SURFACE TENSION

Pascal's Law

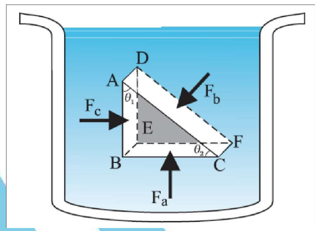
The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. This fact may be demonstrated in a simple way.

$F_b \sin\theta = F_c$, $F_b \cos\theta = F_a$ (by equilibrium)

$A_b \sin\theta = A_c$, $A_b \cos\theta = A_a$ (by geometry)

Thus,

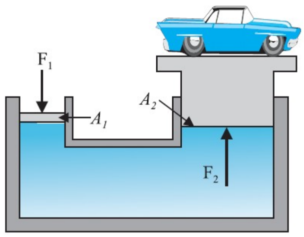
$$\frac{F_b}{A_b} = \frac{F_c}{A_c} = \frac{F_a}{A_a} \quad ; \qquad P_b = P_c = P_a$$



Hence, pressure exerted is same in all directions in a fluid at rest.

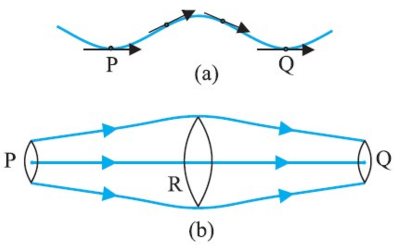
Hydraulic Machines

whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undimin ished and equally in all directions. This is the Pascal's law for transmission of fluid pressure and has many applications in daily life. A number of devices such as hydraulic lift and hydraulic brakes are based on the Pascal's law. In these devices fluids are used for transmitting pressure. In a hydraulic lift as shown in Fig. 10.6 two pistons are separated by the space filled with a liquid. A piston of small cross section A_1 is used to exert a force F_1 directly on the liquid. The pressure $P = \frac{F_1}{A_1}$ is transmitted throughout the liquid to the larger cylinder attached with a larger piston of area A_2, which results in an upward force of $P \times A_2$. Therefore, the piston is capable of supporting a large force (large weight of, say a car, or a truck, placed on the platform) $F_2 = PA_2 = \frac{F_1A_2}{A_1}$ By changing the force at A1, the platform can be moved up or down. Thus, the applied force has been increased by a factor of $\frac{A_2}{A_1}$ A and this factor is the mechanical advantage of the device. The example below clarifies it.



STREAMLINE FLOW

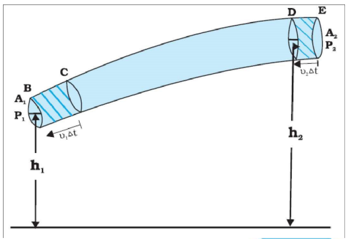
So far we have studied fluids at rest. The study of the fluids in motion is known as fluid dynamics. The flow of the fluid is said to be **`steady`** if at any given point, the velocity of each passing fluid particle remains constant in time.



The path taken by a fluid particle under a steady flow is a streamline. It is defined as a curve whose tangent at any point is in (a) The direction of the fluid velocity at that point. Consider the path of a particle as shown in the curve describes how a fluid particle moves with time. The curve PQ is like a permanent map of fluid flow, indicating how the fluid streams. No two streamlines can cross, for if they do, an oncoming fluid particle can go either one way or the other and the flow would not be steady. Hence, in steady flow, the map of flow is stationary in time. How do we draw closely spaced streamlines ? If we

intend to show streamline of every flowing particle, we would end up with a continuum of lines. Czonsider planes perpendicular to the direction of fluid flow e.g., at three points P, R and Q in (b).The plane pieces are so chosen that their boundaries be determined by the same set of streamlines. This means that number of fluid particles crossing the surfaces as indicated at P, R and Q is the same. If area of cross-sections at these points are A_P,A_R and A_Q and speeds offluid particles are υ_P, υ_R and υ_Q, then mass of fluid Δm_P crossing at A_P in a small interval of time Δt is $\rho_P A_P \upsilon_P\ \Delta t$. Similarly mass of fluid Δm_R flowing or crossing at A_R in a small interval of time At is $\rho_R A_R \upsilon_R$ At and mass of fluid Δm_Q is $\rho_Q A_Q \upsilon_Q$ At crossing at A_Q. The mass of liquidflowing out equals the mass flowing in, holds in all cases. Therefore,
$\rho_P A_P \upsilon_P \Delta t = \rho_R A_R \upsilon_R \Delta t = \rho_Q A_Q \upsilon_Q \Delta t$. For flow of incompressible fluids
$\rho_P = \rho_R = \rho_Q$ reduces to $\Delta_P \upsilon_P = \Delta_R \upsilon_R = \Delta_Q \upsilon_Q$, which is called the equation of continuity and it is a statement of conservation of mass in flowof incompressible fluids. In general Av = constant.

