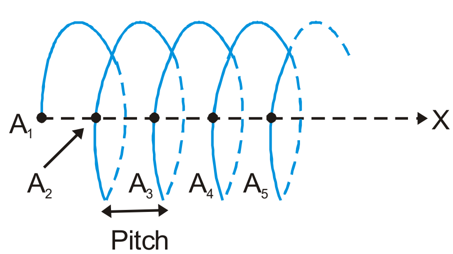
If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components – $v_{\parallel}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The components $v_{\parallel}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Complete analysis

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.







The particle starts from point A_1.

It completes its one revolution at A_2 and 2^{nd} revolution at A_3 and so on. X-axis is the tangent to the helix points A_1, A_2, A_3,..........all are on the x-axis.

distance $A_1A_2 = A_3A_4 = ... = v \cos\theta.\ T$ = pitch

where T = Time period

Let the initial position of the particle be (0,0,0) and v sin θ in +y direction. Then

in x : $F_x = 0,\ a_x = 0,\ v_x = \text{constant} = v\cos\theta,\ x = (v\cos\theta)t$

In y-z plane

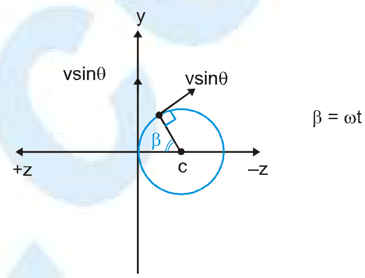
From figure it is clear that

$y = R\sin\beta,\ v_y = v\sin\theta\cos\beta$

$z = -(R - R\cos\beta)$

$v_z = v\sin\theta\sin\beta$

acceleration towards centre = $(v\sin\theta)^2/R = \omega^2 R$



$\therefore \quad a_y = -\omega^2 R\sin\beta,\ a_z = -\omega^2 R\cos\beta$

At any time : the position vector of the particle

(or its displacement w.r.t. initial position)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}\text{, x,y,z already found}$$

velocity $\quad \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, v_x, v_y, v_z already found

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, a_x, a_y, a_z already found

Radius $\quad q(v\sin\theta)B = \dfrac{m(v\sin\theta)^2}{R} \Rightarrow R = \dfrac{mv\sin\theta}{qB} \Rightarrow \omega = \dfrac{v\sin\theta}{R} = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f.$

Ex. A charged particle P leaves the origin with speed $v = v_e$, at some inclination with the x-axis. There is uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. Find the condition so that P will also strike if you change magnetic field and speed.

Sol. Let d = distance of the tangent T from the point of projection. P will strike T if d an integral multiple of the pitch.

Pitch

$$\left(2\pi\frac{m}{qB_0}\right)v_0\cos\theta = N\left(2\pi\frac{m}{qB}\right)v\cos\theta$$

Here N is a natural number.

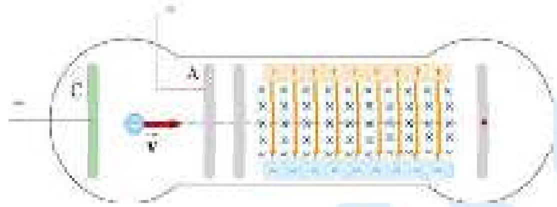
Lorentz Force

In the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force on a charged particle is

$$\vec{F} = q(\vec{E} + \vec{v} + \vec{B})$$

This is known as the Lorentz force.

Velocity Selector



By combining the two fields, particles which move with a certain velocity can be selected. This was the principle used by J.J. Thomson to measure the charge-to-mass ratio of the electrons. In figure the schematic diagram of Thomson's apparatus is depicted.

The electrons with charge $q = -e$ and mass m are emitted from the cathode C and then accelerated toward slit A. Let the potential difference between A and C be $V_A - V_C = \Delta V$. The change in potential energy is equal to external work done in accelerating the electrons : ΔU - $W_{ext} = q\Delta V = -e\Delta V$. By energy conservation, the kinetic energy gained is $\Delta K = -\Delta U = mv^2/2$. Thus, the speed of the electrons is given by

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

If the electrons further pass through a region where there exists a downward uniform electric field, the electrons, being negatively charged, will be deflected upward. However, if in addition to the electric field, a magnetic field directed into the page is also applied, then the electrons will experience an additional downward magnetic force $-e\vec{v} \times \vec{B}$. When the two forces exactly cancel, the electrons will move in a straight path. From Eq., we see that when the condition for the cancellation of the two forces is given by eE = evB. which implies

$$v = \frac{E}{B}$$

In other words, only those particles with speed v = E/B will be able to move in a straight line. Combining the two equations, we obtain $\frac{e}{m} = \frac{E^2}{2(\Delta V)B^2}$

By measuring, E, ΔV and B, the charge-to-mass ratio can be readily determined. The most precise measurement to date is $e/m = 1.758820174(71) \times 10^{11}$ C/kg.

Mass Spectrometer

Various methods can be used to measure the mass of an atom. One possibility is through the use of a mass spectrometer. The basic feature of a Bainbridge mass spectrometer is illustrated in Figure. A particle carrying a charge +q is first sent through a velocity selector.

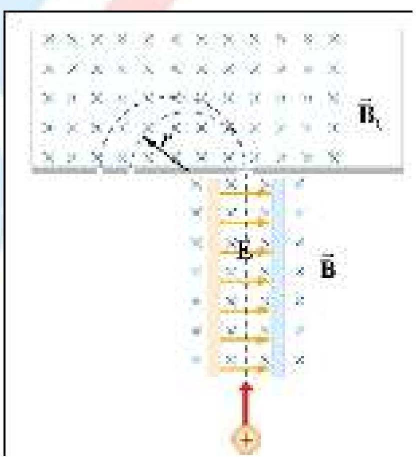


Figure : A Bainbridge mass spectrometer

The applied electric and magnetic field satisfy the relation E = vB so that the trajectory of the particle is a straight line. Upon entering a region where a second magnetic field $\vec{B}_0$ pointing into the page has been applied, the particle will move in a circular path with radius r and eventually strike the photographic plate. Using Eq., we have

$$r = \frac{mV}{qB_0}$$

Since v = E/B, the mass of the particle can be written as $m = \frac{qB_0 r}{v} = \frac{qB_0 Br}{E}$

Hall's Effect

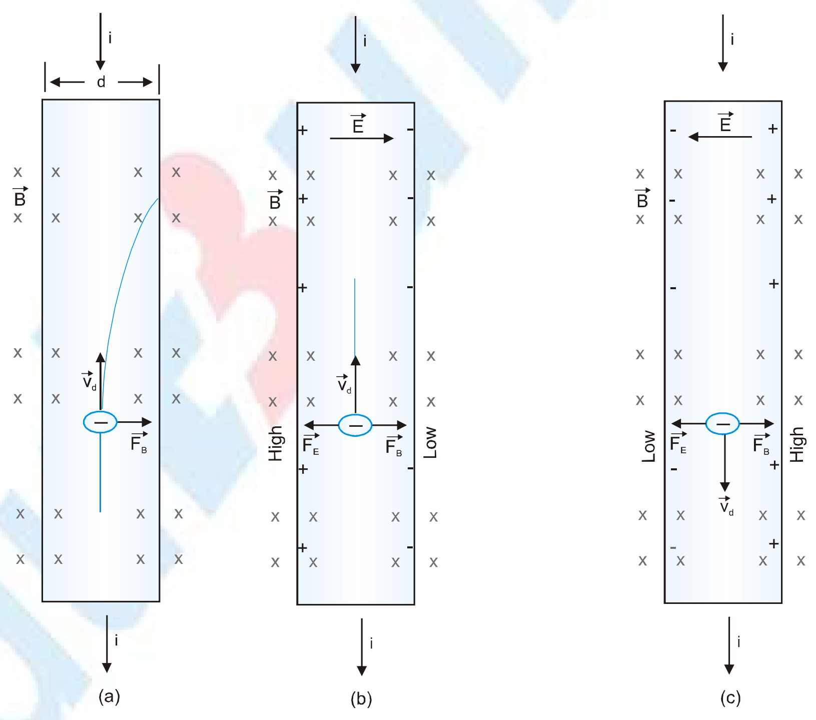
In 1879, Edwin H.Hall, then a 24 -year-old graduate student at the Johns Hopkins University, showed that they can drift electrons in copper wire in presence of magnetic field. This Hall effect allows us to find

(a) If charge carries in a conductor are positively or negatively charged.

