Alternating Current

7

ELECTRIC CURRENT

When a source of emf is connected to a circuit, the free electrons drift an a definite direction in addition to their randonms motion. In this way charge flows in the circuit. This drift continues as long as the emf acts.

* The charge flowing through a point in a circuit in a unit time, i.e., the rate of flow of charge is called current.
* The flow of charge is due to negatively charged electrons and they move from cathode to the anode. But according to classical concepts the direction of current is assumed to be direction of flow of positive charge, i.e., current is assumed to flow from anode to cathode.

If a charge δQ flow during a time interval δt, then current $I = \left(\frac{\delta Q}{\delta t}\right)_{\delta t \to 0} = \frac{dQ}{dt}$

* Practical unit (rationalised M.K.S. unit) of current is ampere and it is equivalent to coulomb per second. When one coulomb charge flows in one second, the current is called one ampere.
* National Bureau of standards has defined ampere by magnetic effect of current. According to this definition.

One ampere current is equivalent to that current which flowing through two thin infinitely long, straight parallel wires placed at a distance of 1 m apart produce a force of 2×10^{-7} N/m between them.

* The charge on one electron is 1.6×10^{-19} coulomb. Thus for one ampere current 6.25×10^{18} electrons would flow in one second through a point
* For generalization of units, current A is also taken as a fundamental unit along with mass M, length L and time T. Thus

(a) Dimensions of current : $M^0L^0T^0A^1$

(b) Dimensions of charge : $M^0L^0T^1A^1$

(c) Dimensions of voltage : $M^1L^2T^{-3}A^{-1}$

(work per unit charge)

(d) Dimensions of resistance : $M^1L^2T^{-3}A^{-2}$

(resistance = voltage/current)

* Electric currents are of two types

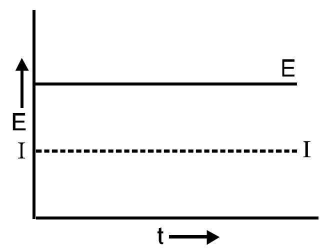
(a) Direct current (dc)

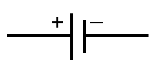
(b) Alternating current (ac)

DIRECT CURRENT (dc)

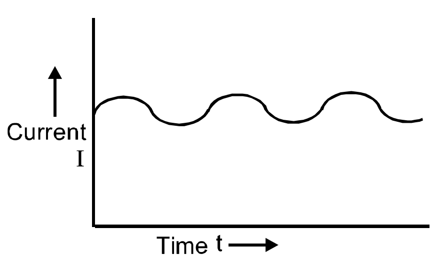
* This current flows in a definite direction.
* The magnitude of current or voltage remains constant, i.e. it does not change with time.
* The graph drawn between the magnitude of this direct current I or voltage E and the time (t) is a straight line parallel to time axis.
* It obeys Ohm's law i.e., $V \propto I$ or $\frac{V}{I} = R$ or $V = IR$
* It obeys Joule's law, i.e.,

  Heat product $H \propto I^2 RT$



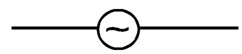
* Direct current is produced by a battery or cell.
* Its frequency is zero.
* Direct current is represented by a definite direction in a circuit.
* Direct current source is shown in the circuit by the following symbol. 

* If we study the current received from special device such as rectifiers, then we will find that the direction of current remains unchanged with time but its value change periodically as shown in the figure. Such current is called fluctuating direct current. In real sense this fluctuating current is a mixture of direct current and periodically varying current. The shapes of ripples or fluctuations present in the current are different in different circuits.



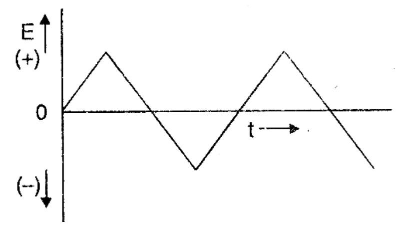
ALTERNATING CURRENT (ac)

* This current does not flow in a definite direction.
* In ac the value of current and voltage does not remain constant but changes periodically with time.
* This current also obeys Ohm's law.
* This current also obeys Joule's law.
* This current is produced when a coil is rotated in a magnetic field. The production of this current is based on electromagnetic induction.
* Its frequency is not zero.
* In ac circuits the direction of current is not indicated.
* Source of ac is represented by the following symbol.

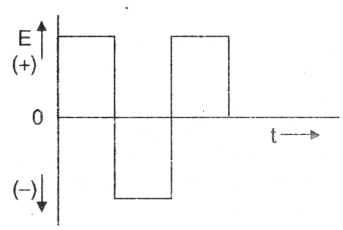


* Only periodic change is necessary for alternating emf or current. According to wave form they can be of many types. Some of them are as follows :

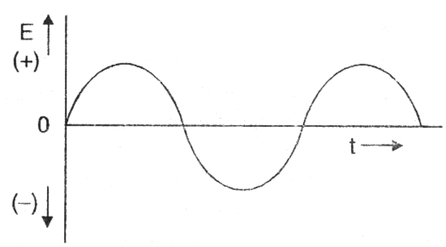
(i) Triangular Wave



(ii) Square Wave



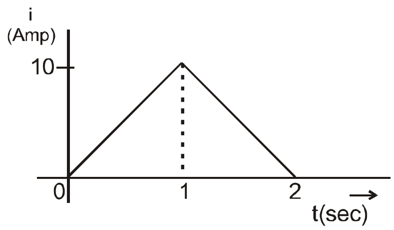
(iii) Sinusoidal Wave

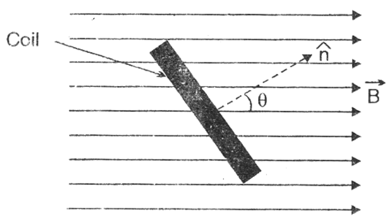


Sinusoidal wave is the simplest alternating emf or current. Its value changes simple harmonically. Such type of change can be represented by sine or cosine functions.

Solved Examples

Ex. 1. Find the average value of current shown graphically,
from t = 0 to t = 2 sec.





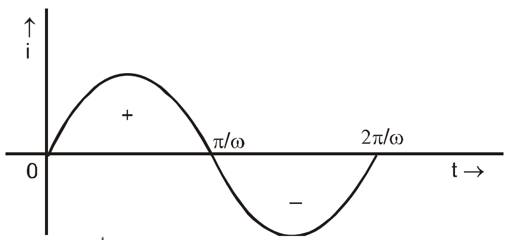
Sol. : From the i – t graph, area from t = 0 to t = 2 sec

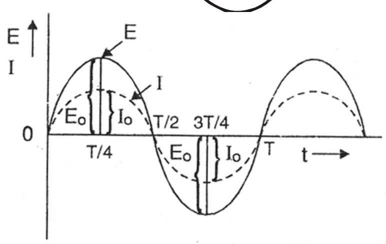
$= \frac{1}{2} \times 2 \times 10 = 10$ Amp. sec.

$\therefore$ Average Current $= \frac{10}{2} = 5$ Amp.

Ex. 2. Find the average value of current from t = 0 to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

Sol. : $\langle i \rangle = \dfrac{\int_0^{\frac{2\pi}{\omega}} I_m \sin \omega t dt}{\frac{2\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0$





It can be seen graphically that the area of i – t graph of one cycle is zero.

$\therefore \langle i \rangle$ in one cycle = 0.

