PERCENTAGE AND ITS APPLICATIONS

8

CONTENTS

We can compare two quantities by two methods.

1. **By finding the differences of their magnitudes :**
   When we want to see how much more or less one quantity is than the other, we find the difference of their magnitudes and such a comparison is known as comparison by difference.

2. **By finding the division of their magnitudes:**
   If we want to see how many times more (or less) one quantity is than the other, we find the ratio (or division) of their magnitudes and such a comparison is known as the comparison by division.

➢ RATIO

Ratio is the comparison by division of same kind of quantities or the ratio of two quantities of same kind and in same units is a fraction that shows how many times the one quantity is of the other.

The ratio a is to b is the fraction $\frac{a}{b}$, and is written as a : b.

We call 'a' as the first term or antecedent and 'b' the second term or consequent.

Note :

1. A ratio remains unchanged if both of its terms are multiplied by the same non-zero quantity. Let $k \neq 0$, then clearly,
   (i) $\frac{a}{b} = \frac{ka}{kb}$ and therefore a : b = ka : kb
   (ii) $\frac{a}{b} = \frac{a/k}{b/k}$ and therefore $a : b = \left(\frac{a}{k} : \frac{b}{k}\right)$
2. The ratio a : b is said to be in simplest form if HCF of a and b is 1.

❖ EXAMPLES ❖

Ex.1 Express 60 : 90 in its simplest form.

Sol. In order to express the given ratio in its simplest form we divide its first and second term by their HCF.

We have $60 = 2 \times 2 \times 3 \times 5$

$90 = 2 \times 3 \times 3 \times 5$

So, HCF of 60 and 90 is $2 \times 3 \times 5$ i.e., 30.

$$\therefore \quad 60 : 90 = \frac{60}{90} = \frac{60 \div 30}{90 \div 30} = \frac{2}{3} = 2 : 3$$

Hence, the simplest form of 60 : 90 is 2 : 3.

◈ Comparison of Ratios

In order to compare two given ratios, we express each of them in simplest form and then compare these fractions by making their denominators equal.

Ex.2 Compare 5 : 12 and 3 : 5

Sol. Writing, the given ratio as fractions, we have

$$5 : 12 = \frac{5}{12} \text{ and } 3 : 5 = \frac{3}{5}$$

LCM of 12 and 5 is 60.

Making the denominator of each fraction equal to 60, we have

$$\frac{5}{12} = \frac{5 \times 5}{12 \times 5} = \frac{25}{60} \text{ and } \frac{3}{5} = \frac{3 \times 12}{5 \times 12} = \frac{36}{60}$$

Clearly, 36 > 25.

$$\therefore \quad \frac{36}{60} > \frac{25}{60} \Rightarrow \frac{3}{5} > \frac{5}{12}$$

➢ EQUIVALENT RATIO

A ratio obtained by multiplying or dividing the numerator and denominator of a given ratio by the same non zero number is called an equivalent ratio.

❖ EXAMPLES ❖

Ex.3 Find two equivalent ratio of 12 : 8.

Sol. We have $\frac{12}{8} = \frac{12 \div 4}{8 \div 4} = \frac{3}{2}$

$\therefore$ 3 : 2 is an equivalent ratio of 12 : 8.

Also, $\frac{12}{8} = \frac{12 \times 2}{8 \times 2} = \frac{24}{16}$

So, 24 : 16 is an equivalent ratio of 12 : 8.

Hence, 3 : 2 and 24 : 16 are two equivalent ratio of 12 : 8.

◈ Unitary Method

Ex.4 If 12 bowls cost ₹ 72, What will be the cost of 20 such bowl ?

Sol. $\because$ cost of 12 bowl = ₹ 72

$\therefore$ cost of 1 bowl = $₹ \frac{72}{12} = ₹ 6$

Hence, cost of 20 bowl = ₹ 6 × 20 = ₹ 120

➢ PROPORTION

Four numbers a, b, c, d are said to be in proportion, if a : b = c : d and we write a : b :: c : d or in other words we can say that an equality of two ratios is called a proportion.

(i) The first and fourth terms are called extreme terms, second and third terms are called mean terms.

If product of means = product of extremes, then given numbers are in proportion.

(ii) d is called the fourth proportional to a, b, c.

❖ EXAMPLES ❖

Ex.5 Are 25, 15, 6, 5 in proportion ?

Sol. We have a = 25, b = 15, c = 6, d = 5

a : b = 25 : 15 = 5 : 3

c : d = 6 : 5

as a : b ≠ c : d

$\therefore$ 25, 15, 6, 5 are not in proportion.

Alternative method

Product of extemes = ad = 25 × 5 = 125

Product of means = bc = 15 × 6 = 90.

as ad ≠ bc.

$\Rightarrow$ 25, 15, 6, 5 are not in proportion.

Ex.6 Find the ratio of

(i) ₹ 5 to 50 paise (ii) 15 kg to 210 gm

(iii) 9 m to 27 cm (iv) 30 days to 36 hours.

Sol. (i) ₹ 5 to 50 paise

= 5 × 100 paise to 50 paise

= 500 : 50

= 10 : 1 (dividing first and second term by their HCF i.e. by 50)

(ii) 15 kg to 210 gm

= 15 × 1000 gm to 210 gm

= 15 × 1000 : 210 = 15000 : 210

= 500 : 7 (HCF of 15000 and 210 is 30, so dividing I and II terms by 30)

(iii) 9 m to 27 cm

= 9 × 100 cm to 27 cm

= 900 : 27

= 100 : 3 (dividing I and II terms by the HCF of 900 and 27 which is 9)

(iv) 30 days to 36 hours

= 30 × 24 hours to 36 hours

= 30 × 24 : 36 = 720 : 36

= 20 : 1 (dividing I and II terms by the HCF of 720 and 36 which is 36)

Ex.7 In a computer lab, there are 3 computer for every 6 students. How many computer will be needed for 24 students ?

Sol. 6 students have = 3 computers

1 student has = $\frac{3}{6}$ computers

24 students have = $\frac{3}{6} \times 24$ computers

= 12 computers

Hence, 24 students will be needed 12 computers

Ex.8 Population of Rajasthan is 570 lakh and population of UP is 1660 lakh. Area of Rajasthan is 3 lakh km^2 and area of UP is 2 lakh km^2.