(b) The number of charge carries per unit volume of the conductor.

Consider a strip of current carrying wire kept in external magnetic field. Let the wire has width d, Cross-sectional area A, and charge carries per unit volume as n.

Figure **(a)** shows a copper strip of width d, carrying a current i whose conventional direction is from the top of the figure to the bottom. The charge carries are electrons and, as we know, they drift (with drift speed vd) in the opposite direction, from bottom to top. At the instant shown in figure, an external magnetic field B, pointing into the plane of the figure, has just been turned on. We see that a magnetic force will act on each drifting electron, pushing it towards the right edge of the strip.



As time goes on, electrons move to the right, mostly piling up on the right edge of the strip, leaving uncompensated positive charges in fixed position at the left edge. The separation of positive charges on the left edge and negative charges on the right edge produces an electric field E within the strip, pointing from left to right in Fig. b. This field exerts an electric force FE on each electron, tending to push it to the left. Thus, this electron force on the electrons, which opposes the magnetic force on them, begins to build up.

Equilibrium quickly develops in which the electric force on each electron has increased enough to match the magnetic force. When this happens, as Fig. b shows, the force due to B and the force due to E are in balance. The drifting electrons then move along the strip toward the top of the page at velocity vd with no further collection of electrons on the right edge of the strip and thus no further increase in the electric field E.

$$eE = ev_dB \qquad \text{------ (2)}$$

$$v_d = \frac{J}{ne} = \frac{i}{neA} \qquad \text{------ (2)}$$

A Hall potential difference V is associated with the electric field across strip width d.

$$V = Ed \qquad \text{-------- (3)}$$

From **(2)**, (2) and **(3)**

$$\frac{E}{B} = \frac{V}{Bd} = \frac{i}{neA}$$

$$\therefore n = \frac{idB}{eAV} \qquad \& \qquad v_d = \frac{i}{neV} = \frac{V}{Bd}$$

By connecting a voltmeter across the width, we can measure the potential difference between the two edges of the strip. Moreover, the voltmeter can tell us which edge is at higher potential. For the situation of Fig. B, we would find that the left edge is at higher potential, which is consistent with our assumption that the charge carries are negatively charged.

It is also possible to use the Hall effect to measure directly the drift speed vd of the charge carries, which you may recall is of the order of centimeters per hour. In this clever experiment, the metal strip is moved mechanically through the magnetic field in a direction opposite that of the drift velocity of the charge carries. The speed of the moving strip is then adjusted until the Hall potential difference vanishes.

As this condition, with no hall effect, the velocity of the charge carries with respect to the laboratory frame must be zero, so the velocity of the strip must be equal in magnitude but opposite the direction of the velocity of the negative charge carries.

For a moment, let us make the opposite assumption, that the charge carries in current i are positively charged (Fig. c).

Convince yourself that as these charge carries move from top to bottom in the strip, they are pushed to the right edge by F_B and thus that the right edge is at higher potential. Because that last statement is contradicted by our voltmeter reading, the charge carries must be negatively charged.

Ex. A non-relativistic proton beam passes without deviation through the region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with E = –120 kV/m and B = 50mT. Then the beam strikes a grounded target. Find the force with which the beam acts on the target of the beam current is equal to I =0.80 mA.

Sol. $F = \frac{dp}{dt} = v\frac{dm}{dt} = v\frac{dm}{dq}\frac{dq}{dt} = \frac{E}{B}\frac{m}{q}I = 20\ \mu\text{N}$

Ex. A particle of mass m and charge q is released from the origin in a region occupied by electric field E and magnetic field B,

$$B = B_0\hat{j}, E = E_0\hat{i}$$

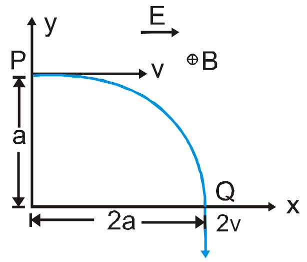
Find the speed of the particle.

Sol. Since the magnetic field does not perform any work, therefore, whatever has been gain in kinetic energy it is only because of the work done by electric field. Applying work-energy theorem.

$$W_E = \Delta K$$

$$qE_0 = \frac{1}{2}mv^2 - 0 \qquad \text{or} \qquad v = \sqrt{\frac{2qE_0}{m}}$$

Ex. A particle of charge +q and mass m moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$ follows a trajectory from P to Q as shown in figure. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{i}$. Find **(a)** E **(b)** rate of work done by the electric field at P. **(C)** rate of work done by each the fields at Q.



Sol. Increase in Kinetic energy of particle

$$= \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{3}{2}mv^2$$

Work done by the uniform electric field, E, in going from P to Q = (qE) × 2a = 2qEa

Hence, $2qEa = \frac{3}{2}mv^2$ or $E = \frac{3mv^2}{4qa}$

Rate of work done by the electric field at

P_{at} P = F. v = qE . v

$$= qE\hat{i}.v\hat{i} = qEv = q.\frac{3mv^2}{4qa}.v = \frac{3}{4}\frac{mv^2}{a}$$

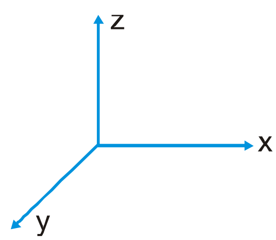
Q is

$$P_{at}.Q = qE\hat{i}.(-2v\hat{j}) = 0$$

At Q, rate of work done by both the fields is zero.

Ex. A particle of mass 1 × 10^{-26} kg and charge 1.6 × 10^{-19} C travelling with a velocity 1.28 × 10^6 ms^{-1} in the +x direction enters a region in which a uniform electric field E and uniform magnetic field of induction B are present such that $E_x = E_y = 0$, $E_x = -102.4$ kVm^{-1} and $B_x = B_z = 0$. $B_y = 8 \times 10^{-2}$ Wbm^{-2}. The particle enters this region at the origin at time t = 0. Determine the location (x, y and z coordinates) of the particle at t = 5 × 10^{-4} s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at t = 7.45 × 10^{-6} s ?

Sol. Let $\hat{i}$, $\hat{j}$ and $\hat{k}$ be unit vector along the positive directions of x, y and z axes. Q = charge on the particle = 1.6 × 10^{-19} C $\vec{v}$ = velocity of the charged particle = (1.28 × 10^6) ms^{-1} $\hat{i}$



$\vec{E}$ = electric field intensity :

$= (-102.4 \times 10^3\ Vm^{-1})\ \hat{k}$

$\vec{B}$ = magnetic induction of the magnetic field

$= (8 \times 10^{-2}\ Wbm^{-2})\ \hat{j}$

∴ $\vec{F}_e$ = electric force on the charge $= q\vec{E}$ = [1.6 × 10^{-19} (–102.4 × 10^3) Nj $\hat{k}$

= 163.84 × 10^{-16} N($-\hat{k}$)

$\vec{F}_m$ = magnetic force on the charge = $q\vec{v} \times \vec{B}$

= [1.6 × 10^{-19} (1.28 × 10^6) (8 × 10^{-2}) N] ($\hat{i} \times \hat{j}$) = (163.84 × 10^{-16} N)($\hat{k}$)

The two forces $\vec{F}_e$ and $\vec{F}_m$ are along z-axis and equal, opposite and collinear.

The net force on the charge is zero and hence the particle does not get deflected and continues to travel along x–axis.

(a) At time t = 5 × 10^{-6} s

$$x = (5 \times 10^{-6})\,(1.28 \times 10^{6}) = 6.4 \text{ m}$$

∴ Coordinates of the particle = (6.4 m, 0, 0)

(b) When the electric field is switched off, the particle is in the uniform magnetic field perpendicular to its velocity only and has a uniform circular motion in the x–z plane (i.e. the plane of velocity and magnetic force), anticlockwise as seen along +y axis.

