CO-ORDINATE GEOMETRY

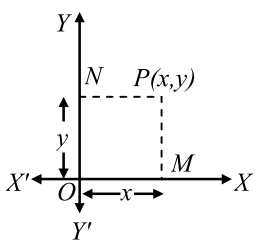
CONTENTS

CO-ORDINATE GEOMETRY

It is a branch of geometry which sets up a definite correspondence between the position of a point in a plane and a pair of algebraic numbers, called co-ordinates.

CARTESIAN CO-ORDINATES (RECTANGULAR CO-ORDINATES)

In Cartesian co-ordinates the position of a point P is determined by knowing the distances from two perpendicular lines passing through the fixed point. Let O be the fixed point called the origin and XOX′ and YOY′, the two perpendicular lines through O, called Cartesian or Rectangular co-ordinates axes.



Draw PM and PN perpendiculars on OX and OY respectively. OM (or MP) is called the y co-ordinate or the ordinate of the point P.

Axes of Co-ordinates

In the figure OX and OY are called as x-axis and y-axis respectively and both together are known as axes of co-ordinates.

Origin

It is point O of intersection of the axes of co-ordinates.

Co-ordinates of the Origin

It has zero distance from both the axes so that its abscissa and ordinate are both zero. Therefore, the coordinates of origin are (0, 0).

Abscissa

The distance of the point P from y-axis is called its abscissa. In the figure, OM is the Abscissa.

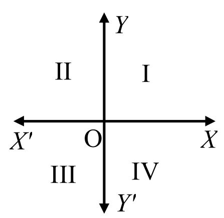
Ordinate

The distance of the point P from x-axis is called its ordinate. ON is the ordinate in the figure.

Quadrant

The axes divide the plane into four parts. These four parts are called quadrants. So, the plane consists of axes and quadrants. The plane is called the cartesian plane or the coordinate plane or the xy-plane. These axes are called the co-ordinate axes.

A quadrant is $\frac{1}{4}$ part of a plane divided by co-ordinate axes.



(i) XOY is called the first quadrant

(ii) YOX′ the second.

(iii) X′OY′ the third.

(iv) Y′OX the fourth

as marked in the figure.

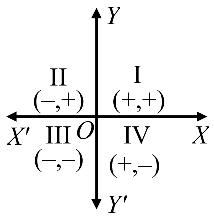
➤ RULES OF SIGNS OF CO-ORDINATES

(i) In the first quadrant, both co-ordiantes i.e., abscissa and ordinate of a point are positive.

(ii) In the second quadrant, for a point, abscissa is negative and ordinate is positive.

(iii) In the third quadrant, for a point, both abscissa and ordinate are negative.

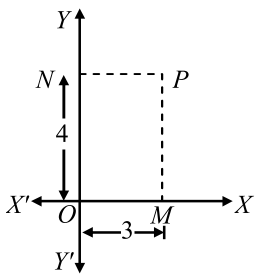
(iv) In the fourth quadrant, for a point, the abscissa is positive and the ordinate is negative.



Quadrant	x-co-ordinate	y-co-ordinate	Point
First quadrant	+	+	(+,+)
Second quadrant	–	+	(–,+)
Third quadrant	–	–	(–,–)
Fourth quadrant	+	–	(+,–)

❖ EXAMPLES ❖

Ex.1 From the adjoining figure find



(i) Abscissa

(ii) Ordinate

(iii) Co-ordinates of a point P

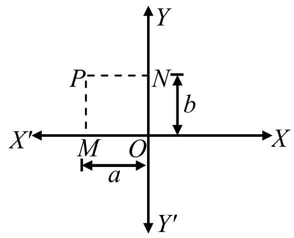
Sol. (i) Abscissa = PN = OM = 3 units

(ii) Ordinate = PM = ON = 4 units

(iii) Co-ordinates of the point

P = (Abscissa, ordinate) = (3, 4)

Ex.2 Determine (i) Abscissa (ii) ordinate (iii) Co-ordinates of point P given in the following figure.



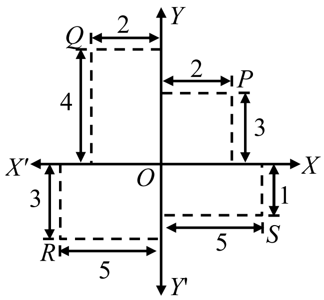
Sol. (i) Abscissa of the point P = – NP = – OM = – a

(ii) Ordinate of the point P = MP = ON = b

(iii) Co-ordinates of point P = (abscissa, ordinate)

= (–a, b)

Ex.3 Write down the (i) abscissa (ii) ordinate (iii) Co-ordinates of P, Q, R and S as given in the figure.



Sol. **Point P**

Abscissa of P = 2; Ordinate of P = 3

Co-ordinates of P = (2, 3)

Point Q

Abscissa of Q = – 2; Ordinate of Q = 4

Co-ordinate of Q = (–2, 4)

Point R

Abscissa of R = – 5; Ordinate of R = – 3

Co-ordinates of R = (–5, –3)

Point S

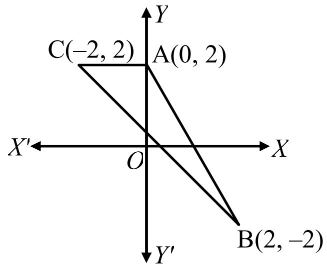
Abscissa of S = 5; Ordinate of S = – 1

Co-ordinates of S = (5, – 1)

Ex.4 Draw a triangle ABC where vertices A, B and C are (0, 2), (2, – 2), and (–2, 2) respectively.

Sol. Plot the point A by taking its abscissa O and ordinate = 2.

Similarly, plot points B and C taking abscissa 2 and –2 and ordinates – 2 and 2 respectively. Join A, B and C. This is the required triangle.

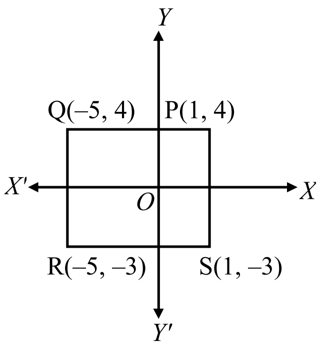


Ex.5 Draw a rectangle PQRS in which vertices P, Q, R and S are (1, 4), (–5, 4), (–5, –3) and (1, – 3) respectively.

Sol. Plot the point P by taking its abscissa 1 and ordinate – 4.

Similarly, plot the points Q, R and S taking abscissa as –5, –5 and 1 and ordinates as 4, – 3 and –3 respectively.

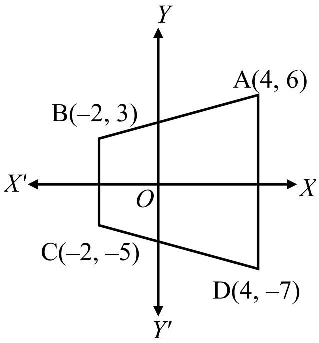
Join the points PQR and S. PQRS is the required rectangle.



Ex.6 Draw a trapezium ABCD in which vertices A, B, C and D are (4, 6), (–2, 3), (–2, –5) and (4, –7) respectively.

Sol. Plot the point A taking its abscissa as 4 and ordinate as 6.

Similarly plot the point B, C and D taking abscissa as – 2, –2 and 4 and ordinates as 3, – 5, and –7 respectively. Join A, B, C and D ABCD is the required trapezium.



➢ DISTANCE BETWEEN TWO POINTS

Theorem : The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{i.e., } PQ = \sqrt{(\text{Diff. of abscissa})^2 + (\text{Diff. of ordinates})^2}$$

Note : If O is the origin and P(x, y) is any point, then from the above formula, we have

$$OP = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

❖ EXAMPLES ❖

Ex.7 Find the distance between the points

(i) P(–6, 7) and Q(–1, –5)

(ii) R(a + b, a – b) and S(a – b, –a – b)

(iii) $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

Sol. (i) Here,

$x_1 = -6$, $y_1 = 7$ and $x_2 = -1$, $y_2 = -5$

$$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow PQ = \sqrt{(-1 + 6)^2 + (-5 - 7)^2}$$

$$\Rightarrow PQ = \sqrt{25 + 144} = \sqrt{169} = 13$$

(ii) We have,

$$RS = \sqrt{(a - b - a - b)^2 + (-a - b - a + b)^2}$$

$$\Rightarrow RS = \sqrt{4b^2 + 4a^2} = 2\sqrt{a^2 + b^2}$$

(iii) We have,

$$AB = \sqrt{(at_2{}^2 - at_1{}^2)^2 + (2at_2 - 2at_1)^2}$$

$\Rightarrow AB = \sqrt{a^2(t_2-t_1)^2(t_2+t_1)^2 + 4a^2(t_2-t_1)^2}$

$\Rightarrow AB = a(t_2 - t_1)\sqrt{(t_2+t_1)^2 + 4}$

Ex.8 If the point (x, y) is equidistant from the points (a + b, b – a) and (a – b, a + b), prove that bx = ay.