(i) How many people are there per km^2 in both these state ?

(ii) Which state is less populated ?

Sol. (i) Population of Rajasthan = 570 lakh

Area of Rajasthan = 3 lakh km^2.

$\therefore$ Number of people in per $km^2 = \frac{570}{3} = 190$

and population of U.P. = 1660 lakh

Area of U.P. = 2 lakh km^2.

$\therefore$ Number of people in per $km^2 = \frac{1660}{2} = 830.$

(ii) As population of Rajasthan per km^2 is less than the population of U.P. per km^2 so Rajasthan state is less populated.

Ex.9 The daily pocket expenses of X and Y are ₹ 45 and ₹ 90 respectively. What is the ratio of their expenses in simplest form ?

Sol. HCF of 45 and 90 = 45

Required ratio = 45 : 90

$$= \frac{45}{90} = \frac{45 \div 45}{90 \div 45} = \frac{1}{2}$$

Hence, required ratio is 1: 2.

Ex.10 Are 63 42, 33, 22 in proportion ?

Sol. Let a = 63, b = 42, c = 33, d = 22.

As product of extremes = 63 × 22 = 1386

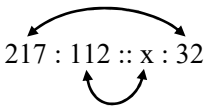
Product of means = 33 × 42 = 1386.

So, Product of extremes = Product of means

Hence, 63, 42, 33, 22 are in proportion.

Ex.11 The first, second and fourth terms of a proporiton are 217, 112, 32. Find the third term.

Sol. Let the third term of the proportion be x.



We know that if numbers in proportion, then product of means = product of extremes

$\Rightarrow$ 112 × x = 217 × 32

$\Rightarrow$ $x = \frac{217 \times 32}{112}$; x = 62

Hence, the third term of the given proportion is 62.

Ex.12 Express the ratio (i) 24 to 48 (ii) 12 cm to 1 m in their simplest form.

Sol. (i) 24 to 48 = $\frac{24}{48} = \frac{1}{2}$

(dividing both the numbers by 24)

(ii) before comparing 12 cm and 1 m they must be expressed in the same unit.

$\therefore$ $\frac{12cm}{1m} = \frac{12cm}{1 \times 100cm} = \frac{12}{100} = \frac{3}{25}$

So 12 cm : 1 m = 3 : 25

Ex.13 Express the following ratios in their simplest form :

(i) $2 : \frac{3}{4}$

(ii) $\frac{6}{7} : \frac{15}{14}$

Sol. (i) $2 : \frac{3}{4} = 2 \times 4 : \frac{3}{4} \times 4$

(Multiplying both the numbers by 4)

= 8 : 3

(ii) $\frac{6}{7} : \frac{15}{14} = \frac{6}{7} \div \frac{15}{14} = \frac{6}{7} \times \frac{14}{15} = \frac{4}{5}$

$\therefore$ $\frac{6}{7} : \frac{15}{14} = \frac{4}{5} = 4 : 5$

Ex.14 Which ratio is greater, 5 : 4 or 7 : 6 ?

Sol. To compare 5 : 4 and 7 : 6 we need to compare $\frac{5}{4}$ and $\frac{7}{6}$ so that we may express both of them with the same denominator.

$\therefore \quad \frac{5}{4} = \frac{5\times6}{4\times6} = \frac{30}{24}$ and $\frac{7}{6} = \frac{7\times4}{6\times4} = \frac{28}{24}$

Clearly, $\frac{30}{24} > \frac{28}{24}$ or 5 : 4 > 7 : 6.

Ex.15 A family has 15 pets of which 6 are cats or kittens, 3 are dogs and the rest are birds. Find the ratio of the numbers of

(i) birds to dogs (ii) birds to pets

Sol. (i) Total no. of pets = 15

No. of cats or kittens = 6

No. of dogs = 3

No. of birds = Total no. of pets

– (No. of cats + No. of dogs)

= 15 – (6 + 3) ⇒ 15 – 9 = 6

So, the no. of birds = 6

There are 6 birds and 3 dogs.

So, the number of birds : number of dogs

= 6 : 3 = 2 : 1

(ii) There are 6 birds and 15 pets

So, the number of birds : number of pets

= 6 : 15 = 2 : 5

Ex.16 Find the missing numbers in the following ratios :

(i) $\square$: 15 = 8 : 10 (ii) $\frac{\square}{4} = \frac{15}{10}$

Ex.17 Two length are in the ratio 3 : 7. The second length is 42 cm. Find the first length.

Sol. Let the first length be x cm. Then we write the ratio of the length as x : 42; but it must be equal to the given ratio 3 : 7

$\therefore \quad 3 : 7 = x : 42 \Rightarrow \frac{x}{42} = \frac{3}{7}$

$x = \frac{3}{7} \times 42 = \frac{3\times6}{1} = 18$

Hence, the first length is 18 cm.

Ex.18 In a class of 60 pupils the ratio of the number of boys to the number of girls is 7 : 8. How many boys and girls are there ?

Sol. Given that 7 are boys and 8 are girls so they are 15 together.

Therefore, boys are 7 out of 15, i.e., $\frac{7}{15}$ of 60.

and girls are 8 out of 15, i.e. $\frac{8}{15}$ of 60.

$\therefore$ The number of boys = $\frac{7}{15}$ of 60

$= \frac{7}{15} \times 60 = 7 \times 4 = 28$

and the number of girls = $\frac{8}{15} \times 60 = 8 \times 4 = 32$

Check : 28 + 32 = 60

Ex.19 Divide ₹2600 amongst three people so that their shares are in the ratio 4 : 5 : 4.