Now, $\frac{mv^2}{r} = qvB$ where r is the radius of the circle.

$$\therefore \qquad r = \frac{mv}{qB} = \frac{(1\times10^{-26})(1.28\times10^{6})}{(1.6\times10^{-19})(8\times10^{-2})}$$

The length of the arc traced by the particle in [(7.45 – 5) × 10^{-6} s]

$$= (v)(t) = (1.28 \times 10^{6})(2.45 \times 10^{-6})$$

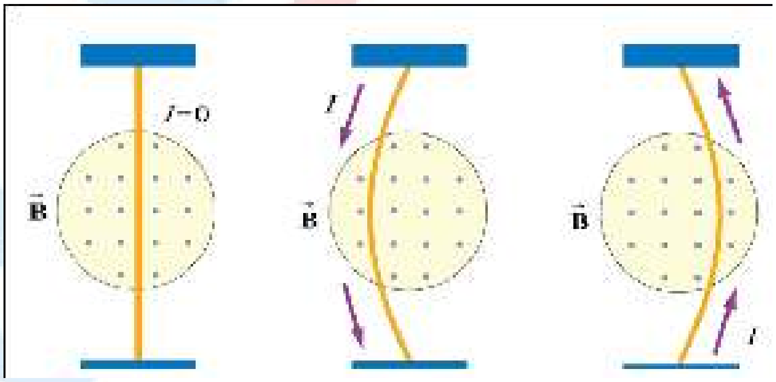
$$= 3.136 \text{ m} = \pi m = \frac{1}{2} \text{ circumference}$$

∴ The particle has the coordinates (6.4, 0, 2m) as (x, y, z)

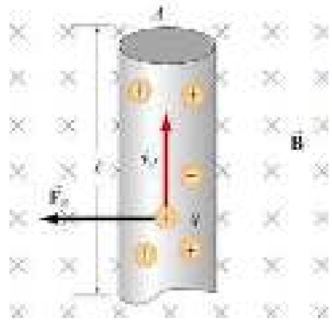
Magnetic Force on a Current-Carrying Wire :

We have just seen that a charged particle moving through a magnetic field experiences a magnetic force $\vec{F}_B$. Since electric current consists of a collection of charged particles in motion, when placed in a magnetic field, a current-carrying wire will also experience a magnetic force.

Consider a long straight wire suspended in the region between the two magnetic poles. The magnetic field points out the page and is represented with dots (.). It can be readily demonstrated that when a downward current passes through, the wire is deflected to the left. However, when the current is upward, the deflection is right-ward, as shown in figure.



To calculate the force exerted on the wire, consider a segment of wire of length ℓ and cross-sectional area A, as shown in Figure. The magnetic field points into the page, and is represented with crosses (X).



The charges move at an average drift velocity $\vec{v}_d$. Since the total amount of charge in this segment is $Q_{tot} = q(nA\ell)$, where n is the number of charges per unit volume, the total magnetic force on the segment is

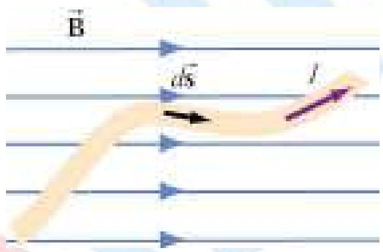
$$\vec{F}_B = Q_{tot}\vec{v}_d \times \vec{B} = qnAl\,(\vec{v}_d \times \vec{B}) = I(\vec{l} \times \vec{B})$$

where $I - nqv_dA$ and $\vec{l}$ is a length vector with a magnitude ℓ and directed along the direction of the electric current.

Special Case - 1 :

Wire of arbitary shape placed in uniform magnetic field

For a wire of arbitary shape, the magnetic force can be obtained by summing over the forces acting on the small segments that make up the wire. Let the differential segment be denoted as $d\vec{s}$ (Figure)

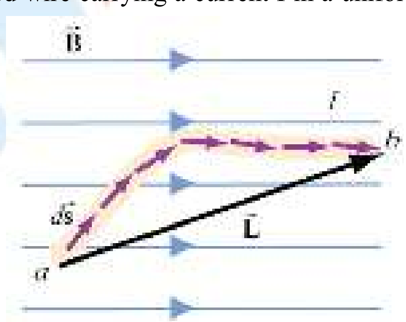


The magnetic force acting on the segment is : $d\vec{F}_B = Id\vec{s} \times B$

Thus, the total force is : $\vec{F}_B = I\int_a^b d\vec{s} \times B$

where a and b represent the endpoints of the wire.

As an example, consider a curved wire carrying a current I in a uniform magnetic field $\vec{B}$, as shown in Figure.



Using the magnetic force on the wire is given by

$$\vec{F}_B = \left(I\int_a^b d\vec{s}\right) \times \vec{B} = I\vec{l} \times \vec{B}$$

where $\vec{l}$ is the length vector directed from a to b. However, if the wire forms a closed loop of arbitary shape (Figure), then the force on the loop becomes

$$\vec{F}_B = I\left(\oint d\vec{s}\right)\times \vec{B}$$

Special Case - 2 :

Magnetic force on a closed loop in uniform magnetic field.

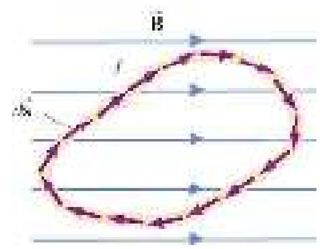
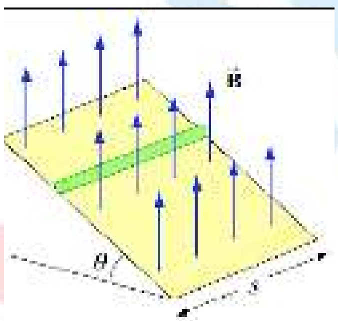


Figure : A closed loop carrying a current I in a uniform magnetic field

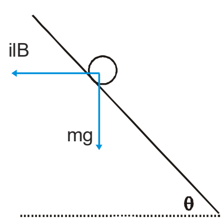
Since the drift of differential length elements $d\vec{s}$ from a closed polygon, and their vector sum is zero, i.e. $\oint d\vec{s} = 0$. The net magnetic force on a closed loop is $\vec{F}_B = \vec{0}$.

Ex. A conducting bar of length is placed on a frictionless inclined plane which is titled at an angle q from the horizontal, as shown in figure.



A uniform magnetic field is applied in the vertical direction. To prevent the bar from sliding down, a voltage source is connected to the ends of the bar with current flowing through. Determine the magnitude and the direction of the current such that the bar will remain stationary.

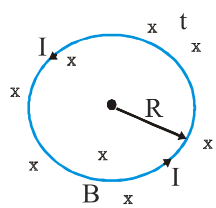
Sol. For equilibrium



$$Il\,B\cos\theta = mg\sin\theta$$

$$\Rightarrow \qquad I = \frac{mg\sin\theta}{l\,B\cos\theta}$$

Ex. A current (I) carrying circular wire of radius R is placed in a magnetic field B perpendicular to its plane. Find the tension T along the circumference of the wire.

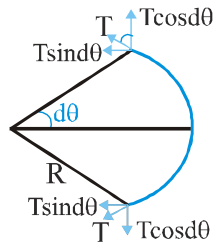


Sol. For small element portion

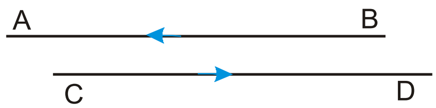
$2T = d\theta\, IB$

$2Td\theta = 2RIBd\theta$

$T = IRB$

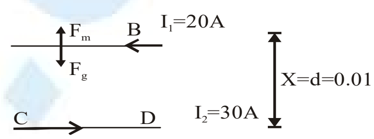


Ex. A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillation.