BERNOULLI'S PRINCIPLE



Consider the flow at two regions 1 (i.e. BC) and 2 (i.e. DE). Consider the fluid initially lying between B and D. In an infinitesimal time interval Δt, this fluid would have moved. Suppose υ_1 is the speed at B and υ_2 at D, thenfluid initially at B has moved a distance $\upsilon_1 \Delta t$ to C ($\upsilon_1 \Delta t$ is small enough to assume constant cross-section along BC). In the same interval Δt the fluid initially at D moves to E, a distance equal to $\upsilon_2 \Delta t$. Pressures P_1 and P_2 act as shown on the plane faces of areas A_1 and A_2 binding the two regions. The work done on the fluid at left end (BC) is $W_1 = P_1 A_1 (\upsilon_1 \Delta t) = P_1 \Delta V$. Since the same volume ΔV passes through both the regions (from the equation of continuity) the work done by the fluid at the other end (DE) is
$W_2 = P_2 A_2 (\upsilon_2 \Delta t) = P_2 \Delta V$ or, the work done on the fluid is $-P_2 \Delta V$. So the total work done on the fluid is
$W_1 - W_2 = (P_1 \Delta\ P_2)\ \Delta V$ Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy. If the density of the fluid is ρ and $\Delta m = \Delta A_1 \upsilon_1 \Delta t = \rho \Delta V$ is the mass passing through the pipe in time Δt, then change in gravitational potential energy is
$\Delta U = \rho g \Delta V\ (h_2 - h_1)$ The change in its kinetic energy is

$$\Delta K = \left(\frac{1}{2}\right) \rho\ \Delta V\ \left(\upsilon_2^2 - \upsilon_1^2\right)$$

We can employ the work – energy theorem to this volume of the fluid and this yields

$$(P_1 - P_2)\ \Delta V = \left(\frac{1}{2}\right) \rho\ \Delta V \left(\upsilon_2^2 - \upsilon_1^2\right) + \rho g \Delta V\ (h_2 - h_1)$$

We now divide each term by ΔV to obtain

$$(P_1 - P_2) = \left(\frac{1}{2}\right) \rho \left(\upsilon_2^2 - \upsilon_1^2\right) + \rho g\ (h_2 - h_1)$$

We can rearrange the above terms to obtain

$$P_1 + \left(\frac{1}{2}\right) \rho \upsilon_1^2 + \rho g h_1 = P_2 + \left(\frac{1}{2}\right) \rho \upsilon_2^2 + \rho g h_2$$

This is **Bernoulli's equation.** Since 1 and 2 refer to any two locations along the pipeline, we may write the expression in general as $P + \left(\frac{1}{2}\right) \rho \upsilon^2 + \rho g h$ = constant

In words, the Bernoulli's relation may be stated as follows: As we move along a streamline the sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{\rho \upsilon^2}{2}\right)$ and the potential energy per unit volume (ρgh) remains a constant.

1. (a) COHESIVE FORCE

The force of attraction between the molecules of the same substance is called cohesive force
In case of solids, the force of cohesin is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The foce of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Example.

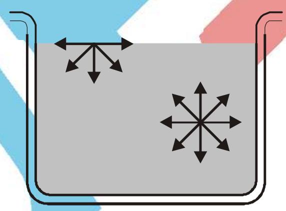
(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.
(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.
(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

1. (b) ADHESIVE FORCE

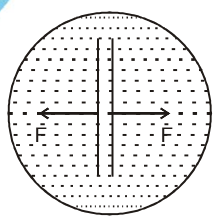
The force at attraction between molecules of different substances is called adhesive force
Examples.
(i) Adhesive force enables us to write on the black board with a chalk.
(ii) Adhesive force helps us to write on the paper with ink.
(iii) Large force of adhesion between cement and bricks helps us in constrution work.
(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

2. SURFACE TENSION

The property of a liquid at rest due to which its free surface tries to have minimum surface area and behaves as if it were under tension somewhat like a stretched elastic membrane is called surface tension.
The molecules of the liquid exert attractive forces on each other. There is zero net force on a molecule inside the volume of the liquid.



But a surface molecules is drawn into the volume. Thus, the liquid tends to minimize its surface area, just as a stretched membrane does.



Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being pependicular to the line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line of length L. then

$$T = (F/L)$$

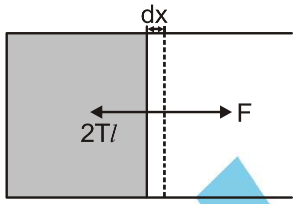
Regarding surface tension it is worth noting that :
(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.
(2) It is a scalar as it has a unique direction which is not to be specified.
(3) It has dimension $[ML^{-2}]$ and SI units N/m while CGS unit dyne/cm, so that one MKS unit of surface tension = 10^3 dyne/cm
(4) Surface tension of a liquid decreases with rise in temperature

(5) The surface tension of a liquid is very sensitive to impurities on the surface (called *contamination*) and decreases with contamination of surface.

(6) In case of soluble impurities surface tension may increase or decrease depending on the nature of impurity. Usually highly soluble salt such as sodium chloride increases surface tension while sparingly soluble salt such as soap decreases surface tension.

2.1 SURFACE ENERGY

When the surface area of a liquid is increased, the molecules from the interior rise to the surface. This requires work against force of attraction of the molecules just below the surface. This work is stored in the form of potential energy. Thus, the molecules in the surface have some additional energy due to their positio. This additional energy per unit area of the surface is called 'surface energy'. The surface energy is related to the surface tension as discussed below :



Let a liquid flim be formed on a wire frame and a straight wire of length l can slide on this wire frame as shown in figure. The film has two surface and both the surface are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force Tl. Thus, net force on the wire due to both the surface is 2Tl. One has to apply an external force F equal and opposite to it to keep the wire in equilibrium. Thus,

$$F = 2Tl$$

Now, suppose the wire is moved through a small distance dx, the work done by the force is,

$$dW = F\,dx = (2Tl)\,dx$$

But (2l) (dx) is the total increase in area of both the surface of the film. Let is be dA. Then,

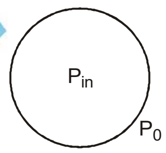
$$dW = T\,dA$$

or $$T = \frac{dW}{dA}$$

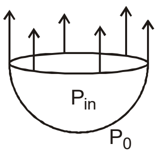
Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity.

