SEQUENCE AND SERIES

1. DEFINITION

When the terms of a sequence or series are arranged under a definite rule then they are said to be in a Progression. Progression can be classified into 5 parts as-

(i) Arithmetic Progression (A.P.)
(ii) Geometric Progression (G.P.)
(iii) Arithmetic Geometric Progression (A.G.P.)
(iv) Harmonic Progression (H.P.)
(v) Miscellaneous Progression

2. ARITHMETIC PROGRESSION (A.P.)

Arithmetic Progression is defined as a series in which difference between any two consecutive terms is constant throughout the series. This constant difference is called common difference. If 'a' is the first term and 'd' is the common difference, then an AP can be written as
$a + (a+ d) + (a + 2d) + (a + 3d) + ...$

Note: If a,b,c, are in AP $\Leftrightarrow$ $2b = a + c$

General Term of an AP General term (n^{th} term) of an AP is given by
$T_n = a + (n-1) d$

Note:

(i) General term is also denoted by ℓ (last term)
(ii) n (No. of terms) always belongs to set of natural numbers.
(iii) Common difference can be zero, + ve or – ve.
(iv) n^{th} term from end is given by
$= m - (n-1) d$
or $= (m - n + 1)^{th}$ term from beginning where m is total no. of terms.

Sum of n terms of an A.P.

The sum of first n terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a + (n-1) d] \quad \text{or}$$

$$S_n = \frac{n}{2}[a + T_n]$$

3. ARITHMETIC MEAN (A.M.)

If three or more than three terms are in A.P., then the numbers lying between first and last term are known as Arithmetic Means between them.i.e.

The A.M. between the two given quantities a and b is A so that a, A, b are in A.P.

i.e. $A - a = b - A \Rightarrow A = \frac{a+b}{2}$

Note : A.M. of any n positive numbers $a_1, a_2 .....a_n$ is

$$A = \frac{a_1 + a_2 + a_3 + ...a_n}{n}$$

n AM's between two given numbers

If in between two numbers 'a' and 'b' we have to insert n AM $A_1, A_2, ...A_n$ then a, $A_1, A_2, A_3....A_n$, b will be in A.P. The series consist of (n+2) terms and the last term is b and first term is a.

$\Rightarrow a + (n + 2 - 1) d = b$

$\Rightarrow d = \frac{b-a}{n+1}$

$A_1 = a + d$, $A_2 = a + 2d,....$
$A_n = a + nd$ or $A_n = b - d$

4. SUPPOSITION OF TERMS IN A.P.

(i) When no. of terms be odd then we take three terms are as: $a - d, a, a+ d$
five terms are as– $2d, a - d, a, a+ d, a + 2d$
Here we take middle term as 'a' and common difference as 'd'.

(ii) When no. of terms be even then we take 4 term are as : $a - 3d, a- d, a + d, a + 3d$
6 term are as = $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$
Here we take '$a - d, a + d$' as middle terms and common difference as '2d'.

5. SOME PROPERTIES OF AN A.P.

(i) If each term of a given A.P. be increased, decreased, multiplied or divided by some non zero constant number then resulting series thus obtained will also be in A.P.

(ii) In an A.P., the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last term.

(iii) Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term i.e.

$$a_n = \frac{1}{2}\left(a_{n-k} + a_{n+k}\right),\ k < n$$

(iv) If in a finite AP, the number of terms be odd, then its middle term is the AM between the first and last term and its sum is equal to the product of middle term and no. of terms.

Some standard results

(i) Sum of first n natural numbers

$$\Rightarrow \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

(ii) Sum of first n odd natural numbers

$$\Rightarrow \sum_{r=1}^{n} (2r-1) = n^2$$

(iii) Sum of first n even natural numbers

$$= \sum_{r=1}^{n} 2r = n\,(n+1)$$

(iv) Sum of squares of first n natural numbers

$$= \sum_{r=1}^{n} r^2 = \frac{n(n+1)\,(2n+1)}{6}$$

(v) Sum of cubes of first n natural numbers

$$= \sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$$

(vi) If r^{th} term of an A.P. $T_r = Ar^3 + Br^2 + Cr + D$, then sum of n term of AP is

$$S_n = \sum_{r=1}^{n} T_r = A\sum_{r=1}^{n} r^3 + B\sum_{r=1}^{n} r^2 + C\sum_{r=1}^{n} r + D\sum_{r=1}^{n} 1$$

(vii) If for an A.P. p^{th} term is q, q^{th} term is p then m^{th} term is $= p + q - m$

(viii) If for an AP sum of p terms is q, sum of q terms is p, then sum of $(p + q)$ term is $-(p + q)$.

(ix) If for an A.P. sum of p terms is equal to sum of q terms then sum of $(p + q)$ terms is zero.