Sol. Let P(x, y), Q(a + b, b – a) and R (a – b, a + b) be the given points. Then

PQ = PR (Given)

$\Rightarrow \sqrt{\{x-(a+b)\}^2 + \{y-(b-a)\}^2}$

$= \sqrt{\{x-(a-b)\}^2 + \{y-(a+b)\}^2}$

$\Rightarrow \{x - (a + b)\}^2 + \{y - (b - a)\}^2$

$= \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow x^2 - 2x(a + b) + (a + b)^2$

$+ y^2 - 2y(b - a) + (b - a)^2$

$= x^2 + (a - b)^2 - 2x(a - b)$

$+ y^2 - 2(a + b) + (a + b)^2$

$\Rightarrow -2x(a + b) - 2y(b - a)$

$= -2x(a - b) - 2y(a + b)$

$\Rightarrow ax + bx + by - ay = ax - bx + ay + by$

$\Rightarrow 2bx = 2ay \Rightarrow bx = ay$

Ex.9 Find the value of x, if the distance between the points (x, – 1) and (3, 2) is 5.

Sol. Let P(x, – 1) and Q(3, 2) be the given points, Then,

PQ = 5 (Given)

$\Rightarrow \sqrt{(x-3)^2 + (-1-2)^2} = 5$

$\Rightarrow (x - 3)^2 + 9 = 5^2$

$\Rightarrow x^2 - 6x + 18 = 25 \Rightarrow x^2 - 6x - 7 = 0$

$\Rightarrow (x - 7)(x + 1) = 0 \Rightarrow x = 7$ or $x = -1$

Ex.10 Show that the points (a, a), (–a, –a) and $(-\sqrt{3}\,a, \sqrt{3}\,a)$ are the vertices of an equilateral triangle. Also find its area.

Sol. Let A (a, a), B(–a, –a) and $C(-\sqrt{3}\,a, \sqrt{3}\,a)$ be the given points. Then, we have

$AB = \sqrt{(-a-a)^2 + (-a-a)^2} = \sqrt{4a^2 + 4a^2} = 2\sqrt{2}\,a$

$BC = \sqrt{(-\sqrt{3}a + a)^2 + (\sqrt{3}a + a)^2}$

$\Rightarrow BC = \sqrt{a^2(1-\sqrt{3})^2 + a^2(\sqrt{3}+1)^2}$

$\Rightarrow BC = a\sqrt{1 + 3 - 2\sqrt{3} + 1 + 3 + 2\sqrt{3}}$

$= a\sqrt{8} = 2\sqrt{2}\,a$

and, $AC = \sqrt{(-\sqrt{3}a - a^2) + (\sqrt{3}a - a)^2}$

$\Rightarrow AC = \sqrt{a^2(\sqrt{3}+1) + a^2(\sqrt{3}-1)^2}$

$\Rightarrow AC = a\sqrt{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}$

$= a\sqrt{8} = 2\sqrt{2}\,a$

Clearly, we have

$AB = BC = AC$

Hence, the triangle ABC formed by the given points is an equilateral triangle.

Now,

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(\text{side})^2$

$\Rightarrow$ Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times AB^2$

$\Rightarrow$ Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times (2\sqrt{2}\,a)^2$ sq. units

$= 2\sqrt{3}\,a^2$ sq. units

Ex.11 Show that the points (1, – 1), (5, 2) and (9, 5) are collinear.

Sol. Let A (1, –1), B (5, 2) and C (9, 5) be the given points. Then, we have

$AB = \sqrt{(5-1)^2 + (2+1)^2} = \sqrt{16+9} = 5$

$BC = \sqrt{(5-9)^2 + (2-5)^2} = \sqrt{16+9} = 5$

and, $AC = \sqrt{(1-9)^2 + (-1-5)^2} = \sqrt{64+36} = 10$

Clearly, AC = AB + BC

Hence, A, B, C are collinear points.

Ex.12 Show that four points (0, – 1), (6, 7), (–2, 3) and (8, 3) are the vertices of a rectangle. Also, find its area.

Sol. Let A (0, –1), B(6, 7), C(–2, 3) and D (8, 3) be the given points. Then,

$AD = \sqrt{(8-0)^2 + (3+1)^2} = \sqrt{64+16} = 4\sqrt{5}$

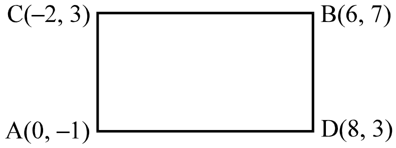
$BC = \sqrt{(6+2)^2 + (7-3)^2} = \sqrt{64+16} = 4\sqrt{5}$

$AC = \sqrt{(-2-0)^2 + (3+1)^2} = \sqrt{4+16} = 2\sqrt{5}$

and, $BD = \sqrt{(8-6)^2 + (3-7)^2} = \sqrt{4+16} = 2\sqrt{5}$

$\therefore$ AD = BC and AC = BD.

So, ADBC is a parallelogram,



Now, $AB = \sqrt{(6-0)^2 + (7+1)^2} = \sqrt{36+64} = 10$

and $CD = \sqrt{(8+2)^2 + (3-3)^2} = 10$

Clearly, $AB^2 = AD^2 + DB^2$ and $CD^2 = CB^2 + BD^2$

Hence, ADBC is a rectangle.

Now, Area of rectangle ADBC = AD × DB

$= (4\sqrt{5} \times 2\sqrt{5})$ sq. units = 40 sq. units

Ex.13 If P and Q are two points whose coordinates are $(at^2, 2at)$ and $\left(\frac{a}{t^2}, \frac{2a}{t}\right)$ respectively and S is the point (a, 0). Show that $\frac{1}{SP} + \frac{1}{SQ}$ is independent of t.

Sol. We have, $SP = \sqrt{(at^2 - a)^2 + (2at - 0)^2}$

$= a\sqrt{(t^2-1)^2 + 4t^2} = a(t^2 + 1)$

and $SQ = \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{2a}{t} - 0\right)^2}$

$\Rightarrow SQ = \sqrt{\frac{a^2(1-t^2)^2}{t^4} + \frac{4a^2}{t^2}}$

$\Rightarrow SQ = \frac{a}{t^2} = \sqrt{(1-t^2)^2 + 4t^2} = \frac{a}{t^2}\sqrt{(1+t^2)^2}$

$= \frac{a}{t^2}(1 + t^2)$

$\therefore \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(t^2+1)} + \frac{t^2}{a(t^2+1)}$

$\Rightarrow \frac{1}{SP} + \frac{1}{SQ} = \frac{1+t^2}{a(t^2+1)} = \frac{1}{a}$,

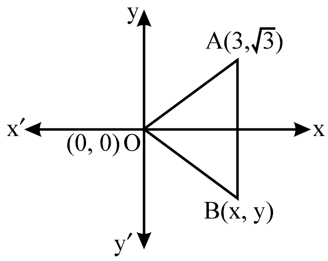
which is independent of t.

Ex.14 If two vertices of an equilateral triangle be (0, 0), $(3, \sqrt{3})$, find the third vertex.

Sol. O(0, 0) and $A(3, \sqrt{3})$ be the given points and let B(x, y) be the third vertex of equilateral ΔOAB. Then,

OA = OB = AB

$\Rightarrow OA^2 = OB^2 = AB^2$



We have, $OA^2 = (3-0)^2 + (\sqrt{3} - 0)^2 = 12$,

$OB^2 = x^2 + y^2$

and, $AB^2 = (x-3)^2 + (y - \sqrt{3})^2$

$\Rightarrow AB^2 = x^2 + y^2 - 6x - 2\sqrt{3}y + 12$

$\therefore OA^2 = OB^2 = AB^2$

$\Rightarrow OA^2 = OB^2$ and $OB^2 = AB^2$

$\Rightarrow x^2 + y^2 = 12$

and, $x^2 + y^2 = x^2 + y^2 - 6x - 2\sqrt{3}y + 12$

$\Rightarrow x^2 + y^2 = 12$ and $6x + 2\sqrt{3}y = 12$

$\Rightarrow x^2 + y^2 = 12$ and $3x + \sqrt{3}y = 6$

$\Rightarrow x^2 + \left(\frac{6-3x}{\sqrt{3}}\right)^2 = 12$

$$\left[\because 3x + \sqrt{3}y = 6 \therefore y = \frac{6-3x}{\sqrt{3}}\right]$$

$\Rightarrow 3x^2 + (6 - 3x)^2 = 36$

$\Rightarrow 12x^2 - 36x = 0$

$\Rightarrow x = 0, 3$

$\therefore x = 0 \Rightarrow \sqrt{3}\, y = 6$

$\Rightarrow y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ $\left[\begin{array}{c}\text{Putting } x = 0 \text{ in} \\ 3x + \sqrt{3}y = 6\end{array}\right]$

and, $x = 3 \Rightarrow 9 + \sqrt{3}\, y = 6$

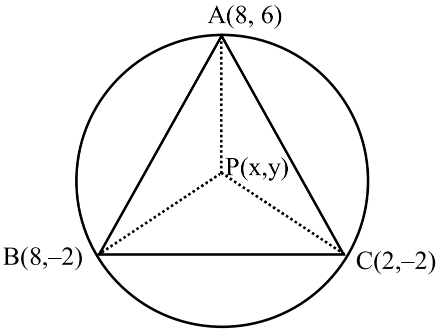
$\Rightarrow y = \frac{6-9}{\sqrt{3}} = -\sqrt{3}$ $\left[\begin{array}{c}\text{Putting } x = 3 \text{ in} \\ 3x + \sqrt{3}y = 6\end{array}\right]$

Hence, the coordinates of the third vertex B are $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$.