Sol. Let m be the mass per unit length of wire AB. At a height x about the wire CD, magnetic force per unit length on wire AB will be given by

$$F_m = \frac{\mu_0 i_1 i_2}{2\pi x} \text{ (upwards)} \qquad \text{..... (i)}$$



Wt. per unit of wire AB is

$F_g = mg$ (downwards)

At x = d, wire is in equilibrium

i.e., $F_m = F_g \Rightarrow \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$

$$\Rightarrow \frac{\mu_0 i_1 i_2}{2\pi d^2} = \frac{mg}{d} \qquad \text{..... (ii)}$$

When AB is depressed, x decreases therefore, F_m will increase, while F_g remains the same. Let AB is displaced by dx downwards. Differentiating equation **(i)** w.r.t. x, we get

$$dF_m = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2}.dx \qquad \text{..... (iii)}$$

i.e., restoring force, F = d $\quad F_m \propto -dx$

Hence the motion of wire is simple harmonic. From equation **(ii)** and **(iii)**, we can write

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad (x = d)$$

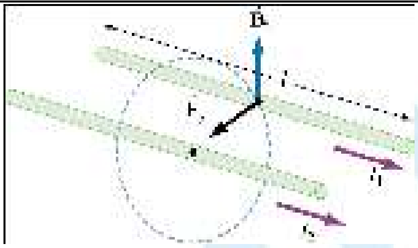
$\therefore$ Acceleration of wire, $a = -\left(\frac{g}{d}\right).dx$

Hence period of oscillations

$$T = 2\pi\sqrt{\frac{dx}{a}} = 2\pi\sqrt{\left|\frac{\text{disp.}}{\text{acc.}}\right|}$$

$$\Rightarrow T = 2\pi\sqrt{d/g} = 2\pi\sqrt{\frac{0.01}{9.8}} \Rightarrow T = 0.2 \text{ s}$$

Force Between Two Parallel Wires :



We have already seen that a current-carrying wire produces a magnetic field. In addition, when placed in a magnetic field, a wire carrying a current will experience a net force. Thus, we expect two current-carrying wires to exert forces on each other. Consider two parallel wires separated by a distance a and carrying currents I_1 and I_2 in the $+x$-direction, as shown in Figure.

The magnetic force $\vec{F}_{12}$, exerted by wire 2 on wire 1 may be computed as follows. Using the result from the previous example, the magnetic field lines due to I_2 going in the $+x$ direction are circles concentric with wire 2, with the field $\vec{B}_2$ pointing in the tangential direction. Thus, at an arbitrary point P on wire 1, we have $\vec{B}_2 = -(\mu_0 I_2 / 2\pi a)\hat{j}$ which points in the direction perpendicular to wire 1, as depicted in Figure. Therefore,

$$\vec{F}_{12} = I_1 \vec{l} \times \vec{B}_2 = (l\hat{i}) \times \left(-\frac{\mu_0 I_2}{2\pi a}\hat{j}\right) = -\frac{\mu_0 I_1 I_2 l}{2\pi a}\hat{k}$$

Clearly $\vec{F}_{12}$ points toward wire 2. The conclusion we can draw from this simple calculation is that two parallel wires carrying currents in the same direction will attract each other. On the other hand, if the currents flow in opposite directions, the resultant force will be repulsive.

Definition of ampere

Consider two parallel wires separated by 1 m and carrying a current of 1A each. Then $i_1 = i_2 = 1$A and d = 1m, so that from equation

$$\frac{dF}{dl} = 2 \times 10^{-7}\ \text{N/m}$$

This is used to formally define the unit 'ampere' of electric current. If two parallel, long wires, kept 1 m apart in vacuum, carry equal currents in the same direction and there is a force of attraction of 2×10^{-7} newton per metre of each wire, the current in each wire is said to be 1 ampere.

Magnetic Moment

Magnetic field (at large distances) due to current in circular current loop is very similar in behaviour to the electric field of an electric dipole. We know that the magnetic field on the axis of a circular loop, of a radius R, carrying a steady current I.

$$B = \frac{\mu_0 I (2\pi a^2)}{4\pi (a^2 + x^2)^{3/2}}$$

its direction is along the axis and given by the right-hand thumb rule. Here, x is the distance along the axis from the centre of the loop.

For x > > R, we may drop the R^2 term in the denominator. Thus

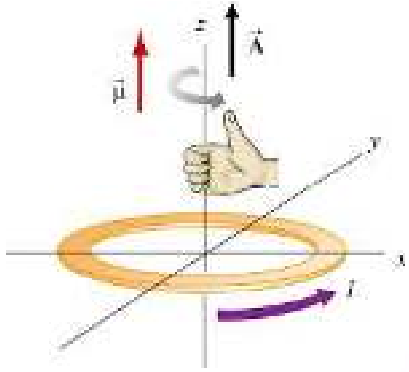
$$B = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IA}{x^2}\right)$$

Where $A = \pi R^2$ = area of the loop

The expression is very similar to an expression obtained earlier for the electric field of a dipole. The similarly may be seen if we can define the magnetic dipole moment $\vec{\mu}$ as

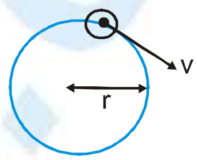
$$\vec{\mu} = I\vec{A}$$

The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule (Figure). The SI unit for the magnetic dipole moment is ampere-meter^2 (A.m^2).



Ex. Find the magnetic moment of an electron orbiting in a circular orbit of radius r with a speed v.

Sol. Magnetic moment $\mu = iA$



we can write $\quad i = \dfrac{e}{T} = \dfrac{e}{\frac{2\pi r}{v}} = \dfrac{ev}{2\pi r}$

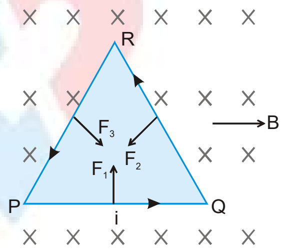
A = area of the loop = πr^2

$$\Rightarrow \quad \mu = (I)\left(\frac{ev}{2\pi r}\right)(\pi r^2)$$

$$\Rightarrow \quad \mu = \frac{evr}{2}$$

Ex. A wire is bent in the form of an equilateral triangle PQR of side 20 cm and carries a current of 2.5 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

Sol. : Suppose the field and the current have directions as shown in figure. The force on PQ is



$\vec{F}_1 = i\vec{\ell} \times \vec{B}$ or, $\quad F_1 = 2.5\text{ A} \times 20\text{ cm} \times 2.0\text{ T} = 1.0\text{ N}$

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

Ex. Figure shows two long metal rails placed horizontally and parallel to each other at a separation y. A uniform magnetic field B exists in the vertically upward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.

(a) What soluble the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the previous part

Sol. **(a)** The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

or, $$F = iyB$$

It acts towards right tin the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to μ_0 mg. Thus,

$$\mu_0 mg = iyB$$

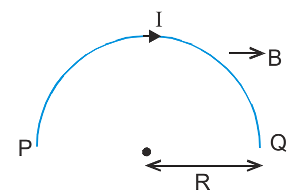
or, $$\mu_0 = \frac{iyB}{mg}$$

(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{iyB}{2mg}$, the wire will slide towards right. The frictional force by the rails is

$$f = \mu mg = \frac{iyB}{2} \text{ towards left.}$$

The resultant force is $iyB - \frac{iyB}{2} = \frac{iyB}{2}$ towards right. The acceleration will be $a = \frac{iyB}{2m}$. The wire will slide towards right with this acceleration.

Ex. In the figure shown a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.

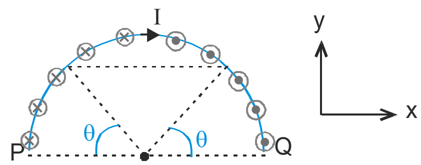


Sol. : The wire is equivalent to

$\because$ $\theta = 0$

$\therefore$ $F_{res} = 0$ **Ans.**

forces on individual parts are marked in the figure by $\otimes$ and $\odot$.
By symmetry their will be pair of forces forming couples.

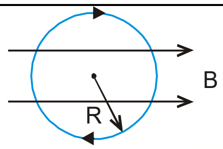


$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta).2R\cos\theta$$

$$\tau = \frac{i\pi R^2}{2}B$$

$\Rightarrow$ $$\vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$$ **Ans.**

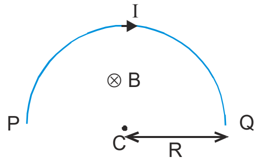
Ex. Find the resultant magnetic force and torque on the loop.