6. GEOMETRICAL PROGRESSION (G.P.)

Geometric Progression is defined as a series in which ratio between any two consecutive terms is constant throughout the series. This constant ratio is called as a common ratio. If 'a' is the first term and 'r' is the common ratio, then a GP can be written as $a + ar + ar^2 + \ldots..$

Note : a, b, c are in G.P. if $\Leftrightarrow b^2 = ac$

General Term of a G.P. :

General term (n^{th} term) of a G.P. is given by $T_n = ar^{n-1}$

Sum of n terms of a G.P.

The sum of first n terms of an A.P. is given by

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a-rT_n}{1-r} \quad \text{when } r < 1$$

$$\text{or } S_n = \frac{a(r^n-1)}{r-1} = \frac{rT_n - a}{r-1} \quad \text{when } r > 1$$

$$\text{and } S_n = nr \quad \text{when } r = 1$$

Sum of an infinite G.P.

The sum of an infinite G.P. with first term a and common ratio r ($-1 < r < 1$ i.e. $|r| < 1$) is

$$S_\infty = \frac{a}{1-r}$$

7. GEOMETRICAL MEAN (G.M.)

If three or more than three terms are in G.P. then all the numbers lying between first and last term are called Geometrical Means between them.i.e.

The G.M. between two given quantities a and b is G, so that a, G, b, are in G.P.

$$\text{i.e. } \frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab \Rightarrow G = \sqrt{ab}$$

n GM's between two given numbers

If in between two numbers 'a' and 'b', we have to insert n GM $G_1, G_2 \ldots. G_n$ then a, $G_1, G_2, \ldots.$ G_n, b will be in GP. The series consist of (n+2) terms and the last term is b and first term is a.

$$\Rightarrow ar^{n+2-1} = b$$

$\Rightarrow \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_1 = ar,\ G_2 = ar^2 \ldots\ldots G_n$
$= ar^n$ or $G_n = b/r$

8. SUPPOSITION OF TERMS IN A G.P.

(i) When no. of term be odd. then we take three terms as $a/r,\ a,\ ar$

Five terms as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Here we take middle term as 'a' and common ratio as 'r'.

(ii) When no. of terms be even then we take

4 terms as : $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$ 6 terms as :

$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

Here we take $\frac{a}{r}$, ar as middle terms and common ratio as r^2.

9. SOME PROPERTIES OF A G.P.

(i) If each term of a G.P. be multiplied or divided by the same non zero quantity, then resulting series is also a G.P.

(ii) In an G.P., the product of two terms which are at equidistant from the first and the last term, is constant and is equal to product of first and last term.

(iii) If each term of a G.P. be raised to the same power, then resulting series is also a G.P.

(iv) In a G.P. every term (except first) is GM of its two terms which are at equidistant from it.

i.e. $T_r = \sqrt{T_{r-k}\,T_{r+k}} \quad k < r$

(v) In a finite G.P. , the number of terms be odd then its middle term is the G.M. of the first and last term.

(vi) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(vii) If $a_1, a_2, a_3 \ldots a_n$ is a G.P. of non zero , non negative terms, then $\log a_1, \log a_2, \ldots\ldots \log a_n$ is an A.P. and vice-versa.

(viii) If $a_1, a_2, a_3 \ldots\ldots$ and $b_1, b_2, b_3 \ldots\ldots$ are two G.P.'s then $a_1 b_1, a_2b_2, a_3b_3 \ldots\ldots$ is also in G.P.

10. ARITHMETICO- GEOMETRICAL PROGRESSION (A.G.P.) :

If each term of a Progression is the product of the corresponding terms of an A.P. and a G.P., then it is called arithmetic- geometric progression (A. G.P.)

e.g. a, (a + d) r, (a + 2d) r^2 , ...

The general term (n^{th} term) of an A.G.P. is

$T_n = [a + (n-1)\,d]\, r^{n-1}$

To find the sum of n terms of an A.G.P. we suppose its sum S, multiply both sides by the common ratio of the corresponding G.P. and than subtract as in following way and we get a G.P. whose sum can be easily obtained.

$S_n = a + (a + d)\, r + (a + 2d)\, r^2 + \ldots [a + (n-1)\, d]\, r^{n-1}$ $rS_n =$

$ar + (a+d)\, r^2 + \ldots\ldots + [a+(n-1)\, d]\, r^n$

After subtraction we get

$S_n\,(1-r)$

$= a + r.d + r^2 d + \ldots dr^{n-1} - [a + (n-1)d]\, r^n$

After solving

$$S_n = \frac{a}{1-r} + \frac{r.d\left(1-r^{n-1}\right)}{(1-r)^2}$$

and $$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

SOLVED PROBLEMS

Ex.1 If for an A.P. T_3 = 18 and T_7 = 30 then find S_{17}

Sol. Let first term = a, common difference = d

Then T_3 = a + 2d = 18 and

T_7 = a + 6d = 30

Solving these , a= 12, d= 3

$\therefore \quad S_{17} = \frac{17}{2}[2a + (17-1)d]$

$= \frac{17}{2}[24 + 16 \times 3] = 612$

Ex.2 Find The sum of integers in between 1 and 100 which are divisible by 2 or 5

Sol. Required sum = (sum of integers divisible by 2) + (sum of integers divisible by 5) – (sum of integers divisible by 2 and 5)