Ex.15 Find the coordinates of the circumcentre of the triangle whose vertices are (8, 6), (8, – 2) and (2, – 2). Also, find its circum radius.

Sol. Recall that the circumcentre of a triangle is equidistant from the vertices of a triangle. Let A (8, 6), B(8, –2) and C(2, – 2) be the vertices of the given triangle and let P (x, y) be the circumcentre of this triangle. Then,

$PA = PB = PC \Rightarrow PA^2 = PB^2 = PC^2$



Now, $PA^2 = PB^2$

$\Rightarrow (x - 8)^2 + (y - 6)^2 = (x - 8)^2 + (y + 2)^2$

$\Rightarrow x^2 + y^2 - 16x - 12y + 100$

$= x^2 + y^2 - 16x + 4y + 68$

$\Rightarrow 16y = 32 \Rightarrow y = 2$

and, $PB^2 = PC^2$

$\Rightarrow (x - 8)^2 + (y + 2)^2 = (x - 2)^2 + (y + 2)^2$

$\Rightarrow x^2 + y^2 - 16x + 4y + 68 = x^2 + y^2 - 4x + 4y + 8$

$\Rightarrow 12x = 60 \quad \Rightarrow x = 5$

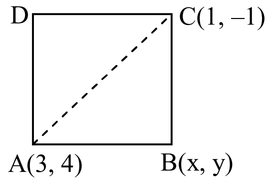
So, the coordinates of the circumcentre P are (5, 2).

Also, Circum-radius = PA = PB = PC

$= \sqrt{(5-8)^2 + (2-6)^2} = \sqrt{9+16} = 5$

Ex.16 If the opposite vertices of a square are (1, – 1) and (3, 4), find the coordinates of the remaining angular points.

Sol. Let A(1, – 1) and C(3, 4) be the two opposite vertices of a square ABCD and let B(*x*, *y*) be the third vertex.



Then, $AB = BC$

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (x - 1)^2 + (y + 1)^2 = (3 - x)^2 + (4 - y)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 + 2y + 1$

$= 9 - 6x + x^2 + 16 - 8y + y^2$

$\Rightarrow x^2 + y^2 - 2x + 2y + 2 = x^2 + y^2 - 6x - 8y + 25$

$\Rightarrow 4x + 10y = 23$

$\Rightarrow x = \frac{23-10y}{4}$ ….(1)

In right-angled triangle ABC, we have

$AB^2 + BC^2 = AC^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2 + (x - 1)^2 + (y + 1)^2$

$= (3 - 1)^2 + (4 + 1)^2$

$\Rightarrow x^2 + y^2 - 4x - 3y - 1 = 0$ .... (2)

Substituting the value of x from (1) and (2), we get

$\left(\frac{23-10y}{4}\right)^2 + y^2 - (23 - 10y) - 3y - 1 = 0$

$\Rightarrow 4y^2 - 12y + 5 = 0 \Rightarrow (2y - 1)(2y - 5) = 0$

$\Rightarrow \quad y = \frac{1}{2}$ or $\frac{5}{2}$

Putting $y = \frac{1}{2}$ and $y = \frac{5}{2}$ respectively in (1) we get

$$x = \frac{9}{2} \text{ and } x = \frac{-1}{2} \text{ respectively.}$$

Hence, the required vertices of the square are $\left(\frac{9}{2}, \frac{1}{2}\right)$ and $\left(-\frac{1}{2}, \frac{5}{2}\right)$.

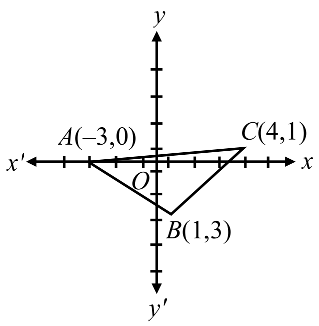
Ex.17 Prove that the points (–3, 0), (1, –3) and (4, 1) are the vertices of an isosceles right angled triangle. Find the area of this triangle.

Sol. Let A (–3, 0), B (1, –3) and C (4, 1) be the given points. Then,

$$AB = \sqrt{\{1-(-3)\}^2 + (-3-0)^2} = \sqrt{16+9} = 5 \text{ units.}$$

$$BC = \sqrt{(4-1)^2 + (1+3)^2} = \sqrt{9+16} = 5 \text{ units}$$

$$\& \; CA = \sqrt{(4+3)^2 + (1-0)^2} = \sqrt{49+1} = 5\sqrt{2} \text{ units}$$



Clearly, AB= BC. Therefore, ΔABC is isosceles.

Also, $AB^2 + BC^2 = 25 + 25 = (5)^2 = CA^2$

$\Rightarrow$ ΔABC is right-angled at B.

Thus, ΔABC is a right-angled isosceles triangle.

Now, Area of $\Delta ABC = \frac{1}{2}(\text{Base} \times \text{Height})$

$$= \frac{1}{2}(AB \times BC)$$

$\Rightarrow$ Area of $\Delta ABC = \left(\frac{1}{2} \times 5 \times 5\right)$ sq. units

$$= \frac{25}{2} \text{ sq. units.}$$

Ex.18 If P (2, – 1), Q(3, 4), R(–2, 3) and S(–3, –2) be four points in a plane, show that PQRS is a rhombus but not a square. Find the area of the rhombus.

Sol. The given points are P(2, –1), Q(3, 4), R(–2, 3) and S(–3, –2).

We have,

$$PQ = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{1^2 + 5^2} = \sqrt{26} \text{ units}$$

$$QR = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$RS = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{1+25} = \sqrt{26} \text{ units}$$

$$SP = \sqrt{(-3-2)^2 + (-2+1)^2} = \sqrt{26} \text{ units}$$

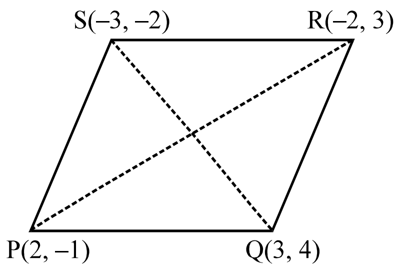
$$PR = \sqrt{(-2-2)^2 + (3+1)^2} = \sqrt{16+16} = 4\sqrt{2} \text{ units}$$

and, $QS = \sqrt{(-3-3)^2 + (-2-4)^2}$

$$= \sqrt{36+36} = 6\sqrt{2} \text{ units}$$

$\therefore \quad PQ = QR = RS = SP = \sqrt{26}$ units

and, $PR \neq QS$



This means that PQRS is a quadrilateral whose sides are equal but diagonals are not equal.

Thus, PQRS is a rhombus but not a square.

Now, Area of rhombus PQRS = $\frac{1}{2}$ × (Product of lengths of diagonals)

$\Rightarrow$ Area of rhombus PQRS = $\frac{1}{2}$ × (PR × QS)

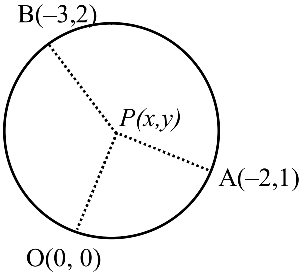
$\Rightarrow$ Area of rhombus PQRS

$$= \left(\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}\right) \text{sq. units} = 24 \text{ sq. units}$$

Ex.19 Find the coordinates of the centre of the circle passing through the points (0, 0), (–2, 1) and (–3, 2). Also, find its radius.