Sol. Given ratio is 4 : 5 : 4

Now sum of the rations = 4 + 5 + 4 = 13

Therefore, the share of first person is 4 out of 13.

i.e., $\frac{4}{13} \times$ ₹2600 = 4 × ₹200 = ₹800

Similarly, the share of the second person is 5 out of 13.

i.e., $\frac{5}{13} \times$ ₹2600 = 5 × ₹200 = ₹1000

and the share of the third person is 4 out of 13

i.e., $\frac{4}{13} \times$ ₹2600 = 4 × ₹200 = ₹800

Check : ₹800 + ₹1000 + ₹800 = ₹2600

Alternative Method :

Let the shares be 4x, 5x and 4x

Now the sum of shares = 4x + 5x + 4x = 13x

According to the questions 13x = ₹2600

$\Rightarrow x = \frac{₹2600}{13} =$ ₹200

Hence the share of first person is

4x = 4 × 200 = ₹800

Share of second person

= 5x = 5 × ₹200 = ₹1000

and share of the third person

= 4x = 4 × ₹200 = ₹800

Check : Sum of shares

= ₹800 + ₹1000 + ₹800 = ₹2600

➢ PERCENTAGE

When we take 100 as the denominator of fractions, the numerators are called percentages. For convenience, the symbol % is used for percent.

Or

''A percentage is simply a ratio in which the second term is arranged to be 100''. Also percent is an abbreviation of the Latin phrase per centum, meaning per hundred or hundredths.

(i) A fraction may be converted into a percentage by multiplying that fraction by 100%. This does not change its value, since 100% is 1.

(ii) A decimal may be converted into a percentage by multiplying it by 100%.

❖ EXAMPLES ❖

Ex.20 Express $\frac{7}{20}$ as a percentage.

Sol. $\frac{7}{20} = \frac{7}{20} \times 100\% = 35\%$

Ex.21 Express 0.625 as a percentage.

Sol. $0.625 = 0.625 \times 100\% = 62.5\%$

Ex.22 Write (a) $\frac{1}{4}$ (b) $\frac{22}{44}$ (c) $\frac{4}{25}$ as percent.

Sol. (a) We have $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = \left(\frac{100}{4}\right)\% = 25\%$

(b) $\frac{22}{44} = \left(\frac{22}{44} \times 100\right)\% = 50\%$

(c) $\frac{4}{25} = \left(\frac{4}{25} \times 100\right)\% = 16\%$

Ex.23 Out of 50 students in a class, 15 like to play cricket. What is percentage of students who like to play cricket ?

Sol. Total students = 50

Students who like to play cricket = 15

So, % age of students who like to play cricket

$= \left(\frac{15}{50} \times 100\right)\% = 30\%$.

Ex.24 Convert the given decimals to percent :

(a) 0.6 (b) 0.75

(c) 0.08 (d) 0.56

Sol. We have

(a) $0.6 = (0.6 \times 100)\% = 60\%$

(b) $0.75 = (0.75 \times 100)\% = 75\%$

(c) $0.08 = (0.08 \times 100)\% = 8\%$

(d) $0.56 = (0.56 \times 100)\% = 56\%$

Ex.25 Convert a percentage into fraction

(i) 45% (ii) 65% (iii) 42.5%

Sol. We have

(i) $45\% = \frac{45}{100} = \frac{9}{20}$

(ii) $65\% = \frac{65}{100} = \frac{13}{20}$

(iii) $42.5\% = \frac{42.5}{100} = \frac{425}{1000} = \frac{85}{200} = \frac{17}{40}$.

Ex.26 Convert each of the following into decimal fraction :

(a) 53% (b) 0.38% (c) 4.7%

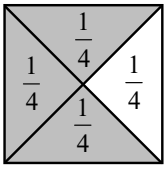
Sol. (a) $53\% = \frac{53}{100} = 0.53$

(b) $0.38\% = \frac{0.38}{100} = 0.0038$

(c) $4.7\% = \frac{4.7}{100} = \frac{47}{1000} = 0.047$.

Ex.27 What percentage of the adjoining figure is shaded and what percentage is unshaded ? Find it.

Sol. First we find the fraction of the figure that is shaded or unshaded. From this fraction we will find the percentage of the shaded and unshaded regions.



So, shaded region $= \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) = \frac{3}{4}$

Now, percentage of shaded region

$= \left(\frac{3}{4} \times 100\right)\% = 75\%$

Unshaded region $= \frac{1}{4}$

Now, percentage of unshaded region

$= \left(\frac{1}{4} \times 100\right)\% = 25\%$

◈ Uses of Percentages

1. Interpreting percentages.
2. Converting percentage to 'How many'.
3. Converting ratio to percentage.
4. Increase or decrease as percent.

Eg. : Raju invests 10% of his pocket money in buying toffees means ₹ 10 out of ₹ 100 are invested by Raju in buying the toffees.

Eg. : A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win ?

Here, the total number of matches played are 20. Out of these 25% are won by the team.

I method (direct). Out of 100, 25 matches are won by the team. So, out of 20, number of matches won by the team

$= \frac{25}{100} \times 20$

= 5 matches.

II method (using percentage).

$25\%\ \text{of}\ 20 = \frac{25}{100} \times 20 = 5$.

❖ EXAMPLES ❖

Ex.28 Convert each of the following ratios into a percentage :

(i) 15 : 45

(ii) 3 : 5

Sol. We have,

(i) $15 : 45 = \frac{15}{45} = \left(\frac{15}{45} \times 100\right)\%$

$= \left(\frac{3}{9} \times 100\right)\%$

$= \left(\frac{1}{3} \times 100\right)\%$

$= \frac{100}{3}\% = 33\frac{1}{3}\%$

(ii) $3 : 5 = \left(\frac{3}{5} \times 100\right)\% = 60\%$

Ex.29 Arun bought a car for ₹ 3,50,000. The next year, the price went upto ₹ 3,70,000. What was the percentage of price increase?

Sol. Original price = ₹ 3,50,000

Change in price = ₹ 3,70,000 – ₹ 3,50,000 = ₹ 20,000.

Percentage increase

$= \frac{\text{Amount of change in price}}{\text{Original price}} \times 100$

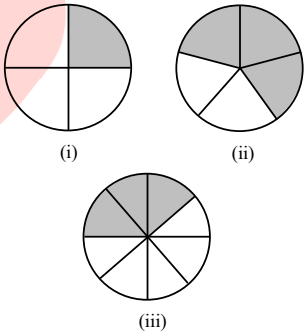
$= \frac{20,000}{3,50,000} \times 100 = \frac{2}{35} \times 100$

$= \frac{2}{7} \times 20 = \frac{40}{7} = 5\frac{5}{7}$

Percentage increase $= 5\frac{5}{7}\%$

Hence, percentage increase in price of car $= 5\frac{5}{7}\%$

Ex.30 Estimate what region of the following figures is shaded and hence find percentage of that shaded region.