= (2 + 4+ 6 +....+ 100) + (5 + 10 + 15 +.....+ 100)– (10 + 20 +....+ 100)

$= \frac{50}{2}[2 \times 2 + (50-1) \times 2] + \frac{20}{2}$

$[2 \times 5 + (20-1) \times 10] - \frac{10}{2}$

$[2 \times 10 + (10-1) \times 10]$

= 50 [2 + 49] + 10 [10 + 95] – 5 [20 + 90]

= 51 × 50 + 105 × 10 – 110 × 5 = 3050

Ex.3 If $a_1, a_2, a_3.....a_n$ are in AP where $a_i > 0$ then find the value of

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ...+ \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

Sol. Let d be the c.d. of the A.P. Now L.H.S.

$$= \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + ...+ \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}$$

(Note)

$$= -\left(\frac{\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+.....\sqrt{a_n}}{d}\right)$$

$$= -\frac{\left(\sqrt{a_1}-\sqrt{a_n}\right)}{d} = \frac{1}{d}\frac{(a_n-a_1)}{\sqrt{a_n}+\sqrt{a_1}}$$

$$= \frac{(n-1)d}{d\left[\sqrt{a_n}+\sqrt{a_1}\right]} \quad [\because \ a_n = a_1 + (n-1)d]$$

$$= \frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$$

Ex.4 If a^2, b^2, c^2 are in A.P. then prove that

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+d}$ are in A.P.

Sol. $\because$ a^2 , b^2, c^2 are in A.P.

$\therefore$ a^2 + ab + bc + ca, b^2 + bc + ca + ab, c^2 + ca + ab + bc ..... are also in A.P.

[adding ab + bc + ca]

or (a+c) (a+ b), (b+ c) (a+b), (c+ a) (b+ c) .. are also in A.P.

or $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+d}$ are in A.P.

[dividing by (a + b) (b + c) (c + a)]

Ex.5 If the sum of first 6 terms of a G.P. is nine times of the sum of its first three terms, then find its common ratio

Sol. $\frac{a(1-r^6)}{1-r} = 9\frac{a(1-r^3)}{1-r}$

$\Rightarrow \quad 1 - r^6 = 9(1 - r^3) \qquad (\because r \neq 1)$

$\Rightarrow \quad 1 + r^3 = 9$

$\therefore \quad r = 2$

Ex.6 If p^{th}, q^{th} and r^{th} terms of an A.P. are equal to corresponding terms of a G.P. and these terms are respectively x,y,z, then find $x^{y-z}. y^{z-x}. z^{x-y}$

Sol. Let first term of an A.P. be a and c.d. be d and first term of a G.P. be A and c.r. be R, then a + (p–1) d = AR^{p-1} = x

$\Rightarrow$ p – 1 = (x– a) /d ...(1)

a + (q – 1) d = AR^{q-1} = y

$\Rightarrow$ q – 1 = (y–a)/d ...(2)

a + (r– 1) d= AR^{r-1} = z

$\Rightarrow$ r – 1 = (z–a) / d ...(3)

$\therefore$ Given expression

$= (AR^{p-1})^{y-z}, (AR^{q-1})^{z-x}, (AR^{r-1})^{x-y}$

$= A^0 \ R^{(p-1)(y-z)+(q-1)(z-x)+(r-1)+(x-y)}$

$= A^0 R^{[(x-a)(y-z)+(y-a)(z-x)+(z-a)(x-y)]/d} = 1$

Ex.7 If x, y, z are in G.P. and $a^x = b^y = c^z$ then prove that $\log_b a = \log_c b$

Sol. x,y,z are in G.P. $\Rightarrow$ y^2 = xz ...(i)

We have, $a^x = b^y = c^z = \lambda$ (say)

$\Rightarrow$ x log a = y log b = z log c = log λ

$\Rightarrow \quad x = \frac{\log \lambda}{\log a}, y = \frac{\log \lambda}{\log b}, z = \frac{\log \lambda}{\log c}$

putting x,y,z in (i) , we get

$$\left(\frac{\log \lambda}{\log b}\right)^2 = \frac{\log \lambda}{\log a} \cdot \frac{\log \lambda}{\log c}$$

$(\log b)^2 = \log a . \log c$

or $\log_a b = \log_b c \Rightarrow \log_b a = \log_c b$

Ex.8 If a, b, c, d are in G.P., then prove that $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$, $(c^3 + d^3)^{-1}$ are in G.P.