Ex. 3. Show graphically that the average of sinusoidally varying current in half cycle may or may not be zero

Sol. :

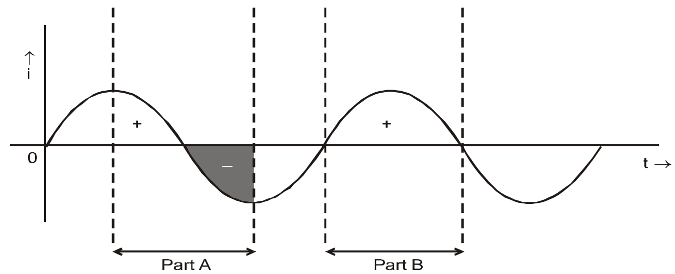


Figure shows two parts A and B, each half cycle. In part A we can see that the net area is zero

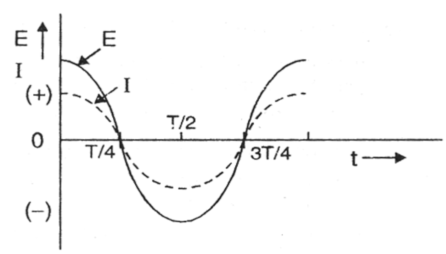
$\therefore \langle i \rangle$ in part A is zero.

In part B, area is positive hence in this part $\langle i \rangle \neq 0$.

Ex. 4. Find the average value of current $i = I_m \sin \omega t$ from (i) t = 0 to $t = \frac{\pi}{\omega}$ (ii) $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$.

Sol. : (i) $\langle i \rangle = \dfrac{\int_0^{\frac{\pi}{\omega}} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{\pi}{\omega}\right)}{\frac{\pi}{\omega}} = \frac{2I_m}{\pi}$

(ii) $\langle i \rangle = \dfrac{\int_{\frac{\pi}{2\omega}}^{\frac{3\pi}{2\omega}} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = 0.$



Ex. 5. Current in an A.C. circuit is given by $i = 2\sqrt{2} \sin(\pi t + \pi/4)$, then the average value of current during time t = 0 to t = 1 sec is:

Sol. : $\langle i \rangle = \dfrac{\int_0^1 i\, dt}{1} = \mathbf{2}\sqrt{2} \int_0^1 \sin\left(\pi t + \frac{\pi}{4}\right) = \frac{4}{\pi}$ **Ans.**

EQUATIONS OF ALTERNATING EMF AND CURRENT AND THEIR GRAPHICAL REPRESENTATION

* When a coil is rotated rapidly in a strong magnetic field magnetic flux linked with the coil changes. As a result an emf is induced in the coil and induced current flows through the circuit. The magnitude and the direction of the induced emf and current change with the orientation of the coil. Such type of current is called alternating current and voltage is called alternating voltage,
* Alternating current or voltage becomes zero two times in one cycle and is maximum two times in one cycle.
* The time taken in completing one cycle is called period (T). The time taken in changing the current or voltage from zero to maximum value or vice-versa is T/4 second.
* The number of cycles completed in one second is called frequency of the alternating current or voltage.
* If a graph is plotted between the induced emf (voltage) or current in the coil and the angle of rotation of the coil, then for one compete rotation of the coil curve similar to the curve shown in the figure is obtained. From this it is found that variation of alternating emf (voltage) or current is periodic with time similar to sine curve. So alternating emf (voltage) or current is called sinusoidal.
* At any instant t, the alternating emf E and current I can be represented by the following relations :

  $E = E_0 \sin \omega t$

  $I = I_0 \sin \omega t$

  where E and I are the instantaneous value of emf and current, E_0 and I_0 are the maximum or peak value of emf and current respectively, and ωt is the angle covered by the coil in t second. The values of E_0 and I_0 in terms of coil parameters are

  $$E_0 = NB\omega A \text{ and } I_0 = \frac{NB\omega A}{R}$$

  where N is number of turns in the coil, B is intensity of magnetic field, A is area of the coil and R is resistance of the circuit including the coil.
* If the frequency of ac is n and period is T, then

  $$\omega = 2\pi n = \frac{2\pi}{T}$$

  Thus the equations of ac voltage and current can be written as :

  $$E = E_0 \sin 2\pi nt = E_0 \sin \frac{2\pi}{T} t \text{ and } I = I_0 \sin 2\pi nt = I_0 \sin \frac{2\pi}{T} t$$

* Alternating emf and current can also be represented in terms of cosine function in place of sine function. Its form will be same but there will be phase difference of $\frac{\pi}{2}$.

  At any instant t the alternating emf and current in terms of cosine function will be $E = E_0 \cos\omega t$

  $$= E_0 \cos 2\pi nt = E_0 \cos \frac{2\pi}{T} t \text{ and } I = I_0 \cos \omega t$$

  $$I_0 \cos 2\pi nt = I_0 \cos \frac{2\pi}{T} t$$

  The graphs representing alternating emf and current in terms of the cosine function are as shown :

(a) The angular speed of current or voltage in one full cycle of ac is 360º or 2π radian and in half cycle it is π radian.

(b) Change of poles occurs two times in one cycle i.e., the direction of current changes two times in one cycle.

(c) The voltage or current becomes zero 2n times in one second and the direction of current changes 2n times in one second, n is the frequency of alternating current.

(d) The rate of change of alternating voltage or current in maximum when they change their direction, i.e. positive (+) to negative (–) or negative (–) to positive (+).

(e) The rate of change of alternating voltage or current is minimum when they are at their peak values.

* **The frequency of alternating current produced by a generator :** If alternating current is produced by an electric generator of many poles, then its frequency is

  $$f = \frac{\text{number of poles} \times \text{rotations per second}}{2} = \frac{N \times n}{2}$$

  where N is the number of poles and n is the rotational frequency of the coil.

INSTANTANEOUS, PEAK, MEAN AND ROOT MEAN SQUARE VALUES

* Different related values of alternating voltage or current in an ac circuit are :

(a) Instantaneous value

(b) Mean value

(c) Peak value

(d) Root mean square value

* **Instantaneous values :**

(a) **At any instant the value of emf (voltage) or current in an ac circuit is called instantaneous value.** The instantaneous value of emf (voltage) is represented by E and that of current by I.

(b) Instantaneous value depend upon time and vary simple harmonically in the circuit.

(c) Its value may be zero .

* **Peak Value :**

(a) The positive or negative maximum value of current in one cycle of alternating current is called peak value of current.

(b) Peak value of voltage is represented by E_0 and that of current by I_0 .

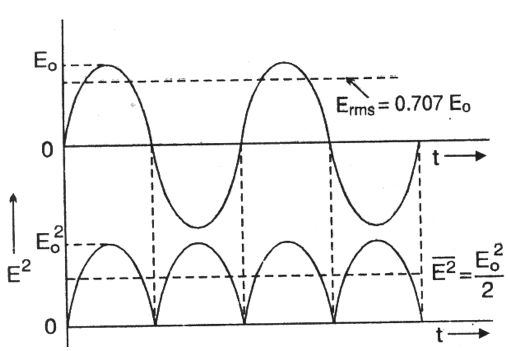
* **Mean value or average value :**

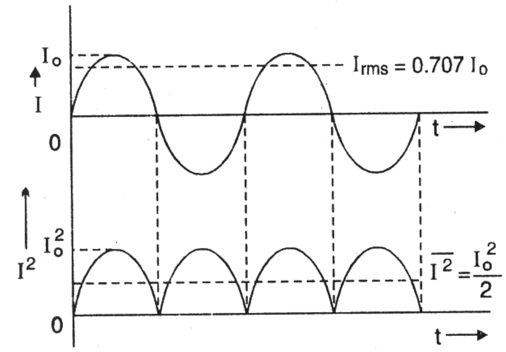
(a) The average of instantaneous value of current or voltage in one cycle is called its mean value.