Sol. Let P (x, y) be the centre of the circle passing through the points O(0, 0), A(–2,1) and B(–3,2). Then,

OP = AP = BP



Now, OP = AP $\Rightarrow$ $OP^2 = AP^2$

$\Rightarrow$ $x^2 + y^2 = (x + 2)^2 + (y – 1)^2$

$\Rightarrow$ $x^2 + y^2 = x^2 + y^2 + 4x – 2y + 5$

$\Rightarrow$ $4x – 2y + 5 = 0$ ....(1)

and, OP = BP $\Rightarrow$ $OP^2 = BP^2$

$\Rightarrow$ $x^2 + y^2 = (x + 3)^2 + (y – 2)^2$

$\Rightarrow$ $x^2 + y^2 = x^2 + y^2 + 6x – 4y + 13$

$\Rightarrow$ $6x – 4y + 13 = 0$ ....(2)

On solving equations (1) and (2), we get

$x = \frac{3}{2}$ and $y = \frac{11}{2}$

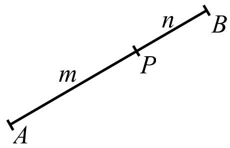
Thus, the coordinates of the centre are $\left(\frac{3}{2}, \frac{11}{2}\right)$

Now, Radius = OP $= \sqrt{x^2 + y^2} = \sqrt{\frac{9}{4} + \frac{121}{4}}$

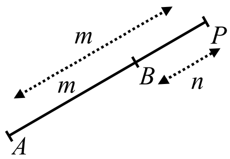
$= \frac{1}{2}\sqrt{130}$ units.

➤ SECTION FORMULAE

Let A and B be two points in the plane of the paper as shown in fig. and P be a point on the segment joining A and B such that AP : BP = m : n. Then, the point P divides segment AB internally in the ratio m : n.

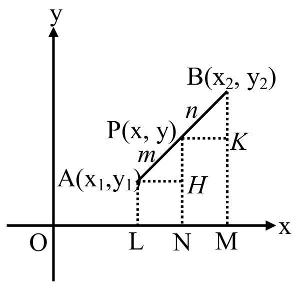


If P is a point on AB produced such that AP : BP = m : n, then point P is said to divide AB externally in the ratio m : n.



The coordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m : n are given by

$$\left(x = \frac{mx_2 + nx_1}{m + n}, y = \frac{my_2 + ny_1}{m + n}\right)$$



The coordinates of P are

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

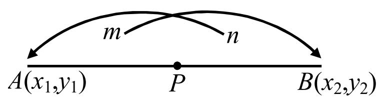
Note 1 :

If P is the mid-point of AB, then it divides AB in the ratio 1 : 1, so its coordinates are

$$\left(\frac{1 . x_1 + 1.x_2}{1+1}, \frac{1.y_1 + 1.y_2}{1+1}\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Note 2 :

Fig. will help to remember the section formula.



Note 3 :

The ratio m : n can also be written as $\frac{m}{n}$: 1, or λ : 1, where $\lambda = \frac{m}{n}$.

So, the coordinates of point P dividing the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right) = \left(\frac{\frac{m}{n}x_2 + x_1}{\frac{m}{n}+1}, \frac{\frac{m}{n}y_2 + y_1}{\frac{m}{n}+1}\right)$$

$$= \left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}\right)$$

❖ EXAMPLES ❖

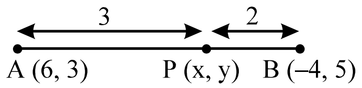
Type I : On finding the section point when the section ratio is given

Ex.20 Find the coordinates of the point which divides the line segment joining the points (6, 3) and (– 4, 5) in the ratio 3 : 2 internally.

Sol. Let P (x, y) be the required point. Then,

$$x = \frac{3\times(-4)+2\times6}{3+2} \text{ and } y = \frac{3\times5+2\times3}{3+2}$$

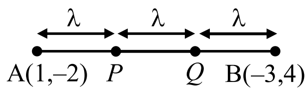
$$\Rightarrow \quad x = 0 \text{ and } y = \frac{21}{5}$$



So, the coordinates of P are (0, 21/5).

Ex.21 Find the coordinates of points which trisect the line segment joining (1, –2) and (–3, 4).

Sol. Let A(1, –2) and B(–3, 4) be the given points. Let the points of trisection be P and Q. Then, AP = PQ = QB = λ (say).



$\therefore$ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = $\lambda : 2\lambda$ = 1 : 2 and

AQ : QB = $2\lambda : \lambda$ = 2 : 1

So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1. Thus, the coordinates of P and Q are

$$P\left(\frac{1\times(-3)+2\times1}{1+2}, \frac{1\times4+2\times(-2)}{1+2}\right) = P\left(\frac{-1}{3}, 0\right)$$

$$Q\left(\frac{2\times(-3)+1\times1}{2+1}, \frac{2\times4+1\times(-2)}{2+1}\right) = Q\left(\frac{-5}{3}, 2\right)$$

respectively

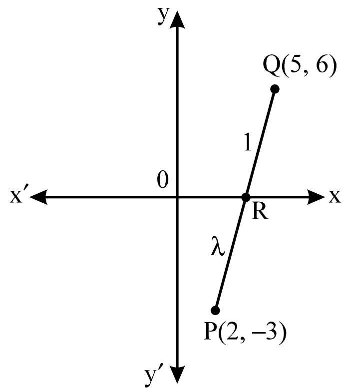
Hence, the two points of trisection are (–1/3, 0) and (–5/3, 2).

Type II : On Finding the section ratio or an end point of the segment when the section point is given

Ex.22 In what ratio does the x-axis divide the line segment joining the points (2, –3) and (5, 6) ? Also, find the coordinates of the point of intersection.

Sol. Let the required ratio be λ : 1. Then, the coordinates of the point of division are,

$$R\left(\frac{5\lambda+2}{\lambda+1}, \frac{6\lambda-3}{\lambda+1}\right)$$



But, it is a point on x-axis on which y-coordinates of every point is zero.

$$\therefore \quad \frac{6\lambda-3}{\lambda+1} = 0$$

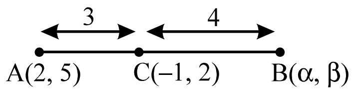
$$\Rightarrow \lambda = \frac{1}{2}$$

Thus, the required ratio is $\frac{1}{2}$: 1 or 1 : 2.

Putting $\lambda = 1/2$ in the coordinates of R, we find that its coordinates are (3, 0).

Ex.23 If the point C (–1, 2) divides internally the line segment joining A (2, 5) and B in ratio 3 : 4, find the coordinates of B.

Sol. Let the coordinates of B be (α, β). It is given that AC : BC = 3 : 4. So, the coordinates of C are



$$\left(\frac{3\alpha+4\times 2}{3+4}, \frac{3\beta+4\times 5}{3+4}\right) = \left(\frac{3\alpha+8}{7}, \frac{3\beta+20}{7}\right)$$

But, the coordinates of C are (–1, 2)

$$\therefore \quad \frac{3\alpha+8}{7} = -1 \text{ and } \frac{3\beta+20}{7} = 2$$

$\Rightarrow \quad \alpha = -5$ and $\beta = -2$

Thus, the coordinates of B are (–5, –2).

Ex.24 Determine the ratio in which the line $3x+y-9=0$ divides the segment joining the points (1, 3) and (2, 7).

Sol. Suppose the line $3x + y - 9 = 0$ divides the line segment joining A (1, 3) and B(2, 7) in the ratio k : 1 at point C. Then, the coordinates of C are

$$\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$$

But, C lies on $3x + y - 9 = 0$. Therefore,

$$3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0$$

$\Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow k = \frac{3}{4}$

So, the required ratio is 3 : 4 internally.

Type III : On determination of the type of a given quadrilateral

Ex.25 Prove that the points (–2, –1), (1, 0), (4, 3) and (1, 2) are the vertices of a parallelogram. Is it a rectangle ?

Sol. Let the given point be A, B, C and D respectively. Then,

Coordinates of the mid-point of AC are

$$\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = (1, 1)$$

Coordinates of the mid-point of BD are

$$\left(\frac{1+1}{2}, \frac{0+2}{2}\right) = (1, 1)$$

Thus, AC and BD have the same mid-point. Hence, ABCD is a parallelogram.

Now, we shall see whether ABCD is a rectangle or not.

We have, AC = $\sqrt{(4-(-2))^2 + (3-(-1))^2} = 2$

and, BD = $\sqrt{(1-1)^2 + (0-2)^2} = 2$

Clearly, AC $\neq$ BD. So, ABCD is not a rectangle.

Ex.26 Prove that (4, – 1), (6, 0), (7, 2) and (5, 1) are the vertices of a rhombus. Is it a square ?

Sol. Let the given points be A, B, C and D respectively. Then,

Coordinates of the mid-point of AC are

$$\left(\frac{4+7}{2}, \frac{-1+2}{2}\right) = \left(\frac{11}{2}, \frac{1}{2}\right)$$

Coordinates of the mid-point of BD are

$$\left(\frac{6+5}{2}, \frac{0+1}{2}\right) = \left(\frac{11}{2}, \frac{1}{2}\right)$$

Thus, AC and BD have the same mid-point.

Hence, ABCD is a parallelogram.

Now, AB = $\sqrt{(6-4)^2 + (0+1)^2} = \sqrt{5}$,

BC = $\sqrt{(7-6)^2 + (2-0)^2} = \sqrt{5}$

$\therefore$ AB = BC

So, ABCD is a parallelogram whose adjacent sides are equal.

Hence, ABCD is a rhombus.