Sol. Let $b = ar$, $c = ar^2$ and $d = ar^3$. Then,

$$\frac{1}{a^3+b^3} = \frac{1}{a^3(1+r^3)}, \frac{1}{b^3+c^3} = \frac{1}{a^3r^3(1+r^3)}$$

and, $$\frac{1}{c^3+d^3} = \frac{1}{a^3r^3(1+r^3)}$$

Clearly, $(a^3 + b^3)^{-1}$,$(b^3 + c^3)^{-1}$ and $(c^3 + d^3)^{-1}$ are in G.P. with common ratio $1/r^3$.

Ex.9 If x,y,z are in A.P. and x,y, t are in G.P. then prove that x, x– y, t– z are in G.P.

Sol. x,y,z are in A.P.

$\Rightarrow$ $2y = x + z$

or $2xy = x^2 + xz$ (multiplying with x)

$\Rightarrow$ $x^2 - 2xy = - xz$ ...(1)

x,y, t are in G.P.

$\Rightarrow$ $y^2 = xt$ ...(2)

or $(x^2 - 2xy + y^2) = - xz + xt$

or $(x-y)^2 = x (t- z)$

x, x–y, t– z are in G.P.

Ex.10 Find the sum of the series $a - (a + d) + (a + 2d) - (a + 3d) + \ldots$ upto (2n + 1) terms

Sol. The given series is an A.G.P. with common ratio

$S = a-(a + d) + (a + 2d) - (a + 3d)+\ldots.+ (a + 2nd)$

$\Rightarrow$ $- S = - a + (a + d) - (a + 2d) + \ldots+ (a+ (2n - 1)d) - (a + 2nd)$

$\therefore$ $2S = a + \{- d + d - d + d\ldots$upto 2n terms$\} + (a+2nd)$

$\Rightarrow$ $2S = 2a + 2nd$ $S = a + nd$

Ex.11 Find the sum to n terms of the series

$$1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + \ldots.$$

Sol. Let S be the sum of n terms of the given series and $x = 1 + 1/n$, Then,

$S = 1 + 2x + 3x^2 + 4x^3 + \ldots.+ n x^{n-1}$

$\Rightarrow$ $xS = x + 2x^2+ 3x^3 +\ldots.+ (n-1) x^{n-1} + nx^n$

$\therefore$ $S - xS = 1 + [x + x^2 + \ldots+ x^{n-1}] -nx^n$

$\Rightarrow$ $S (1-x) = \frac{1-x^n}{1-x} - n x^n$

$\Rightarrow$ $S (-1/n) = - n[1-(1+1/n)^n] - n (1+1/n)^n$

$\Rightarrow$ $\frac{1}{n} . S = n [1- (1+1/n)^n + (1+1/n)^n] = n$

$\Rightarrow$ $S = n^2$

Ex.12 Find three numbers a,b,c between 2 and 18 such that - (i) Their sum is 25 (ii)The numbers 2, a,b are consecutive terms of an A.P. (iii) The numbers b,c, 18 are three consecutive terms of a G.P.

Sol. $a + b + c = 25$ ...(1)

2, a,b are in A.P. $\Rightarrow$ $a = \frac{2+b}{2}$ ...(2)

and b,c, 18 are in G.P. $\Rightarrow c^2 = 18 b$ ...(3)

Eliminating a and b from (1) ,(2) and (3), gives the following equation

$c^2 + 12 c - 288 = 0$

$(c- 12) (c+24) = 0$ $c = 12, - 24$

[Leaving c = – 24 because this is not in between 2 and 18]

$\therefore$ c = 12, from (3) b = 8 and from (1) a = 5

Hence a,b,c = 5,8,12

Ex.13 Find the sum of n terms of the series $8 + 88 + 888 + \ldots.$

Sol. Sum = $\frac{8}{9}$[9 + 99 + 999 + ...n terms]

$= \frac{8}{9}$ [(10–1) + (100–1) + (1000–1) + ... n terms]

$= \frac{8}{9}$ $[(10 + 10^2 + 10^3 + \ldots.+ 10^n) - n]$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{10-1} - n\right] = \frac{8}{81}[10^{n+1} - 9n - 10]$$

EXERCISE

Q.1 Write first 4 terms in each of the sequences:

(i) $a_n = 5n + 2$ (ii) $a_n = \frac{(2n-3)}{2}$

(iii) $a_n = (-1)^{n-1} \times 2^{n+1}$

Q.2 Find first five terms of the sequence, defined by $a_1 = 1$, $a_n = a_{n-1} + 3$ for $n \geq 2$

Q.3 Find first 5 terms of the sequence, defined by

$$a_1 = -1,\ a_n = \frac{a_{n-1}}{n} \text{ for } n \geq 2.$$

Q.4 Find 23rd term of the AP 7, 5, 3, 1, ...