Sol. We have,

(i) Shaded region $= \frac{1}{4}$

% of shaded region $= \left(\frac{1}{4} \times 100\right)\% = 25\%$

(ii) Shaded region $= \frac{3}{5}$

% of shaded region $= \left(\frac{3}{5} \times 100\right)\% = 60\%$

(iii) Shaded region $= \frac{3}{8}$

% of shaded region $= \left(\frac{3}{8} \times 100\right)\%$

$= \left(\frac{3}{2} \times 25\right)\% = \frac{75}{2}\% = 37.5\%$

Ex.31 Find

(i) 15% of 250 (ii) 1% of 1 hour

(iii) 20% of ₹ 2500 (iv) 75% of 1kg

Sol. (i) $15\%\text{ of }250 = \frac{15}{100} \times 250 = \frac{15}{4} \times 10 = 37.5$

(ii) $1\%\text{ of 1 hour} = \left(\frac{1}{100} \times 1\right)\text{hour}$

$= \left(\frac{1}{100} \times 60\right) = \frac{3}{5}\text{ min or } \left(\frac{3}{5} \times 60\right)\text{ sec}$

$= \frac{3}{5}\text{ min or 36 sec}$

(iii) $20\%\text{ of } ₹ 2500 = \frac{20}{100} \times 2500 = \frac{1}{5} \times 2500$

$= ₹ 500$

(iv) $75\%\text{ of 1 kg} = \left(\frac{75}{100} \times 1\right)\text{kg} = 0.75\text{ kg}$

Ex.32 Convert given percents to decimal fractions and also to fractions in simplest form :

(i) 25% (ii) 150% (iii) 20% (iv) 5%

Sol.

S.No.	Percentage	Fraction	*Decimal*
(i)	25%	$\frac{25}{100} = \frac{1}{4}$	*0.25*
(ii)	150%	$\frac{150}{100} = \frac{3}{2}$	*1.50*
(iii)	20%	$\frac{20}{100} = \frac{1}{5}$	*0.20*
(iv)	*5%*	$\frac{5}{100} = \frac{1}{20}$	*0.05*

Ex.33 Convert each of the ratio to percentage :

(i) 3 : 1 (ii) 2 : 3 : 5

Sol. (i) Given, 3 : 1

Total = 3 + 1 = 4

Which shows in fractions : $\frac{3}{4}$ and $\frac{1}{4}$

Also, $\frac{3}{4} = \left(\frac{3}{4} \times 100\right)\% = 75\%$

$\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%.$

(ii) Given 2 : 3 : 5

Total = 2 + 3 + 5 = 10

Which shows in fraction : $\frac{2}{10}, \frac{3}{10}, \frac{5}{10}$.

Also $\frac{2}{10} = \left(\frac{2}{10} \times 100\right)\% = 20\%$

$\frac{3}{10} = \left(\frac{3}{10} \times 100\right)\% = 30\%$

$\frac{5}{10} = \left(\frac{5}{10} \times 100\right)\% = 50\%$

Ex.34 The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.

Sol. Percentage decrease

$= \frac{\text{Decrease in population}}{\text{Initial population}} \times 100$

$= \frac{25000 - 24500}{25000}$

$= \left(\frac{500}{25000} \times 100\right)\% = 2\%$

Ex.35 In a city 30% are females, 40% are males and remaining the children. What % are children ?

Sol. Percentage of females = 30%

Percentage of males = 40%

Percentage of children = (100 – 30 – 40)% = 30%

Ex.36 (i) Chalk contains calcium, carbon and oxygen in the ratio 10 : 3 : 12. Find the percentage of carbon in chalk.

(ii) If in a stick of chalk, carbon is 3g, what is the weight of the chalk stick ?

Sol. (i) As chalk contains,

Calcium : Carbon : Oxygen = 10 : 3 : 12

Total = 10 + 3 + 12 = 25

$$\text{Carbon in chalk} = \frac{3}{25}$$

percentage of carbon in chalk

$$= \left(\frac{3}{25} \times 100\right)\% = 12\%$$

(ii) As ratio of Calcium, Carbon and oxygen in chalk is

10 : 3 : 12

Total = 10 + 3 + 12 = 25

If Carbon = 3g

$$3\text{gm} = \frac{3}{25} \times \text{chalk}$$

$$\text{Chalk} = \frac{3 \times 25}{3}\text{ gm}$$

So, weight of chalk = 25 gm

Ex.37 If in a school 45% are girls, what percentage are boys ?

Sol. If 45% are girls, then (100 – 45)% are boys i.e., 55% are boys

Ex.38 In a particular town if 85% houses have a telephone, what percentage do not have

Sol. All house (i.e. 100% of houses) either have or do not have a telephone.

If 85% have a telephone, then (100 – 85)% do not, i.e., 15% do not have telephone

Ex.39 Express 15 cm as a percentage of 3 m.

Sol. First express 3m in cm to bring both quantities to the same unit.

$\therefore$ 3 m = 3 × 100 cm = 300 cm

Then the first quantity as a percentage of the second quantity is

$$\frac{15}{300} \times 100\% = \frac{1}{20} \times 100\% = 5\%$$

Ex.40 Express 33.6 g as a percentage of 80g.

Sol. The first quantity as a percentage of the second quantity is

$$\frac{33.6}{80} \times 100\% = \frac{3360}{80}\% = \frac{336}{8}\% = 42\%$$

Ex.41 Find the value of

(i) 44% of 650 km (ii) $3\frac{1}{8}$% of 64 kg

Sol. (i) $44\%\text{ of }650\text{ km} = \frac{44}{100} \times 650 = \frac{44 \times 65}{10} = 286\text{ km}$

(ii) $3\frac{1}{8}\%$ of 64 kg $= \frac{25}{8}\%$ of 64 kg

$$= \frac{25}{8} \times \frac{1}{100} \times 64 = \frac{200}{100} = 2\text{ kg}$$

Ex.42 Find the value of $82\frac{1}{2}\%$ of 16 mm.