We have,

AC = $\sqrt{(7-4)^2 + (2+1)^2} = 3\sqrt{2}$ and

BD = $\sqrt{(6-5)^2 + (0-1)^2} = \sqrt{2}$

Clearly, $AC \neq BD$.

So, ABCD is not a square.

Type IV : On finding the unknown vertex from given points

Ex.27 The three vertices of a parallelogram taken in order are (–1, 0), (3, 1) and (2, 2) respectively. Find the coordinates of the fourth vertex.

Sol. Let A(–1, 0), B(3, 1), C(2, 2) and D(x, y) be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

$\therefore$ Coordiantes of the mid-point of AC

= Coordinates of the mid-point of BD

$$\Rightarrow \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$$

$$\Rightarrow \left(\frac{1}{2}, 1\right) = \left(\frac{3+x}{2}, \frac{y+1}{2}\right)$$

$$\Rightarrow \frac{3+x}{2} = \frac{1}{2} \text{ and } \frac{y+1}{2} = 1$$

$\Rightarrow$ x = – 2 and y = 1

Hence, the fourth vertex of the parallelogram is (–2, 1).

Ex.28 If the points A (6, 1), B (8, 2), C(9, 4) and D (p, 3) are vertices of a parallelogram, taken in order, find the value of p.

Sol. We know that the diagonals of a parallelogram bisect each other. So, coordinates of the mid-point of diagonal AC are same as the coordinates of the mid-point of diagonal BD.

$$\therefore \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$$

$$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\Rightarrow \frac{15}{2} = \frac{8+p}{2} \Rightarrow 15 = 8 + p$$

$\Rightarrow$ p = 7

Ex.29 If A(–2, –1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram, find the values of a and b.

Sol. We know that the diagonals of a parallelogram bisect each other. Therefore, the coordinates of the mid-point of AC are same as the coordinates of the mid-point of BD i.e.,

$$\left(\frac{-2+4}{2}, \frac{-1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$$

$$\Rightarrow \left(1, \frac{b-1}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$

$$\Rightarrow \frac{a+1}{2} = 1 \text{ and } \frac{b-1}{2} = 1$$

$\Rightarrow$ a + 1 = 2 and b – 1 = 2

$\Rightarrow$ a = 1 and b = 3

Ex.30 If the coordinates of the mid-points of the sides of a triangle are (1, 2) (0, –1) and (2, –1). Find the coordinates of its vertices.

Sol. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC. Let D (1, 2), E (0, –1), and F(2, –1) be the mid-points of sides BC, CA and AB respectively. Since D is the mid-point of BC.

$$\therefore \frac{x_2+x_3}{2} = 1 \text{ and } \frac{y_2+y_3}{2} = 2$$

$\Rightarrow x_2 + x_3 = 2$ and $y_2 + y_3 = 4$ .... (1)

Similarly, E and F are the mid-point of CA and AB respectively.

$$\therefore \frac{x_1+x_3}{2} = 0 \text{ and } \frac{y_1+y_3}{2} = -1$$

$\Rightarrow x_1 + x_3 = 0$ and $y_1 + y_3 = -2$ .... (2)

and $\frac{x_1+x_2}{2} = 2$ and $\frac{y_1+y_2}{2} = -1$

$\Rightarrow x_1 + x_2 = 4$ and $y_1 + y_2 = -2$ .... (3)

From (1), (2) and (3), we get

$(x_2 + x_3) + (x_1 + x_3) + (x_1 + x_2) = 2 + 0 + 4$ and,

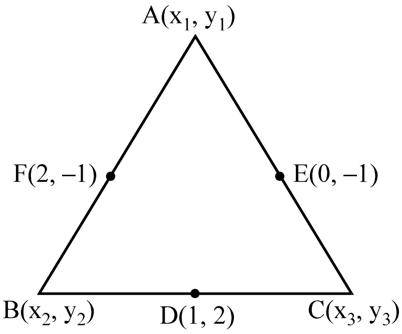
$(y_2 + y_3) + (y_1 + y_3) + (y_1 + y_2) = 4 - 2 - 2$

$\Rightarrow 2(x_1 + x_2 + x_3) = 6$ and

$2(y_1 + y_2 + y_3) = 0$ .... (4)

$\Rightarrow x_1 + x_2 + x_3 = 3$

and $y_1 + y_2 + y_3 = 0$



From (1) and (4), we get

$x_1 + 2 = 3$ and $y_1 + 4 = 0$

$\Rightarrow$ $x_1 = 1$ and $y_1 = -4$

So, the coordinates of A are (1, – 4)

From (2) and (4), we get

$x_2 + 0 = 3$ and $y_2 - 2 = 0$

$\Rightarrow$ $x_2 = 3$ and $y_2 = 2$

So, coordinates of B are (3, 2)

From (3) and (4), we get

$x_3 + 4 = 3$ and $y_3 - 2 = 0$

$\Rightarrow$ $x_3 = -1$ and $y_3 = 2$

So, coordinates of C are (–1, 2)

Hence, the vertices of the triangle ABC are A(1, – 4), B(3, 2) and C(–1, 2).

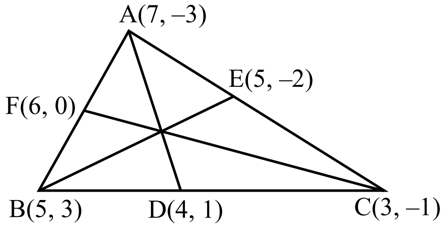
Ex.31 Find the lengths of the medians of a ΔABC whose vertices are A(7, –3), B(5,3) and C(3,–1).

Sol. Let D, E, F be the mid-points of the sides BC, CA and AB respectively. Then, the coordinates of D, E and F are

$$D\left(\frac{5+3}{2}, \frac{3-1}{2}\right) = D(4, 1),$$

$$E\left(\frac{3+7}{2}, \frac{-1-3}{2}\right) = E\,(5, -2)$$

and, $$F\left(\frac{7+5}{2}, \frac{-3+3}{2}\right) = F\,(6, 0)$$



$$\therefore\ AD = \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = 5 \text{ units}$$

$$BE = \sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0+25} = 5 \text{ units}$$

$$\text{and, } CF = \sqrt{(6-3)^2 + (0+1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units.}$$

Ex.32 If A (5, –1), B(–3, –2) and C(–1, 8) are the vertices of triangle ABC, find the length of median through A and the coordinates of the centroid.

Sol. Let AD be the median through the vertex A of ΔABC. Then, D is the mid-point of BC. So, the coordinates of D are $\left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$ i.e.,(–2, 3).

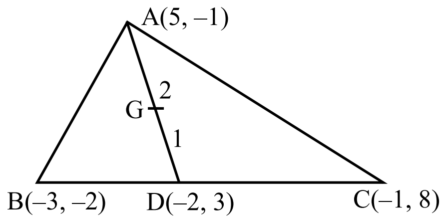
$$\therefore\ AD = \sqrt{(5+2)^2 + (-1-3)^2}$$

$$= \sqrt{49+16} = \sqrt{65} \text{ units}$$

Let G be the centroid of ΔABC. Then, G lies on median AD and divides it in the ratio 2 : 1. So, coordinates of G are

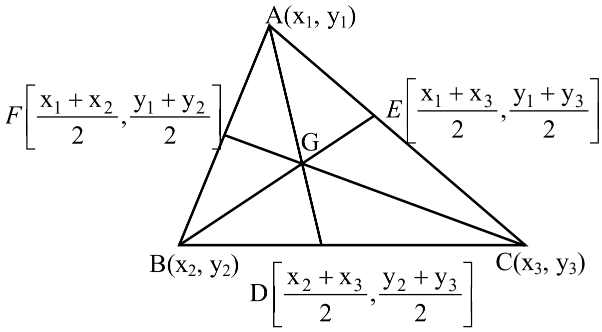
$$\left(\frac{2\times(-2)+1\times 5}{2+1}, \frac{2\times 3+1\times(-1)}{2+1}\right)$$

$$= \left(\frac{-4+5}{3}, \frac{6-1}{3}\right) = \left(\frac{1}{3}, \frac{5}{3}\right)$$



➢ APPLICATION OF SECTION FORMULA

Theorem : The coordinates of the centroid of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$



❖ **EXAMPLES** ❖

Ex.33 Find the coordinates of the centroid of a triangle whose vertices are (–1, 0), (5, –2) and (8, 2).

Sol. We know that the coordinates of the centroid of a triangle whose angular points are (x_1, y_1), (x_2, y_2), (x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

So, the coordiantes of the centroid of a triangle whose vertices are (–1, 0), (5, –2) and (8, 2) are

$$\left(\frac{-1+5+8}{3}, \frac{0-2+2}{3}\right) \text{ or, } (4, 0)$$

Ex.34 If the coordinates of the mid points of the sides of a triangle are (1, 1), (2, – 3) and (3, 4) Find its centroid.