Q.5 Find the 20th term of the AP

$\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$, ...

Q.6 Find the nth term of the AP

8, 3, –2, –7, ...

Q.7 Which term of the AP 5, 8, 11, 14, ...Is 320?

Q.8 Which term of the AP 64, 60, 56, ... is 0?

Q.9 How many terms are there in the AP 10, 13, 16, ... , 43 ?

Q.10 How many terms are there in the AP $\frac{5}{6}, 1, 1\frac{1}{6}, ..., 3\frac{1}{3}$?

Q.11 Is 302 a term of the AP 3, 8, 13, 18, ... ?

Q.12 The 5th and 13th terms of an AP are 5 and –3 respectively. Find this AP and obtain its 16th term.

Q.13 The 2nd, 31st and the last terms of an A.P. are $7\frac{3}{4}, \frac{1}{2}$ and $-6\frac{1}{2}$ respectively. Find the first term.

Q.14 If the 9th term of an A.P. is 0, prove that its 29th term is double the 19th term.

Q.15 How many two-digit numbers are divisible by 7?

Q.16 The 4th term of an AP is three times the first and the 7th term exceeds twice the third term by 1. Find the first term and the common difference.

Q.17 Find the 15th term from the end of the AP 3, 5, 7, 9,....., 201.

Q.18 If 7 times the 7th term of an AP is equal to 11 times its 11th term, show that its 18th term is 0.

Q.19 Three numbers are in AP. If their sum is 27and the product 648, find the numbers.

Q.20 The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 165. Find these terms.

Q.21 The angles of a quadrilateral are in AP whose common difference is 10°. Find the angles.

Q.22 Divide 32 into four parts which are in AP such that the product of the extremes is to the product of means as 7 : 15.

Q.23 Find the sum of 24 terms of the AP 1, 3, 5, 7, ...

Q.24 Find the sum of 10 terms of the AP $6, 5\frac{1}{3}, 4\frac{2}{3}, ...$.

Q.25 Find the sum of 20 terms of the AP $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$, ...

Q.26 Find the sum of 100 terms of AP 0.7, 0.71, 0.72, ...

Q.27 Find the sum of the series 101 + 99 + 97 + ...+ 47.

Q.28 How many terms of the AP 26, 21, 16, 11, ... are needed to give the sum 11?

Q.29 How many terms of the AP 18, 16, 14, 12,....are needed to give the sum 78? Explain the double answer.

Q.30 Find the sum of 32 terms of an AP whose third term is 1 and the 6th term is –11.

Q.31 In an AP, the first term is 2 and the sum of first is one-fourth of the next five terms. Show that its 20th term is –112.

Q.32 If the sum of certain number of terms of the AP 25, 22, 19, ... Is 115, find the last term.

Q.33 The sum of n term of two arithmetic progressions are in the ratio (3n + 8) : (7n + 15). Find the ratio of their 12th terms.

Q.34 The sum of n term of two arithmetic progressions are in the ratio (7n – 5) : (5n + 17). Show that their 6th terms are equal.

Q.35 If the m th term of an AP is a and its n th term is b, show that the sum of its (m + n) terms is

$\frac{(n+m)}{2}\left\{a+b+\frac{(a-b)}{(m-n)}\right\}$.

Q.36 If S_1, S_2, S_3, …., S_m are the sums of n terms of m APs whose first terms are 1, 2, 3, …., m and common differences are 1, 3, 5, …., (2m –1) respectively, show that

$(S_1 + S_2 + S_3 + …..+ S_m) = \frac{1}{2}mn(mn + 1)$.

Q.37 The sum of first 7 terms of an AP is 10 and that of next 7 terms is 17. Find the AP.

Q.38 If the sum of n terms of an AP is given by $S_n = (3n^2 + 4n)$, find its rth term.

Q.39 If the sum of n terms of an AP is $(3n^2 + 5n)$ and its m^{th} term is 164, find the value of m.

Q.40 Arun buys a scooter for Rs.22000. He pays Rs 4000 in cash and agrees to pay the balance in annual instalments of Rs l000 each plus 10% interest on the unpaid amount. How much will the scooter cost him?

Q.41 Ashok buys an old scooter for Rs l2000. He pays Rs 6000 in cash and agrees to pay the balance in annual instalments of Rs 500 each plus 12% interest on the unpaid amount. How much will the scooter cost him?

Q.42 A man repays a loan of Rs 3250 by paying Rs 20 in the first month and then increasing it by Rs 15 every month. How long will it take him to clear the loan?

Q.43 A man saved Rs 16500 in 10 years. In each year after the first year he saved Rs 100 more than he did in the preceding year. How much did he save in the first year?