Sol. $82\frac{1}{2}\%$ of 16 mm $= \frac{165}{2}\%$ of 16mm

$$= \frac{165}{200} \times 16\text{ mm}$$

$$= \frac{165 \times 2}{25}\text{ mm} = \frac{33 \times 2}{5}\text{ mm} = \frac{66}{5}\text{ mm} = 13.2\text{ mm}$$

Ex.43 William travelled a distance of 10 km. He covered 70% of the distance by bus and the remaining on foot. What distance did he travel by bus ? How much distance did he cover on foot ?

Sol. Distance covered by bus = 70% of 10 km

$$= \frac{70}{100} \times 10 = 7\text{ km}$$

Distance covered on foot = 10 km – 7 km

= 3 km

Ex.44 55% of the population of a town is male. If the total population of the town is 128200, find the female population of the town.

Sol. Male population of the town = 55% of 128200

$$= \frac{55}{100} \times 128200 = 70510$$

Therefore, the female population of the town

= 128200 – 70510 = 57690

Ex.45 A person donates 6% of his total savings to the Prime Minister's Relief Fund. He divides the remaining money equally between his one son and one daughter. If the total saving of the person is ₹ 1500000, find the amount donated to the Prime Minister's Relief Fund. Find the amounts received by his son and daughter respectively

Sol. Amount donated to the Prime Minister's Relief Fund = 6% of ₹ 15,00,000

$$= \frac{6}{100} \times ₹\ 15,00,000$$

$$= ₹\ 90000$$

Amount of savings left after donation to P.M.'s Relief Fund = ₹ 15,00,000 – ₹ 90,000

= ₹ 14,10,000

Amount of received by his son

= ₹ 14,10,000 ÷ 2 = ₹ 7,05,000

because the remaining amount of his savings has been divided between his son and daughter equally. So amount of received by his daughter = ₹ 7,05,000.

Ex.46 There are 800 students in a school, out of which 560 are girls. Find the percentage of girl students in the school.

Sol. Required percentage of girl students

$$= \frac{560}{800} \times 100 = 70\%$$

Ex.47 Out of an income of ₹ 15000, Hardik spends ₹ 10200. What percentage of his income does be save ?

Sol. Hardik's total income is ₹ 15000.

Hardik's spending is ₹ 10200.

His saving is ₹ (15000 – 10200)

= ₹ 4800

Therefore, required percentage of his saving

$$= \frac{4800}{15000} \times 100 = 32\%$$

Ex.48 The population of India is 113 crore. If it increases by 1.7% every year, Find India's population after one year.

Sol. India's population = 113 crore

Increased by 1.7%

$$= 113 \text{ crore} + \left(\frac{1.7}{100} \times 113\right) \text{crore}$$

$$= 113 \text{ crore} + \frac{192.1}{100} \text{crore}$$

= 113 crore + 1.921 crore

= 14.921 crore

➢ PROFIT AND LOSS

◈ Cost Price

The price that a person spends to purchase or manufacture some goods is called the cost price. In short, we write C.P. for cost price.

◈ Selling Price

The price at which a shopkeeper or a person sells his good is called the selling price. In short, we write S.P. for selling price

In case of profit	In case of Loss
• Profit = S.P. – C.P.	• Loss = C.P. – S.P.
• S.P. = Profit + C.P.	• C.P. = Loss + S.P.
• C.P. = S.P. – Profit	• S.P. = C.P.– Loss

❖ EXAMPLES ❖

Ex.49 Find the profit or loss :

(i) C.P. = ₹ 176.50 ; S.P. = ₹ 215.80

(ii) C.P. = ₹ 499 ; S.P. = ₹ 357

(iii) C.P. = ₹ 44,450 ; S.P. = ₹ 38,578

Sol. (i) Here S.P. > C.P., therefore

Profit = S.P. – C.P. = ₹ 215.80 – ₹ 176.50

= ₹ 39.30

(ii) Here S.P. < C.P., therefore

Loss = C.P. – S.P.

= ₹ 499 – ₹ 357 = ₹ 142

(iii) Here C.P. > S.P.

So, Loss = C.P. – S.P.

= ₹ 44,450 – ₹ 38,578

= ₹ 5,872

Ex.50 A trade purchased 10 quintals of wheat from a farmer for ₹ 8,750. He sold it at ₹ 11.50 per kg. Find the amount of profit/ loss of the trader.

Sol. We know that 1 quintal = 100 kg

$\therefore$ 10 quantials = 10 × 100 kg = 1000 kg

So, the cost price of 1000 kg wheat = ₹ 8,750

Also the selling price of 1kg wheat = 11.50

Therefore, the S.P. of 1000 kg wheat

= 1000 × 11.50

= ₹ 11,500.00

Since S.P. > C.P.

So, the profit = S.P. – C.P.

= ₹ 11,500 – ₹ 8750

= ₹ 2750

Thus, the profit of the trader is ₹ 2750.

Ex.51 A shopkeeper earns a profit of ₹ 325.75 on each sewing machine. If the C.P. of a machine is ₹ 2018.50, what is the selling price ?

Sol. Profit = ₹ 325.75, Cost Price = ₹ 2018.50

$\therefore$ S.P. = Profit + C.P.

= ₹ 325.75 + ₹ 2018.50

= ₹ 2344.25

Ex.52 A milkman buys 20 litres of milk from a dairy for ₹ 370. He sells it at the rate of ₹ 21.50 per litre. Find his profit or loss.

Sol. C.P. of 20 litre milk = ₹ 370

S.P. of 1 litre milk = ₹ 21.50

Therefore, S.P. of 20 litres milk

= ₹ 21.50 × 20

= 430

Clearly, S.P. > C.P., so profit

= ₹ 430 – ₹ 370

= ₹ 60

Ex.53 A girl purchased 12 packet for ₹ 156. Each packet contains 10 pencils. She sold all the pencils at a price of ₹ 2 per pencil. Find the profit or loss.

Sol. 12 packets have 12 × 10 = 120 pencils.

C.P. for 120 pencils = ₹ 156

Selling price for 1 pencil = ₹ 2

Therefore, the S.P. of 120 pencil

= 120 × 2₹ = ₹ 240

Since S.P. > C.P., therefore, there will be the profit.