Sol. Let P (1, 1), Q(2, –3), R(3, 4) be the mid-points of sides AB, BC and CA respectively of triangle ABC. Let A (x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of triangle ABC. Then,

P is the mid-point of BC

$\Rightarrow \frac{x_1 + x_2}{2} = 1, \frac{y_1 + y_2}{2} = 1$

$\Rightarrow x_1 + x_2 = 2$ and $y_1 + y_2 = 2$ ...(1)

Q is the mid-point of BC

$\Rightarrow \frac{x_2 + x_3}{2} = 2, \frac{y_2 + y_3}{2} = -3$

$\Rightarrow x_2 + x_3 = 4$ and $y_2 + y_3 = -6$ ...(2)

R is the mid-point of AC

$\Rightarrow \frac{x_1 + x_3}{2} = 3$ and $\frac{y_1 + y_3}{2} = 4$

$\Rightarrow x_1 + x_3 = 6$ and $y_1 + y_3 = 8$ ...(3)

From (1), (2) and (3), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$

and,$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$...(4)

The coordinates of the centroid of ΔABC are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right)$$

$$= \left(2, \frac{2}{3}\right) \quad \text{[Using (4)]}$$

Ex.35 Two vertices of a triangle are (3, –5) and (–7, 4). If its centroid is (2, –1). Find the third vertex.

Sol. Let the coordinates of the third vertex be (x, y). Then,

$$\frac{x + 3 - 7}{3} = 2 \text{ and } \frac{y - 5 + 4}{3} = -1$$

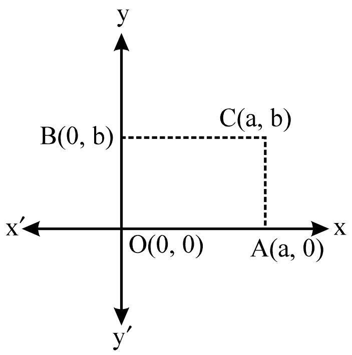
$\Rightarrow x - 4 = 6$ and $y - 1 = -3$

$\Rightarrow x = 10$ and $y = -2$

Thus, the coordinates of the third vertex are (10, –2).

Ex.36 Prove that the diagonals of a rectangle bisect each other and are equal.

Sol. Let OACB be a rectangle such that OA is along x-axis and OB is along y-axis. Let OA = a and OB = b.



Then, the coordinates of A and B are (a, 0) and (0, b) respectively.

Since, OACB is a rectangle. Therefore,

AC = Ob ⇒ AC = b

Thus, we have

OA = a and AC = b

So, the coordiantes of C are (a, b).

The coordinates of the mid-point of OC are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Also, the coordinates of the mid-points of AB are $\left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Clearly, coordinates of the mid-point of OC and AB are same.

Hence, OC and AB bisect each other.

Also, $OC = \sqrt{a^2 + b^2}$ and

$AB = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$\therefore$ OC = AB

AREA OF A TRIANGLE

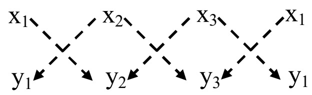
Theorem : The area of a triangle, the coordinates of whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\frac{1}{2} \mid x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) \mid$$

Remark : The area of ΔABC can also be computed by using the following steps :

Step I : Write the coordinates of the vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ in three coloums as shown below and augment the coordinates of $A(x_1, y_1)$ as fourth coloumn.

Step II : Draw broken parallel lines pointing down wards from left to right and right to left.



Step III : Compute the sum of the products of numbers at the ends of the lines pointing downwards from left to right and subtract from this sum the sum of the products of numbers at the ends of the lines pointing downward from right to left i.e., compute

$$(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)$$

Step IV : Find the absolute of the number obtained in step III and take its half to obtain the area.

Remark : Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear iff

Area of $\Delta ABC = 0$ i.e., $x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

❖ EXAMPLES ❖

Type I : On finding the area of a triangle when coordinates of its vertices are given.

Ex.37 Find the area of a triangle whose vertices are A(3, 2), B (11, 8) and C(8, 12).

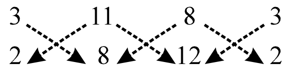
Sol. Let $A = (x_1, y_1) = (3, 2)$, $B = (x_2, y_2) = (11, 8)$ and $C = (x_3, y_3) = (8, 12)$ be the given points. Then,

$$\text{Area of } \Delta ABC = \frac{1}{2} |\{x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\}|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} |\{3(8-12) + 11\,(12-2) + 8\,(2-8)\}|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} |(-12 + 110 - 48)| = 25 \text{ sq. units}$$

ALTER We have,



$$\therefore \text{Area of } \Delta ABC = |(3 \times 8 + 11 \times 12 + 8 \times 2) - (11 \times 2 + 8 \times 8 + 3 \times 12)|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} |(24 + 132 + 16) - (22 + 64 + 36)|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} |\ 172 - 122\ | = 25 \text{ sq. units}$$

Ex.38 Prove that the area of triangle whose vertices are (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

Sol. Let $A = (x_1, y_1) = (t, t-2)$, $B (x_2, y_2) = (t+2, t+2)$ and $C = (x_3, y_3) = (t+3, t)$ be the vertices of the given triangle. Then,

$\therefore$ Area of ΔABC

$$= \frac{1}{2} \mid x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} |\{t(t + 2 - t) + (t + 2)\,(t - t + 2)$$

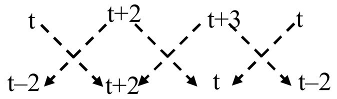
$+ (t + 3) (t - 2 - t - 2)\}|$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}|\{2t+2t+1-4t-12\}| = |-4|$

= 4 sq. untis

Clearly, area of ΔABC is independent f t.

ALTER We have,



$\therefore$ Area of ΔABC

$$= \frac{1}{2}\left|\begin{matrix}\{t(t+2)+(t+2)t+(t+3)(t-2)\}-\{(t+2)(t-2) \\ +(t+3)(t+2)+t\times t\}\end{matrix}\right|$$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}\ |(t^2 + 2t + t^2 + 2t + t^2 + t - 6)$

$- (t^2 - 4 + t^2 + 5t + 6 + t^2)|$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}|\ (3t^2 + 5t - 6) - (3t^2 + 5t + 2)|$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}|\ (-6 - 2)|$

$\Rightarrow$ Area of ΔABC = 4 sq. units

Hence, Area of ΔABC is independent of t.

Ex.39 Find the area of the triangle formed by joining the mid-point of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of area of the triangle formed to the area of the given triangle.

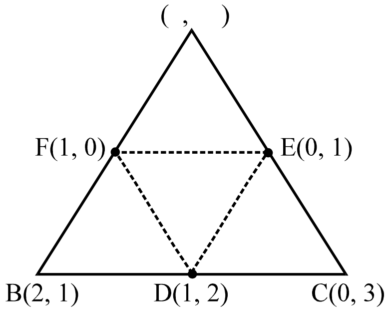
Sol. Let A (0, –1), B(2, 1) and C(0, 3) be the vertices of ΔABC. Let D, E, F be the mid-points of sides BC, CA and AB respectively. Then, the coordinates of D, E and F are (1, 2), (0, 1) and (1, 0) respectively.

Now,

$$\text{Area of } \Delta ABC = \frac{1}{2}|\ x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3 (y_1 - y_2)|$$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}|\ 0(1 - 3) + 2\ (3 - (-1)) + 0(0 - 1)|$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}\ |\ 0 + 8 + 0\ |$ = 4 sq. units



$$\text{Area of } \Delta DEF = \frac{1}{2}|x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$$

$\Rightarrow$ Area of $\Delta DEF = \frac{1}{2}|1\ (1 - 0) + 0\ (0 - 2) + 1\ (2 - 1)|$

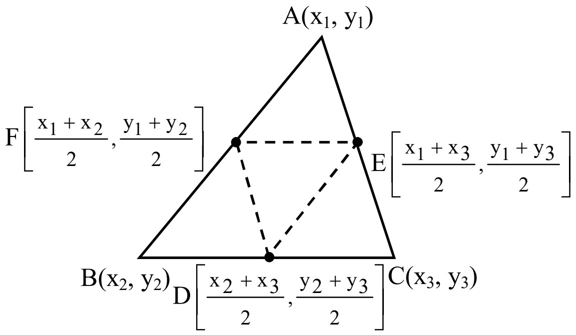
$\Rightarrow$ Area of $\Delta DEF = \frac{1}{2}|1 + 1|$ = 1 sq. units

$\therefore$ Area of ΔDEF : Area of ΔABC = 1 : 4

Ex.40 If D, E and F are the mid-points of sides BC, CA and AB respectively of a ΔABC, then using coordinate geometry prove that

$$\text{Area of } \Delta DEF = \frac{1}{4}(\text{Area of } \Delta ABC)$$

Sol. Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of ΔABC. Then, the coordinates of D, E and F are $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$ $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$ and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ respectively.