(b) The mean value of alternating voltage or current in one full cycle is zero, i.e.,

$$\bar{E} = \frac{1}{T}\int_0^T E\,dt = 0 \text{ and } \bar{I} = \frac{1}{T}\int_0^0 I\,dt = 0$$

* **Mean square value** $(\overline{E^2} \text{ or } \overline{I^2})$





(a) The average of square of instantaneous value in one cycle is called mean square value

(b) Mean square value of alternating voltage or current in one full cycle is always positive.

(c) Mean square value of voltage is

$\overline{E^2} = \frac{1}{T}\int_0^T E^2 dt = \frac{E_0^2}{2}$ and mean square value of current is $\overline{I^2} = \frac{1}{T}\int_0^T I^2\,dt = \frac{I_0^2}{2}$

* **Root mean square (rms) value :**

(a) Root of mean of square of emf or current in an ac circuit for one full cycle is called root mean square value (rms value). The rms value of emf (voltage) is represented by E_{rms} and rms value of current is represented by I_{rms}.

Thus $E_{rms} = (\overline{E^2})^{1/2}$ and $I_{rms} = (\overline{I^2})^{1/2}$

(b) The square of instantaneous value of voltage or current is always positive. Therefore rms value is not zero.

(c) The rms value of voltage for one full cycle is

$$E_{rms} = (\overline{E^2})^{1/2} = \left(\frac{E_0^2}{2}\right)^{1/2} = \frac{E_0}{\sqrt{2}} = 0.707\ E_0$$

and rms value of current for one full cycle is I_{rms}

$$= (\overline{I^2})^{1/2} = \left(\frac{I_0^2}{2}\right)^{1/2} = \frac{I_0}{\sqrt{2}} = 0.707\ I_0$$

In general rms value $= \frac{\text{peak value}}{\sqrt{2}}$

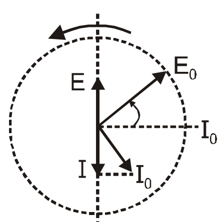
(d) The rms value of alternating current is also called virtual value or effective value. In general when values of voltage or current for alternating circuit are given, these are rms values.

(e) If direct current of strength I_{rms} is passed through a resistance R, then heat generated per second in the resistance will be $I_{rms}^2 R$ and if direct voltage of value E_{rms} is applied across a resistance R, then the heat generated per second in the resistance will also be E_{rms}^2/R. Thus, the effective values I_{rms} and E_{rms} of ac are equivalent dc values in reference to their capability of generating heat. I_{rms} E_{rms} are also called virtual current and virtual voltage.

(f) The virtual current or rms value of ac is equal to that value of dc which generates the same amount of heat in one second across a resistance as the given ac generates.

(g) Similarly the virtual voltage or rms value of alternating voltage is equal to the direct voltage which generates the same amount of heat in one second when applied across resistance R as the given alternating voltage generates.

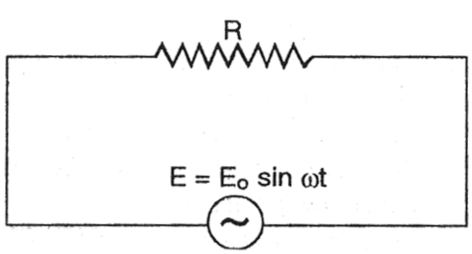
(h) The amount of heat generated by a current in a definite time does not depend on the direction of flow.



* **Peak to peak value :** It is equal to the sum of the magnitude of positive and negative peak values.

$$\therefore \text{ Peak to peak value} = E_0 + E_0 = 2E_0 = 2\sqrt{2}\,E_{rms} = 2.828\ E_{rms}$$

Different values of ac are shown in the following figure :



FORM FACTOR

* Form factor for a sinusoidal current is defined as:

$$\text{Form factor} = \frac{\text{rms value of ac}}{\text{Average value of positive half cycle}}$$

$$= \frac{I_{rms}}{2I_0/\pi} = \frac{I_0}{\sqrt{2}}\cdot\frac{\pi}{2I_0} = \frac{\pi}{2\sqrt{2}}$$

* Similarly form factor for a sinusoidal voltage :

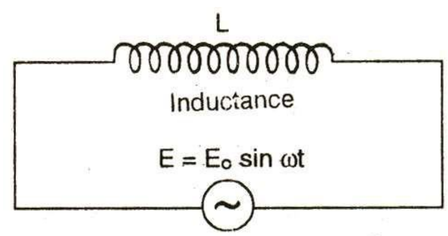
$$F = \frac{\text{rms value of alternating voltage}}{\text{Average value of positive half cycle}} = \frac{\pi}{2\sqrt{2}}$$

Solved Examples

Ex. 6. Find the rms value of current from $t = 0$ to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

Sol. : $$i_{rms} = \sqrt{\frac{\int_0^{2\pi/\omega} I_m^2 \sin^2 \omega t\,dt}{\frac{2\pi}{\omega}}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

Ex. 7. Find the rms value of current $i = I_m \sin \omega t$ from (i) $t = 0$ to $t = \frac{\pi}{\omega}$ (ii) $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$.



Sol. : (i) $$i_{rms} = \sqrt{\frac{\int_0^{\pi/\omega} I_m^2 \sin^2 \omega t\,dt}{\frac{\pi}{\omega}}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

(ii) $$\langle i \rangle = \sqrt{\frac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m^2 \sin^2 \omega t\,dt}{\frac{\pi}{\omega}}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

Note:

* The r m s values for one cycle and half cycle (either positive half cycle or negative half cycle) is same.

* From the above two examples note that for sinusoidal functions **rms value** (Also called **effective value**) $= \dfrac{\text{peak value}}{\sqrt{2}}$ or $\mathbf{I_{rms} = \dfrac{I_m}{\sqrt{2}}}$

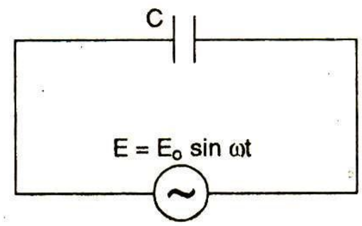
Ex. 8. Find the effective value of current $i = 2 \sin 100\pi t + 2 \cos(100\pi t + 30^o)$.

Sol. : The equation can be written as $i = 2 \sin 100\pi t + 2 \sin(100\pi t + 120^o)$

so phase difference $\phi = 120^o$

$I_m)_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$

$= \sqrt{4 + 4 + 2 \times 2 \times 2\left(-\frac{1}{2}\right)} = 2$, so effective value or rms value $= 2/\sqrt{2} = \sqrt{2}$ A



MEASUREMENT OF ALTERNATING CURRENT AND VOLTAGE

* The readings of ac ammeter and voltmeter are in terms of the virtual values and directly give rms values of current and voltage respectively.
* The ammeter and the voltmeter used for measuring ac are called hot wire ammeter or voltmeter. These devices are based on heating effect of ac.
* Dc ammeter and voltmeter measure average value of current and voltage respectively. They are based on the principle of torque acting on a current carrying coil placed in a magnetic field. These are modified moving coil galvanometers.
* If ac is passed through a direct current galvanometer (ammeter or voltmeter), no deflection will be observed in the galvanometer because direction of alternating current changes so rapidly that the pointer of the galvanometer can not follows the changes due to its inertia. Thus the pointer remains in its mean position and shows the mean value only. The average or mean value of alternating current or voltage for a full cycle is zero. Therefore the pointer of the galvanometer will remain at zero position.
* If ac ammeter or voltmeter are connected in dc circuit, then they will show deflection and will measure the actual value of current or voltage.
* The scale of these instruments (ammeter and voltmeter) measuring alternating current is not linear and the distance between the divisions increases as the magnitude of current or voltage increases while the divisions of the scale of dc instruments are equidistant.