Profit = ₹ 240 – ₹ 156

= ₹ 84.

Ex.54 Bela purchased a second hand car for ₹ 89,000. She spent ₹ 21,000 on its repair and sold it to Aman for ₹ 1,10,000. Find her profit or loss in this transaction.

Sol. The amount at which the Bela purchased the car = ₹ 89,000

The amount he spent on repair = ₹ 21000

Therefore, the cost price

= ₹ 89000 + ₹ 21,000

= ₹ 1,10,000

Note : Total C.P. = actual cost price + overheads.

Since S.P. = 1,10,000

$\Rightarrow$ S.P. = C.P.

Therefore Bela neither suffered loss nor gained any profit.

➢ PROFIT OR LOSS PERCENT

In order to calculate profit or loss in percent, we use the following formulae :

1 (i) $\text{Profit \%} = \frac{\text{Amount of profit}}{\text{C.P.}} \times 100$

i.e. $\text{Profit\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100$

(ii) $\text{Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$

2. Profit or loss percent is always calculated on the C.P.

Also we can find

$$\text{S.P.} = \frac{\text{C.P.} \times (100 + \text{Profit\%})}{100}$$

In case of profit %

$$\text{S.P.} = \frac{\text{C.P.} \times (100 - \text{Loss\%})}{100}$$

In case of Loss %

$$\text{C.P.} = \frac{\text{S.P} \times 100}{(100 + \text{Profit\%})}$$

In case of Profit %

$$\text{C.P.} = \frac{\text{S.P.} \times 100}{(100 - \text{Loss\%})}$$

In case of Loss %

❖ EXAMPLES ❖

Ex.55 Find the profit or loss percent if :

(i) C.P. = ₹ 500; S.P. = ₹ 600

(ii) C.P. = ₹ 600; S.P. = ₹ 500

Sol. (i) We have, C.P. = ₹ 500, S.P. = ₹ 600

Clearly S.P. > C.P.

Therefore profit = 600 – 500 = 100

Hence, **Profit percent** = $\frac{\text{Profit}}{\text{C.P.}} \times 100$

$= \frac{100}{500} \times 100 = 20\%$

So, Profit percent = 20%

(ii) We have, C.P. = ₹ 600, S.P. = ₹ 500

Clearly C.P. > S.P.

Loss = C.P. – S.P. = 600 – 500 = ₹ 100

Therefore, **Loss percent**

$= \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{100}{600} \times 100 = \frac{50}{3} = 16\frac{2}{3}$

So Loss percent = $16\frac{2}{3}\%$

Ex.56 Karim bought 150 dozens of pencils at ₹ 20 a dozen. He sold them at ₹ 2.50 per pencil. Find the profit or loss percent.

Sol. C.P. of one dozen of pencils = ₹ 20

C.P. of 150 dozens of pencils

$= 20 \times 150 =$ ₹ 3000

Now, S.P. of 1 pencil = ₹ 2.50

S.P. of 1 dozen (i.e. 12) pencils

$= 2.50 \times 12 =$ ₹ 30

Therefore, S.P. of 150 dozen pencils

$= 150 \times 30 =$ ₹ 4500

Profit = S.P. – C.P. = ₹ (4500 – 3000)

= ₹ 1500

Profit (%) = $\frac{1500}{3000} \times 100 = 50\%$

Ex.57 Neelu bought 2400 bananas at ₹ 15 a dozen. She sold 1350 of them at ₹ 4 for 2 and remaining at ₹ 8 for 5. Find her gain or loss percent.

Sol. C.P. of 12 bananas = ₹ 15

C.P. of 1 banana = $\frac{15}{12}$

C.P. of 2400 bananas = $\frac{15}{12} \times 2400 =$ ₹ 3000

S.P. of 2 bananas = ₹ 4

S.P. of 1 banana = $\frac{4}{2}$

S.P. of 1350 bananas = $\frac{4}{2} \times 1350 =$ ₹ 2700

Remaining bananas = 2400 – 1350 = 1050

S.P. of 5 remaining bananas = ₹ 8

S.P. of 1 remaining bananas = $\frac{8}{5}$

S.P. of 1050 remaining bananas = $\frac{8}{5} \times 1050$

= ₹ 1,680

Total S.P. = ₹ 2700 + ₹ 1680 = ₹ 4380

Gain = ₹ (4380 – 3000) = ₹ 1380

Gain (%) = $\frac{1380}{3000} \times 100 = 46\%$

Ex.58 A book wholesaler sold 300 copies of a book at a profit of 15%. If C.P. of a book is ₹ 48, find the selling price of the books.

Sol. C.P. of 1 copy of the book = ₹ 48

C.P. of 300 copies of the book = 300 × 48

= ₹ 14400

Profit (%) = 15%, Profit = $\frac{15}{100} \times 14400$

= ₹ 2160

Therefore, S.P. of books = 14400 + 2160

= ₹ 16560

Ex.59 A horse bought for ₹ 8000 was sold at a loss of 6%. At what price was the horse sold ?

Sol. C.P. = ₹ 8000, Loss (%) = 6%

$$\text{Loss} = \frac{6}{100} \times 8000 = ₹\ 480$$

Therefore, S.P. = C.P. – Loss

$$= 8000 - 480 = ₹\ 7520$$

Ex.60 Shweta bought 1200 eggs at ₹ 16 a dozen. At what price per hundred must she sell the eggs so as to earn a profit of 15% ?

Sol. C.P. of a dozen i.e. 12 eggs = ₹ 16

$$\text{Therefore, C.P. of 1200 eggs} = \frac{16}{12} \times 1200$$

$$= ₹\ 1600$$

Profit (%) = 15%

$$\text{Profit} = \frac{15}{100} \times 1600 = ₹\ 240$$

S.P. = ₹ 1600 + ₹ 240 = ₹ 1840

Therefore, S.P. of 1200 eggs = ₹ 1840

$$\text{S.P. of 1 egg} = ₹\ \frac{1840}{1200}$$

$$\text{S.P. of 100 eggs} = ₹\ \frac{1840}{1200} \times 100$$

$$= \frac{460}{3} = ₹\ 153\frac{1}{3}.$$

Ex.61 An article is sold for ₹ 420 at a profit of 12%. Find the C.P.