2.2 Excess Pressure Insider a liquid drop :

Consider a liquid drop of radius 'R' and surface tension 'T' A liquid drop has only one surface film, hence the surface tension force is $T(2\pi R)$



lower half shows in figure



Force due to inside pressure (P_{in}) is P_{in} × area i.e. $P_{in}\,\pi R^2$ similarly force due to outiside pressure (P_0) is $P_0\pi R^2$ since each half of the liquid drop is in equilibrium

$$P_0\pi R^2 + T\,(2\pi R) = P_{in}\,(\pi R^2)$$

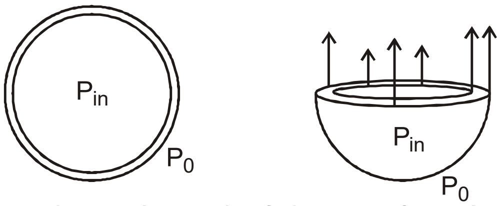
$$P_{in} - P_0 = \frac{2T}{R} = \text{Excess Pressure}$$

2.3 Excess pressure inside a bubble

Consider a bubble of radius 'R' and surface tension 'T'. A bubble consists of two spherical surface films with a thin layer of liquid between them.

The total surface tension force for each surface inner and outer $T\,(2\pi R)$ for a total of $(2T)\,(2\pi R)$

Force due to inside pressure (P_{in}) is $P_{in}\,\pi R^2$ and due to outside pressure (P_0) is $P_0\pi R^2$

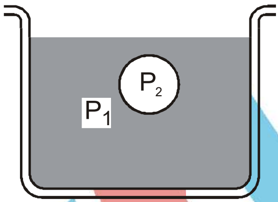


Since each half of bubble is in equilibrium (lower half shown in figure)

$$P_0\,\pi R^2 + 2T(2\pi R) = P_{in}\,\pi R^2$$

$$P_{in} - P_0 = \frac{4T}{R} = \text{Excess pressure}$$

Note : (1) If we have an air bubble inside a liquid, a single surface is formed. There is air on the concave side and liquid on the convex side. The pressure in the concave side (that is in the air) is greater than the pressure in the convex side (that is in the liquid) by an amount $\frac{2T}{R}$.



$$\therefore \quad P_2 - P_1 = \frac{2T}{R}$$

The above expression has been written by assuming P_1 to be constat from all sides of the bubble. For small size bubbles this can be assumed.

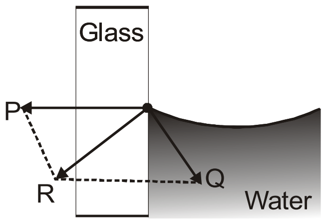
(2) From the above discussion, we can make a general statement. The pressure on the concave side of a spherical liquid surface is greater than the convex side by $\frac{2T}{R}$.

(3) For any curved surface excess pressure on the concave side = $T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ where R_1 & R_2 are radius of curvature of the surface in two perpendicular direction of instead of liquid surface, liquid film is given then above exression will be

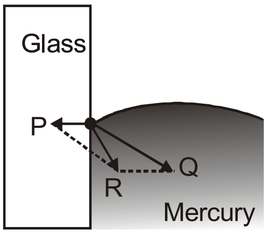
$P = 2T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ For spherical curved surface R_1, R_2

3. CONTACT ANGLE AND SHAPE OF LIQUID SURFACE

The surface of a liquid when meets a solid, such as the wall of a container, it usually curves up or down near the solid surface. The angle which the tangent to the is called the contact angle. The curved liquid surface at the pt. of surface of the liquid is called meniscus. The shape of the meniscus contact of liquid surface with (convex or concave) is determined by the relative strengths of solid cohesive and adhesive forces surface with the solid surface inside the liquid.



When the adhesive force (P) between solid and liquid molecules is more than the cohesive force (Q) between liquid-liquid molecules (as with water and glass), shape of the meniscus is concave and the angle of contact θ is less than 90º. In this case the liquid wets or adheres to the solid surface. The resultant (R) of P and Q passes through the solid.



On the other hand when P < Q (as with glass and mercury), shape of the meniscus is convex and the angle of contact θ > 90º. The resultant (R) of P and Q in this case passes through the liquid.

Let us now see why the liquid surface bends near the contact with a solid. A liquid in equilibrium can not sustain trangential stress. The resultant force on any small part of the surface layer must be perpendicular to the surface at that point. Basically three forces are acting on a small part of the liquid surface near its contact with solid. These forces are,

(i) P, attraction due to the molecule of the solid surface near it i.e. adhesive force which acts outwards at right angle to the wall of tube.

(ii) Q, attraction due to liquid molecules near this part and i.e. cohesive force which acts at an angle of 45º to the vertical.

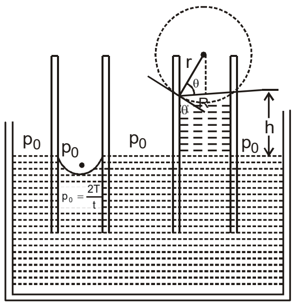
We have considered very small part, so weight of that part can be ignored for better understanding. As we have seen in the last figures, to make the resultant (R) of P and Q perpendicular to the liquid surface the surface becomes curved (convex or concave).