Δ_1 = Area of $\Delta ABC = \frac{1}{2}|x_1 (y_2 - y_3) + x_2 (y_3 - y_1)$

$+ x_3 (y_1 - y_2)|$

Δ_2 = Area of ΔDEF

$$= \frac{1}{2}\left|\left(\frac{x_2+x_3}{2}\right)\left(\frac{y_1+y_3}{2}-\frac{y_1+y_2}{2}\right)+\left(\frac{x_1+x_3}{2}\right)\right.$$

$$\left.\left(\frac{y_1+y_2}{2}-\frac{y_2+y_3}{2}\right)+\left(\frac{x_1+x_2}{2}\right)\left(\frac{y_2+y_3}{2}-\frac{y_1+y_3}{2}\right)\right|$$

$$\Rightarrow \Delta_2 = \frac{1}{8}|(x_2+x_3)(y_3-y_2)+(x_1+x_3)(y_1-y_3) + (x_1+x_2)(y_2-y_1)|$$

$$\Rightarrow \Delta_2 = \frac{1}{8}|x_1(y_1-y_3+y_2-y_1)+x_2(y_3-y_2+y_2-y_1) + x_3(y_3-y_2+y_1-y_3)|$$

$$\Rightarrow \Delta_2 = \frac{1}{8}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

$$\Rightarrow \Delta_2 = \frac{1}{4}(\text{Area of } \Delta ABC) = \frac{1}{4}\Delta_1$$

Hence, Area of $\Delta DEF = \frac{1}{4}$ (Area of ΔABC)

Ex.41 The vertices of ΔABC = are A (4, 6), B(1, 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of ΔADE and compare it with the area of ΔABC.

Sol. We have, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} = 4$$

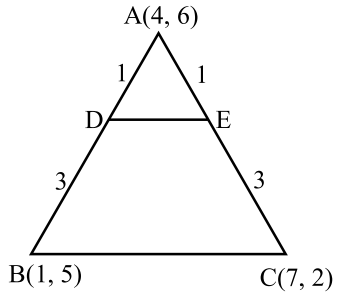
$$\Rightarrow \frac{AD+DB}{AD} = \frac{AE+EC}{AE} = 4$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} = 4$$

$$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE} = 3 \Rightarrow \frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

$\Rightarrow$ AD : DB = AE : EC = 1 : 3

$\Rightarrow$ D and E divide AB and AC respectively in the ratio 1 : 3.

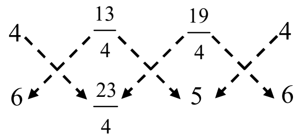


So, the co-ordinates of D and E are

$$\left(\frac{1+12}{1+3},\frac{5+18}{1+3}\right)=\left(\frac{13}{4},\frac{23}{4}\right) \text{ and } \left(\frac{7+12}{1+3},\frac{2+18}{1+3}\right)=\left(\frac{19}{4},5\right)$$

respectively.

We have,



$\therefore$ Area of ΔADE

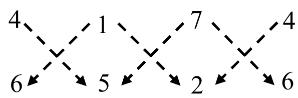
$$=\frac{1}{2}\left|\left(4\times\frac{23}{4}+\frac{13}{4}\times 5+\frac{19}{4}\times 6\right)-\left(\frac{13}{4}\times 6+\frac{19}{4}\times\frac{23}{4}+4\times 5\right)\right|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\left|\left(\frac{92}{4}+\frac{65}{4}+\frac{114}{4}\right)-\left(\frac{78}{4}+\frac{437}{16}+20\right)\right|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\left|\frac{271}{4}-\frac{1069}{16}\right|$$

$$= \frac{1}{2} \times \frac{15}{16} = \frac{15}{32} \text{ sq. untis.}$$

Also, we have



$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}|(4\times 5+1\times 2+7\times 6) - (1\times 6+7\times 5+4\times 2)|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}|(20+2+42)-(6+35+8)|$$

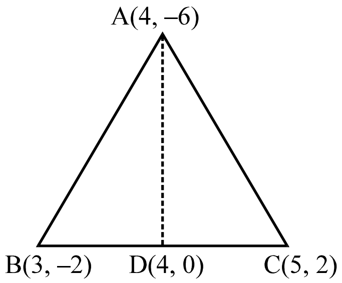
$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\ |64-49| = \frac{15}{2} \text{ sq. units}$$

$\therefore \quad \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{15/32}{15/2} = \frac{1}{16}$

Hence, Area of ΔADE : Area of ΔABC = 1 : 16.

Ex.42 If A(4, –6), B(3, –2) and C(5, 2) are the vertices of ΔABC, then verify the fact that a median of a triangle ABC divides it into two triangle of equal areas.

Sol. Let D be the mid-point of BC. Then, the coordinates of D are (4, 0).



We have,

$$\begin{matrix} 4 & 3 & 5 & 4 \\ -6 & -2 & 2 & -6 \end{matrix}$$

$\therefore$ Area of ΔABC = $\frac{1}{2}$ |(4 × (– 2) + 3 × 2 + 5 × (– 6))

– (3 × (– 6) + 5 × (–2) + 4 × 2)|

$\Rightarrow$ Area of ΔABC = $\frac{1}{2}$ |(–8 + 6 – 30) – (–18 – 10 + 8)|

$\Rightarrow$ Area of ΔABC = $\frac{1}{2}$ | –32 + 20 | = 6 sq. units

Also, We have

$$\begin{matrix} 4 & 3 & 4 & 4 \\ -6 & -2 & 0 & -6 \end{matrix}$$

$\therefore$ Also of ΔABD = $\frac{1}{2}\left|\begin{matrix}\{(4\times(-2)+3\times 0+4\times(-6))\} \\ -\{3\times(-6)+4\times(-2)+4\times 0\}\end{matrix}\right|$

$\Rightarrow$ Area of ΔABD = $\frac{1}{2}$ |(–8 + 0 + 26) – (–18 – 8 + 0)|

$\Rightarrow$ Area of ΔABD = $\frac{1}{2}$ |(–32 + 26)| = 3 sq. units

$\Rightarrow \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ABD} = \frac{6}{3} = \frac{2}{1}$

$\Rightarrow$ Area of ΔABC = 2 (Area of ΔABD)

TypeII : On finding the area of a quadrilateral when coordinates of its vertices are given

Ex.43 Find the area of the quadrilateral ABCD whose vertices are respectively A(1, 1), B(7, –3), C(12, 2) and D(7, 21).

Sol. Area of quadrilateral ABCD

= | Area of ΔABC | + | Area of ΔACD |

We have,

$$\begin{matrix} 1 & 7 & 12 & 1 \\ 1 & -3 & 2 & 1 \end{matrix}$$

$\therefore$ Area of ΔABC = $\frac{1}{2}$ |(1× –3 + 7 × 2 + 12 × 1)

– (7 × 1 + 12 × (–3) + 1× 2)|

$\Rightarrow$ Area of ΔABC = $\frac{1}{2}$ |(–3 + 14 + 12) – (7 – 36 + 2)|

$\Rightarrow$ Area of ΔABC = $\frac{1}{2}$ |23 + 27| = 25 sq. units

Also, we have

$$\begin{matrix} 1 & 12 & 7 & 1 \\ 1 & 2 & 21 & 1 \end{matrix}$$

$\therefore$ Area of ΔACD = $\frac{1}{2}$ |(1 ×2 + 12 × 21 + 7 × 1)

– (12 × 1 + 7 × 2 + 1 × 21)|

$\Rightarrow$ Area of ΔACD = $\frac{1}{2}$ |(2 + 252 + 7) – (12 + 14 + 21)|

$\Rightarrow$ Area of ΔACD = $\frac{1}{2}$ |261 – 47| = 107 sq. units

$\therefore$ Area of quadrilateral ABCD = 25 + 107 = 132 sq. units

Type III : On collinearity of three points

FORMULA:

Three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear iff

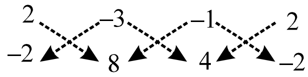
$$x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$$

or, $(x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_3y_2 + x_2y_1) = 0$

Ex.44 Prove that the points (2, – 2), (–3, 8) and (–1, 4) are collinear.

Sol. Let Δ be the area of the triangle formed by the given points.

We have,



$\therefore\ \Delta = \frac{1}{2}|\{2 \times 8 + (-3) \times 4 + (-1) \times (-2)\}$

$- \{(-3) \times (-2) + (-1) \times 8 + 2 \times 4\}|$

$\Rightarrow\ \Delta = \frac{1}{2}\ |(16 - 12 + 2) - (6 - 8 + 8)|$

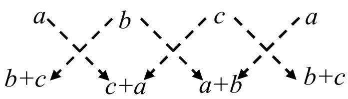
$\Rightarrow\ \Delta = \frac{1}{2}\ |6 - 6| = 0$

Hence, given points are collinear.

Ex.45 Prove that the points (a, b + c), (b, c + a) and (c, a + b) are collinear.

Sol. Let Δ be the area of the triangle formed by the points (a, b + c), (b, c + a) and (c, a + b).