Sol. Let the cost price of the article be 100

Given Profit = ₹ 12

S.P. = ₹ 100 + ₹ 12 = 112.

Using unitary method, we have

When S.P. is ₹ 112, C.P. = ₹ 100

$$\text{When S.P. is ₹ 1, C.P.} = ₹\ \frac{100}{112}$$

$$\text{When S.P. is ₹ 420, C.P.} = ₹\ \frac{100}{112} \times 420$$

$$= ₹\ 375$$

Hence, C.P. = ₹ 375.

Ex.62 An old bike bought for ₹ 2000 is sold for ₹ 2200. Find the profit and the profit % (or Gain%).

Sol. C.P. of the old bike = ₹ 2000

S.P. of the old bike = ₹ 2200

Clearly, S.P. > C.P.

So, Profit = S.P. – C.P.

$$= ₹\ 2200 - ₹\ 2000 = ₹\ 200$$

Therefore, gain %

$$= \left(\frac{\text{Gain}}{\text{C.P.}} \times 100\right)\% = \left(\frac{200}{2000} \times 100\right)\% = 10\%$$

Ex.63 If a man were to sell his hand cart for ₹ 720, he would loss 25%. What must be the selling price if he were to gain 25% ?

Sol. Given S.P. of the hand-cart = ₹ 720,

Loss = 25%

$$\text{C.P.} = \frac{\text{S.P.} \times 100}{(100 - \text{Loss}\%)}$$

$$\text{So, C.P.} = \frac{720 \times 100}{100 - 25} = \frac{720 \times 100}{75} = ₹\ 960$$

Desired gain = 25%

$$\text{In this case, S.P.} = \frac{\text{C.P.} \times (100 + \text{Gain}\%)}{100}$$

$$= \frac{960 \times (100 + 25)}{100} = \frac{960 \times 125}{100}$$

$$= ₹\ 1200$$

Ex.64 Nandan sells a quintal of wheat for ₹ 308 thereby, gaining a profit of 12%.

By selling a quintal of rice for the same amount, he losses 12%. Find the C.P. of both rice and wheat. Also his total gain or loss.

Sol. Given S.P. of wheat = ₹ 308, Gain = 12%

We know, $C.P. = \frac{S.P. \times 100}{(100 + gain\%)}$

Therefore, $C.P. = \frac{308 \times 100}{100 + 12} = \frac{308 \times 100}{112}$

$= ₹ 275$

Now, S.P. of rice = ₹ 308

Loss = 12%

We know, $C.P. = \frac{100 \times S.P.}{(100 - Loss\%)}$

Therefore, C.P. of rice $= \frac{100 \times 308}{(100 - 12)}$

$= \frac{100 \times 308}{88} = ₹ 350$

Total C.P. of wheat and rice

$= ₹ (275 + 350) = ₹ 625$

Total S.P. $= ₹ (308 \times 2) = ₹ 616$

We can see that S.P. < C.P.

Loss = ₹ 625 – ₹ 616 = ₹ 9

➢ SIMPLE INTEREST

Interest : Interest is the amount paid in lieu of using some money which is not owned by us.

- The amount of money deposited, lent or borrowed is called principal (P).
- The additional money given at the end of a period for using the principal is called interest.
- The total money we receive or pay is called the amount due at that time. Thus the sum of principal and interest is called amount.

  i.e. amount = principal + interest
- The time for which the money is kept in the bank or for which the loan has been borrowed is called the time period.

To find the simple interest on a certain amount of money we need to know three quantities.

(i) Amount deposited or borrowed is called principal (P)

(ii) Rate of interest (R)

(iii) Time period (T)

So, simple Interest $= \frac{P \times R \times T}{100}$

Note : If the rate of interest is given per annum then the time period must be expressed in terms of year.

For Example

Time period T = 3 month should be written as

$$\frac{3}{12} = \frac{1}{4} \text{ year}$$

T = 6 month should be written as

$$\frac{6}{12} = \frac{1}{2} \text{ year}$$

T = 9 month should be written as

$$\frac{9}{12} = \frac{3}{4} \text{ year}$$

❖ EXAMPLES ❖

Ex.65 Find the simple interest when; Principal = ₹ 600, Rate = 2% per annum and Time = 20 months.

Sol. We have, P = Principal = ₹ 600, R = Rate percent per annum = 2

And T = Time = 20 months $= \frac{20}{12}$ year

Therefore, simple interest (S.I.)

$$= \frac{P \times R \times T}{100} = ₹ \left(\frac{600 \times 2 \times 20}{100 \times 12}\right)$$

Thus S.I. = ₹ 20.

Ex.66 Find the principal when Simple Interest = ₹ 72, Rate = 3% per annum and Time = 3 months.

Sol. We have, SI = ₹ 72, R = 3%,

$$T = 3 \text{ months} = \frac{3}{12} = \frac{1}{4} \text{ year}$$

Therefore, $Principal\ (P) = \frac{100 \times S.I.}{R \times T}$

$$P = ₹ \left(\frac{100 \times 72 \times 4}{3 \times 1}\right) = ₹ (100 \times 24 \times 4)$$

$= ₹ 9600$

Ex.67 Find the rate when Principal = ₹ 700, Simple Interest = ₹ 168 and Time = 16 months

Sol. We have, P = ₹ 700, SI = ₹ 168,

$$T = 16 \text{ months} = \frac{16}{12} \text{ year}$$

$$\text{Therefore, Rate} = \frac{100 \times S.I.}{P \times T}\%$$

$$\text{Rate} = \frac{168 \times 100 \times 12}{700 \times 16}\% = \left(\frac{168 \times 12}{7 \times 16}\right)\%$$

$$= \frac{2016}{112}\% = 18\%$$

Ex.68 Find the time when principal = ₹ 640, Rate $=12\frac{1}{2}\%$ per annum and Simple Interest = ₹ 40.

Sol. We have, P = ₹ 640,

$$R = 12\frac{1}{2}\% = \frac{25}{2} \text{ per annum, SI} = ₹ 40$$

$$\text{Therefore, } T = \frac{S.I \times 100}{P \times R} = \frac{40 \times 100 \times 2}{640 \times 25} = \frac{1}{2}$$

$$\text{Thus, } T = \frac{1}{2} \text{ year or 6 months.}$$

Ex.69 Neeraj borrowed a sum of money at $10\frac{1}{2}\%$ per annum from a bank. If he paid ₹ 1863.75 as interest for $2\frac{1}{2}$ years, find the sum.