Note : The angle of contact between water and clean glass is zero.

4. CAPILLARY RISE

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called **capillarily.**

In order to calculate the height to which a liquid will rise in a capaillary, consider a glass capillary of radius R dipped in water as shown in Fig. shown. As the meniscus is concave and nearly spherical, the pressure below the meniscus will be [p_0 – (2T/r)] with p_0 as atmospheric pressure and r as radius of meniscus. Now as liquid flows from higher to lower pressure and at same level in a liquid pressure must be same (this is because a liquid cannot sustain tangential stress), so the liquid will ascends in the capillary till hydrostatic pressure of the liquid compensates for the decrease in pressure. i.e.,



$$p_0 = \left[p_0 - \frac{2T}{r}\right] + h\rho g \quad \text{or} \quad h = \frac{2T}{r\rho g} \qquad ...(1)$$

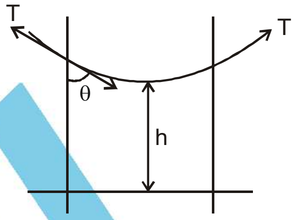
But from figure shown it is clear that radius of meniscus r is related to the radius of capillary through the relation

$(R/r) = \cos\theta$, i.e., $r = R/\cos\theta$ ...(2)

where θ is the anlge of contact. *So substituting the value of from Eqn. (2) in (1), we get

$$h = \frac{2T}{r\rho g} = \frac{2T\cos\theta}{R\rho g} \qquad ...(3)$$

- ***Alternate Method***

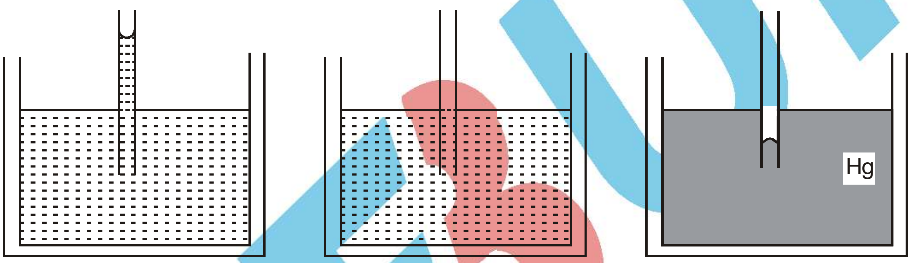


As it can be seen from figure that $T\sin\theta$ cancels out :
The force due to $T\cos\theta$ balances the weight of liquid ($mg = \rho\, vg$)
vol. of the curve is negligible
$\therefore$ vol. of liquid in $\pi r^2 h$

$$T\cos\theta = 2\pi r = \pi r^2 hg \Rightarrow h = \frac{2T\cos\theta}{r\rho g}$$

This is the desired result and from this it is clear that :

1. The capillarity depends on the nature of liquid and solid both, i.e., on T, ρ, θ and R. If $\theta > 90°$, i.e., meniscus is convex, h will be negative, i.e., the liquid will descends in the capillary as actually happens in case of mercury in a



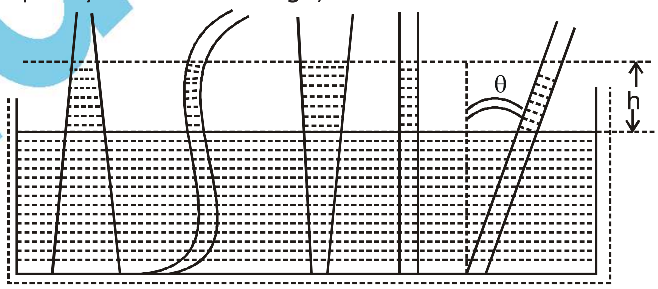
(A) (B) (C)

θ 90°;h = +ve $\qquad \theta = 90°; h = 0 \qquad \theta > 90°; h = -ve$

glass tube. However, if $\theta = 90°$, i.e., meniscus is plane, h = 0 and so no capillarity.

2. For a given liquid and solid at a given place as ρ, T, θ and g are constant, (figure shown)
hr = constant
$\therefore$ lesser the radius of capillary greater will be the rise and vice-versa. (figure shown)

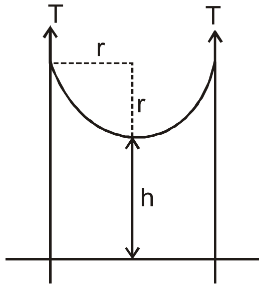
3. Here it is important to note that in equilibrium the height h is independent of the shape of capillary if the radius of meniscus remains the same. This is why the vertical height h of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same and also the vertical height of the liquid in a capillary does not change, when it is inclined to the vertical. (figure shown)



(a) (b) (c) (d) (e)

4. Capillarity has large number of applications in our daily life, e.g.,
(a) The oil in the wick of a lamp rise due to capillary action of threads in the wick.
(b) Action of towel in soaking up moisture from the body is due to capillary action of cotton in the towel.
(c) Water is retained in a piece of sponge on account of capillarity.
(d) A blotting paper soaks ink by capillary action of the pores in the blotting paper.
(e) The root-hairs of plants drawn water from the soil through capillary action.