Sol. : $\vec{F}_{res} = 0$, ($\because$ loop)

and $\vec{\tau} = i\pi R^2 B(-\hat{j})$ using the above method

Ex. In the figure shown find the resultant magnetic force and torque about 'C', and 'P'.



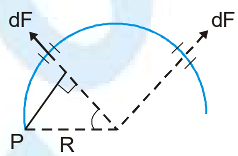
Sol. $\vec{F}_{nett} = I \cdot 2R \cdot B$

$\because$ wire is equivalent to

$$P \xrightarrow{\quad 2R \quad} Q$$

Force on each element is radially outward : $\tau_c = 0$

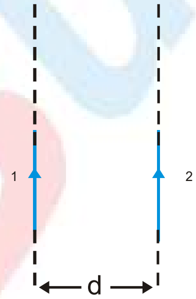
point about



$$P = \int_0^{\pi} [i(Rd\theta)B\sin 90^0]\ R\sin\theta$$

$= 2IBR^2$ **Ans.**

Ex. Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2/2\pi d$. Here it is attractive also.



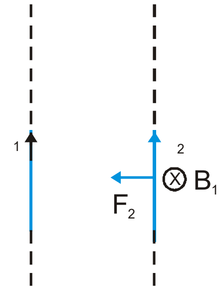
Sol. On (2), B due to **(i)** is $= \dfrac{\mu_0 I_1}{2\pi d} \otimes$

$\therefore$ F on (2) on 1m length

$= I_2 \cdot \dfrac{\mu_0 I_1}{2\pi d} \cdot 1$ towards left it is attractive

$= \dfrac{\mu_0 I_1 I_2}{2\pi d}$ (hence proved)

Similarly on the other wire also.



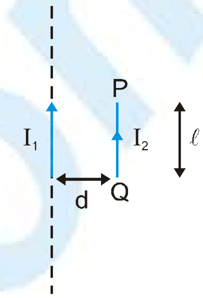
Note :

- Definition of ampere (fundamental unit of current) using the above formula.
  If $I_1 = I_2 = 1A$, $d = 1m$ then $F = 2 \times 10^{-7}$ N

$\therefore$ "When two very long wires carrying equal currents and separated by 1m distance exert on each other a magnetic force of 2×10^{-7} N on 1m length then the current is 1 ampere."

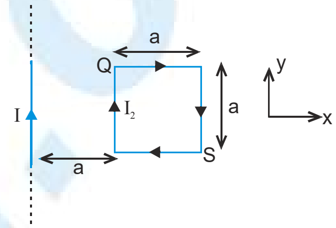
- The above formula can also be applied if to one wire is infinitely long and the other is of finite length. In this case the force per unit length on each wire will not be same.

  Force per unit length on PQ = $\dfrac{\mu_0 I_1 I_2}{2\pi d}$ (attractive)



- If the currents are in the opposite direction then the magnetic force on the wires will be repulsive.

Ex. Find the magnetic force on the loop 'PQRS' due to the loop wire.

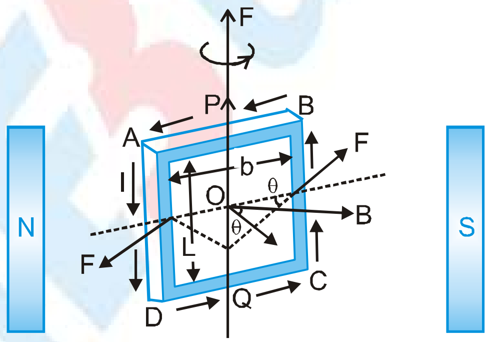


Sol. : $F_{res} = \dfrac{\mu_0 I_1 I_2}{2\pi a}\, a\,(-\hat{i}) + \dfrac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i})$

$= \dfrac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$

Torque on a current loop :

When a current-carrying coil is placed in a uniform magnetic field the net force on it is always zero. However, as its different parts experience forces in different directions so the loop may experience a torque (or couple) depending on the orientation of the loop and the axis of rotation. For this, consider a rectangular coil in a uniform field B which is free to rotate about a vertical axis PQ and normal to the plane of the coil making an angle θ with the field direction as shown in figure **(A)**.



Axis of rotation
(A)

The arms AB and CD will experience forces B(NI)b vertically up and down respectively. These two forces together will give zero net force and zero torque (as are collinear with axis of rotation), so will have no effect on the motion of the coil.

Now the forces on the arms AC and BD will be BINL in the direction out of the page and into the page respectively, resulting in zero net force, but an anticlockwise couple of value

$\tau = F \times \text{Arm} = BINL \times (b \sin\theta)$

i.e. $\tau = BIA \sin\theta$ with $A = NLb$ ....**(i)**

Now treating the current–carrying coil as a dipole of moment $\vec{M} = I\vec{A}$ Eqn. **(i)** can be written in vector form as

$$\vec{\tau} = \vec{M} \times \vec{B} \qquad [\text{with } \vec{M} = I\vec{A} = NIA\hat{n} \text{ ....(ii)}]$$

This is the required result and from this it is clear that :

(2) Torque will be minimum (= 0) when sinθ = min = 0, i.e., θ = 0º, i.e. 180º i.e., the plane of the coil is perpendicular to magnetic field i.e. normal to the coil is collinear with the field [fig. **(A)** and **(C)**]

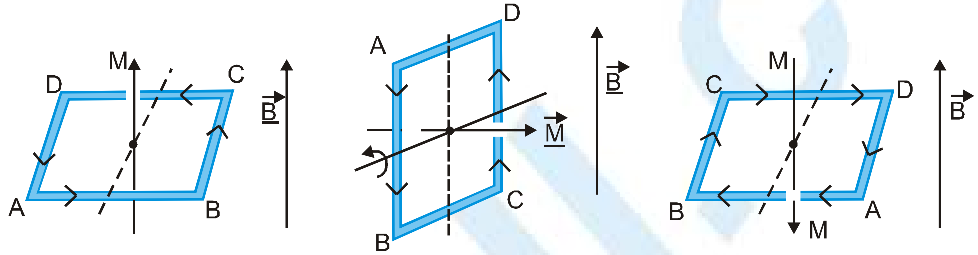
(2) Torque will be maximum (= BINA) when sinθ = max = 1, i.e., θ = 90º i.e. the plane of the coil is parallel to the field i.e. normal to the coil is perpendicular to the field. [fig.**(B)**].

(3) By analogy with dielectric or magnetic dipole in a field, in case of current–carrying in a field.

$$U = -\vec{M} \bullet \vec{B} \qquad \text{with} \qquad F = -\frac{dU}{dr}$$

and $\quad W = MB(1 - \cos\theta)$

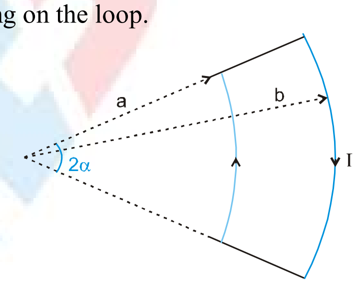
The values of U and W for different orientations of the coil in the field are shown in fig.



θ = 0º ($\vec{M}$ is parallel to $\vec{B}$)	θ = 90º ($\vec{M}$ is ⊥ to $\vec{B}$)	θ = 180º ($\vec{M}$ is antiparallel to $\vec{B}$)
τ = 0 = min	τ = MB = max	τ = 0
W = 0 = min	W = MB	W = 2MB = max
U = –MB = min	U = 0	U = MB = max
Stable equilibrium	No equilibrium	Unstable equilibrium
(A)	(B)	(C)

(4) Instruments such as electric motor, moving coil galvanometer and tangent galvanometers etc. are based on the fact that a current–carrying coil in a uniform magnetic field experiences a torque (or couple).

Ex. A loop with current I is in the field of a long straight wire with current I_0. The plane of the loop is perpendicular to the straight wire. Find torque acting on the loop.