Sol. We have, $R = 10\frac{1}{2}\% = \frac{21}{2}$, S.I.= ₹ 1863.75

and $T = 2\frac{1}{2}$ years $= \frac{5}{2}$ years.

We have to find the sum.

$$\text{Now, Principal (P)} = \frac{S.I. \times 100}{R \times T}$$

$$= \frac{1863.75 \times 100 \times 2 \times 2}{21 \times 5} = 1775 \times 4$$

= ₹ 7100

Hence, the required sum = ₹ 7100

Ex.70 A sum of money becomes $\frac{7}{4}$ of itself in 6 years at a certain rate of interest. Find the rate of interest.

Sol. Let the Principal be ₹ P. Then amount = ₹ $\frac{7}{4}$P

We have, principal = ₹ P, Amount = ₹ $\frac{7}{4}$P,

T = 6 years.

We have to find the rate (R)

Then, Amount = Principal + S.I.

$$\frac{7P}{4} = P + S.I.$$

$$S.I. = \frac{7P}{4} - P = \frac{7P - 4P}{4} = \frac{3P}{4}$$

We know that,

$$S.I. = \frac{P \times R \times T}{100}$$

$$\frac{3P}{4} = \frac{P \times R \times 6}{100}$$

$$3P \times 100 = 4 \times P \times R \times 6$$

$$300\ P = 24\ P \times R$$

Therefore, Rate (R)

$$= \frac{300P}{24P}\% \text{ or } R = \frac{300P}{24P} = \frac{300}{24}\%$$

$$\Rightarrow R = \frac{300 \div 12}{24 \div 12}\% = \frac{25}{2}\% = 12\frac{1}{2}\%$$

Hence, required rate percent

$$= 12\frac{1}{2}\% \text{ per annum}$$

Ex.71 If Meena gives an interest of ₹ 45 for one year at 9% rate p.a., what is the sum she has borrowed ?

Sol. S.I. = 45, R = 9%, T = 1 year

$$S.I. = \frac{P \times R \times T}{100}$$

$$45 = \frac{P \times 9 \times 1}{100}$$

$$P = \frac{45 \times 100}{9} = 500$$

Hence, Meena has borrowed ₹ 500

Ex.72 What rate gives ₹ 280 as interest on a sum of ₹ 56,000 in 2 years ?

Sol. We have, P = ₹ 56000, T = 2, R = ?

S.I. = ₹ 280

$$S.I. = \frac{P \times R \times T}{100}$$

$$280 = \frac{56000 \times R \times 2}{100} \Rightarrow R = \frac{280 \times 100}{56000 \times 2}$$

Hence Rate (R) = 0.25%

Ex.73 Find the amount to be paid at the end of 3 years in each case :
(i) Principal = ₹ 1200 at 12% p.a.
(ii) Principal = ₹ 7500 at 5% p.a.

Sol. (i) We have, P = ₹ 1200, R = 12%, T = 3 years

$$S.I. = \frac{P \times R \times T}{100} = \frac{1200 \times 12 \times 3}{100}$$

S.I. = ₹ 432.
A = P + S.I. = ₹ (1200 + 432)
A = ₹ 1632.

(ii) We have, P = ₹ 7500, R = 5%, T = 3 years

$$S.I. = \frac{P \times R \times T}{100} = \frac{7500 \times 5 \times 3}{100}$$

S.I. = 1125
A = P + S.I. = 7500 + 1125
A = ₹ 8625

Ex.74 Amina buys a book for ₹ 275 and sells it at a loss of 15%. How much does she sell it for ?

Sol. We have
C.P. = ₹ 275
Loss % = 15%

$$Loss\% = \frac{Loss}{C.P.} \times 100$$

$$15 = \frac{Loss}{275} \times 100$$

$$Loss = \frac{15 \times 275}{100}$$

= ₹ 41.25.
S.P. = C.P. – Loss = 275 – 41.25
Hence, S.P. = ₹ 233.75

Ex.75 Juhi sells a washing machine for ₹ 13,500. She loses 20% in the bargain. What was the price at which she bought it ?

Sol. We have
S.P. = ₹ 13500

$$Loss\% = \frac{Loss}{C.P.} \times 100 \qquad \frac{Loss\%}{100} = \left(\frac{C.P. - S.P.}{C.P.}\right)$$

$$C.P. = \frac{100 \times S.P.}{(100 - loss\%)} = \frac{100 \times 13500}{100 - 20}$$

$$= \frac{100 \times 13500}{80}$$ Hence C.P. = ₹ 16,875

Ex.76 I bought a T.V. for ₹ 10,000 and sold it at a profit of 20%. How much money do I get for it?

Sol. We have, C.P. = ₹ 10,000
Profit % = 20%

$$Profit\% = \frac{Profit}{C.P.} \times 100$$

$$Profit = \frac{Profit\% \times C.P.}{100} = \frac{20 \times 10,000}{100}$$

Profit = ₹ 2000
S.P. = C.P. + Profit
= ₹ (10,000 + 2000)
= ₹ 12,000
Hence I got ₹ 12000 for T.V.

Ex.77 An article was bought for ₹ 400 and sold for ₹ 350. Find the loss and loss percent.

Sol. We have
C.P = ₹ 400
S.P = ₹ 350
As C.P. > S.P.
Loss = C.P. – S.P.
= ₹ (400 – 350)
Loss = ₹ 50

$$Loss\% = \frac{Loss}{C.P} \times 100$$

$$= \frac{50}{400} \times 100$$

Loss % = 12.5%

Ex.78 An article was purchased for ₹ 500 and sold for ₹ 550. Find the gain and gain percent.

Sol. We have, C.P. = ₹ 500
S.P. = ₹ 550
As S.P. > C.P.
∴ Profit = ₹ 50

$$Profit\% = \frac{Profit}{C.P.} \times 100$$

$$= \frac{50}{500} \times 100 = 10\%$$

Hence, Profit % = 10%