PHASOR DIAGRAM

It is a diagram in which AC voltages and current are represented by rotating vectors. The phasor represented by a vector of magnitude proportional to the peak value rotate counter clockwise with an angular frequency ω about the origin. The projection of the phasor on vertical axis gives the instantaneous value of the alternating quantity involved. For fig.

$E = E_0 \sin \omega t$

$I = I_0 \sin(\omega t - \pi/2) = -I_0 \cos\omega t$

CURRENT IN AC CIRCUITS CONTAINING DIFFERENT COMPONENTS

* **AC circuits containing pure resistance only :**

(a) A pure resistance R is connected to an alternating source of emf as shown. Suppose at time t emf applied to the circuit is

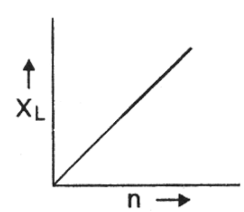
$E = E_0 \sin \omega t$

(b) According to Ohm's law, at time t the current in the circuit due to this emf will be

$I = \frac{E}{R} = \frac{E_0}{R} \sin\omega t$ or $I = I_0 \sin \omega t$

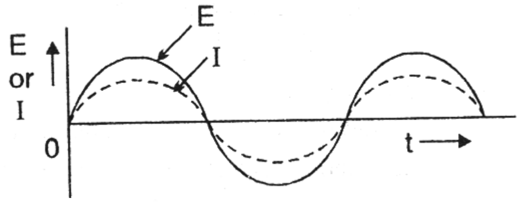
where $I_0 = \frac{E_0}{R}$ is the maximum or peak value of current.

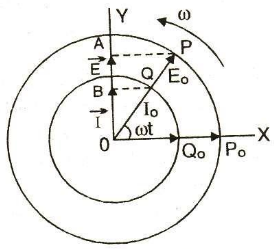
(c) By comparing the values of E and I it is found that the phase angle of the emf and the current are same ($= \omega t$) in the circuit, i.e., E and I in resistance are always in the same phase or the phase difference is zero, i.e., $\phi = 0$



(d) When emf in the resistance is maximum, current is also maximum at the same instant and when emf is zero, current is also zero.

(e) Graphical and vector representations of E and I are shown below :





* **AC circuit containing pure inductance only :**

(a) A coil of inductance L and negligible resistance (such a coil is considered to be pure inductor) is connected to a source of alternating emf as shown in the figure:

(b) Let the applied emf at any time t be $E = E_0 \sin \omega t$

An ac I flows in the circuit due to the alternating emf E such that

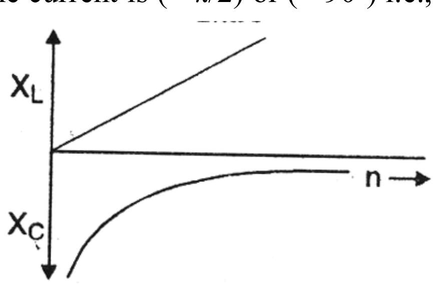
$$E - L\frac{dI}{dt} = 0 \quad \text{or} \quad \frac{dI}{dt} = \frac{E}{L} = \frac{E_0}{L}\sin \omega t$$

$$\therefore\ I = -\frac{E_0}{\omega L}\cos \omega t \quad \text{or} \quad I = -I_0 \cos \omega t$$

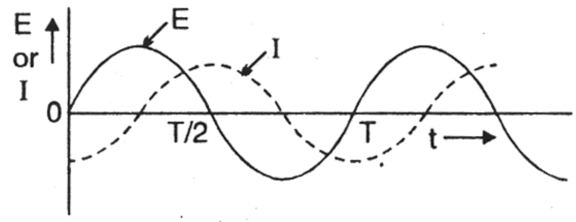
but $-\cos\omega t = \sin(\omega t - \pi/2)$

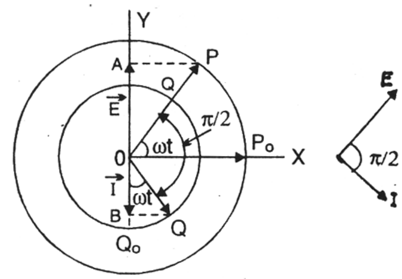
$$\therefore\ I = I_0 \sin(\omega t - \pi/2)$$

(c) By comparing the equations of E and I it is found that current is also alternating in nature and its frequency is equal to that of applied emf. But at any time t the phase angle of the current is $(\omega t - \pi/2)$ which lags behind the applied emf by $\pi/2$ or 90º. Thus, the phase difference of the voltage with respect to the current is $(+\pi/2)$ or (+ 90º) i.e., $\phi = +\pi/2$



(d) Graphical and vector respresentations of E and I are shown in the following figures :





* **AC circuit containing pure capacitance only :**

(a) A capacitor of capacitance C is connected to a source of alternating emf $E = E_0 \sin \omega t$ as shown in the figure :

(b) Since applied emf is periodic, so that charge stored on the plates of the capacitor will also be periodic, as a result an alternating current flows through the capacitor,

$$q = CE = CE_0 \sin \omega t$$

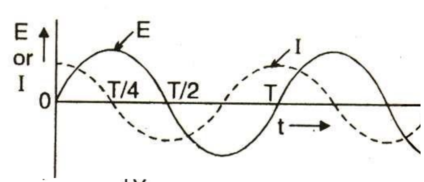
$$\therefore\ I = \frac{dq}{dt} = \omega CE_0 \cos \omega t \quad \text{or} \quad I = I_0 \cos \omega t$$

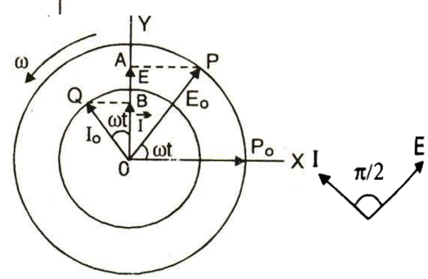
but $\cos \omega t = \sin(\omega t + \pi/2)$

$$\therefore\ I = I_0 \sin(\omega t + \pi/2)$$

(c) Comparing the relations for E and I at any time t the phase angle of the current $(\omega t + \pi/2)$ leads the emf by $\pi/2$ or 90º or the phase of the emf legs behind the current by $\pi/2$ angle, i.e., $\phi = -\pi/2$

(d) The graphical and vector representations of E and I are shown in the following figures :



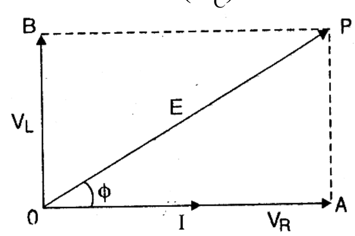


REACTANCE

* The inductor and the capacitor both obstruct the flow of alternating current. A phase difference between the emf and the current also occurs due to both of them.
* There is no loss of power due to the obstruction caused by the inductor and the capacitor, i.e., both these components are different in behaviour from an ordinary resistance in which there is always loss of power.
* The impedance offered by the inductor or the capacitor to the flow of ac is called reactance.
* The magnitude of reactance is equal to the ratio of the potential difference and the current flowing in the circuit:
* The reactance is represented by X and X $= \frac{E}{I} = \frac{E_{rms}}{I_{rms}} = \frac{E_0}{I_0}$
* Its unit is ohm.
* There are two types of reactances :

(a) Inductive Reactance (X_L)

(b) Capacitive Reactance (X_C)



1. INDUCTIVE REACTANCE (X_L)

* It is due to pure inductor
* Inductive reactance $X_L = \omega L = 2\pi nL$ where n is the frequency of the ac.
* Inductive reactance depends upon the frequency and the self inductance of the coil, i.e,

  $X_L \propto n$ and $X_L \propto L$
* The graph between X_L and n is as shown :
* Inductive coil provides a resistanceless path for dc but provides resistance to the flow of ac.