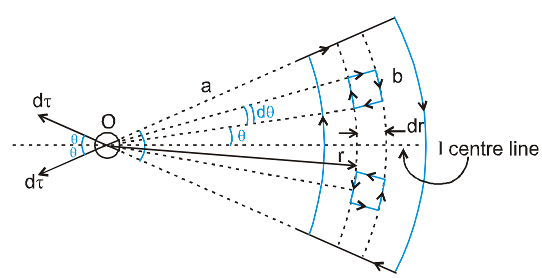


Sol. : $d\vec{s} = (rd\theta\, dr)$ (inwards)

$d\vec{M} = (rI\, d\theta\, dr)$ (inwards)

$\vec{B} = \dfrac{\mu_0 I_0}{2\pi r}$ (tangential clockwise)

$d\tau = |\, d\vec{M} \times \vec{B}\,| = \dfrac{\mu_0 I I_0\, d\theta\, dr}{2\pi}$ (towards centre)



$$\therefore \qquad \tau = \int_{-\alpha}^{\alpha} \int_{a}^{b} d\tau \cos\theta$$

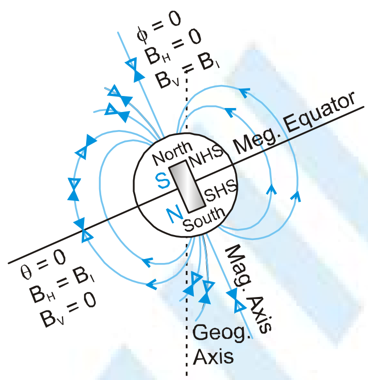
$$= \frac{\mu_0 I I_0}{2\pi} \int_{-\alpha}^{\alpha} \int_{a}^{b} \cos\theta \, d\theta \, dr$$

$$= \frac{\mu_0 I I_0 (b-a)\sin\alpha}{\pi} \qquad \text{(to the left)}$$ **Ans.**

Terrestrial Magnetism (Earth's Magnetism) :

Introduction :

The idea that earth is magnetised was first suggested towards the end of the sixteenth century by Dr William Gilbert. The origin of earth's magnetism is still a mater of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.5º) to the earth's axis of rotation with its south pole pointing north. The lines of force of earth's magnetic field are shown in figure which are parallel to the earth's surface near the equator and perpendicular to it near the poles. While discussing magnetism of the earth one should keep in mind that:



(a) The **magnetic meridian** at a place is not a line but a vertical plane passing through the axis of a freely suspended magnet, i.e., it is a plane which contains the place and the magnetic axis.

(b) The **geographical meridian** at a place is a vertical plane which passes through the line joining the geographical north and south, i.e., it is a plane which contains the place and earth's axis of rotation, i.e., geographical axis.

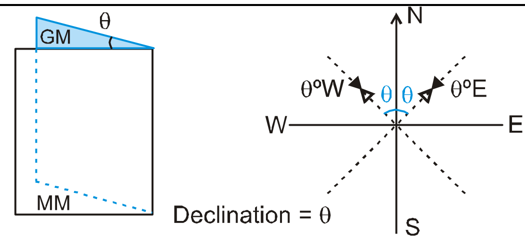
(c) The **magnetic Equator** is a great circle (a circle with the centre at earth's centre) on earth's surface which is perpendicular to the magnetic axis. The magnetic equator passing through Trivandrum in South India divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called the northern hemisphere (NHS) while the other, the southern hemisphere (SHS).

(d) The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

Elements of the Earth's Magnetism :

The magnetism of earth is completely specified by the following three parameters called elements of earth's magnetism :

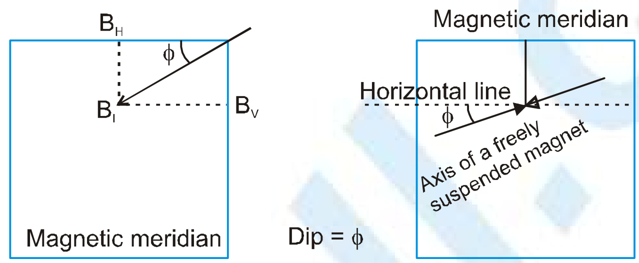
(a) **Variation or Declination θ :** At a given place the angle between the geographical meridian and the magnetic meridian is called declination, i.e., at a given place it is the angle between the geographical north south direction and the direction indicated by a magnetic compass needle, Declination at a place is expressed at θº E or θº W depending upon whether the north pole of the compass needle lies to the east (right) or to the west (left) of the geographical north-south direction. The declination at London is 10ºW means that at London the north pole of a compass needle points 10ºW, i.e., left of the geographical north.



(A) (B)

(b) Inclination or Angle of Dip ϕ : It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place. Actually it is the angle which the axis of a freely suspended magnet (up or down) subtends with the horizontal in magnetic meridian at a given place.

Here, it is worthy to note that as the northern hemisphere contains south polarity of earth's magnetism, in it the north pole of a freely suspended magnet (or pivoted compass needle) will dip downwards, i.e., towards the earth while the opposite will take place in the southern hemisphere.



(A) (B)

Angle of dip at a place is measured by the instrument called Dip-Circle in which a magnetic needle is free to rotate in a vertical plane which can be set in any vertical direction. Angle of dip at Delhi is 42º.

(c) Horizontal Component of Earth's Magnetic Field BH : At a given place it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by B_H and is measured with the help of a vibration or deflection magnetometer. At Delhi the horizontal component of the earth's magnetic field is 35 μT, i.e., 0.35 G.

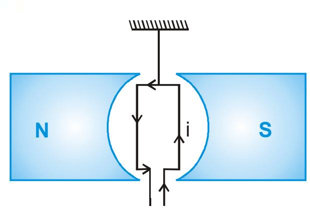
If at a place magnetic field of earth is B_I and angle of dip ϕ, then in accordance with figure **(a)**.

$$B_H = B_I \cos\phi \qquad \text{and} \qquad B_v = B_I \sin\phi \qquad \text{....(2)}$$

So that,
$$\tan\phi = \frac{B_V}{B_H} \qquad \text{and} \qquad I = \sqrt{B_H^2 + B_V^2} \qquad \text{....(2)}$$

Moving coil galvanometer

A galvanometer is used to detect the current and has moderate resistance.



Principle

When a current carrying coil is placed in a magnetic field, it experiences a torque given by τ = NiAB sin θ where θ is the angle between normal to plane of coil and direction of magnetic field. In actual arrangement the coil is suspended between the cylindrical pole pieces of a strong magnet.

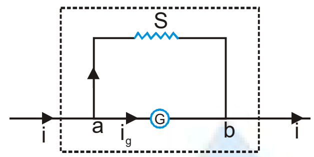
The cylindrical pole pieces give the field radial such that sin θ=1 (always). So torque τ=NiAB If C is torsional rigidity (i.e., restoring couple per unit twist of the suspension wire), then for deflection θ of coil τ=Cθ. In equilibrium we have external couple = Restoring couple i.e.Cθ=NiAB or $\theta = \frac{NAB}{C}$ i i.e., $\theta \propto i$

In words the deflection produced is directly proportional to current in the coil.

The quantity $\frac{\theta}{i} = \frac{NAB}{C}$ is called the current sensitivity of the galvanometer. Obviously for greater sensitivity of galvanometer the number of turns N, area of coil A and magnetic field B produced by pole pieces should be larger and torsional rigidity C should be smaller. That is why the suspension wire is used of phosphor bronze for which torsional rigidity C is smaller.

Conversion of Galvanometer into Ammeter

An ammeter is a low resistance galvanometer; used to measure current directly in amperes and is always connected in series with the circuit. To convert a galvanometer into ammeter, a low resistance, called shunt, is connected in parallel to the galvanometer as shown in figure.



Let i_g be the current in galvanometer for its full scale deflection and G the resistance of galvanometer. Let i is the range of ammeter and i_s the current in shunt S. Then potential difference across a and b is

$$V_{ab} = i_g G = i_S S. \quad \text{...(i)}$$

At junction a, $i = i_S + i_g$ i.e., $i_S = i - i_g$

Therefore from **(i)** $i_g G = (i - i_g)S$ or $i_g(S+G) = iS$ i.e., $i_g = \frac{S}{S+G} i$ ...**(ii)**

This is the working equation for conversion of galvanometer into ammeter. Here $i_g < i$.