5. In Case of glass and water $\theta = 0'$
here force due to surface tension balances the weight of the liquid ($\rho \times v \times g$)



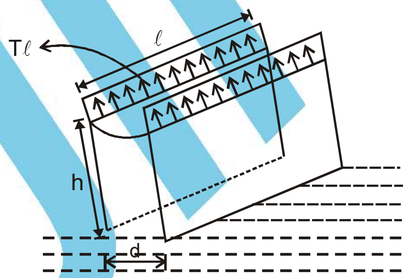
volume of the liquid = $\pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3$

where $\pi r^3 - \frac{2}{3}\pi r^3$ is the volume of the curve which is not negligible in this case

$\therefore$ $T.2\pi r = \rho\,(\pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3)g$

$2T = rh\,\rho g + \frac{1}{3}\pi r^2\,\rho g$

6. If two parallel plates with the spacing 'd' are placed in water reservoir, then height or rise.

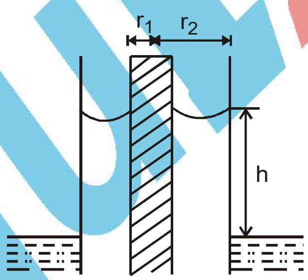


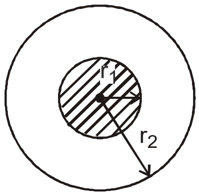
$2T\ell = \rho\ell h d g \Rightarrow h = \dfrac{2T}{\rho d g}$

7. If two concentric tubes of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise?

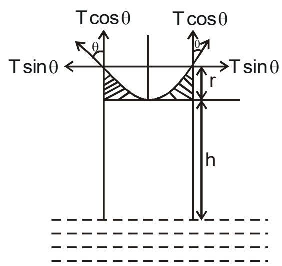
$\Rightarrow$ $T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g$

$h = \dfrac{2T}{(r_2 - r_1)\rho g}$





8. If weight of the liquid in the meniscus is to be consider :



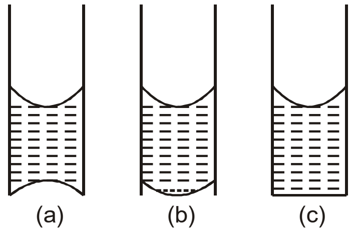
$T\cos\theta \times 2\pi r = [\pi r^2 h + \frac{1}{3}\,\pi r^2 \times r]\,\rho\,g$

$\left[h + \dfrac{r}{3}\right] = \dfrac{2T\cos\theta}{r\rho g}$

9. When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = \dfrac{2T}{R_1}$

where R_1 = radius of curvature of upper meniscus.
The hydrostatic pressure $p_2 = h\,\rho\,g$ is always directed downwards.
If $p_1 > p_2$ i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus

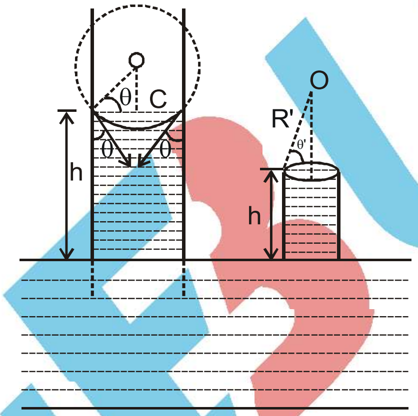
should be downward. This makes lower meniscus concave downward (fig a). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$



If $p_1 < p_2$ i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (fig. b) The radius of lower meniscus can be given by $\frac{2T}{R_2} = p_2 - p_1$

If $p_1 = p_2$, then is no resulting pressure. then, $p_1 - p_2 = \frac{2T}{R_2} = 0$ or, $R_2 = \infty$ i.e. lower surface will be FLAT (fig c)

4.1 CAPILLARY RISE IN A TUBE OF INSUFFICIENT HEIGHT



We know, the height through which a liquid rises in the capillary tube of radius r is given by

$$\therefore \quad h = \frac{2T}{R\rho g} \text{ or } h\,R = \frac{2T}{\rho g} = \text{constant}$$

When the capillary tube is cut an its length is less then h (i.e. h'), then the liquid rises upto the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus increases and it becomes more flat so that hR = h'R' = Constant. Hence the liquid does not overflow.

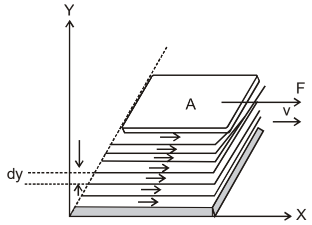
If h' < h then R' > R or $\frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$

$\Rightarrow \cos\theta' < \cos\theta \quad \Rightarrow \quad \theta' > \theta$

5. VISCOSITY AND NEWTON'S LAW OF VISCOUS FORCE

In case of steady flow of a fluid when a layer of fluid slips or tends to slip on adjacent layer in contact, the two layers exert tangential force on each other which tries to destroy the relative motion between them. The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion *viscous force.* A briskly stirred fluid comes to rest after a short while because of viscosity.

As a result of large number of experiments Newton found that viscous force F acting on any layer of a fluid is directly proportional to its area A and to the velocity gradient (dv/dy)* at the layer i.e.,



$$F \propto A\frac{dv}{dy} \quad \text{or} \quad F = -\eta A\frac{dv}{dy} \qquad ...(1)$$

when η is a constant called coefficient of viscosity or simply viscosity of the fluid. The negative sign shows that viscous force on a liquid layer acts in a direction opposite to the relative velocity of flow of fluid. The Eq. (1) is known as Newton's law of viscous force. Here y is taken from the layer of which velocity is zero. Regarding viscosity of fluid it is worth noting that :

(1) It depends only on the nature of fluid and is independent of area considered or velocity gradient.