  $X_L = 2\pi nL$

  For dc, n = 0, $X_L = 0$

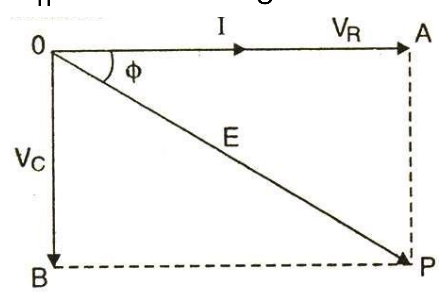
  For ac, n ≠ 0, $X_L = 2\pi nL > 0$
* The voltage across the inductor is ahead of current by a phase angle of $\pi/2$. Thus the reactance of the inductor can be expressed as :

  $X_L = \omega L \underline{|+\pi/2}$ ohm.

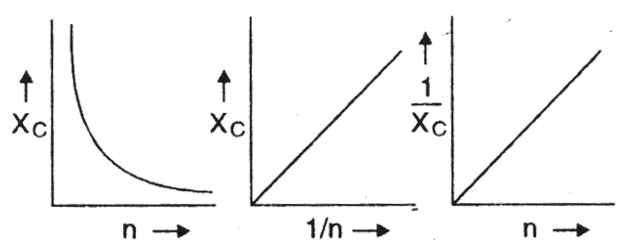
2. CAPACITIVE REACTANCE (X_C)

* It is due to a pure capacitor.
* Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi nC}$
* It depends upon the frequency and the capacitance of the capacitor as

  $X_C \propto \frac{1}{n}$ and $X_C \propto \frac{1}{C}$



* The graphs between X_C and n are



* A capacitor offers infinite resistance to the flow of dc but it provides a path for ac.

  $X_C = \frac{1}{2\pi nC}$

  For dc, n = 0, $X_C = \infty$

  For ac, n ≠ 0, $X_C = \frac{1}{2\pi nC}$ = a finite value
* The voltage across the capacitor always lags behind the current by a phase angle ($-\pi/2$). Thus the reactance of the capacitor is

  $X_C = \frac{1}{\omega C}\underline{|-\pi/2}$

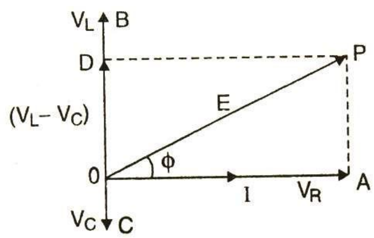
IMPEDANCE (Z)

* When both resistance and reactive components are present in a circuit, they all jointly obstruct the flow of current. This obstruction is called impedance.
* The impedance of the circuit is represented by Z. It is equal to the ratio of magnitudes of the alternating voltage and the current. Thus

$$|Z| = \frac{|E|}{|I|} = \frac{E_{rms}}{I_{rms}} = \frac{E_0}{I_0}$$

* If the phase angle of E with respect to I is θ, then

$$|Z| = \frac{|E|}{|I|}\underline{|\theta} = \frac{E_{rms}}{I_{rms}}\underline{|\theta} = \frac{E_0}{I_0}\underline{|\theta}$$

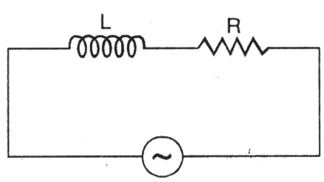


* Its unit is ohm.
* The impedance of an ac circuit depends upon the nature of the components connected in the circuit and the frequency of the alternating emf or the current.
* The impedance of the circuit and the phase angle can be determined with the help of a vector diagram.

IMPEDANCE OF SOME AC CIRCUITS

* **Series L-R circuit :**

(a) An alternating voltage E is applied in a circuit containing an inductance L and resistance R in series, as shown in figure.



$E = E_0 \sin \omega t$

(b) Let the voltage across the inductor L be V_L & the voltage across the resistor R be V_R. Same current I will flow through the resistor R and the inductor L because these are connected in series. Thus

$\therefore\ V_R = IR$

and $\quad V_L = IX_L = I\omega L$

(c) Voltage V_R and current are in phase but the voltage V_L leads the current by an angle $\pi/2$. Thus the phase difference between V_L and V_R will be $\pi/2$ or 90º, i.e., they will be perpendicular to each other when represented as vectors. The vector diagram is shown in the figure :

(d) Resultant vector OP represents the voltage on L and R which is equal to the applied voltage E.

(e) From right angle triangle OAP

$E^2 = V_R^{\,2} + V_L^{\,2}$

(f) If the impedance of the circuit is Z_{RL}, then $E = I Z_{RL}$

$\therefore\ (I\,Z_{RL})^2 = (I\,R)^2 + (I\,X_L)^2$

or $Z^2_{RL} = R^2 + X_L^{\,2}$

Hence $\quad Z_{RL} = \sqrt{R^2 + X_L^{\,2}} \quad = \sqrt{R^2 + \omega^2 L^2}$

(g) If the phase difference between the applied voltage E and the current is ϕ, then vector E will make an angle ϕ with the vector I.

Thus $\quad \tan\phi = \frac{V_L}{V_R} = \frac{I\omega L}{IR} = \frac{\omega L}{R}$

or $\ \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Solved Examples

Ex. 9. A $\frac{9}{100\pi}$ H inductor and a 12 ohm resistance are connected in series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

Sol. : Here $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{9}{100\pi} = 9\ \Omega$

So, $Z = \sqrt{R^2 + X_L^{\,2}} = \sqrt{12^2 + 9^2} = 15\ \Omega$

So (a) $\quad I = \frac{V}{Z} = \frac{225}{15} = 15$ A $\qquad$ **Ans**

and (b) $\quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right)$

$= \tan^{-1} 3/4 = 37^o$

i.e., the current will lag the applied voltage by 37º in phase. **Ans**

Ex. 10. When an inductor coil is connected to an ideal battery of emf 10 V, a constant current 2.5 A flows. When the same inductor coil is connected to an AC source of 10 V and 50 Hz then the current is 2A. Find out inductance of the coil .

Sol. : When the coil is connected to dc source, the final current is decided by the resistance of the coil .

$\therefore \quad r = \frac{10}{2.5} = 4 \ \Omega$

When the coil is connected to ac source, the final current is decided by the impedance of the coil .