We have,



$\therefore\ \ \Delta = \frac{1}{2}|\{a\ (c + a) + b\ (a + b) + c\ (b + c)\}$

$- \{b\ (b + c) + c\ (c + a) + a\ (a + b)\}|$

$\Rightarrow\ \Delta = \frac{1}{2}|(ac + a^2 + ab + b^2 + bc + c^2)$

$- (b^2 + bc + c^2 + ca + a^2 + ab)|$

$\Rightarrow\ \Delta = 0$

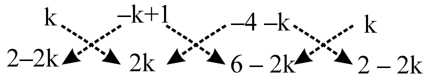
Hence, the given points are collinear.

Type IV : On Finding the desired result or unknown when three points are collinear

Ex.46 For what value of k are the points (k, 2 – 2k) (–k + 1, 2k) and (–4 – k, 6 – 2k) are collinear ?

Sol. Given points will be collinear, if area of the triangle formed by them is zero.

We have,



i.e.,

$|\{2k^2 + (-k + 1)\ (6 - 2k) + (-4 - k)\ (2 - 2k)\}$

$- \{(-k + 1)\ (2 - 2k) + (-4 - k)\ (2k)$

$+ k\ (6 - 2k)\}| = 0$

$\Rightarrow\ |(2k^2 + 6 - 8k + 2k^2 + 2k^2 + 6k - 8)$

$- (2 - 4k + 2k^2 - 8k - 2k^2 + 6k - 2k^2)|$

$\Rightarrow\ (6k^2 - 2k - 2) - (-2k^2 - 6k + 2) = 0$

$\Rightarrow\ 8k^2 + 4k - 4 = 0$

$\Rightarrow\ 2k^2 + k - 1 = 0 \Rightarrow (2k - 1)\ (k + 1) = 0$

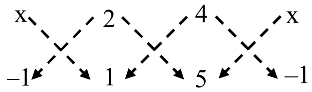
$\Rightarrow\ k = 1/2$ or, $k = -1$

Hence, the given points are collinear for

$k = 1/2$ or, $k = -1$.

Ex.47 For what value of x will the points (x, –1), (2, 1) and (4, 5) lie on a line ?

Sol. Given points will be collinear if the area of the triangle formed by them is zero.



$\therefore$ Area of the triangle = 0

$\Rightarrow\ |\{x \times 1 + 2 \times 5 + 4 \times (-1)\}$

$- \{(2 \times (-1) + 4 \times 1 + x \times 5\}| = 0$

$\Rightarrow\ (x + 10 - 4) - (-2 + 4 + 5x) = 0$

$\Rightarrow\ (x + 6) - (5x + 2) = 0$

$\Rightarrow\ -4x + 4 = 0$

$\Rightarrow x = 1$

Hence, the given points lies on a line, if x = 1.

Type V : Mixed problems based upon the concept of area of a triangle

Ex.48 If the coordinates of two points A and B are (3, 4) and (5, – 2) respectively. Find the coordniates of any point P, if

PA = PB and Area of $\Delta PAB = 10$.

Sol. Let the coordinates of P be (x, y). Then,

PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$

$\Rightarrow x - 3y - 1 = 0$ ....(1)

Now, Area of $\Delta PAB = 10$

$$\begin{matrix} x & 3 & 5 & x \\ y & 4 & -2 & y \end{matrix}$$

$\Rightarrow \frac{1}{2}|(4x + 3\times(-2) + 5y) - (3y + 20 - 2x)| = 10$

$\Rightarrow |(4x + 5y - 6) - (-2x + 3y + 20)| = 20$

$\Rightarrow |6x + 2y - 26| = \pm 20 \Rightarrow 6x + 2y - 26 = \pm 20$

$\Rightarrow 6x + 2y - 46 = 0$ or, $6x + 2y - 6 = 0$

$\Rightarrow 3x + y - 23 = 0$ or, $3x + y - 3 = 0$

Solving $x - 3y - 1 = 0$ and $3x + y - 23 = 0$ we get $x = 7, y = 2$.

Solving $x - 3y - 1 = 0$ and $3x + y - 3 = 0$, we get $x = 1, y = 0$.

Thus, the coordinates of P are (7, 2) or (1, 0).

Ex.49 The coordinates of A, B, C are (6, 3), (–3, 5) and (4, – 2) respectively and P is any point (x, y). Show that the ratio of the areas of triangle PBC and ABC is $\left|\frac{x+y-2}{7}\right|$.

Sol. We have,

$$\begin{matrix} x & -3 & 4 & x \\ y & 5 & -2 & y \end{matrix}$$

$\therefore$ Area of $\Delta PBC = \frac{1}{2}|(5x+6+4y)-(-3y+20-2x)|$

$\Rightarrow$ Area of $\Delta PBC = \frac{1}{2}|5x+6+4y+3y-20+2x|$

$\Rightarrow$ Area of $\Delta PBC = \frac{1}{2}|7x+7y-14|$

$\Rightarrow$ Area of $\Delta PBC = \frac{7}{2}|x+y-2|$

$\Rightarrow$ Area of $\Delta PBC = \frac{7}{2}|6+3-2|$

$\left[\begin{array}{l}\text{Replacing x by 6 and y = 3} \\ \text{in Area of } \Delta PBC\end{array}\right]$

$\Rightarrow$ Area of $\Delta ABC = \frac{49}{2}$

$\therefore \frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{\frac{7}{2}|x+y-2|}{\frac{49}{2}}$

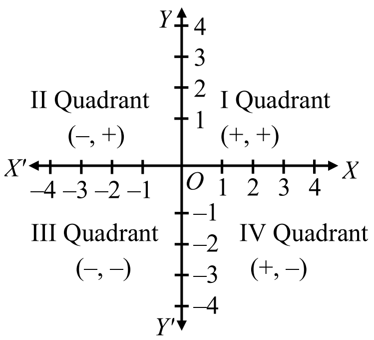
$= \frac{|x+y-2|}{7} = \left|\frac{x+y-2}{7}\right|$

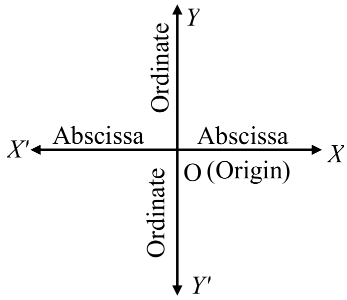
IMPORTANT POINTS TO BE REMEMBERED

1. The point of intersection of two perpendicular lines is known as origin O(0, 0) .
2. The Horizontal line XOX' is called x-axis and the vertical line YOY' is call y-axis.
3. The plane on which these lines are graphed is known as cartesian Plane and lines are called the co-ordinate axes.
4. These two mutually perpendicular lines divide the cartesian plane into four parts which are called quadrants I, II, III and IV.
5.

Quadrant	Abscissa(x)	Ordinate(y)	Co-ordinate(x,y)
1st	+	+	(+, +)
2nd	–	+	(–, +)
3rd	–	–	(–, –)
4th	+	–	(+, –)

6. In a point A(x, y), x-cordinate is known as Abscissa and y-coordinate is known as ordinate.
7. Equation of x-axis is y = 0
8. Equation of y-axis is x = 0
9. Graph of the line x = a is always parallel to y-axis.
10. Graph of the line y = b is always parallel to x-axis.
11. Any point on x-axis is (x, 0) and any point on y-axis is (0, y)
12. Abscissa is +ve to the right of the origin and negative to the left of the origin.
13. Ordinate (y) is +ve above x-axis and –ve below x-axis.
14. Sign convention- While marking a point on the squared paper, the sign convention must be kept in mind.





15. Distance between two points (x_1, y_1) and $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
16. Distance of a point P(x, y) from the origin O(0,0) is-

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

17. Section formula : the co-ordinates of the point P(x, y) which divide the join of the points A(x, y) and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

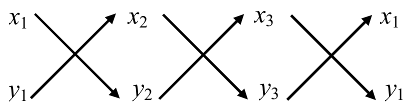
18. The mid-point of a line segment divides the line segment in the ratio 1:1. Therefore, the co-ordinates of the point P of the join of the points $A(x_1, y_1)$ and $B(x_2,y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

19. Area of ΔABC whose vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are -

$$\text{area of } \Delta\, ABC = \frac{1}{2}\,[x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

OR



OR

$$\text{area of } \Delta ABC = \frac{1}{2}\,[(x_1y_2 + x_2y_3 + x_3y_1)$$

$$- (x_1y_3 + x_3y_2 + x_2y_1)]$$

20. The points A, B, C are collinear if

area of $\Delta ABC = 0$.

21. Centroid : $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

22. AREA OF QUADRILATERAL

If A (x_1, y_1), B(x_2, y_2), C(x_3, y_3) and D(x_4, y_4) be the vertices of quadrilateral ABCD then area of the quadrilateral ABCD

$$= \frac{1}{2} [x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_4 - x_4y_3 + x_4y_1 - x_1y_4]$$