(2) Its dimensions are $[ML^{-1} T^{-1}]$ and SI unit poiseuille (PI) while CGS unit dyne-s/cm^2 called poise (P) with

1 Pl = 10 poise

(3) Viscosity of liquids is much greater (say about 100 times more) than that of gases

i.e., $\eta_L > \eta_G$

6. STOKES LAW

When a body moves through a fluid, the flui in contact with the body is dragged with it. This establishes relative motion in fluid layers near the body, due to which viscous force starts operating. The fluid exerts viscous force on the body to oppose its motion. The magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the fluid. Stokes established that if a sphere of radius r moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

$$F = 6\,\pi\,\eta\,rv$$

7. TERMINAL VELCOITY (V_T)

Consider a small sphere falling from rest through a large column of viscous fluid. The forces acting on the shere are,

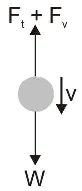
(i) Weight W of the sphere acting vertically downwards

(ii) Upthrust F_t acting vertically upwards

(iii) Viscous force F_v acting vertically upwards, i.e., in a direction opposite to velocity of the sphere.

Initially, $F_v = 0$

and $W > F_t$



and the sphere accelerates downwards. As the velocity of the sphere increases, F_v increases, Eventually a stage is reached when

$$W = F_t + F_v$$

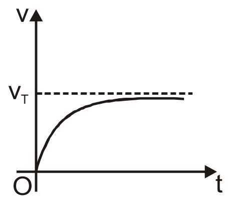
After this net force on the sphere is zero and it moves downwards with a constant velocity called terminal velocity (v_T).

Substituting proper values in Eq. (i) we have,

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_T$$

Here, ρ = density of sphere, σ = density of fluid

and η = coefficient of viscosity of fluid



From Eq. (ii), we get $$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Figure shows the variation of the velocity v of the sphere with time.

SOLVED EXAMPLE

Ex.1 The two thigh bones (femurs), each of cross-sectional area10 cm² support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs.

Ans. Total cr×oss-sectional area of the femurs is $A = 2 \times 10\ cm^2 = 20 \times 10^{-4}\ m^2$. The force acting on them is F = 40 kg wt = 400 N (taking $g = 10\ m\ s^{-2}$). This force is acting vertically down and hence, normally on the femurs. Thus, the average pressure is

$$P_{av} = \frac{F}{A} = 2 \times 10^5 Nm^{-2}$$

Ex.2 The density of the atmosphere at sea level is 1.29 kg/m³. Assume that it does not change with altitude. Then how high would the atmosphere extend?

Ans. We use $P = P_a + \rho gh$

$= 1.29\ kg\ m^{-3} \times 9.8\ m\ s^2 \times h\ m = 1.01 \times 10^5$ Pa

$\therefore$ h = 7989 m . 8 km

Ex.3 At a depth of 1000 m in an ocean (a) what is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintainedat sea-level atmospheric pressure. (The density of sea water is 1.03 × 10³ kg m⁻³, g = 10m s⁻².)

Ans. Here h = 1000 m and ? = $1.03 \times 10^3\ kg\ m^{-3}$.(a) From Eq. (10.6), absolute pressure

$P = P_a + \rho gh$

$= 1.01\ 10^5$ Pa+ $1.03 \times 10^3\ kg\ m^{-3} \times 10\ m\ s^{-2} \times 1000$ m

$= 104.01 \times 10^5$ Pa × 10^4 atm

(b) Gauge pressure is P + Pa = ρgh = Pg

$Pg = 1.03 \times 10^3\ kg\ m^{-3} \times 10\ ms^2 \times 1000$ m

$= 103 \times 10^5\ P_a + 103$ atm

(c) The pressure outside the submarine is P= Pa + ρgh and the pressure inside it is Pa. Hence, the net pressure acting on the window is gauge pressure, Pg = ρgh. Since the area of the window is A = 0.04 m², the force acting on it is

$F = Pg\ A = 103 \times 10^5\ P_a \times 0.04\ m^2 = 4.12 \times 10^5$ N

Ex.4 Two syringes of different cross sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diametersof the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively. (a) Findthe force exerted on the larger piston whena force of 10 N is applied to the smaller piston. (b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

Ans. (a) Since pressure is transmitted undiminished throughout the fluid, $F_2 \frac{A_2}{A_1} F_1 \ \frac{3/2 10^{-2} m^2}{1/2 10^{-2} m^2} 10\ N = 90\ N$

(b) Water is considered to be perfectly incompressible. Volume covered by the movement of smaller piston inwards is equal to volume moved outwards due to the larger piston. $L_1 A_1 = L_2 A_2$

$\cong 0.67 \times 10^{-2}$ m = 0.67 cm

Note, atmospheric pressure is common to both pistons and has been ignored.

Ex.5 In a car lift compressed air exerts a force F1 on a small piston having a radius of 5.0 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, calculate F1. What is the pressure necessary to accomplish this task? (g=9.8 ms⁻²).

Ans. Since pressure is transmitted undiminished throughout the fluid,

$$F_1 \frac{A_1}{A_2} F_2 \cdot \frac{\pi(5 \times 10^{-2} m^2)}{\pi(15 \times 10^{-2} m^2)} (1350\ N\ 9.8\ ms^{-2}) = 1470\ N$$

$\approx 1.5 \times 10^3$ N

The air pressure that will produce this force is

$$P \frac{F_1}{A_1} \frac{1.5 \times 10^3 N}{5 \times 10^{-2} N} 1.9 \times 10^5\ Pa$$

This is almost double the atmospheric pressure. Hydraulic brakes in automobiles also work on the same principle.