Q.44 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped out on second day, four more workers dropped out on third day, and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

Q.45 A manufacturer of TV sets produced 6000 units in the third year and 7000 units in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production (i) in the first year,(ii)in the l0 year (iii) in 7 years

Q.46 Two cars start together in the same direction from the same place. The first goes with uniform speed of 40 km/hr. The second goes at a speed of 36 km/hr in the first hour and increases the speed by 1km each succeeding hour. After how many hours will the second car overtake the first car if both cars go non-stop?

Q.47 There are 30 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardner waters all the trees separately starting from the well and return to the well after watering each trees to get water for the next. Find the total distance the gardner will cover in order to water all the trees.

Q.48 A sum of Rs 62400 is paid off in 30 instalments such that each instalment is Rs 100 more than the preceding instalment. Calculate the first instalment.

Q.49 A circle is completely divided into n sectors in such a way that the angles of the sectors are in AP. If the smallest of these angles is 8^o and the largest is 72^o, calculate n and the angle in the fifth sector.

Q.50 The interior angles of a polygon are in AP. The smallest angle is 52^o and the common difference is 8^o. Find the number of sides of the polygon.

Q.51 The digits of a three-digit number are in AP and their sum is 15. The number obtained by reversing the digits is 594 less then the original number. Find the number.

Q.52 Find AM between : 9 and 19

Q.53 Insert three numbers between 3 and 19 such that the resulting sequence is an AP.

Q.54 Insert four numbers between 4 and 29 such that the resulting sequence is an AP.

Q.55 Insert five numbers between 8 and 26 such that the resulting sequence is an AP.

Q.56 Between 1 and 31. m numbers have been inserted in such a way that the ratio of 7^{th} and (m – 1)th numbers is 5 : 9. Find the value of m.

Q.57 If a, b, c are in AP, prove that:
$(a – c)^2 = 4(a – b)(b – c)$

Q.58 If a, b, c are in AP, show that (b + c – a), (c + a – b), (a + b – c) are in AP.

Q.59 If a, b, c are in AP, show that
(a+2b–c)(2b+c–a)(c+a–b) = 4abc.

Q.60 If a^2, b^2, c^2 are in AP, show that
$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP.

Q.61 If $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP, prove that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

Q.62 If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP, prove that $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP.

Q.63 If $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in AP, show that either a, b, c are in AP or (ab + bc + ca) = 0.

Q.64 Find the 10th and nth terms of the GP $12, 4, \frac{4}{3}, \frac{4}{9}, \ldots$.

Q.65 Find the 17th and nth terms of the GP $2, 2\sqrt{2}, 4, 8\sqrt{2}, \ldots$.

Q.66 Find the 7th and nth terms of the GP 0.4, 0.8, 1.6, ….. .

Q.67 Find the 10th and nth terms of the GP $\frac{-3}{4}, \frac{1}{2}, \frac{-1}{3}, \frac{2}{9}, \ldots$.

Q.68 Which term of the GP 3, 6, 12, 24, … is 3072?

Q.69 Which term of the GP $\frac{1}{4}, \frac{-1}{2}, 1, \ldots$ is -128?

Q.70 Which term of the GP $\sqrt{3}$, 3, $3\sqrt{3}$, … is 729?

Q.71 Find the geometric series whose 5th and 8th terms are 80 and 640 respectively.

Q.72 Find the GP whose 4th and 7th terms are $\frac{1}{18}$ and $\frac{-1}{486}$ respectively.

Q.73 For what values of x are the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ in GP?

Q.74 For what values of x are the numbers (x + 9), (x – 6) and 4 in GP?

Q.75 The 5th, 8th, and 11th, terms of a GP are a, b, c respectively. Show that $b^2 = ac$.

Q.76 The first term of a GP is –3 and the square of the second term is equal to its 4th term. Find its 7th term.

Q.77 The sum of three numbers in GP is $\frac{39}{10}$ and their product is 1. Find the numbers.

Q.78 The sum of three numbers in GP is 21 and the sum of their squares is 189. Find the numbers.

Q.79 Find the 6th term from the end of the GP 8, 4, 2, …., $\frac{1}{1024}$.

Q.80 Find the 4th term from the end of the GP $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots, 162$.

Q.81 Find the sum of the GP 1 + 3 + 9 + 27 +…to 12 terms

Q.82 Find the sum of the GP $\sqrt{7}+\sqrt{21}+3\sqrt{7}$ +… to n terms

Q.83 The first term of a GP is 27 and its 8th term is $\frac{1}{81}$. Find the sum of its first 10 terms.

Q.84 The 2nd and 5th terms of a GP are $\frac{-1}{2}$ and $\frac{1}{16}$ respectively. Find the sum of the GP up to 8 terms.