$\therefore \quad Z = \frac{10}{2} = 5\ \Omega$

But $Z = \sqrt{(r)^2 + (X_L)^2}$ $\qquad X_L{}^2 = 5^2 - 4^2 = 9$

$X_L = 3\ \Omega$

$\therefore \quad \omega L = 2\pi f L = 3$

$\therefore \quad 2\pi\, 50\, L = 3$

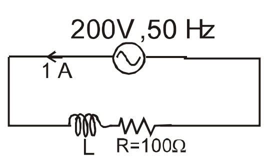
$\therefore \quad L = 3/100\pi$ Henry

Ex. 11. A bulb is rated at 100 V,100 W , it can be treated as a resistor .Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

Sol. : From the rating of the bulb , the resistance of the bulb is $R = \frac{V_{rms}{}^2}{P} = 100\ \Omega$

For the bulb to be operated at its rated value the rms current through it should be 1A

Also, $\qquad I_{rms} = \frac{V_{rms}}{Z}$



$\therefore \quad 1 = \frac{200}{\sqrt{100^2 + (2\pi 50 L)^2}} \qquad L = \frac{\sqrt{3}}{\pi} H$

Ex. 12. A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5 Ω when running of 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both cases.

Sol. : As for lamp $V_R = IR = 10 \times 5 = 50$ V, so when it is connected to 160 V ac source through a choke in series,

$V^2 = V_R{}^2 + V_L{}^2, \quad V_L = \sqrt{160^2 - 50^2} = 152$ V

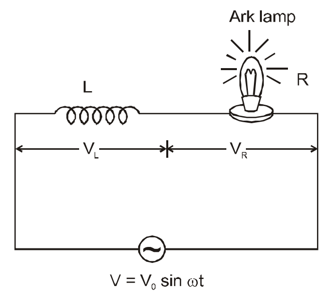
and as, $V_L = IX_L = I\omega L = 2\pi f L I$

So, $L = \frac{V_L}{2\pi f I} = \frac{152}{2 \times \pi \times 50 \times 10} = 4.84 \times 10^{-2}$ H

Ans.

Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

$V = I(R + r)$, i.e., $160 = 10(5 + r)$



i.e., $\qquad r = 11\ \Omega$ $\qquad$ **Ans.**

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

$P = I^2 R = 10^2 \times 5 = 500$ W

However, in case of dc as resistance r is to be used instead of choke, the power loss in the resistance r will be.

$PL = 10^2 \times 11 = 1100$ W

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

* **Series R-C circuit** :

(a) An alternating emf $E = E_0 \sin \omega t$ is applied to a capacitor of capacitance C and a non-inductive resistor R connected in series, as shown in figure:

(b) Let the voltage across the resistor R and the capacitor C be V_R and V_C respectively.

Then $\qquad V_R = IR$

and $\qquad V_C = IX_C = \frac{1}{\omega C}$

(c) V_R and I are in phase while V_C will lag behind the current I by $\pi/2$ or 90°. Their vector diagram is shown in figure. In this fig. V_R is represented by OA & V_C by OB. Vector OP will represent the resultant or applied voltage E on the combination

(d) From triangle OAP

$$E^2 = V_R^2 + V_C^2$$

(e) If the impedance of the circuit is Z_{RC}, then $E = IZ_{RC}$

or $(IZ_{RC})^2 = (IR)^2 + (IX_C)^2$

or $Z^2_{RC} = R^2 + X_C^2 = R^2 + \dfrac{1}{\omega^2 C^2}$

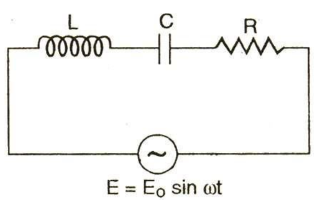
Thus $\quad Z_{RC} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}$

If the phase difference between the applied voltage E and the current I is ϕ, then

$$\tan\phi = \frac{V_C}{V_R} = \frac{I}{\omega C}/IR = \frac{1}{\omega CR}$$

* **Series LCR circuit :**

(a) Suppose a coil of inductance L, a capacitor of capacitance C and a resistor of resistance R are connected in series and an alternating emf $E = E_0 \sin \omega t$ is applied to this combination, as shown in figure :



(b) Let the potential difference across inductance L, capacitance C and resistance R be V_L, V_C and V_R respectively. The current flowing through theses components L, C and R will be same due to series combination and the vector sum of the potential difference across these components will be equal to the applied emf. If the current flowing in the circuit is I, then

$V_R = IR$

$V_L = IX_L = I\omega L$

$V_C = IX_C = I/\omega C$

(c) V_R and I are the phase, voltage V_L leads the current I by an angle $\pi/2$, voltage V_C lags behind the current by an anlge $\pi/2$. Thus V_L and V_C will be opposite in phase. Their vector diagram is shown in the figure. In this figure V_R, V_L and V_C are represented by OA, OB and OC vectors respectively, V_L and V_C are opposite in direction. Let $V_L > V_C$. The magnitude of their resultant voltage will be equal to their difference and is represented by OD where OD = OB – OC

(d) The diagonal of rectangle OAPD represents the resultant voltage E of V_R, V_L and V_C.

(e) From the figure :

$$E^2 = V_R^2 + (V_L - V_C)^2$$

(f) If the impedance of the circuit is Z, then $E = IZ$

$(IZ)^2 = (IR)^2 + (IX_L - IX_C)^2$

or $Z^2 = R^2 + (X_L - X_C)^2$

or $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Substituting the values of X_L and X_C :

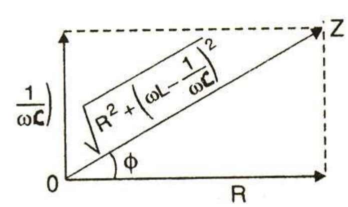
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(g) If the resultant voltage makes an angle ϕ with the current vector, then

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR}]$$

$$= \frac{(X_L - X_C)}{R} = \frac{(\omega L - 1/\omega C)}{R}$$

or $\quad \phi = \tan^{-1}\left(\dfrac{\omega L - \dfrac{1}{\omega C}}{R}\right)$



Case 1 :

If $\omega L > \dfrac{1}{\omega C}$, then $\tan\phi$ will be positive, i.e., ϕ will be positive and the voltage will be ahead of current in the circuit.

Case 2 :

If $\omega L < \frac{1}{\omega C}$, then tan ϕ will be negative, i.e., ϕ will be negative and voltage will be lagging behind the current in the circuit.

Case 3 :

If $\omega L = \frac{1}{\omega C}$, then tan ϕ will be equal to zero, i.e., $\phi = 0$. In this case the voltage and current both will be in phase. This state of circuit is called resonance. In this state the impedance of the circuit is minimum and is equal to R, i.e., circuit behaves like a circuit containing pure resistor only.

ADMITTANCE, SUSCEPTANCE AND CONDUCTANCE

* **Admittance :**

(a) The reciprocal of the impedance of an ac circuit is called admittance. It is represented by Y .

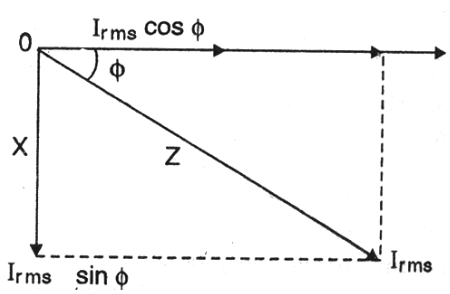
$\therefore$ Admittance $= \frac{1}{\text{Impedance}}$ $\qquad Y = \frac{1}{Z}$

(b) The unit of admittance is $(\text{ohm})^{-1}$ or ohm.

* **Susceptance :**

(a) The reciprocal of the reactance of an ac circuit is called susceptance. It is represented by S.