Ex.6 Blood velocity: The flow of blood in a large artery of an anesthetised dog is diverted through a Venturi meter. The wider part of the meter has a crosssectional area equal to that of the artery. A = 8 mm2. The narrower part has an area a = 4 mm². The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery?

Ans. We take the density of blood to be

1.06×10^3 kg m^{-3}. The ratio of the areas is $\left(\frac{A}{a}\right) = 2$.

Using Eq. we obtain

$$\upsilon_1 = \sqrt{\frac{2 \times 24\text{Pa}}{1060\text{kgm}^{-3} \times (2^3 - 1)}} = 0.125\text{ms}^{-1}$$

$$v = \sqrt{\frac{2p_m gh}{p}}\left[\left(\frac{A}{a}\right)^2\right]^{-1/2}$$

Ex.7 A fully loaded Boeing aircraft has a mass of 3.3 × 10⁵ kg. Its total wing area is 500 m². It is in level flight with a speed of 960 km/h. (a) Estimate the pressure difference between the lower and upper surfaces of the wings (b)Estimate the fractional increase in the speed of the air on the upper surface ofthe wing relative to the lower surface. [Thedensity of air is ? = 1.2 kg m-3]

Ans. (a) The weight of the Boeing aircraft is balanced by the upward force due to thepressure difference

$\Delta P \times A = 3.3 \times 10^5$ kg × 9.8

$\Delta P = (3.3 \times 10^5 \text{ kg} \times 9.8 \text{ m s}^{-2}) / 500 \text{ m}^2$

$= 6.5 \times 10^3$ Nm^{-2}

(b) We ignore the small height difference between the top and bottom sides in The pressure difference between them is then $\Delta P = P/2\,(V_2^2 - V_1^2)$

where υ^2 is the speed of air over the upper surface and $\upsilon 1$ is the speed under the bottomsurface.

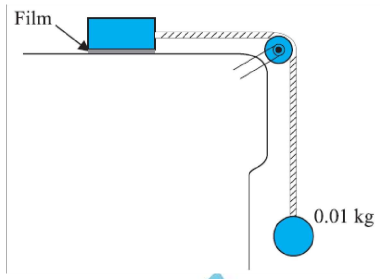
$$\upsilon_2 - \upsilon_1 \frac{2P}{\upsilon_2 - \upsilon_1}$$

Taking the average speed vav = $(\upsilon_2 + \upsilon_1)/2$ = 960 km/h = 267 m s^{-1}, we have

$$\upsilon_2 - \upsilon_1 / \upsilon_{av} = \frac{P}{\upsilon_{av}^2} \approx 0.08$$

The speed above the wing needs to be only 8% higher than that below.

Ex.8 A metal block of area 0.10 m² is connected to a 0.010 kg mass via a stringthat passes over an ideal pulley (considered massless and frictionless), as in Fig. A liquid with a film thickness of 0.30 mm is placed between the block and the table. When released the block moves to the right with a constant speed of 0.085 m s⁻¹. Find the coefficient of viscosity of the liquid. Fig. 10.15 Measurement of the coefficient of viscosity of a liquid.



Ans. The metal block moves to the right because of the tension in the string. The tension T is equal in magnitude to the weight of the suspended mass m. Thus the shear force F is

$F = T = mg = 0.010 \text{ kg} \times 9.8 \text{ m s}^{-2} = 9.8 \times 10^{-2}$ N

Shear stress on the fluid = F/A = $\frac{9.8 \times 10^{-2}}{0.10}$

Strain rate = $\frac{\upsilon}{\ell} = \frac{0.085}{0.030}$

$$\eta = \frac{\text{stress}}{\text{strain rate}}$$

$$\frac{(9.8 \times 10^{-2}\text{N})\,(0.30 \times 10^{-3}\text{N})}{(0.085\text{ms}^{-1})(0.10\text{m}^2)}$$

$= 3.45 \times 10^{-3}$ Pa s

Ex.9 The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20ºC is 6.5 cm s⁻¹. Compute the viscosity of the oil at 20oC. Density of oil is 1.5 × 10³ kg m⁻³, density of copper is 8.9 × 10³ kg m⁻³.

Ans. We have vt = 6.5×10^{-2} ms^{-1}, a = 2×10^{-3} m, g = 9.8 ms^{-2}, ? = 8.9×10^3 kg m^{-3},

? =1.5×10^3 kg m^{-3}.