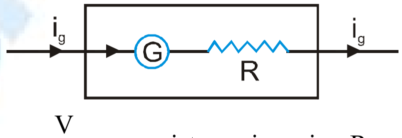
From **(ii)** shunt required $S = \frac{i_g G}{i - i_g}$. If $i_g << i$, $S = \left(\frac{i_g}{i}\right) G$

The resistance of ammeter R_A so formed is given by $\frac{1}{R_A} = \frac{1}{G} + \frac{1}{S} \Rightarrow R_A = \frac{SG}{S+G}$ ....**(iii)**

Note: Equation **(ii)** may also be used to increase the range of given ammeter. Here G will be resistance of given ammeter, S shunt applied, i_g its initial range and i the new range desired.

Conversion of Galvanometer into Voltmeter

A voltmeter is a high resistance galvanometer and is connected between two points across which potential difference, is to be measured i.e., voltmeter is connected in parallel with the circuit. To convert a galvanometer into voltmeter, a high resistance R in series is connected to the galvanometer.



If V is range of voltmeter, then $i_g = \frac{V}{R+G}$ or resistance in series $R = \frac{V}{i_g} - G$ ...**(i)**

This is working equation for conversion of galvanometer into voltmeter.

The resistance of voltmeter so formed is $R_V = R + G$ ...**(ii)**

Note: Equation **(i)** may also be used to increase the range of voltmeter. If V_0 is initial range and V is new range of voltmeter, then $i_g = \frac{V_0}{G} = \frac{V}{R+G}$

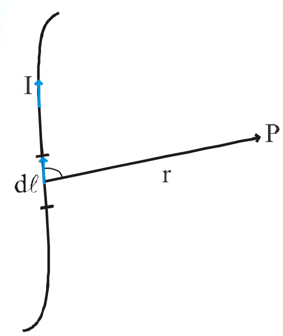
A static charge produced only electric field and only electric field can exert a force on it. A moving charge produced both electric field and magnetic field can exert force on it. A current carrying conductor produces only magnetic field and only magnetic field can exert a force on it.

Magnetic charge (i.e. current), produces a magnetic field. It can not produce electric field as net charge on a current carrying conductor is zero. A magnetic field is detected by its action current carrying conductors (or moving charges) and magnetic needles (compass). The vector quantity $\vec{B}$ known as **MAGNETIC INDUCTION** is introduced to characterise a magnetic field. It is a vector quantity which may be defined is term of the force it produces on electric currents. Lines of magnetic induction may be drawn in the same way as lines of electric field. The number of line per unit area crossing a small area perpendicular to the direction of the induction bring numerically equal to $\vec{B}$. The number of lines of $\vec{B}$ crossing a given area is referred to as the **magnetic flux** linked with that area. For this reason $\vec{B}$ is also called **magnetic flux density.**

1. Magnetic Induction Produced by a Current (Biot-Savart Law) :

The magnetic induction dB produced by an element d*l* carrying a current I are a distance r is given by :

$$dB = \frac{\mu_0\mu_r}{4\pi}\frac{Idl\sin\theta}{r^2} \Rightarrow \vec{dB} = \frac{\mu_0\mu_r}{4\pi}\frac{I\left(\vec{dl}\times\vec{r}\right)}{r^3}$$



here the quantity Idl is called as current element strength.

μ = permeability of the medium = $\mu_0\mu_r$

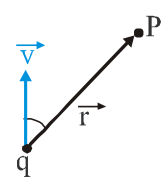
μ_0 = permeability of free space

μ_r = relative permeability of the medium (Dimensionless quantity).

Unit of μ_0 & μ is NA^{-2} or Hm^{-1} ;

$\mu_0 = 4\pi\times10^{-7}\,Hm^{-1}$

2. Magnetic Induction Due to a Moving Charge :

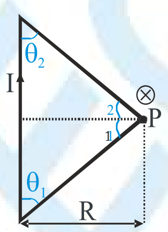


$$dB_p = \frac{\mu_0 qv\sin\theta}{4\pi r^2}$$

In vector form it can be written as $\vec{dB} = \frac{\mu_0}{4\pi}\frac{q\left(\vec{v}\times\vec{r}\right)}{r^3}$

3. Magnetic Induction Due to a Current Carrying Straight Conductor

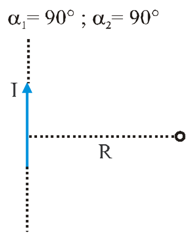
(a) Magnetic induction due to a current carrying straight wire



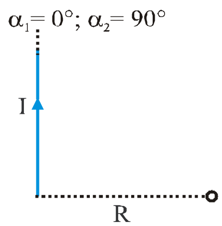
$$B = \frac{\mu_0 I}{4\pi R}\left(\cos\theta_1 + \cos\theta_2\right) = \frac{\mu_0 I}{4\pi R}\left(\sin\alpha_1 + \sin\alpha_2\right)$$

If the wire is very long $\theta_1 \cong \theta_2 \cong 0^\circ$ then , $B = \dfrac{\mu_0 I}{2\pi R}$

(b) **Magnetic induction due to a infinity long wire** $B = \dfrac{\mu_0 I}{2\pi R} \otimes$

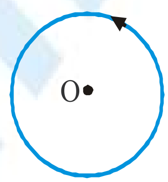


4. Magnetic Induction due to semi infinite straight conductor $B = \dfrac{\mu_0 I}{4\pi R} \otimes$



5. Magnetic Field Due to a Flat Circular Coil Carrying A Current :

(a) At its centre $B = \dfrac{\mu_0 NI}{2R} e$



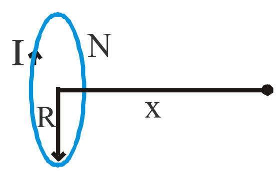
where

N = total number of turns in the coil

I = current in the coil

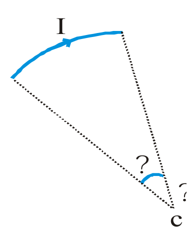
R = Radius of the coil

(b) On the axis $B = \dfrac{\mu_0 NIR^2}{2\left(x^2 + R^2\right)^{3/2}}$



where x = distance of the point from the centre.

It is maximum at the centre $B_C = \dfrac{\mu_0 NI}{2R}$



(c) **Magnetic Induction due to Flat Circular Arc :** $B = \dfrac{\mu_0 I\theta}{4\pi R}$

6. Magnetic field due to infinite long solid cylindrical conductor of radius R

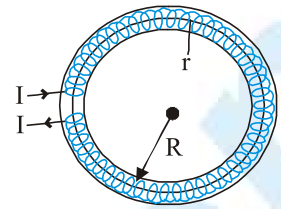
(i) For $r \geq R : B = \dfrac{\mu_0 I}{2\pi r}$ **(ii)** For $r < R : B = \dfrac{\mu_0 Ir}{2\pi rR^2}$

7. Magnetic Induction Due to Solenoid

$B = \mu_0 nI$, direction along axis.

where n $\rightarrow$ number of turns per meter ; I $\rightarrow$ current

8. Magnetic Induction due to Toroid : $B = \mu_0 nI$



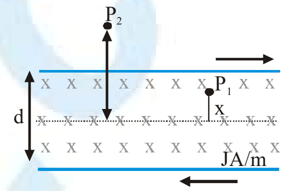
where $n = \dfrac{N}{2\pi R}$ (no. of turns per m)

N = total turns R >> r

9. Magnetic Induction due to Current Carrying Sheet

$B = \dfrac{1}{2}\mu_0 I$ where I = Linear current density (A/m)

10. Magnetic Induction Due to Thick Sheet



At point P_2 $\quad B_{out} = \dfrac{1}{2}\mu_0 Jd$

At point P_1 $\quad B_{in} = \mu_0 Jx$

11. GILBERT'S MAGNETISM (EARTH'S MAGNETIC FIELD) :

(a) The line of earth's magnetic induction lies in a vertical plane coinciding with the magnetic North- South direction at that place. This plane is called the **Magnetic Meridian.** Earth's magnetic axis is slightly inclined to the geometric axis of earth and this angle varies from 10.5^0 to 20^0. The Earth's Magnetic poles are opposite to the geometric poles i.e. at earth's north pole, its magnetic south pole is situated and vice versa.