Q.85 The 4th and 7th terms of a GP are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of the GP.

Q.86 The common ratio of a finite GP is 3 and its last term is 486. If the sum of these terms is 728, find the first term.

Q.87 Find the sum of the geometric series 3 + 6 + 12 + … + 1536.

Q.88 How many terms of the series 2 + 6 + 18 + …. Must be taken to make the sum equal to 728?

Q.89 How many terms of the geometric series $\frac{2}{9}-\frac{1}{3}+\frac{1}{2}-\ldots$ must be taken to make the sum equal to $\frac{55}{72}$?

Q.90 The sum of n terms of a progression is $(2^n - 1)$. Show that it is a GP and find its common ratio.

Q.91 In a GP, the ratio of the sum of first 3 terms is to that of first 6 terms is 125 : 152. Find the common ratio.

Q.92 Find the sum of the series 8 + 88 + 888 + … to n terms.

Q.93 Find the sum of the series 3 + 33 + 333 + … to n terms.

Q.94 Find the sum of the series 0.7 + 0.77 + 0.777 + … to n terms.

Q.95 Evaluate $\sum_{k=1}^{11}(2+3^k)$

Q.96 The inventor of the chessboard suggested a reward of one grain of wheat for the first square; 2 grains for the second; 4 grains for the third, and so on doubling the number of grains for subsequent squares. How many grains would have to be given to the inventor?

Q.97 A man writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they move the chain similarly. Assuming that the chain is not broken and it costs Rs 2 to mail one letter, find the amount spent on postage when 8^{th} set of letters is mailed.

Q.98 A manufacturer reckons that the value of a machine which costs him Rs 31250, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Q.99 Find two positive numbers a and b whose AM and GM are 25 and 20 respectively.

Q.100 Find two positive numbers a and b whose AM and GM are 10 and 8 respectively.

Q.101 Find the GM between 5 and 125

Q.102 Insert two numbers between 9 and 243 so that the resulting sequence is a GP.

Q.103 Inset three numbers between $\frac{1}{3}$ and 432 so that the resulting sequence is a GP.

Q.104 Insert four numbers between 6 and 192 so that the resulting sequence is a GP.

Q.105 The AM between two positive numbers a and b (a > b) is twice their GM. Prove that a : b = (2 + $\sqrt{3}$) : (2 – $\sqrt{3}$).

Q.106 If a, b, c are in AP, x is the GM between a and b; y is the GM between b and c; then show that b^2 is the AM between x^2 and y^2.

Q.107 Show that the product of n geometric means between a and b is equal to the n^{th} power of the single GM between a and b.

Q.108 If a, b, c are in GP, prove that $\frac{1}{(a+b)}, \frac{1}{2b}, \frac{1}{(b+c)}$ are in AP.

Q.109 If a, b, c, d are in GP, prove that (b + c)(b + d) = (c + a)(c + d)

Q.110 If a, b, c, d are in GP, prove that (i) (a + b), (b + c), (c + d) are in GP.

Q.111 If a, b, c, d are in GP, show that $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$ are in GP.

Q.112 If $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP, prove that p, q, r are in GP.

Q.113 If a, b, c are in AP, and a, x, b and b, y, c are in GP then show that x^2, b^2, y^2 are in AP.

Q.114 If a, b, c are in AP and a, b, d are in GP, show that a, (a – b) and (d – c) are in GP.

Q.115 Find the sum of the series whose nth is given by: $(3n^2 + 2n)$

Q.116 Find the sum of the series: $(2^2 + 4^2 + 6^2 + 8^2 + ...$ To n terms)

Q.117 Find the sum of the series: (1 × 2 × 4) + (2 × 3 × 7) + (3 × 4 × 10) + ...to n terms

Q.118 Find the sum of the series: $\frac{1}{(1\times2)}+\frac{1}{(2\times3)}+\frac{1}{(3\times4)}+$ ..... to n terms

Q.119 Find the sum of each of the following infinite series: $8 + 4\sqrt{2} + 4 + 2\sqrt{2} + ... \infty$

Q.120 Prove that $9^{\frac{1}{3}}+9^{\frac{1}{9}}+9^{\frac{1}{27}}+.....\infty = 3$.

Q.121 Using geometric series, express $0.\overline{3}$ as a rational number.

Q.122 Using geometric series express $0.\overline{231}$ as a rational number.

Q.123 Find the value of $0.4\overline{23}$ in the form of a simple fraction.

Q.124 Find the rational number whose decimal from is $0.3\overline{56}$.

Q.125 Find the value of $3.5\overline{2}$ in the form of a simple fraction.

Q.126 The sum of an infinite geometric series is 6. If its first term is 2, find its common ratio.