From $v_t = \frac{2a^2(P - 0)g}{ag}$

$$\mu = \frac{2}{9} \times \frac{(2 \times 10^{-3})\text{m}^2 \times 9.8\text{ms}^{-2}}{6.5 \times 10^{-2}\text{ms}^{-1}} \times 7.4\ 10^3 \text{ kgm}^{-3}$$

$= 9.9 \times 10^{-1}$ kg m^{-1} s^{-1}

Exercise - I UNSOLVED PROBLEMS

Q.1 Explain why

(a) A balloon filled with helium does not rise in air indefinitely but halts after a certain height (Neglect winds)

(b) The force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air

Q.2 Explain why

(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute

(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets the glass while mercury does not)

(c) Surface tension of a liquid is independent of the area of the surface

(d) Detergents should have small angles of contact

(e) A drop of liquid under no external forces is always spherical in shape

Q.3 Fill in the blanks using the word(s) form the list appended with each statement ;

(a) Surface tension of liquid generally.................. With temperature (increases/ decreases)

(b) Viscosity of gases............... with temperature, whereas viscosity of liquids................. with temperature (increases / decreases)

(c) For solid with elastic modulus of rigidity, the shearing force is proportional to........... while for fluids it is proportional to ....(shear strain / rate of shear strain)

(d) For a fluid in steady flow, the increase in flow speed at a constriction follows from........... while the decrease of pressure there follows from............(conservation of mass / Bernoulli's principle)

(e) For the model of a plane in a wind tunnel, turbulence occurs at a.......... speed than the critical speed for turbulence for an actual plane (greater / smaller)

Q.4 Explain why

(a) To keep a piece of paper horizontal, you should blow over, not under, it

(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers

(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection

(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel

(e) A spinning cricket ball in air does not follow a parabolic trajectory

Q.5 A 50 kg girl wearing high heel shoes balances on a single heel. The is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?

Q.6 Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.

Q.7 A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean ? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

Q.8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear ?

Q.9 A U-tube contains water and methylated spirit separated by mercury. The mercury column in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?

Q.10 In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? (Specific gravity of mercury =13.6)

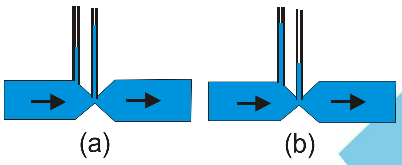
Q.11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river ? Explain.

Q.12 Does it mater if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.

Q.13 Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kg s^{-1}, what is the pressure difference between the two ends of the tube ? (Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Pa s.) [You may also like to check if the assumption of laminar flow in the tube is correct].

Q.14 In a test experiment on a model aeroplane in a wind tunnel, the flow speed on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 ms^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2 ? Take the density of air to be 1.3 kg m^{-3}.

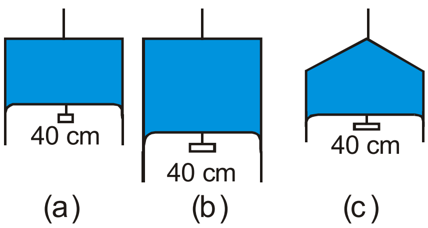
Q.15 Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?



Q.16 The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes ?

Q.17 A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5×10^{-2} N (Which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film ?

Q.18 Figure (a) shows a thin liquid film supporting a small weight = 4.5×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in fig. (b) and (c) ? Explain your answer physically.

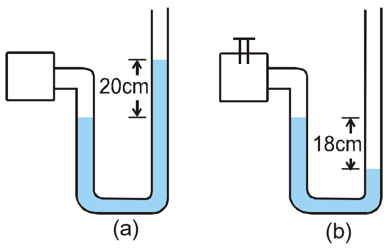


Q.19 What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature ? Surface tension of soap solution at the temperature (20 °C) is 4.65×10^{-1} N m^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.

Q.20 What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is 2.50×10^{-2} N m^{-1} ? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? (1 atmospheric pressure is 1.01×10^5 Pa).

Q.21 A tank with a square base of area 1.0 m^2 is divided by a vertical, partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. compute the force necessary to keep the door close.

Q.22 A manometer reads the pressure of a gas in an enclosure as shown in Fig. (a). When a pump removes some of the gas, the manometer reads as in Fig. (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.
(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.
(b) How should the levels change incase (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer ? (Ignore the small change in the volume of the gas).



Q.23 A spring balance areas 10 kg when a bucket of water is suspended from it. What is the reading on the spring balance when
(a) an ice cube of mass 1.5 kg is put into the bucket
(b) an iron piece of mass 7,8 kg suspended by another string is immersed with half its volume inside the water inthe bucket ? (Relative density of iron =7.8).

Q.24 Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases ? If so, why do the vessels filled with water to that same height give different readings on a weighing scale ?

Q.25 During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ? [Use density of whole blood from Table 10.1].

Q.26 In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change inthe potential and kinetic energy. (a) How does the pressure change as the fluid moves along the tube if dissipative forces are present ? (b) Do the dissipative forces become more important as the fluid velocity increases ? Discuss qualitatively.

Q.27 (a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar ? (b) What is the corresponding flow rate ? (Take viscosity of blood to be 2.084×10^{-3} Pa s).

Q.28 A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 km m^{-3}).

Q.29 In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^{-3} kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa s. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop to air.

Q.30 Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment is 0.465 N m^{-1}. Density of mercury $= 13^{-6} \times 10^3$ kg m^{-3}.

Q.31 Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a u-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube ? Surface tension of water at the temperature os the experiment is 7.3×10^{-2} N m^{-1}. Take the angle of contact to be zero and density of water to be 1.0×10^{-3} kg m^{-3} (g = 9.8 m s^{-1}).

Q.32 (a) It is know that density ρ of air decreases with height y (in meters) as

$$\rho = \rho_0 e^{-y/y_o}$$

where ρ_0 =1.25 kg m^{-3} is the density at sea level, and y_o is a constant. This density variations called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

(b) A large he balloon of volume 1425 m^3 is used to lift a payload of 400 kg. Assume that the balloon maintains constat radius as it rises. How high does it rise ? [Take y_o = *8000* m and ρ_{He} = 0/18 kg m^{-3}].