(b) On the magnetic meridian plane, the magnetic induction vector of the earth at any point, generally inclined to the horizontal at an angle called the **Magnetic Dip** at that place , such that $\vec{B}$ = total magnetic induction of the earth at that point.

$\vec{B}_V$ = the vertical component of $\vec{B}$ in the magnetic meridian plane = B sin θ

$\vec{B}_H$ = the horizontal component of $\vec{B}$ in the magnetic meridian plane = B cos θ.

$$\frac{B_V}{B_H} = \tan\theta$$

(c) At a given place on the surface of the earth, the magnetic meridian and the geographic meridian may not coincide. The angle between them is called **"DECLINATION AT THAT PLACE"**

12. AMPERES LAW

$\oint \vec{B}.\vec{dl} = \mu \sum I$ where $\sum I$ = algebraic sum of all the currents.

13. MOTION OF A CHARGE IN UNIFORM MAGNETIC FIELD :

(a) When $\vec{v}$ is | | to $\vec{B}$: Motion will be in a straight line and $\vec{F} = 0$

(b) When $\vec{v}$ is $\perp$ to $\vec{B}$: Motion will be in circular path with radius

$R = \dfrac{mv}{qB}$ and angular velocity $\omega = \dfrac{qB}{m}$ and F = qvB.

(c) When $\vec{v}$ is at $\angle\theta$ to $\vec{B}$: Motion will be helical with radius

$$R_k = \frac{mv\sin\theta}{qB} \text{ and pitch } P_H = \frac{2\pi mv\cos\theta}{qB} \text{ and } F = qvB\sin\theta.$$

14. LORENTZ FORCE :

An electric charge 'q' moving with a velocity $\vec{v}$ through a magnetic field of magnetic induction $\vec{B}$ experiences a force $\vec{F}$, given by $\vec{F} = q\vec{v}\times\vec{B}$. There fore, if the change moves in a space where both electric and magnetic fields are superposed.

$\vec{F}$ = net electromagnetic force on the charge = $q\vec{E} + q\vec{v}\times\vec{B}$

This force is called the **Lorentz Force**

15. MOTION OF CHARGE IN COMBINED ELECTRIC FIELD & MAGNETIC FIELD

When $\vec{v}\parallel\vec{B}$ & $\vec{v}\parallel\vec{E}$, motion will be uniformly accelerated in straight line as $F_{magnetic} = 0$ and $F_{electrostatic} = qE$

So the particle will be either speeding up or speeding down

When $\vec{v}\parallel\vec{B}$ & $\vec{v}\perp\vec{E}$, motion will be uniformly accelerated in a parabolic path

When $\vec{v}\perp\vec{B}$ & $\vec{v}\perp\vec{E}$, the particle may more undeflected & undeivated with same uniform speed if $v = \frac{E}{B}$ (This is called as velocity selector condition

16. Magnetic Force on a Straight Current Carrying Wire : $\vec{F} = I(\vec{L}\times\vec{B})$

I = current in the straight conductor

$\vec{L}$ = length of the conductor in the direction of the current in it

$\vec{B}$ = magnetic induction. (Uniform throughout the length of conductor)

Note : In general force is $\vec{F} = \int I(d\vec{l}\times\vec{B})$

17. Magnetic Interaction Force Between Two Parallel Long Straight Currents :

When two long straight linear conductors are parallel and carry a current in each, they magnetically interact with each other, one experiences a force.

This force is of :

(i) Repulsion if the currents are anti-parallel (i.e. in opposite direction) or

(ii) Attraction if the currents are parallel (i.e. in the same direction)

This force per unit length on either conductor is given by $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$.

Where r = perpendicular distance between the parallel conductors.

18. Magnetic Torque on a Closed Circuit :

When a plane closed current circuit of 'N' turns and of area 'A' per turn carrying a current I is placed in uniform magnetic field, it experience a zero net force, but experience a torque given by $\vec{\tau} = NI\vec{A}\times\vec{B} = \vec{M}\times\vec{B}$ = BINA sinθ
where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise,

$\vec{B}$ = magnetic induction of the uniform magnetic field

$\vec{M}$ = magnetic moment of the current circuit = IN $\vec{A}$

Note : This expression can be used only if $\vec{B}$ is uniform otherwise calculate will be used.

19. Moving Coil Galvanometer :

It consists of a plane coil of many turns suspended in a radial magnetic field. When a current is passes in the coil it experiences a torque which produces a twist in the suspension.

The deflection is directly proportional to the torque $\therefore$ NIAB = Kθ

$$I = \left(\frac{K}{NAB}\right)\theta \text{ ; K = elastic torsional constant of the suspension}$$

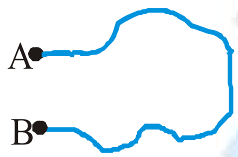
$$I = Cq \quad C = \frac{K}{NAB} = \text{Galvanometer Constant}$$

20. Force Experienced by a Magnetic Dipole in a Non-Uniform Magnetic Field :

$$|\vec{F}| = \left|M\frac{\partial B}{\partial r}\right|$$

where M = magnetic dipole moment.

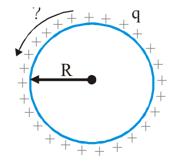
21. Force on a Random shaped conductor in a uniform magnetic field



(a) Magnetic force on a closed loop in a uniform $\overline{B}$ is zero.

(b) Force experienced by a wire of any shape is equivalent to force on a wire joining points A & B in a uniform magnetic field.

22. Magnetic moment of a rotating charge :



If a charge q is rotating at an angular velocity ω, its equivalent current is given as

$$I = \frac{q\omega}{2\pi} \text{ \& its magnetic moment is } M = I\pi R^2 = \frac{1}{2}q\omega R^2 .$$

Note : The ratio of magnetic moment to angular momentum of a uniform rotating object which is charged uniformly is always a constant. Irrespective of the shape of conductor M/L = q/2m

23. Magnetic dipole

(a) Magnetic moment M = m × 2l where m = pole strength of the magnet.

(b) Magnetic field at axial point (or End-on) of dipole $\vec{B} = \frac{\mu_0 2\vec{M}}{4\pi\, r^3}$

(c) Magnetic field at equatorial position (Broad-on) of dipole $\vec{B} = \frac{\mu_0}{4\pi^3}\frac{(-\vec{M})}{r^3}$

Magnetic Potential $V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$

Magnetic field $B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$

(d) Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M} \times \vec{B}$

(e) Potential energy of dipole placed in an uniform field $U = -\vec{M}.\vec{B}$

24. Intensity of magnetisation $I = M / V$

25. Magnetic induction $B = \mu H = \mu_0 (H+1)$

26. Magnetic permeability $\mu = \dfrac{B}{H}$

27. Magnetic suspectibility $\chi_m = \dfrac{1}{H} = \mu - 1$

28. Curie law

- For paramagnetic materials $\chi_m \propto \dfrac{1}{T}$

29. Curie Wires law

- For Ferromagnetic materials $\chi_m \propto \dfrac{1}{T - T_c}$

  Where T_c = curie temperature

30. A charged particle moves perpendicular to magnetic field. Its kinetic energy will remain constant but momentum changes because magnetic force acts perpendicular to velocity of particle.

31. If a unit north pole rotates around a current carrying wire then work has to be done because magnetic field produced by current is always non-conservative in nature.

32. In a conductor, free electrons keep on moving but no magnetic force acts on a conductor in a magnetic field because in a conductor, the average thermal velocity of electrons is zero.

33. Magnetic force between two charges is generally much smaller then the electric force between them because speeds of charges are much smaller than the free space speed of light.

Note : $\dfrac{F_{magnetic}}{F_{electric}} = \dfrac{v^2}{c^2}$