Q.127 One side of an equilateral triangle is 24 cm. The midpoints of its sides are joined to form another triangle whose midpoints, in turn, are joined to form still another triangle. The process continues indefinitely. Find the sum of the perimeters of all the triangles.

Q.128 An equilateral is drawn by joining the midpoints of the sides of a given equilateral triangle. A third equilateral triangle is drawn inside the second triangle in the same manner. This process is repeated indefinitely. If each side of the first equilateral triangle is 6cm, find the sum of the areas of all the triangles.

ANSWER KEY

1. (i) 7, 12, 17, 22 (ii) $\frac{-1}{4},\frac{1}{4},\frac{3}{4},\frac{5}{4}$ (iii) 4,–8,16,–32 **2.** 1, 4, 7, 10, 13 **3.** $-1, \frac{-1}{2},\frac{-1}{6},\frac{-1}{24},\frac{-1}{120}$

4. –37 **5.** $39\sqrt{2}$ **6.** (13 – 5n) **7.** 106th **8.** 17th **9.** 12 **10.** 16 **11.** No

12. 9, 8, 7, 6, ... and $a_{16} = -6$ **13.** a = 8 & n = 59 **15.** 13 **16.** a = 3, d = 2 **17.** 173

19. 6, 9, 12 **20.** 4, 7, 10 **21.** 75°, 85°, 95°, 105° **22.** 2, 6, 10, 14 **23.** 576 **24.** 30 **25.** $210\sqrt{2}$ **26.** 119.5 **27.** 2072 **28.** 11 **29.** 6, 13 **30.** –1696 **31.** x = –2 **32.** x = –2 **33.** 7 : 16 **37.** $1+1\frac{1}{7}+1\frac{2}{7}+.....$

38. 6r + 1 **39.** m = 27 **40.** Rs 39100 **41.** Rs 16680 **42.** 20 months **43.** Rs 1200 **44.** 25 days

45. 5500 units (ii) 7750 units (iii) 43750 units **46.** 9 hours **47.** 4795 m **48.** Rs. 630

49. n = 9, 5th angle = 40° **50.** Three **51.** 852 **52.** 14 **53.** 7, 11, 15 **54.** 9, 14, 19, 24

55. 11, 14, 17, 20, 13 **56.** m = 14 **64.** $\frac{4}{6561},\frac{4}{3^{n-2}}$ **65.** 512, $2^{\frac{1}{2}(n+1)}$ **66.** 25.6, $\frac{2^n}{5}$ **67.** $\frac{128}{6561},\frac{-3}{4}\times\left(\frac{-2}{3}\right)^{n-1}$

68. 11th **69.** 10th **70.** 12th **71.** 5 + 10 + 20 + 40 +..... **72.** $\frac{-3}{2},\frac{1}{2},\frac{-1}{6}$ **73.** –1 or 1

74. 0 or 16 **76.** –2187 **77.** $\left(\frac{5}{2}, 1, \frac{2}{5}\right)$ or $\left(\frac{2}{5}, 1, \frac{5}{2}\right)$ **78.** (3, 6, 12) or (12, 6, 3) **79.** $\frac{1}{32}$

80. 6 **81.** 265720 **82.** $\frac{\sqrt{7}}{2}(\sqrt{3}+1)(3^{n/2}-1)$ **83.** $\frac{81}{2}.\left(1-\frac{1}{3^{10}}\right)$ **84.** $\frac{85}{128}$ **85.** $\frac{3}{2}.\left(1-\frac{1}{3^n}\right)$

86. 2 **87.** 3069 **88.** 6 **89.** 5 **90.** 2 **91.** $\frac{3}{5}$ **92.** $\frac{8}{81}(10^{n+1}-10-9n)$

93. $\frac{1}{27}.(10^{n+1}-9n-10)$ **94.** $\frac{7}{81}\left(9n-1+\frac{1}{10^n}\right)$ **95.** 265741 **96.** $(2^{64}-1)$ **97.** Rs. 131072

98. Rs. 10240 **99.** (a = 40, b = 10) or (a = 10, b = 40) **100.** (a = 4, b = 16) or (a = 16, b = 4)

101. 25 **102.** 27, 81 **103.** (2, 12, 72) or (–2, 12, –72) **104.** 12, 24, 48, 96

115. $\frac{1}{2}n(n+1)(2n+3)$ **116.** $\frac{2}{3}n(n+1)(2n+1)$ **117.** $\frac{n(n+1)(3n^2+19n+14)}{12}$ **118.** $\frac{n}{n+1}$

119. $8\,(2+\sqrt{2})$ **121.** $\frac{1}{3}$ **122.** $\frac{231}{999}$ **123.** $\frac{419}{990}$ **124.** $\frac{353}{990}$ **125.** $\frac{317}{90}$ **126.** $\frac{2}{3}$

127. 144 cm **128.** $12\sqrt{3}$ cm^2