$\therefore$ Susceptance $= \frac{1}{\text{Reactance}}$ or $S = \frac{1}{X}$



(b) The unit of susceptance is $(\text{ohm})^{-1}$ or mho.

(c) The susceptance of a coil of inductance L is called inductive susceptance. It is equal to the reeiprocal of inductive reactance.

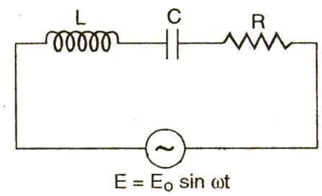
$\therefore$ Inductive susceptance $= \frac{1}{\text{Inductive reactance}}$

(d) The susceptance of a capacitor of capacitance C is called capacitive susceptance. It is equal to the reciprocal of capacitive reactance.

$\therefore$ Capacitive susceptance

$= \frac{1}{\text{Capacitive reactance}}$

$S_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C$ mho



* **Conductance :**

(a) The reciprocal of resistance of a circuit is called conductance. It is represented by G.

$\therefore$ Conductance $= \frac{1}{\text{Resistance}}$ or $G = \frac{1}{R}$

(b) The unit of conductivity is also $(\text{ohm})^{-1}$ or mho.

In the circuit in which different components are connected in parallel and same emf is applied on them its analysis in therms of admittance, susceptance and conductance becomes simpler because current in a component = voltage/(Impedance or Reactance or Resistance) = Voltage × (Admittance or Susceptance or Conductance)

POWER IN AN AC CIRCUIT

* In an electric circuit the rate of dissipation of power or the work done by the current in one second is called power of the circuit. It is equal to the product of the current and the voltage (or emf)

Thus,

electrical power = (current in the circuit) × (voltage in the circuit)

Unit of power is watt or joule/s.

$1 \text{ Watt} = \frac{1 \text{ joule}}{1 \text{ second}}$

1 horse power = 746 watt

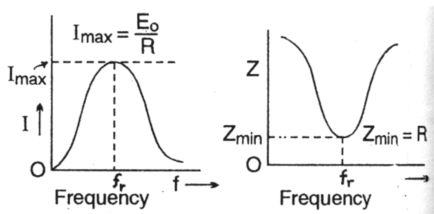
* There is a phase differnce between the voltage (emf) and the current, i.e., they are not in phase. Thus the power in an ac circuit also depends upon the phase difference between the voltage and the current.

There are three terms used for power in an ac circuit.

(a) Instantaneous power,

(b) Average power,

(c) Virtual power,



* **Instantaneous power :**

(a) The power in an ac circuit at an instant is called instantaneous power.

(b) Suppose in a circuit $E = E_0 \sin \omega t$ and $I = I_0 \sin (\omega t - \phi)$ ampere.

(c) Instantanous power

$P_{inst} = EI$

$= (E_0 \sin \omega t)\,[I_0 \sin (\omega t - \phi)]$

$= E_0 I_0 \sin^2 \omega t \cos \phi - E_0 I_0 \cos\omega t \sin \omega t \sin \phi$

$= (E_0 \sin \omega t)\,[(I_0 \cos \phi) \sin \omega t] + (E_0 \sin \omega t)\,[(I_0 \sin \phi) \sin (\omega t - \pi/2)]$

(d) Instantaneous power has two parts

(e) Its value may be positive, neagative or zero.

* **Average power :**

(a) The average of instantaneous power in an ac circuit over a full cycle is called average power.

(b) $P_{av} = \overline{P_{inst}}$

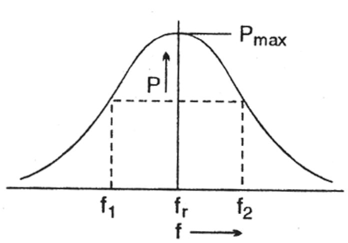
$$= \overline{E_0 I_0 \sin^2 \omega t \cos \omega + E_0 I_0 \cos \omega t \sin \omega t \sin \phi}$$

$$= \frac{1}{2} E_0 I_0 \cos \phi - 0$$

Because $\overline{\sin^2 \omega t} = \frac{1}{2}$ and $\overline{\sin \omega t} = \overline{\cos \omega t} = 0$

$$\therefore \quad P_{av} = \frac{1}{2} E_0 I_0 \cos \phi \quad = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos \phi$$

$= E_{rms}\, I_{rms} \cos \phi$



* **Virtual power :**

The product of virtual voltage and virtual current in the circuit is called virtual power.

Virtual power = $E_{rms}\, I_{rms}$

$$P_v = \frac{E_0 I_0}{2}$$

POWER FACTOR

* The factor $\cos \phi$ present in the relation for average power of an ac circuit is called power factor

so $\qquad \cos \phi = \dfrac{P_{ac}}{E_{rms} I_{rms}} = \dfrac{P_{av}}{P_v}$

Thus, ratio of average power and virtual power in the circuit is equal to power factor.

* Power factor is also equal to the ratio of the resistance and the impedance of the ac circuit.

Thus, $\qquad \cos \phi = \dfrac{R}{Z}$

* Power factor depends upon the nature of the components used in the circuit.

* If a pure resistor is connected in the ac circuit then

$\phi = 0, \cos \phi = 1$

$$\therefore \; P_{av} = \frac{E_0 I_0}{2} = \frac{E_0^2}{2R} = E_{rms} I_{rms}$$

Thus the power loss is maximum and electrical energy is converted in the form of heat.

* If a pure inductor or a capacitor are connected in the ac circuit, then

$\phi = \pm 90^o, \cos \phi = 0$

$\therefore \; P_{av} = 0$ (minimum)

Thus there is no loss of power.

* If a resistor and an inductor or a capacitor are connected in an ac circuit, then

$\phi \neq 0$ or $\pm 90^o$

Thus ϕ is in between 0 & 90º.

* If the components L, C and R are connected in series in an ac circuit, then

$$\tan \phi = \frac{X}{R} = \frac{(\omega L - 1/\omega L)}{R} \quad \text{and} \quad \cos \phi = \frac{R}{Z}$$

$$= \frac{R}{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}}$$

$\therefore$ Power factor $\qquad \cos \phi = \dfrac{R}{Z}$

* Power factor is a unitless quantity .
* If there is only inductance coil in the circuit, there will be no loss of power and energy will be stored in the magnetic field.
* If a capacitor is only connected in the crcuit, even then there will be no loss of power and energy will be stored in the electrostatic field.
* In reality an inductor and a capacitor do have some resistance, so there is always some loss of power.
* In the state of resonance the power factor is one.

WATTLESS CURRENT

* The component of current whose contribution to the average power is nil, is called wattless current.
* The average of wattless power is zero because the average of second component of instantaneous power for a full cycle will be

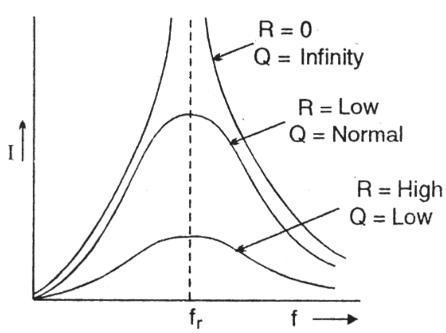
$= \overline{E_0 \sin\omega t (I_0 \sin\phi)\sin(\omega t - \pi/2)} = 0$

* The component of current associated with this part is called wattless current. Thus the current (I_0 sin ϕ) sin ($\omega t - \pi/2$) is a wattless current whose amplitude is $I_0 \sin\phi$.
* If rms value of current in the circuit is I_{rms}, then the rms value of wattless current will be $I_{rms} \sin\phi$. Wattless current lags or leads the emf by an angle $\pi/2$.

Rms value of wattless current

$= I_{rms} \sin\phi = \frac{I_0}{\sqrt{2}} \sin\phi \quad = \frac{I_0}{\sqrt{2}} \frac{X}{Z}$

Since $\sin\phi = \frac{X}{Z}$, where X is the resultant reactance of the circuit.



SERIES L-C-R RESONANT CIRCUIT

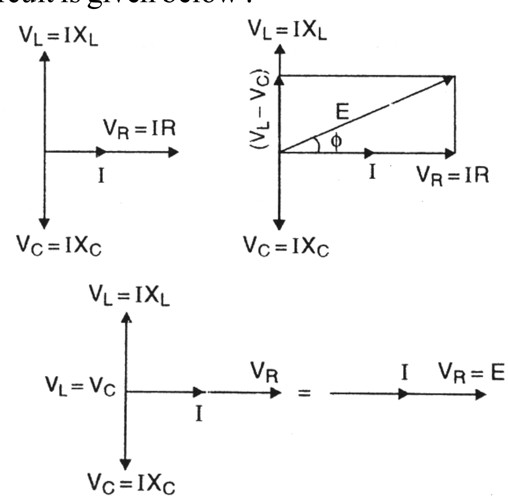
* When an inductor L, a capacitor C and a resistor R are connected in series with an ac circuit, then in general there is phase difference between the voltage and the resultant current due to these components. This phase difference is due to reactance of the circuit.
* If the frequency of the applied emf is increased, the inductive reactance ωL in the circuit increases and the capacitive reactance $\frac{1}{\omega C}$ decreases. On decreasing the frequency inductive reactance decreases and capacitive reactance increases.
* For a definite frequency of the alternating voltage the current flowing in the circuit flowing in the circuit becomes maximum. This state of ac circuit is called **electrical resonance** and the circuit is called sereis resonant circuit. The frequency at which the current in ac circuit becomes maximum, is called resonant frequency.
* In an ac circuit inductive reactance is $X_L = \omega L$ and capacitive reactance is $X_C = 1/\omega C$. In the state of resonance the inductive reactance is equal to the capacitive reactance. Thus

or $X_L = X_C$

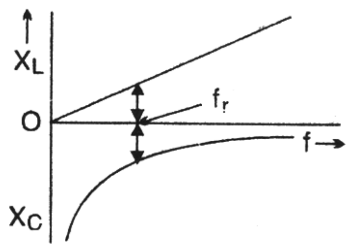
Hence $\quad \omega L = \frac{1}{\omega C} \quad$ or $\quad \omega^2 = \frac{1}{LC}$ or $\omega = \frac{1}{\sqrt{LC}}$

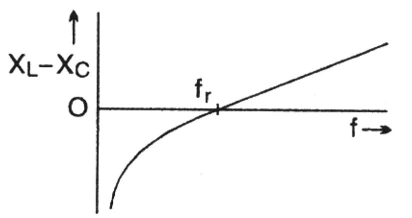
If resonant frequency is f_r, then $f_r = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$ Hz

* Vector diagram showing the resonance state of a circuit is given below :



* The variation of X_L and X_C in the circuit with the frequency f has been shown in the following figures:





* If current I and impedance Z are plotted against frequency f, then the nature of curves are as shown in the following figures for the series circuit. These curves are called resonance curves The current at resonant frequency is larger than the current at any other frequency.
* In the state of resonance following characteristics are found :

(a) The impedance of the circuit is minimum and it is purely resistive in nature. It is equal to the resistance connected in the circuit. Thus $Z = Z_{min} = R$

(b) The applied emf and the current are in same phase, i.e., the phase difference between them is zero or $\phi = 0$.

(c) Total reactance of the circuit is zero, i.e., $X = X_L - X_C = 0$

(d) The current in the circuit is maximum and depends upon the resistance of the circuit.

(e) The potential difference V_R across the resistor R connected in the circuit is equal to the applied emf.

(f) The potential difference across the inductor L is equal and opposite to the potential difference across the capacitor C. Thus the net potential difference across L and C is zero. Hence

$$\vec{V}_L = -\vec{V}_C \quad \text{and} \quad \vec{V}_L + \vec{V}_C = 0$$

(g) The power factor of the circuit is maximum and equal to one

* Series resonant circuit is also called acceptor circuit. The impedance of the circuit is minimum in the state of resonance. If electrical signal or emf of different frequencies are applied to the circuit, then for the signal or emf whose frequency is equal to the resonant or natural frequency of the circuit, maximum current will flow in the circuit and it will favourably receive signal or emf of that particular frequency.

HALF-POWER POINTS OR FREQUENCIES, BAND WIDTH & QUALITY FACTOR OF A SERIES RESONANT CIRCUIT

(A) Half power frequencies

* The frequencies at which the power in the circuit is half of the maximum power (the power at resonance), are called half-power frequencies. Thus at these freuencies

$$P = \frac{P_{max}}{2}$$

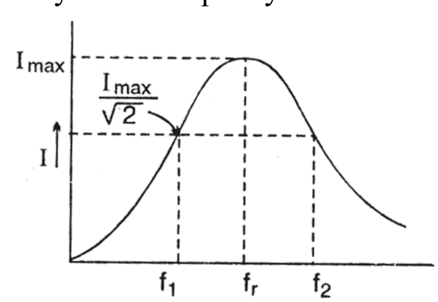
* The current in the circuit at half-power frequencies is $\frac{1}{\sqrt{2}}$ or 0.707 or 70.7% of the maximum current I_{max} (current at resonance).

Thus $$I = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

* There are two half power frequencies f_1 and f_2 :

(a) Lower half power frequency (f_1) :

This half power frequency is less than the resonant frequency. At this frequency the circuit is capacitive.



(b) Upper half power frequency (f_2) :

This half-power frequency is greater than the resonant frequency. At this frequency the circuit is inductive.

(B) Band width (Δf) :

* The difference of half-power frequencies f_1 and f_2 is called band-width (Δf)
* Band width $\Delta f = (f_2 - f_1)$
* For series resonant circuit : $\Delta f = \frac{1}{2\pi}\left(\frac{R}{L}\right)$

(C) Quality factor (Q) :

* In an ac circuit Q is defined by the following ratio:

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation per cycle}}$$

$$= \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

* For an L–C–R series resonant circuit :

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR}$$

* Quality factor in terms of band-width :

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{2\pi f_r}{2\pi(f_2 - f_1)} = \frac{f_r}{(f_2 - f_1)} = \frac{f_r}{\Delta f}$$

* Quality factor $= \frac{\text{Resonant frequency}}{\text{Band width}}$

Thus the ratio of the resonant frequency and the band-width is equal to the quality factor of the circuit.

* In the state of resonance the voltage across the resistor R will be equal to the applied voltage E. The magnitudes of voltage across the inductor and the capacitor will be equal and their values will be equal QE. Thus

$$\therefore\ V_L = I\omega L = \frac{E}{R}\omega L = EQ$$

and $$V_C = I\left(\frac{I}{\omega C}\right) = \frac{E}{\omega CR} = EQ$$

(D) Sharpness of resonance :

* For an ac circuit Q measures the sharpness of resonance.
* When Q is large, the resonance is sharp and when Q is small, the resonance is flat.
* The sharpness of resonance is inversely proportional to the band-width and the resistance R.
* For resonace to be sharp the resistance of the circuit should be small.